CONTROL SYSTEM DESIGN PROJECT

A PROJECT REPORT SUBMITTED

for evaluation of

CONTROL SYSTEMS (EET 3071)

Bachelor of Technology

In

Electronics & Communication Engineering

By

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There are eleven student outcomes (A-K) for the Electrical Engineering B. Tech program.

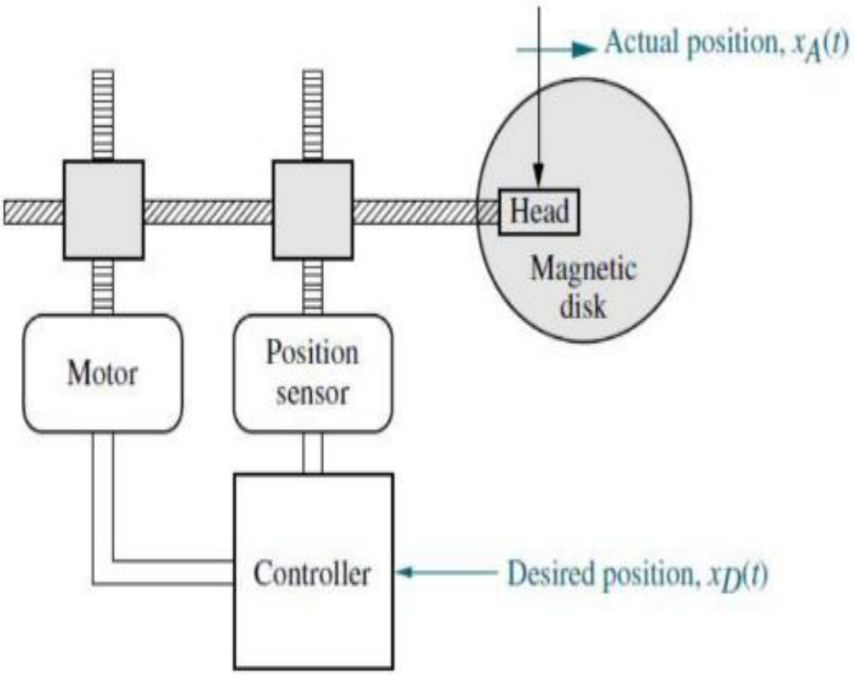
Abet Outcomes	Description of Outcome
A	An Ability to apply knowledge of mathematics, science, and engineering.
B	An Ability to design and conduct experiments, as well as to analyze and interpret data.
C	An ability to design a system, component, or process to meet desired needs within realistic constraints such as economic, environmental, social, political, ethical, health and safety, manufacturability, and sustainability.
D	An ability to function on multidisciplinary teams.
E	An ability to identify, formulate, and solve engineering problems.
F	An understanding of professional and ethical responsibility.
G	An ability to communicate effectively.
H	The broad education necessary to understand the impact of engineering solutions in a global, economic, environmental, and societal context.
I	A recognition of the need for, and an ability to engage in life-long learning.
J	A knowledge of contemporary issues.
K	An ability to use the techniques, skills, and modern engineering tools necessary for engineering practice.

The students will able to satisfy ABET outcomes A,E,B,C,G and K in this subject. They will satisfy outcome A,E through mid-semester, end semester, Quizzes and assignments, where as they will satisfy B,C,G and K through lab tests, projects, reports and viva-voce.

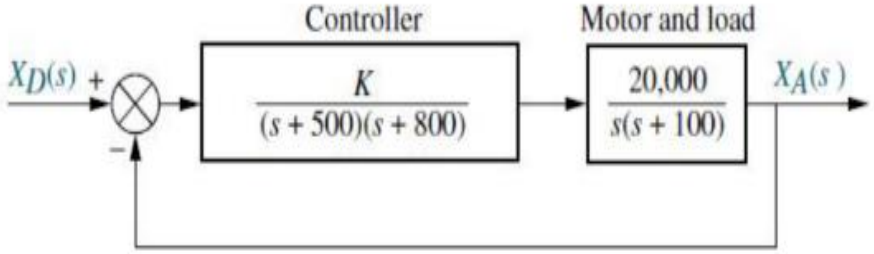
CONTENTS

1. PROBLEM STATEMENT

A disk drive is a position control system in which a read/write head is positioned over a magnetic disk. The system responds to a command from a computer to position itself at a particular track on the disk. A physical representation is shown in below Figure. Find K to yield a settling time of 0.1second. What is the resulting percent overshoot? What is the range of K that keeps the system stable? A head position control system for disk drive was designed to yield a settling time of 0.1 second through gain adjustment alone. Design a lead compensator to decrease the settling time to 0.005 second without changing the percent overshoot.



