Linear Demand and Supply Models

Based on Cowen & Tabarrok MRU Principles of Microeconomics, Ch. 3–4

Professor [Your Name]

2025-10-09

Table of contents

1 Graph Paper Grids

This section overlays demand and supply curves onto a graph-paper style grid (light gray major and minor grid lines) to aid students in reading values directly from the plot. Here's the updated `graph-paper` section with negative supply values filtered out, preventing the flat 0 segments at small P:

```r
# Sample curves for demonstration
demand <- create_demand(120, 4)
supply <- create_supply(-20, 6)
p_vals <- seq(0, 35, 0.5)
df_grid <- tibble(
  P = p_vals,
  Qd = demand(p_vals),
  Qs = supply(p_vals)
)
```

```r
eq_point <- tibble(Q = 64, P = 14)

# Filter supply to positive quantities to remove jagged zero segment
df_supply <- filter(df_grid, Qs > 0)
df_demand <- filter(df_grid, Qd > 0)

ggplot() +
  # Graph-paper grid
  geom_hline(yintercept = seq(0, 35, 5), color = grid_color, size = 0.3) +
  geom_vline(xintercept = seq(0, 140, 20), color = grid_color, size = 0.3) +
  geom_hline(yintercept = seq(0, 35, 1), color = grid_color, linetype = "dotted", size = 0.2
  geom_vline(xintercept = seq(0, 140, 5), color = grid_color, linetype = "dotted", size = 0
  # Demand curve (all Qd >= 0)
  geom_line(data = df_demand, aes(x = Qd, y = P), color = demand_color, size = 1.2) +
  # Supply curve only where Qs > 0
  geom_line(data = df_supply, aes(x = Qs, y = P), color = supply_color, size = 1.2) +
  # Equilibrium point
  geom_point(data = eq_point, aes(x = Q, y = P), color = equilibrium_color, size = 3) +
  annotate("text", x = 64, y = 16, label = "(64, 14)", color = equilibrium_color, fontface =
  labs(
    title = "Supply and Demand with Graph-Paper Grid",
    x = "Quantity (Q)",
    y = "Price (P)"
  ) +
  coord_cartesian(xlim = c(0, 140), ylim = c(0, 35))
```

Supply and Demand with Graph–Paper Grid
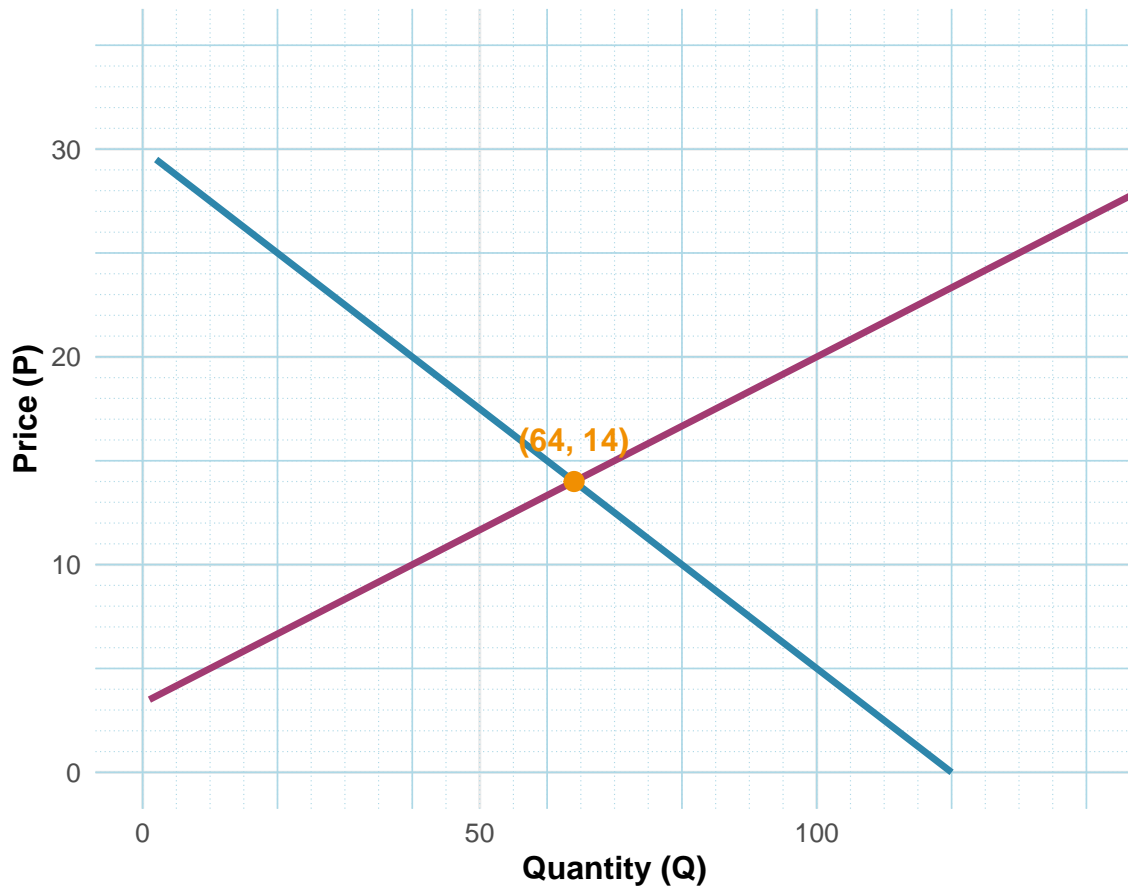
This filters out the jagged zeros of the supply curve for prices where supply is zero or negative.

