



UNIVERSITY  
OF AMSTERDAM

# Robust Bayesian Meta-Regression

## Model-Averaged Moderation Analysis in the Presence of Publication Bias

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with Maximilian Maier, Eric-Jan Wagenmakers, and Tom D. Stanley

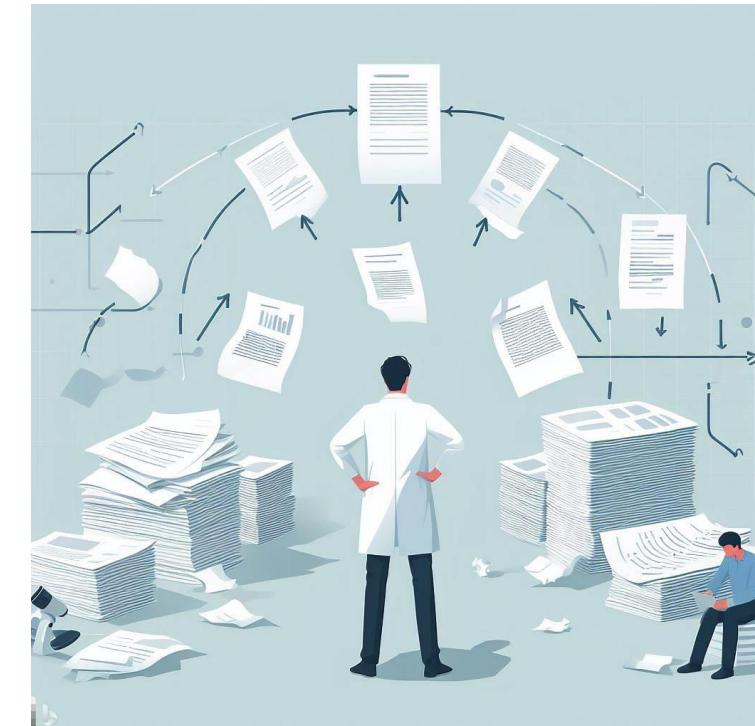


# Outline

- Meta-Analysis
- Publication bias
- Robust Bayesian meta-analysis
- Robust Bayesian meta-regression

# Publication Bias

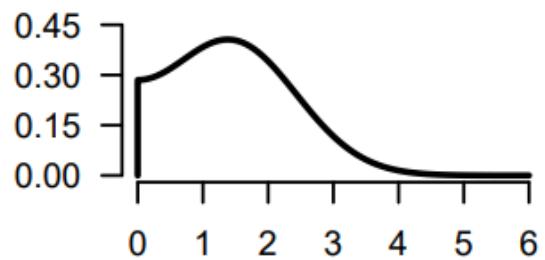
- Most large studies are likely to get published regardless of results
- Some moderately size studies might get loss if not convincing
- Many small studies won't be published unless statistically significant



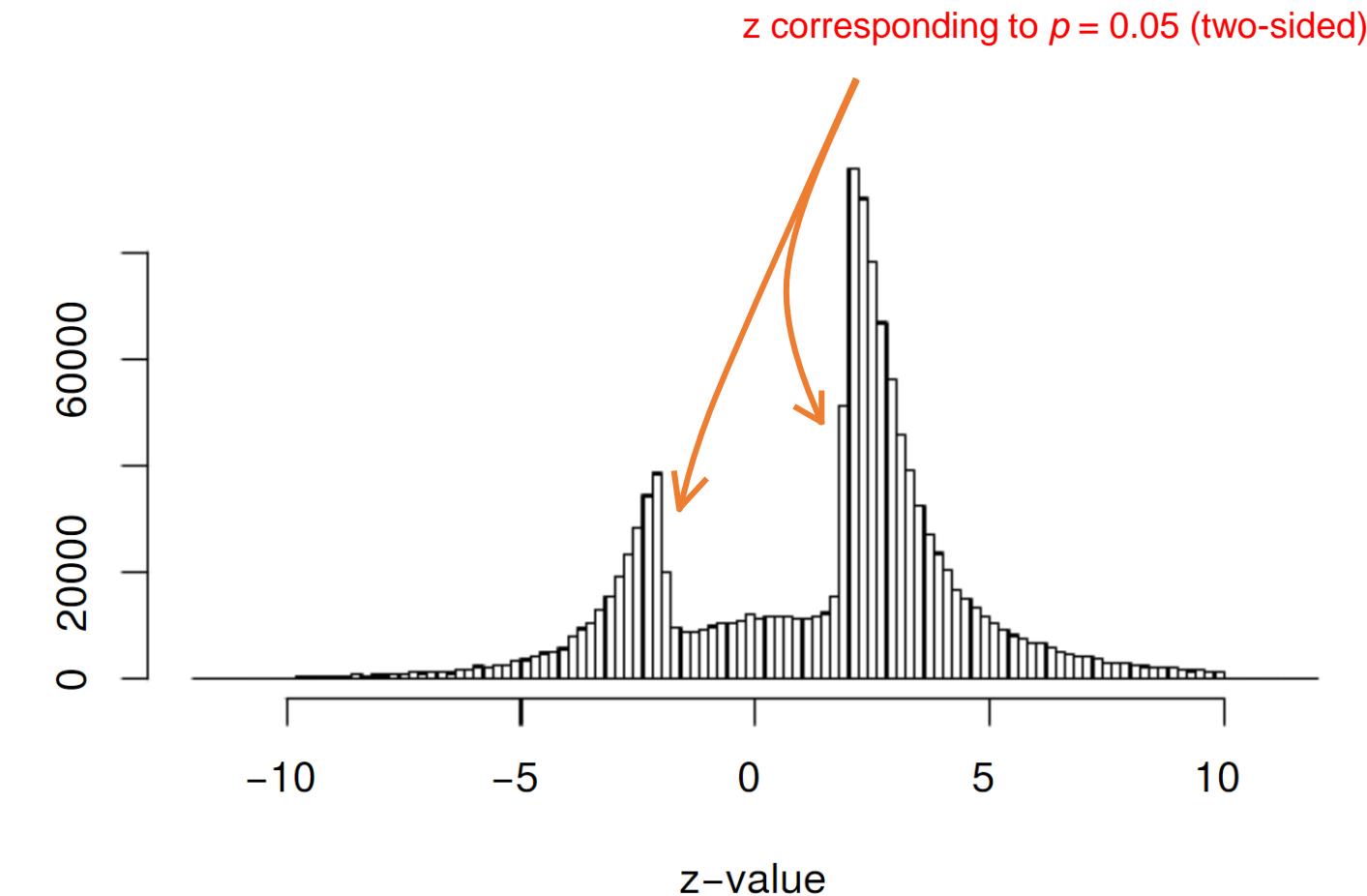
# Publication Bias

Power

0.3



# Publication Bias



**FIGURE 1** The distribution of more than one million  $z$ -values from Medline (1976–2019).

# Meta-Analyses vs. RRR: Kvarven et al. (2020)

- Comparison of:
  - 15 meta-analyses from the field of psychology
  - Registered replication reports of a corresponding experiment
- The registered replication reports do not suffer from publication bias  
=> should provide the best possible estimate of the true effect