(a)



2. BACKGROUND THEORY AND ANALYSIS

Disk Drive Positioning System

A disk drive's read/write head must be positioned accurately and quickly over a magnetic disk for data access. The control system ensures that the head follows input commands precisely, minimizing errors and achieving rapid response. The dynamic behaviour is governed by the system's transfer function, which includes the controller and load dynamics.

Control System Components

The open-loop transfer function G(s) of the system consists of:

1. **Controller Transfer Function**:

$$G_C(s) = \frac{K}{(s + 500)(s + 800)}$$

2. **Motor & Load (Plant) Transfer Function**:

$$G_P(s) = \frac{20000}{s(s + 100)}$$

The Controller & Plant are considered as a whole system as they are connected in cascade so the open-loop transfer function is:

$$G(s) = G_C(s) \cdot G_P(s) = \frac{20000K}{s(s + 100)(s + 500)(s + 800)}$$

The characteristic equation of the system is with unity feedback:

$$0 = 1 + G(s) \cdot H(s) = s(s + 100)(s + 500)(s + 800) + 20000K$$

where H(s) is the unity feedback

Simplifying we get the characteristic equation to be:

$$s^4 + 1400s^3 + 530000s^2 + 40000000 + 20000K = 0$$

System Performance Metrics

1. **Settling Time (T_s)**: The time required for the system to reach and stay within a specific tolerance of the final value.

$$T_s = \frac{4}{\zeta\omega_n} = 4\tau$$

Where
:

ζ = Damping Ratio (Which controls the amplitude of oscillation or overshoot)

ω_n = Natural Frequency (Defines how fast the system oscillates)

τ = Time Constant (Determines how quickly the system responds)

2. **Percent Overshoot (M_P)**: The extent to which the system output exceeds the steady-state value during transients:

$$M_P = e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}} \times 100$$

3. **Stability**: The system remains stable if all poles of the closed-loop transfer function lie in the left-half of the s-plane.

Assuming ζ to be 0.54 for an underdamped system we require T_s to be 0.1 sec.

$$T_s = \frac{4}{\zeta\omega_n}$$

$$0.1 = \frac{4}{0.54 \cdot \omega_n}$$

$$\omega_n = \frac{4}{0.54 \cdot 01} \simeq 74\, rad/sec$$

To ensure the dominant poles satisfy the desired $\omega_n \simeq 74\ rad/sec$ and $\zeta = 0.54$, the dominant poles are located at:

$$s = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$$

Substituting the values we get:

$$s = -39 \pm j62.28$$

Substituting the value of $s = -39 + j62.28$ in the characteristic equation we get

$$-2265770456 - j44629640.7 + 20000K = 0$$

As K is the gain so considering only the real part of s, we obtain the value of K to be

$$K = \frac{2265770456}{20000} \cong 113288$$

Since we considered ζ to be 0.54 we can calculate the Maximum Peak Overshoot as:

$$M_P = e^{\frac{-\pi \cdot 0.54}{\sqrt{1-0.54^2}}} \times 100 = 13.32\%$$

Routh-Hurwitz Stability Criterion

The Routh-Hurwitz stability criterion is a method to determine the stability of a linear time- invariant system by examining its characteristic equation. It involves constructing a Routh Array using the coefficients of the characteristic polynomial.

Key points:

1. **Stability Condition**: The system is stable if all elements in the first column of the Routh array are positive.
2. **Procedure**: The rows of the array are derived recursively using a determinant-like formula based on the coefficients of the polynomial.
3. **Result**: The presence of zero or sign changes in the first column indicates instability or marginal stability.

This criterion avoids solving for the roots of the polynomial explicitly, making it a practical and efficient tool for stability analysis.

Using the characteristics equation we construct a Routh Array

s^4	1	530000	20000K
s^3	1400	40000000	
s^2	501428.5714	20000K	
s^1	40000000 – 55.84K		
s^0	20000K		

As there are no zeros or sign changes making it a stable system and we can say from

$$\text{Row } s^1\text{: } 40000000 - 55.84K > 0$$

Solving for K we get the value to be

$$0 < K < 716326.53$$

Since the value of K = 113288 falls in the range above we can conclude that it is the correct value of K to achieve a settling time of 0.1sec.