Introduction to Linear Demand and Supply

Learning Objectives

By the end of this lecture, you will be able to:

1. **Define and graph** linear demand and supply functions
2. **Calculate** market equilibrium algebraically and graphically
3. **Analyze** the effects of market shifts on equilibrium
4. **Compute** consumer and producer surplus
5. **Evaluate** the impacts of taxes and price controls
6. **Apply** elasticity concepts to linear demand curves

Linear Demand and Supply Functions

The Linear Demand Curve

A **linear demand curve** has the form:

$$Q_D = a - bP$$

where: - $a > 0$ is the quantity intercept (maximum quantity when $P = 0$) - $b > 0$ is the slope parameter (how responsive quantity is to price) - The price intercept occurs at $P = \frac{a}{b}$ (choke price)

1.1 The Linear Supply Curve

A **linear supply curve** has the form:

$$Q_S = c + dP$$

where: - c can be positive or negative (quantity intercept) - $d > 0$ is the slope parameter (positive relationship between price and quantity) - If $c < 0$, the supply curve has a minimum price $P = -\frac{c}{d}$ before positive quantities

---

Market Equilibrium

Finding Equilibrium Algebraically

Market equilibrium occurs where $Q_D = Q_S$:

For our example: $Q_D = 120 - 4P$ and $Q_S = -20 + 6P$

$$120 - 4P = -20 + 6P$$
$$140 = 10P$$
$$P^* = 14$$

Substituting back: $Q^* = 120 - 4(14) = 64$

General Equilibrium Formula

For general linear curves $Q_D = a - bP$ and $Q_S = c + dP$:

$$P^* = \frac{a - c}{b + d}$$

$$Q^* = \frac{ad + bc}{b + d}$$

Market Shifts

Demand Shifts

An **increase in demand** shifts the entire demand curve rightward (parallel shift). This could result from: - Increase in consumer income (normal goods) - Increase in price of substitutes - Decrease in price of complements - Positive change in consumer preferences

---

Consumer and Producer Surplus

Consumer Surplus

Consumer surplus is the area between the demand curve and the market price, representing the net benefit to consumers.

For a linear demand curve, consumer surplus is:

$$CS = \frac{1}{2}(P_{choke} - P^*)Q^*$$

where $P_{choke} = \frac{a}{b}$ is the price intercept of demand.

1.2 Producer Surplus

Producer surplus is the area between the market price and the supply curve, representing the net benefit to producers.

For a linear supply curve, producer surplus is:

$$PS = \frac{1}{2}(P^* - P_{min})Q^*$$

where $P_{min} = -\frac{c}{d}$ is the minimum price for positive supply.

Tax Incidence and Deadweight Loss

Per-Unit Tax

A **per-unit tax** of amount t creates a wedge between the price buyers pay and the price sellers receive. The tax can be imposed on either buyers or sellers with the same final outcome.

1.2.1 Tax on Sellers

When a tax t is imposed on sellers, the supply curve shifts up by t: - Original supply: $Q_S = c + dP_S$ - After-tax supply: $Q_S = c + d(P_B - t) = (c - dt) + dP_B$

Tax Incidence

The **tax incidence** shows how the tax burden is split between buyers and sellers:

- **Buyers' burden**: $\Delta P_B = P_B^* - P^* = \frac{d}{b+d} \cdot t$

- **Sellers' burden**: $\Delta P_S = P^* - P_S^* = \frac{b}{b+d} \cdot t$

    The side of the market with **less elastic** (steeper) curve bears more of the tax burden.

Price Controls

Price Floors

A **price floor** is a minimum legal price. It's only binding when set above equilibrium.

1.3 Price Ceilings

A **price ceiling** is a maximum legal price. It's only binding when set below equilibrium.

Price Elasticity of Demand

Definition and Formula

Price elasticity of demand measures the responsiveness of quantity demanded to price changes:

$$\varepsilon = \frac{\% \text{ change in } Q_D}{\% \text{ change in } P} = \frac{dQ_D/dP \cdot P}{Q_D}$$

For linear demand $Q_D = a - bP$:

$$\varepsilon = -b \cdot \frac{P}{Q}$$

Interpreting Elasticity

- $|\varepsilon| > 1$: **Elastic** (quantity very responsive to price)
- $|\varepsilon| = 1$: **Unit elastic** (proportional response)

- $|\varepsilon| < 1$: **Inelastic** (quantity less responsive to price)

1.4 Elasticity Along Linear Demand

Elasticity varies along a linear demand curve:

Summary and Key Takeaways

Market Mechanics 1. **Linear demand**: $Q_D = a - bP$ (negative slope) 2. **Linear supply**: $Q_S = c + dP$ (positive slope)
3. **Equilibrium**: Where $Q_D = Q_S$, solving for P^* and Q^*

Welfare Analysis - **Consumer Surplus**: Area under demand curve, above price - **Producer Surplus**: Area above supply curve, below price - **Total Surplus**: $CS + PS$ measures market efficiency

1.5 Policy Effects

- **Taxes**: Create deadweight loss, incidence depends on relative elasticities
- **Price floors**: Binding when above equilibrium $\rightarrow$ surplus
- **Price ceilings**: Binding when below equilibrium $\rightarrow$ shortage

1.6 Elasticity Insights

- Measures price responsiveness of demand
- Varies along linear demand curves
- Key for predicting policy impacts and business decisions

This comprehensive treatment of linear demand and supply models provides the foundation for understanding market behavior, policy impacts, and welfare analysis in microeconomics. The mathematical tools and graphical techniques demonstrated here are essential for analyzing real-world economic phenomena.