Oppenheimer et al. (2009)

Tversky & Kahneman (1981)

Husnu & Crisp (2010)

Schwarz et al. (1991)

Hauser et al. (2007)

Critcher & Gilovich (2008)

Graham et al. (2009)

Jostmann et al. (2009)

Monin & Miller (2001)

Schooler  
& Engstler-Schooler (1990)

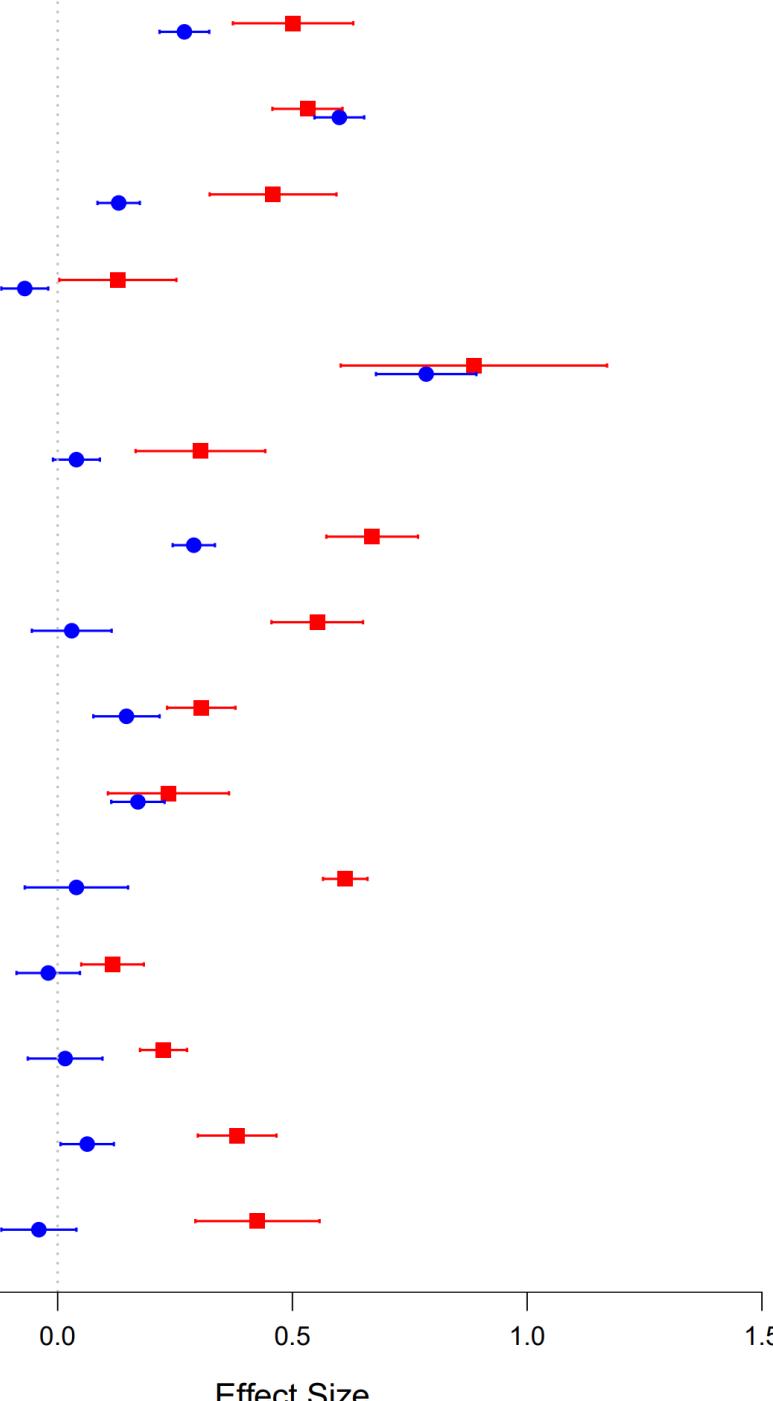
Sripada et al. (2014)

Rand et al. (2012)

Strack et al. (1988)

Srull & Wyer (1979)

Mazar et al. (2008)