Component Selection.

Chosen Component: Lead Compensator

A lead compensator is chosen for this system due to its ability to improve transient response and system stability by introducing phase lead, which shifts the root locus towards the desired pole locations in the left half of the s-plane. This helps achieve faster settling time and better control over the system dynamics.

System Overview

The open-loop transfer function of the system is given by:

$$G(s) = \frac{20000 \cdot 113288}{s(s+100)(s+500)(s+800)}$$

Key characteristics of the system:

3. **Poles:** 0, -100, -500 and -800

    - These poles are located far from each other, with the dominant poles being closer to the origin (0 and -100).
    - The absence of zeros makes it challenging to directly influence the system dynamics using the existing pole structure alone.

4. **No Zeros in the System:**

    - The lack of zeros limits the system's flexibility in shaping the root locus trajectory, requiring the addition of compensator poles and zeros for effective control.

Specification of Lead Compensator

The transfer function of the lead compensator is typically represented as:

$$C(s) = K_c \frac{s + z_c}{s + p_c}$$

Where:

- z_c is the compensator zero.

- p_c is the compensator pole ($p_c > z_c$ for phase lead).
- K_C is the compensator gain.

The placement of z_c and p_c is chosen based on the desired performance metrics, such as:

- Achieving a settling time of 0.005 seconds.
- Maintaining overshoot below a specified limit.

In summary, the lead compensator is selected for its ability to effectively reshape the root locus, improve system performance, and meet design requirements in a system with no zeros and poles at 0, -100, -500, -800

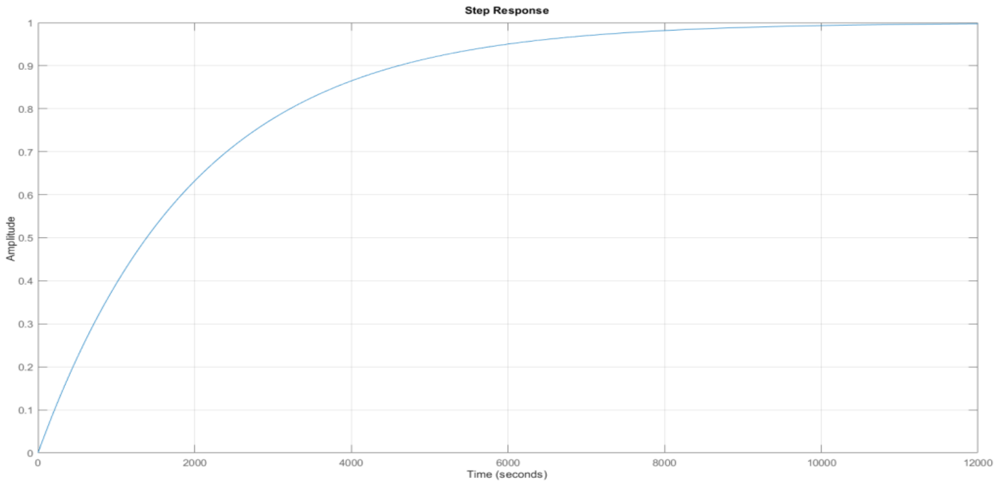
II. Implementation and Verification in MATLAB.

Obtaining the value of K, Range of K, M_P and Settling Time of 0.1Sec Script Code

```
clear all; close all; clc;

%Plant Transfer Function
num1 = [20000];
den1 = [1 100 0];
PlantTF = tf(num1, den1)

%Controller Transfer Function
num2 = [1]; %Assuming the gain to be 1
den2 = [1 1300 400000];
ContTF = tf(num2, den2)

OLtf = PlantTF*ContTF %OpenLoop TF
tf = (PlantTF*ContTF)/(1+(PlantTF*ContTF)) %ClosedLoop TF

step(tf); grid on;
stepinfo(tf)
sisotool(OLtf)
```

Assuming the controller gain K = 1, the step response of the system is simulated, yielding the following output:

We obtain T_s to be 7824 seconds and M_P to be 0.

