

INDIAN INSTITUTE OF TECHNOLOGY MADRAS
Department of Chemical Engineering

CH 2082
Assignment 4 (27/03/2014)

1. Attempt all problems on your own.
2. You may discuss with the TAs for assistance.
3. All variables should be declared or initialized within your program
4. **Comment all your files**
5. Name each program using the following convention using a combination of your roll number, assignment number and question number. i.e., if your roll number is CH12B001, name the program in the first question of the first assignment as follows:
CH12B001_A1_Q1.m
6. Submit a single zipped folder containing .m files for each of the questions and a single pdf file that is published. The zip file should be named as follows: if your roll number is CH12B001, then the zip file name would be CH12B001_A1

Note: All submissions are to be made by 5pm on the same day.

1. The dimensionless concentrations of two species, y_1 and y_2 , in a reactor vary with time as per the following set of ODEs:

$$\begin{aligned}\frac{dy_1}{dt} &= 0.5 - y_1^{1.25} - 0.5y_1 & y_1(0) &= 1 \\ \frac{dy_2}{dt} &= -0.5y_2 + 2y_1^{1.25} - 0.4y_2^2 & y_2(0) &= 0\end{aligned}$$

Obtain the solution for $y_1(t)$ and $y_2(t)$. Plot the results in the same figure.

2. After hitting Irfan Pathan for consecutive sixes that ensured Chennai's entry into the semis of IPL-3, the captain M. S. Dhoni remarked that it was easier to hit sixes in Dharmasala (1.2 km above sea level) than it would be in Mohali (assume sea level for this problem). The overall model for the system, obtained from Newton's laws of motion, is given by

$$x'' = -cvx' \quad \text{and} \quad y'' = -cvy' - g$$

In the above equations, x and y are the horizontal and vertical coordinates in the direction the ball was hit. The velocity magnitude is given by $v = \sqrt{(x')^2 + (y')^2}$; c is the drag constant and g is the gravitational acceleration.

The earth's gravitational acceleration at Mohali is 9.81 m/s² and the drag coefficient is $c = 0.006$.

- If Dhoni hit the ball at 35 m/s with a 45° angle, find the distance ball travels in Mohali.
- What will be the distance travelled by the ball in Dharmasala?

The gravitational acceleration varies with height as,

$$\frac{g_h}{g_{Mohali}} = \left(\frac{R_e}{R_e + h} \right)^2 \quad \text{where } R_e = 6400$$

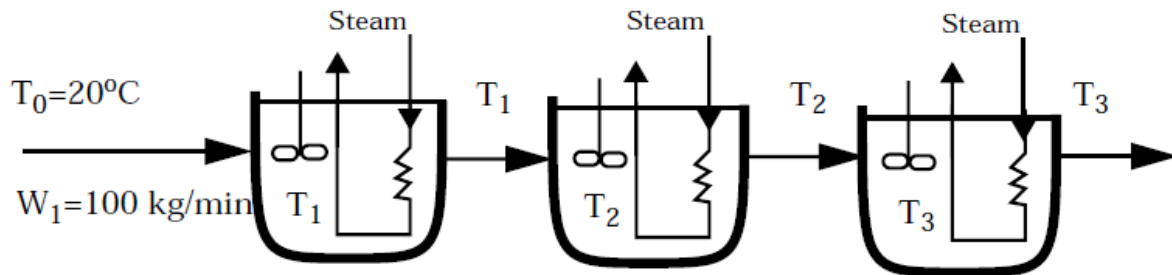
Whereas, the drag constant varies with height as:

$$\frac{c_h}{c_{Mohali}} = \left(1 - \frac{6.5h}{300} \right)^5$$

3. Three tanks in series are used to preheat a multicomponent oil solution before it is fed to a distillation column for separation as shown in Figure 1. Each tank is initially filled with 1000 kg of oil at 20°C. Saturated steam at a temperature of 250°C condenses within coils immersed in each tank. The oil is fed into the first tank at the rate of 100 kg/min and overflows into the second and the third tanks at the same flow rate. The temperature of the oil fed to the first tank is 20°C. The tanks are well mixed so that the temperature inside the tanks is uniform, and the outlet stream temperature is the temperature within the tank. The heat capacity, C_p , of the oil is 2.0 KJ/kg. For a particular tank, the rate at which heat is transferred to the oil from the steam coil is given by the expression:

$$Q = UA(T_{steam} - T)$$

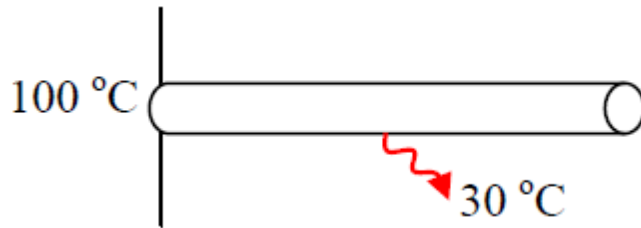
where $UA = 10 \text{ kJ/min} \cdot ^\circ\text{C}$ is the product of the heat transfer coefficient and the area of the coil for each tank, T = temperature of the oil in the tank in $^\circ\text{C}$, and Q = rate of heat transferred in kJ/min.



Heat transfer equation:

$$MC_p \frac{dT_1}{dt} = WC_p T_0 + UA(T_{steam} - T_1) - WC_p T_1$$

4. Consider a metal rod which has its one end connected to a hot equipment and the other end exposed to atmosphere such that it dissipates heat to the surroundings. Such structures are routinely used to manage heat: in car radiators, refrigerators, computer chips etc. We will model just one single rod of the assembly and solve it using an appropriate solver.



With appropriate derivation, the model for this system can be represented as:

$$\frac{d^2T}{dz^2} = 0.1(T - 30) \quad \text{subject to} \quad T_{(z=0)} = 100; T_{(z=5)} = 30$$

We discretize the above equation into 50 intervals. Thus, we will get linear equations in 51 variables. The linear set of equations that you need to solve are as follows:

$$\begin{aligned} T_1 &= 100 \\ 100T_{i-1} - 200.1T_i + 100T_{i+1} &= -3 \quad \text{for } i = 2 \text{ to } 50 \\ T_{51} &= 30 \end{aligned}$$

With this, we define a vector containing all the temperatures as: $x = [T_1, T_2, \dots, T_{51}]^T$

The above equations can then be written in the form: $Ax = b$

Solve the equations to get the steady state temperature profile of the rod.