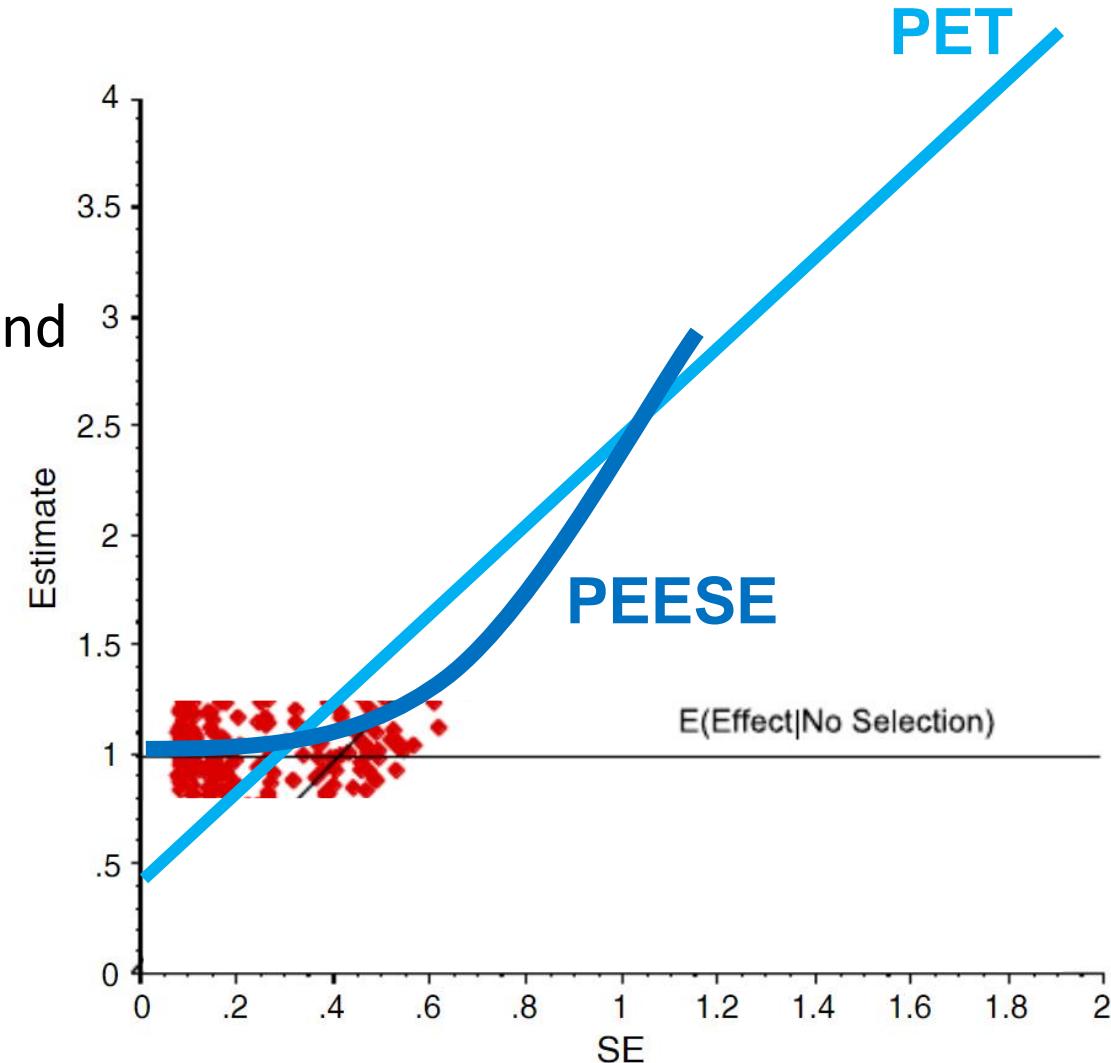


# Publication Bias Adjustment Methods

- Models adjusting for relationship between effect sizes and standard errors
  - Trim and fill (Duval & Tweedie, 2000)
  - PET-PEESE (Stanley & Doucouliagos, 2014)
  - EK (Bom & Rachinger, 2019)
- Selection models of  $p$ -values
  - 3PSM, 4PSM (Vevea & Hedges, 1995)
  - AK1, AK2 (Andrews & Kasy, 2019)
  - $p$ -curve (Simonsohn et al., 2014)
  - $p$ -uniform (Van Assen et al., 2015)

# PET-PEESE

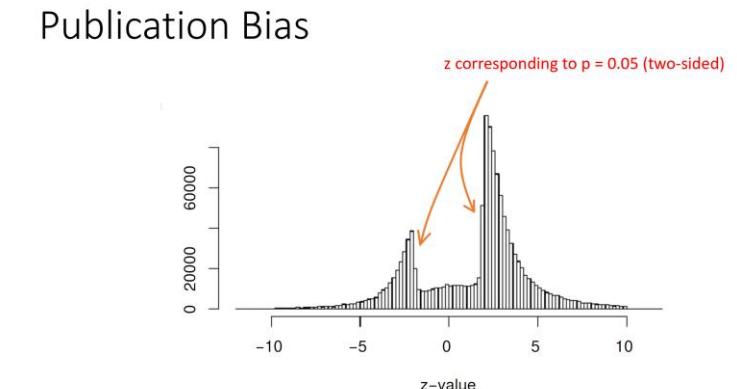
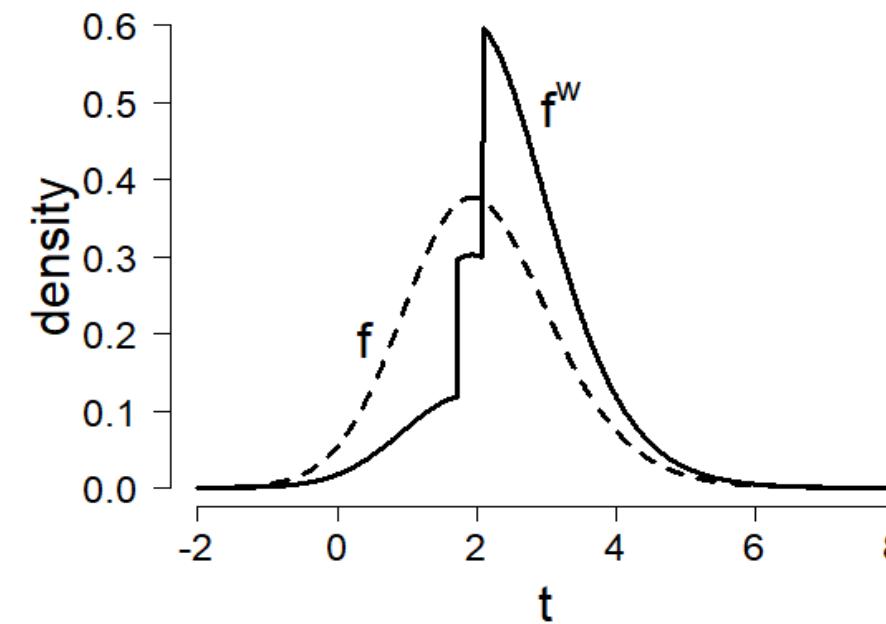
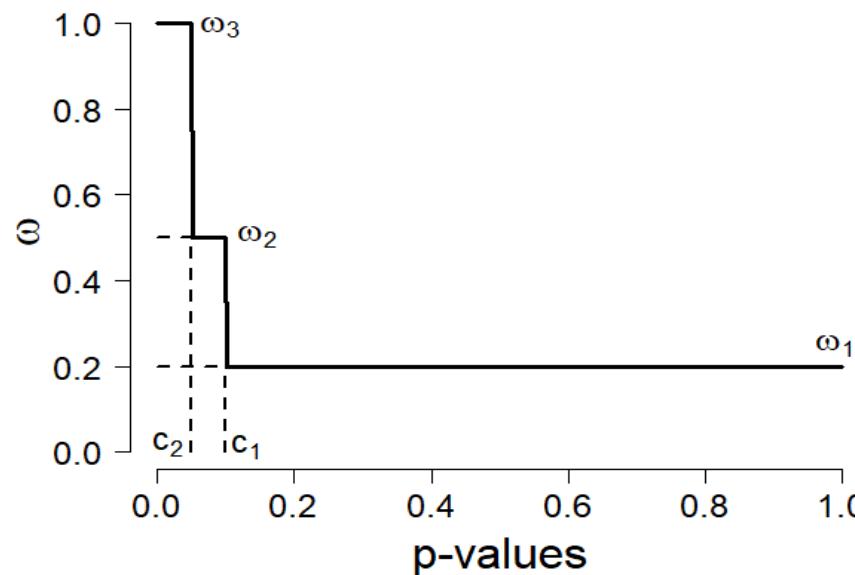
- Conditional meta-regression estimators
- Corrects for relationship between effect sizes and
  - Standard errors (PET)
  - Standard errors<sup>2</sup> (PEESE)
- Effect size estimate is based on
  - PET, if effect size test is not significant on  $\alpha = 0.10$
  - PEESE, if effect size test is significant on  $\alpha = 0.10$



**Figure 1.** Plots 300 randomly generated yet selected effects (vertical axis) against their standard errors.

# Selection Models

- Adjust for publication bias operating on  $p$ -values
- Meta-analytic models with:
  - Mean parameter  $\mu$
  - (Heterogeneity parameter  $\tau$ )
  - Publication bias weights  $\omega$