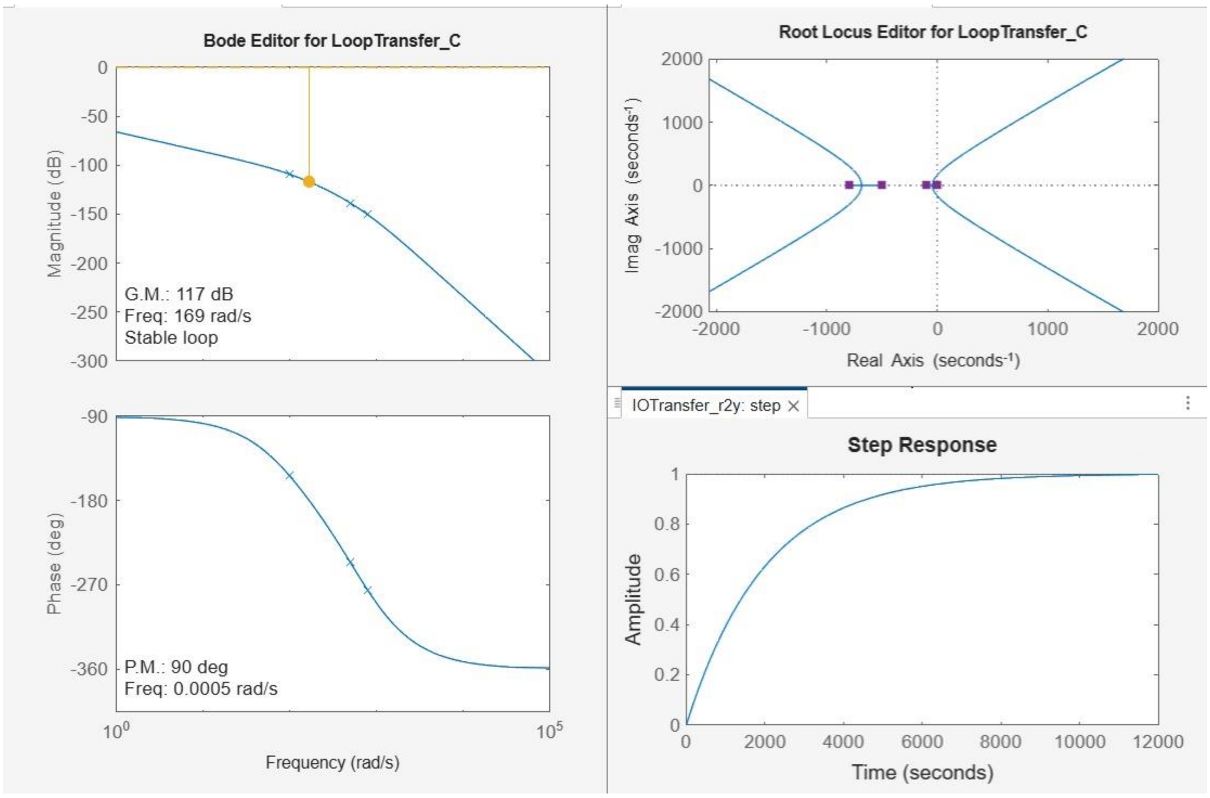


Using the SISOTOOL with the open-loop transfer function, the gain is adjusted to the calculated value of K=113288 on the root locus. Applying the condition for a settling time of
0.1 seconds, the following results are obtained:

COMMAND WINDOW:

```
PlantTF =

     20000
  -----------
  s^2 + 100 s

Continuous-time transfer function.
Model Properties

ContTF =

            1
  ---------------------
  s^2 + 1300 s + 400000

Continuous-time transfer function.
Model Properties

OLtf =

                 20000
  ------------------------------------
  s^4 + 1400 s^3 + 530000 s^2 + 4e07 s

Continuous-time transfer function.
Model Properties
```

```
tf =

                        20000 s^4 + 2.8e07 s^3 + 1.06e10 s^2 + 8e11 s
  -------------------------------------------------------------------------------------------------
  s^8 + 2800 s^7 + 3.02e06 s^6 + 1.564e09 s^5 + 3.929e11 s^4 + 4.24e13 s^3 + 1.6e15 s^2 + 8e11 s

Continuous-time transfer function.
Model Properties

ans =

  struct with fields:

         RiseTime: 4.3944e+03
    TransientTime: 7.8240e+03
     SettlingTime: 7.8240e+03
      SettlingMin: 0.9000
      SettlingMax: 0.9989
        Overshoot: 0
       Undershoot: 0
             Peak: 0.9989
         PeakTime: 1.3659e+04

>> |
```