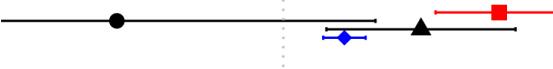
Oppenheimer et al. (2009)



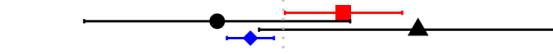
Tversky & Kahneman (1981)



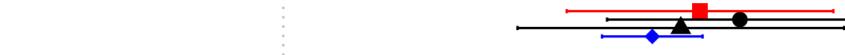
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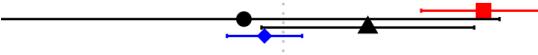
Strack et al. (1988)



Srull & Wyer (1979)



Mazar et al. (2008)



# Limitations of Existing Methods

- Require researchers to decide whether or not to adjust for publication bias in all-or-none fashion
- Cannot quantify evidence against publication bias; a non-significant p-value may indicate evidence of absence or absence of evidence
- Most fail under high between-study heterogeneity
- Poor performance in small samples and convergence issues

# RoBMA – Robust Bayesian Meta-Analysis

- Bayesian model-averaging to base inference on multiple models simultaneously  
(vs. deciding to adjust for publication bias in all-or-none fashion)
- Bayes factors to quantify evidence in favor of the presence or absence of effect/heterogeneity/publication bias  
(vs. rejecting or failing to reject the null hypothesis)
- Prior distributions to regularize the estimates/incorporate prior knowledge  
(vs. convergence problems/highly variable estimates under small sample sizes)
- Bayesian evidence updating independent of sampling plan  
(vs. accumulation bias)

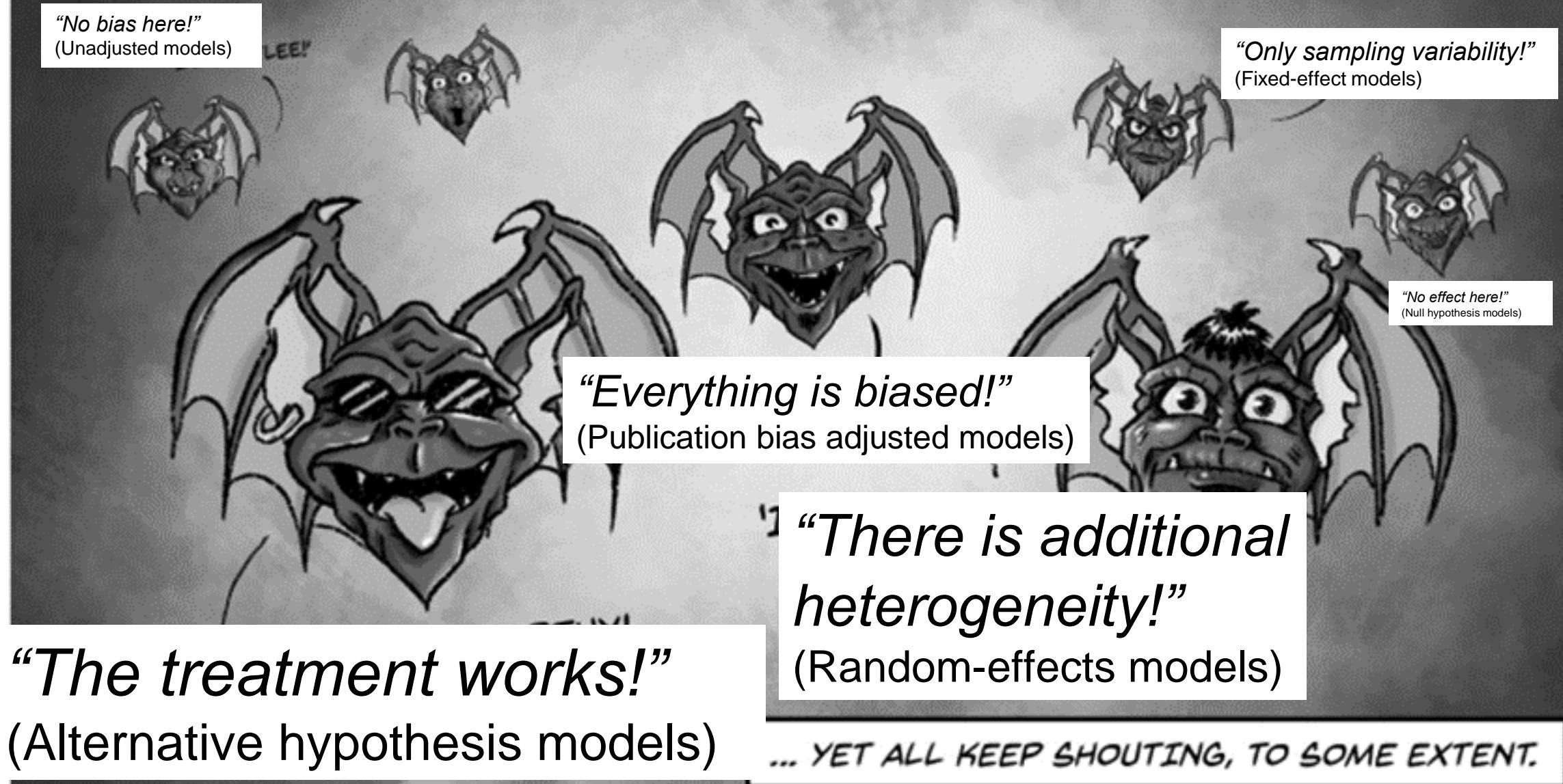
*THE PRIOR MODEL DEMONS ALL SHOUT ...*



*... BUT AFTER THEY GORGE ON DATA ...*



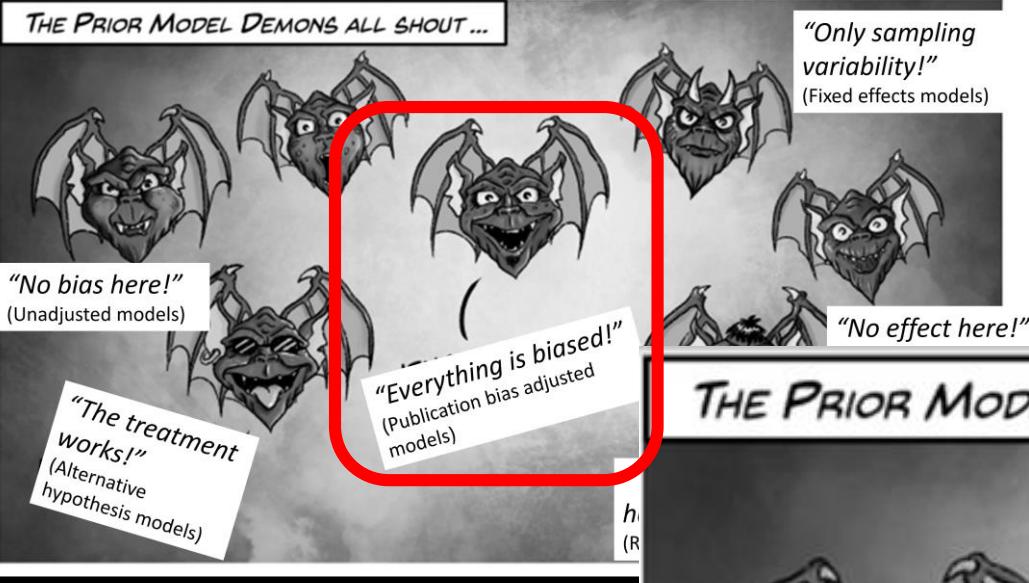
*... SOME POSTERIOR MODEL DEMONS BECOME POWERFUL, AND OTHERS WITHER AWAY ...*



# RoBMA: Model Types

- Absence vs. presence of the:
  - Effect
  - Heterogeneity
  - Publication bias

THE PRIOR MODEL DEMONS ALL SHOUT ...