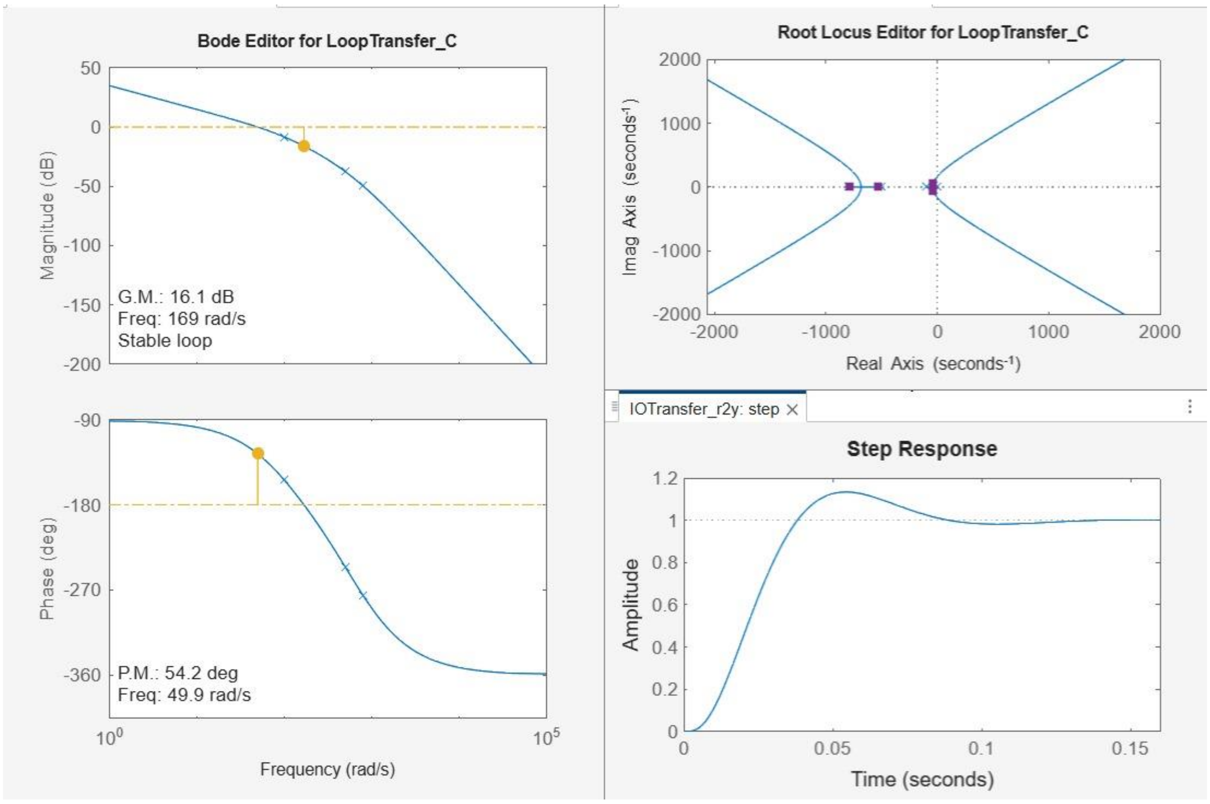
Design of Compensator

Using the previously calculated value of K=112190, we modify the code to design the compensator.

```
clear all; close all; clc;

%Plant Transfer Function
num1 = [20000];
den1 = [1 100 0];
PlantTF = tf(num1, den1)

%Controller Transfer Function
num2 = [112190]; %Obtained Value of K
den2 = [1 1300 400000];
ContTF = tf(num2, den2)

OLtf = PlantTF*ContTF %OpenLoop TF
tf = (PlantTF*ContTF)/(1+(PlantTF*ContTF)) %ClosedLoop TF
sisotool(OLtf)
```