THE PRIOR MODEL DEMONS ALL SHOUT ...

"Use PET model!"

"The relationship with standard errors is quadratic!"

"One-sided selection on significant p-values!"

"Two-sided selection on significant and marginally significant p-values!"

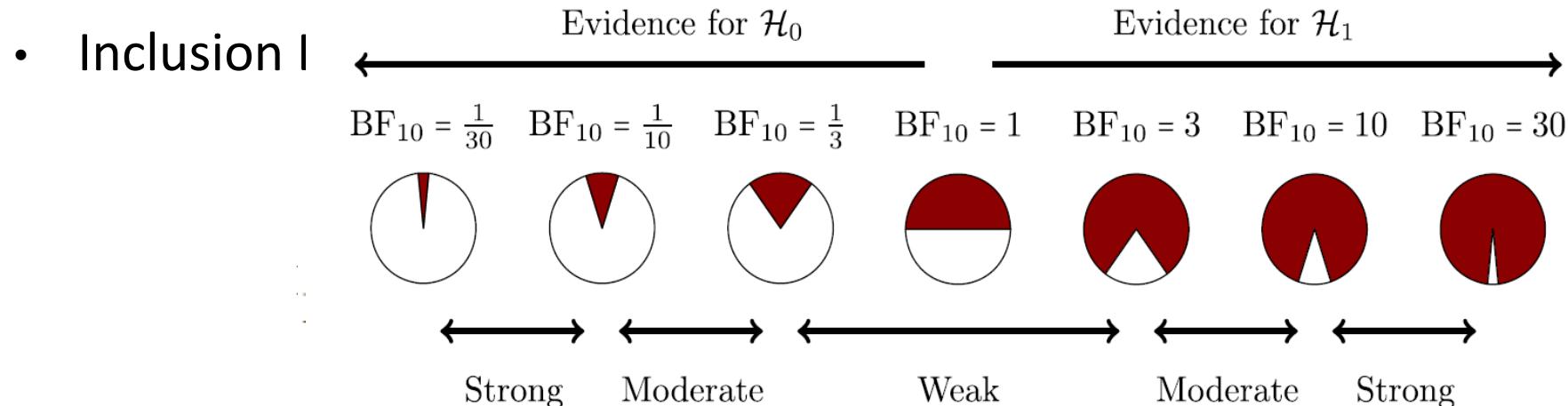
"How about direction of the effect?"

# RoBMA – Evaluating Evidence

- Bayes factors quantify evidence for/against an effect/heterogeneity/publication bias:

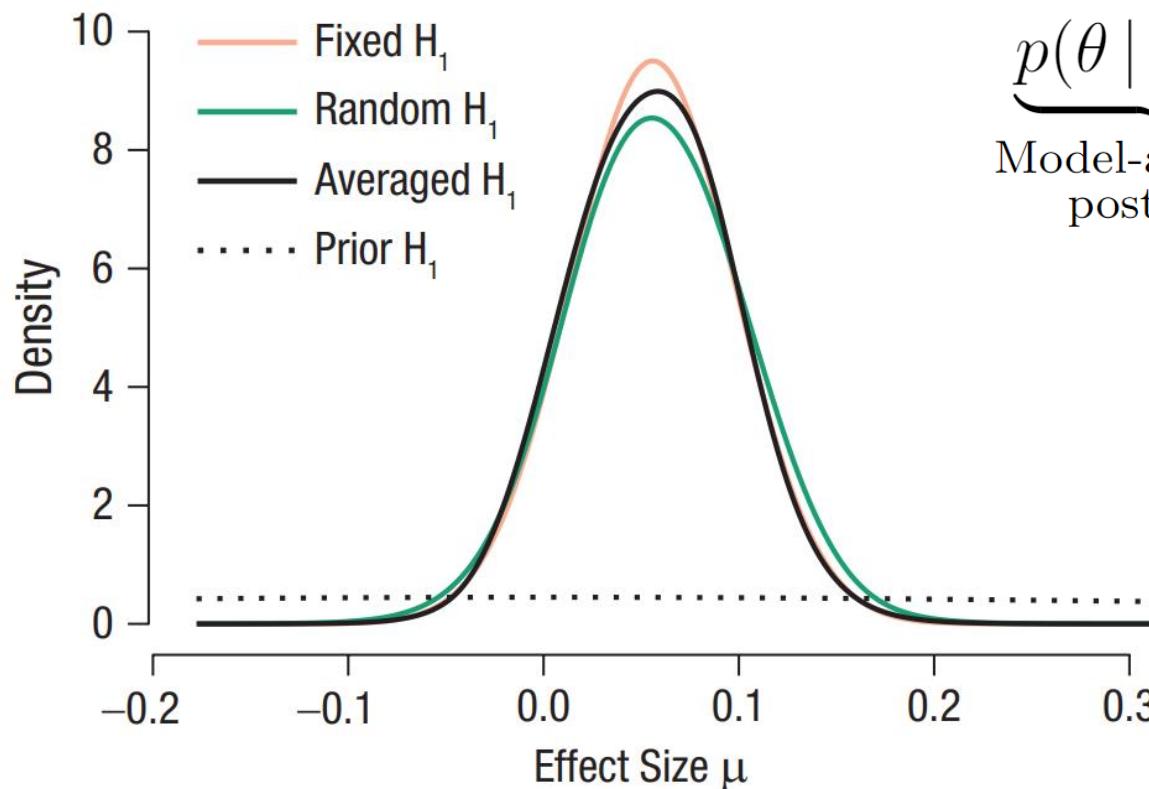
$$BF_{10} = \frac{p(\text{data} | \mathcal{H}_1)}{p(\text{data} | \mathcal{H}_0)}$$

$$\underbrace{\frac{p(\text{data} | \mathcal{H}_1)}{p(\text{data} | \mathcal{H}_0)}}_{\text{Bayes factor}} = \underbrace{\frac{p(\mathcal{H}_1 | \text{data})}{p(\mathcal{H}_0 | \text{data})}}_{\text{Posterior odds}} \Bigg/ \underbrace{\frac{p(\mathcal{H}_1)}{p(\mathcal{H}_0)}}_{\text{Prior odds}}$$



# RoBMA – Estimating Parameters

- Model-averaged posterior distributions account for uncertainty in the selected models



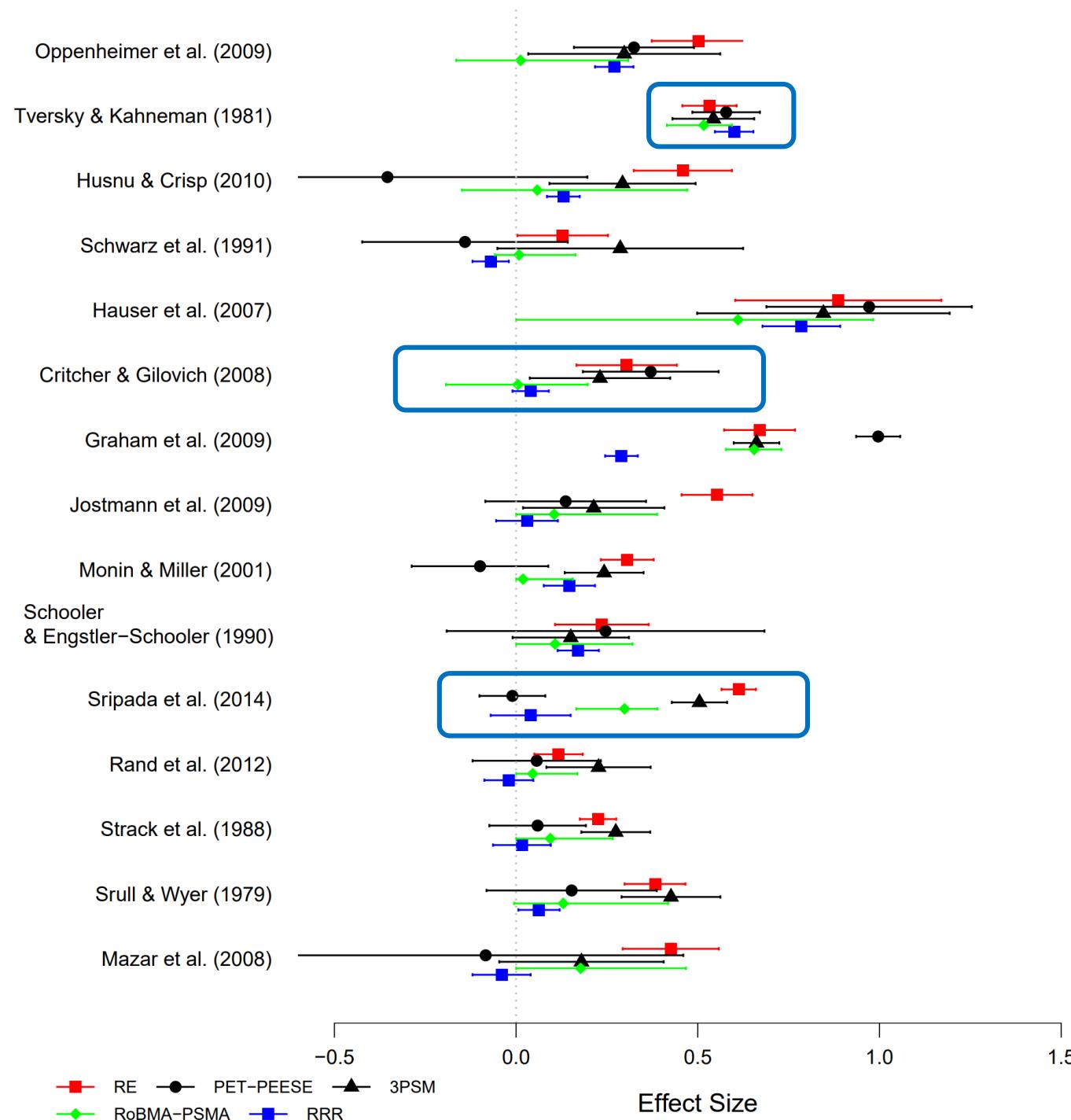
$$\underbrace{p(\theta | \text{data})}_{\text{Model-averaged posterior}} = \sum \underbrace{p(\theta | \mathcal{M}_i, \text{data})}_{\text{Model specific posterior}} \underbrace{p(\mathcal{M}_i | \text{data})}_{\text{Posterior probability of model}}$$

# RoBMA: Publication Bias Adjustment Components

- Models adjusting for relationship between effect sizes and standard errors
  - PET model (regression of effect sizes on standard errors)
  - PEESE model (regression of effect sizes on standard errors square)
- Selection models of  $p$ -values
  - Two-sided selection on significant  $p$ -values
  - Two-sided selection on significant and marginally significant  $p$ -values
  - One-sided selection on significant  $p$ -values
  - One-sided selection on significant and marginally significant  $p$ -values
  - One-sided selection on significant  $p$ -values and effects in expected direction
  - One-sided selection on significant, marginally significant  $p$ -values and effects in expected direction

# Publication Bias Adjustment Methods

- Models adjusting for relationship between effect sizes and standard errors
  - Trim and fill (Duval & Tweedie, 2000)
  - PET-PEESE (Stanley & Doucouliagos, 2014)
  - EK (Bom & Rachinger, 2019)
- Selection models of *p*-values
  - 3PSM, 4PSM (Vevea & Hedges, 1995)
  - AK1, AK2 (Andrews & Kasy, 2019)
  - *p*-curve (Simonsohn et al., 2014)
  - *p*-uniform (Van Assen et al., 2015)



**TABLE 2** Performance of 13 publication bias correction methods for the Kvarven and colleagues<sup>34</sup> test set comprised of 15 meta-analyses and 15 corresponding “Gold Standard” registered replication reports (RRR)