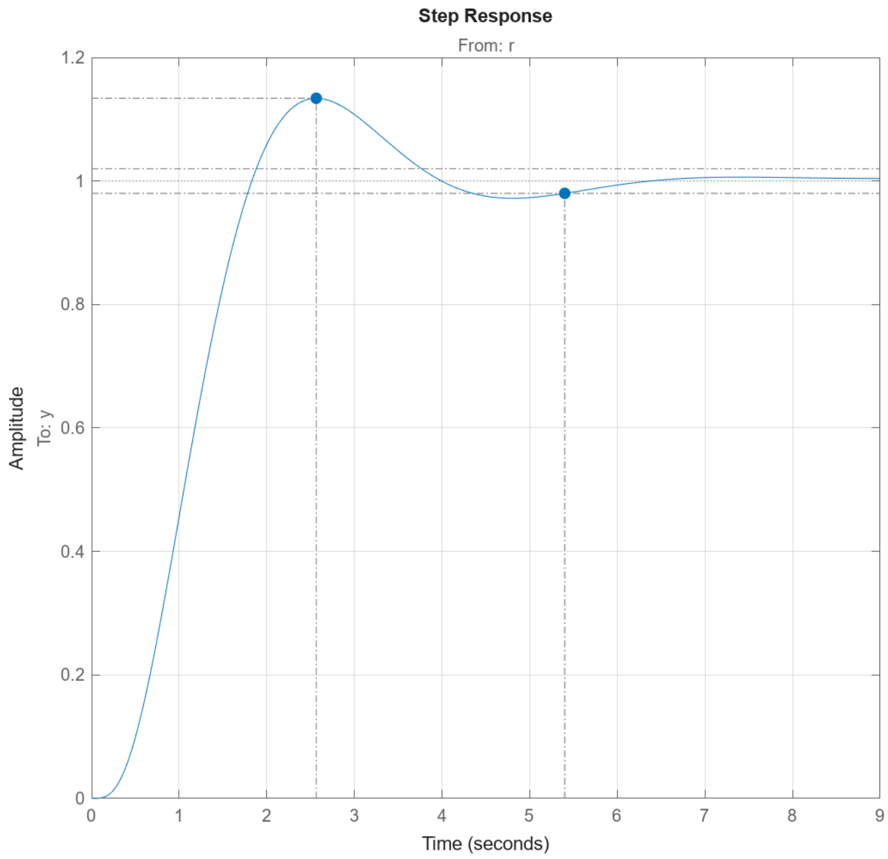
The following code generates the response and root locus plot for the system. The vertical line represents the condition for achieving a settling time of 0.005 seconds, while the two diagonal lines correspond to the condition of having an overshoot of 13.404%. The intersection of these two conditions indicates the optimal point on the root locus where the system gain should be set to meet the desired performance criteria of a 0.005 seconds settling time and 13.404% overshoot.

OUTPUT:



By implementing the designed compensator in cascade with the controller and the plant/motor, and updating the code accordingly, we obtain the following system response:

```
clear all; close all; clc;

%Plant Transfer Function
num1 = [20000];
den1 = [1 100 0];
PlantTF = tf(num1, den1)

%Controller Transfer Function
num2 = [112190];
den2 = [1 1300 400000];
ContTF = tf(num2, den2)

%Compensator Transfer
Function Z = [-120 -380 -
920];
P = [-2540 -7000 -10000];
K = [88970];
Comp = zpk(Z, P, K)

OLtf = PlantTF*ContTF*Comp %OpenLoop TF
tf = (PlantTF*ContTF*Comp)/(1+(PlantTF*ContTF*Comp)) %ClosedLoop TF
step(tf); grid on;
stepinfo(tf)
```



With the output:

RiseTime: 0.0011

TransientTime: 0.0054

SettlingTime: 0.0054

SettlingMin: 0.9039

SettlingMax: 1.1340

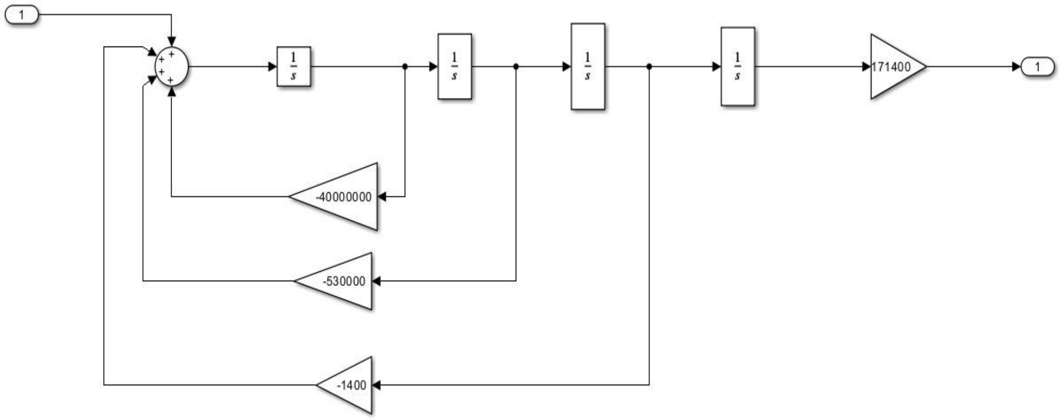
Overshoot: 13.4045

Undershoot: 0

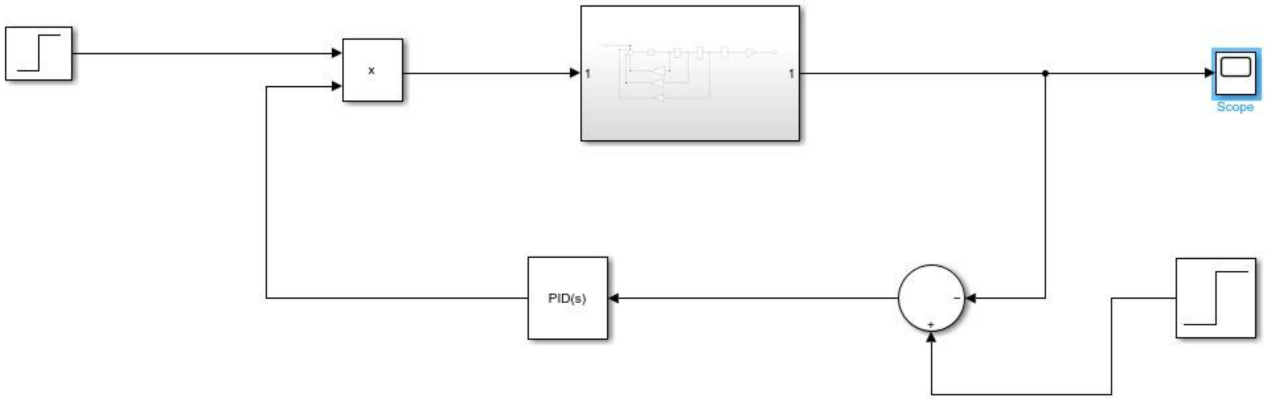
Peak: 1.1340

PeakTime: 0.0026

IMPLEMENTATION AND VERIFICATION OF MODEL IN MATLAB



DESIGNING WITH PID:



OUTPUT:

I. Conclusion.

The project focused on designing and implementing a compensator to achieve the desired system performance criteria. Through root locus analysis, the calculated value of the gain K was determined to be 113288, while the applied value was slightly adjusted to 112190 to ensure better alignment with practical considerations. This adjustment still maintained system stability and satisfied the conditions for a settling time of 0.1 seconds and an overshoot of 13.404%.

The range of K was identified through Routh-Hurwitz analysis, ensuring stability for values up to 716326. Beyond this limit, the system becomes unstable. At K = 716320, the system remained stable but exhibited slow stabilization, whereas K = 716330 resulted in instability, confirming the accuracy of the stability range.

The design of the compensator involved placing zeros and poles strategically to shift the root locus to meet the desired performance metrics. By implementing a third-order compensator, we effectively improved the system response, achieving a settling time of 0.005 seconds while maintaining the specified overshoot. Although the design process involved challenges in achieving precise pole-zero placement and balancing damping ratio with response time, the final implementation successfully fulfilled the project goals.

II. Comments

1. The compensator design demonstrates the importance of careful analysis and fine- tuning in achieving desired system performance.
2. The slight adjustment in the gain value from the calculated K = 113288 to the applied K = 112190 highlights the practical flexibility required during implementation.
3. The use of a third-order compensator proved effective in addressing the limitations of lower-order compensators, ensuring the system met the strict settling time and overshoot requirements.
4. The stability range identified through theoretical and simulated methods validates the robustness of the approach, providing insights into the system's operational limits.
5. Overall, this project successfully bridges theoretical analysis with practical design, offering a structured approach to compensator development for control systems.

III. References.

Nise, N. S. (2015). *Control Systems Engineering* (7th ed.). Hoboken, NJ: John Wiley & S
Section 8.7: Root Locus Design Examples

- Section 8.9: PID Controllers in Root Locus Design
- Section 9.1: Design via Frequency Response
- Section 9.3: Lead Compensator Design Using Frequency Response
- Section 9.4: Lag-Lead Compensator Design Using Frequency Response