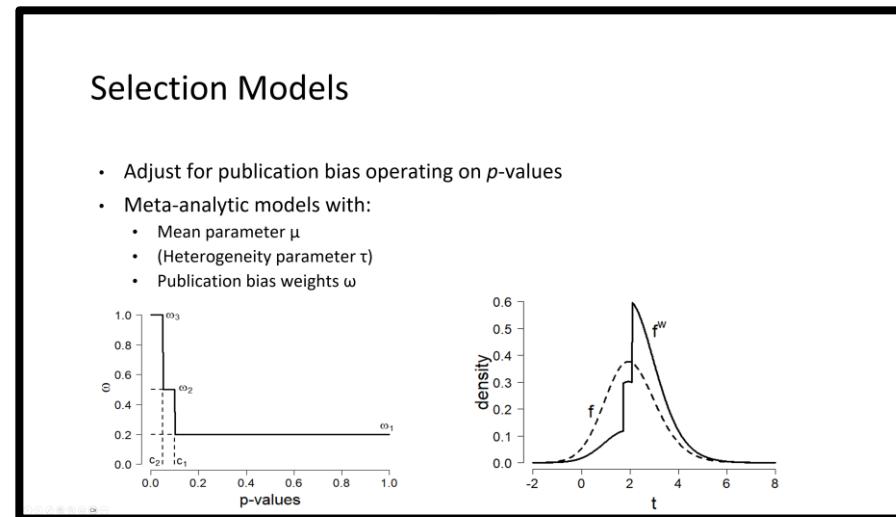
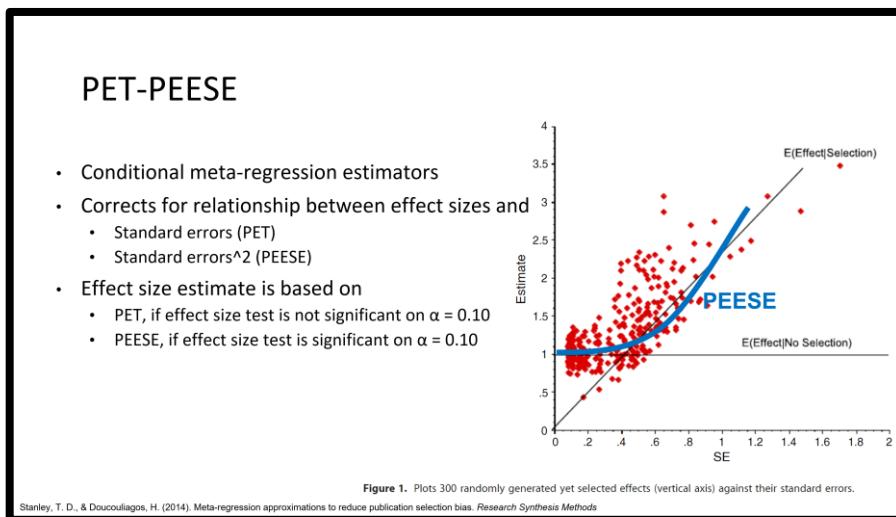
Method	FPR/Uncertain	FNR/Uncertain	OF	Bias	RMSE
RoBMA-PSMA	0.143/0.857	0.000/0.750	1.160	0.026	0.164
AK2	0.000/—	0.250/—	1.043	-0.070	0.268
PET-PEESE	0.143/—	0.500/—	1.307	0.050	0.256
EK	0.143/—	0.500/—	1.399	0.065	0.283
RoBMA-old	0.714/0.286	0.000/0.000	2.049	0.171	0.218
4PSM	0.714/—	0.500/—	1.778	0.127	0.268
3PSM	0.714/—	0.125/—	2.193	0.195	0.245
TF	0.833/—	0.000/—	2.315	0.206	0.259
AK1	0.857/—	0.000/—	2.352	0.221	0.264
p-uniform	0.500/—	0.429/—	2.375	0.225	0.288
p-curve			2.367	0.223	0.289
WAAP-WLS	0.857/—	0.125/—	2.463	0.239	0.295
Random Effects (DL)	1.000/—	0.000/—	2.586	0.259	0.310

# Robust Bayesian Meta-Regression

- Extends RoBMA to moderators
- Bayesian model-averaging to base inference on multiple models simultaneously
- Accounts for uncertainty about the presence vs. absence of the effect/heterogeneity/publication bias/**moderators**
- Quantifies evidence in favor of the presence vs. absence of effect/heterogeneity/publication bias/**moderators**

# Robust Bayesian Meta-Regression

- Uncertainty in model structure



- Under-powered moderation analyses

THE PRIOR MODEL DEMONS ALL SHOUT...



"YOU NEED  
ONLY THIS  
PREDICTOR!"



'INCLUDE ALL  
PREDICTORS!'



'Include no predictor!'



THE PRIOR MODEL DEMONS ALL SHOUT ...



THE PRIOR MODEL DEMONS ALL SHOUT ...

"No bias here!"  
(Unadjusted models)

"The treatment works!"  
(Alternative hypothesis models)

"There is additional heterogeneity!"  
(Random effects models)

"Only sampling variability!"  
(Fixed effects models)

"Everything is biased!"

THE PRIOR MODEL DEMONS ALL SHOUT ...

"Use PET model!"

'YOU NEED  
"Use PESE model!"  
PREDICTOR!'

"One-sided selection on significant p-values!"

"How about direction of the effect?"

"Two-sided selection on significant and marginally significant p-values!"

# RoBMA.reg – Evaluating Evidence

$$\underbrace{\text{BF}_{\text{inclusion}}}_{\text{Inclusion Bayes factor}} = \frac{\underbrace{\sum_{a \in A} p(\mathcal{M}_a | \text{data})}_{\text{Posterior inclusion odds}}}{\underbrace{\sum_{b \in B} p(\mathcal{M}_b | \text{data})}_{\text{Prior inclusion odds}}} / \frac{\underbrace{\sum_{a \in A} p(\mathcal{M}_a)}_{\text{Posterior inclusion odds}}}{\underbrace{\sum_{b \in B} p(\mathcal{M}_b)}_{\text{Prior inclusion odds}}}$$

- Inclusion Bayes factors for the moderation effect

$$\underbrace{\text{BF}_{\text{moderation}}}_{\text{Inclusion Bayes factor for moderation}} = \frac{\underbrace{\sum_{a \in A} p(\mathcal{M}_a | \text{data})}_{\text{Posterior inclusion odds for moderation}}}{\underbrace{\sum_{b \in B} p(\mathcal{M}_b | \text{data})}_{\text{Prior inclusion odds for moderation}}} / \frac{\underbrace{\sum_{a \in A} p(\mathcal{M}_a)}_{\text{Posterior inclusion odds}}}{\underbrace{\sum_{b \in B} p(\mathcal{M}_b)}_{\text{Prior inclusion odds}}}$$

# RoBMA.reg – Estimating Parameters

- Model-averaged posterior distributions account for uncertainty in the selected models

$$\underbrace{p(\theta \mid \text{data})}_{\text{Model-averaged posterior}} = \sum \underbrace{p(\theta \mid \mathcal{M}_., \text{data})}_{\text{Model specific posterior}} \underbrace{p(\mathcal{M}_. \mid \text{data})}_{\text{Posterior probability of model}}$$

# Some Complications

- Parameterization
- Follow-up analyses

# Continuous vs. Categorical Moderators

- Different scaling (continuous moderators) and contrasts coding (factor moderators) corresponds to different hypotheses

## Continuous moderators

- Centering
  - => intercept corresponds to the mean effect  
(prior distribution on the mean effect corresponds to a meta-analysis)
- Scaling
  - => standardized meta-regression coefficients  
(prior distribution on the regression coefficient is scale invariant)

# Continuous vs. Categorical Moderators

## Categorical moderators

- Dummy coding
  - => intercept corresponds to the effect in the default category
  - => individual dummy coefficients test for differences between the default and remaining categories
- (Scaled) Orthonormal contrasts
  - => intercept corresponds to the mean effect  
(prior distribution on the mean effect corresponds to a meta-analysis)
  - => individual orthonormal coefficients on the differences of each category and the mean effect  
(prior distribution on the regression coefficients is label invariant)

# Continuous vs. Categorical Moderators

Default parameter prior distributions (Cohen's d)

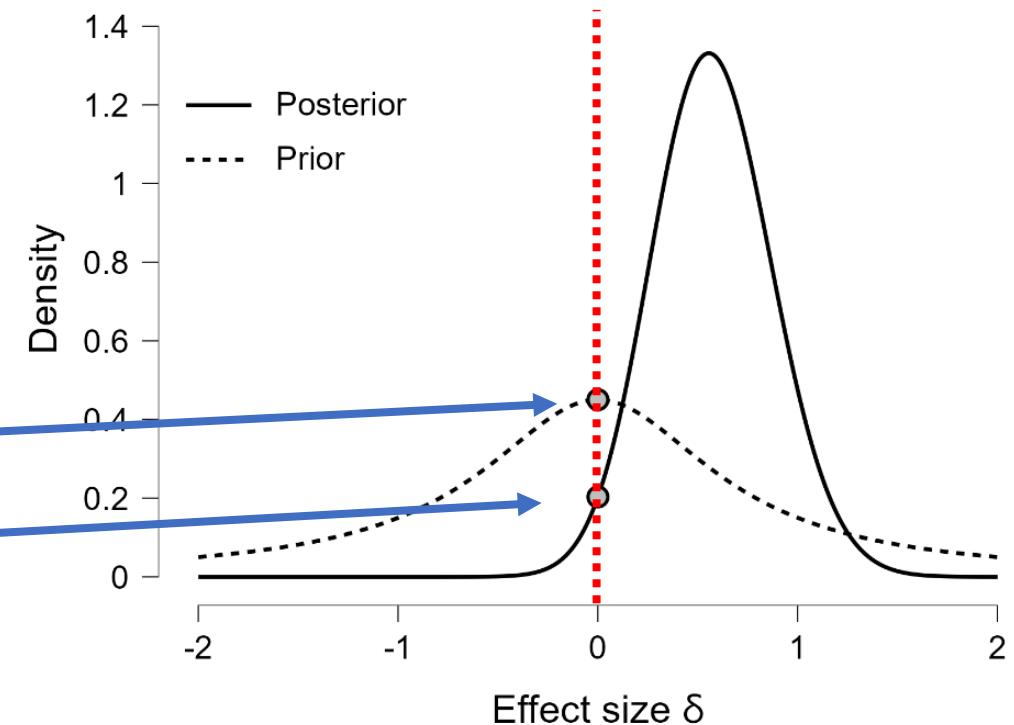
- Standard normal prior distribution on the mean effect
- Normal prior distribution with mean = 0 and standard deviation =  $\frac{1}{4}$  on centered and scaled continuous moderators
- Normal prior distribution with mean = 0 and standard deviation =  $\frac{1}{4}$  on differences from grand mean for each factor level via scaled orthonormal contrasts

# Testing Subgroup Effects

- Categorical predictors: “Is there an effect in group A?”
  - Subgroup analyses (data sub-setting)
  - Savage Dickey density ratio with model-averaged prior/posterior distributions (assuming presence of the effect or moderation)

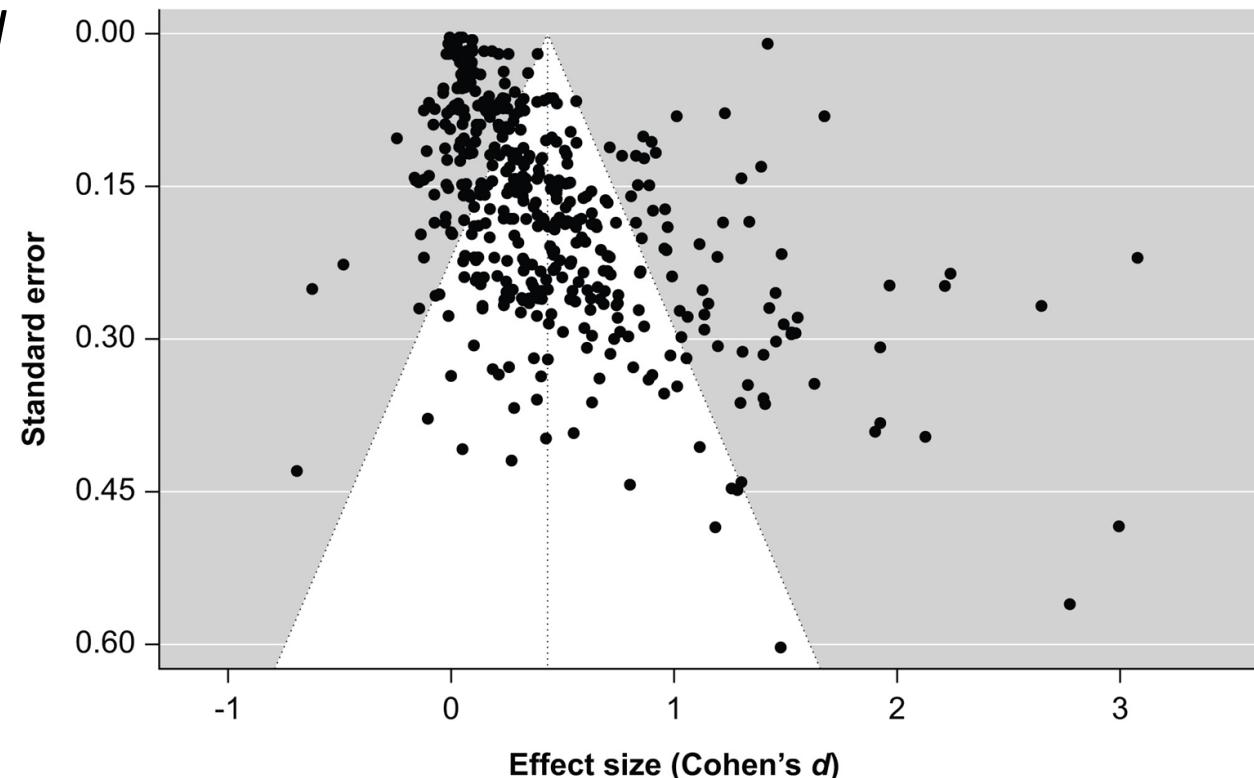
$$BF_{10} = \frac{p(\text{data} \mid \mathcal{H}_1)}{p(\text{data} \mid \mathcal{H}_0)}$$

$$BF_{10} = \frac{p(\theta_g = 0 \mid A)}{p(\theta_g = 0 \mid \text{data}, A)}$$



# Example: No Evidence for Nudging

- Mertens and colleagues (2022) conducted large meta-analysis on nudging  
*“choice architecture is an effective and widely applicable behaviour change tool” (p. 8)*
- Effect moderated based on domain and category of nudge

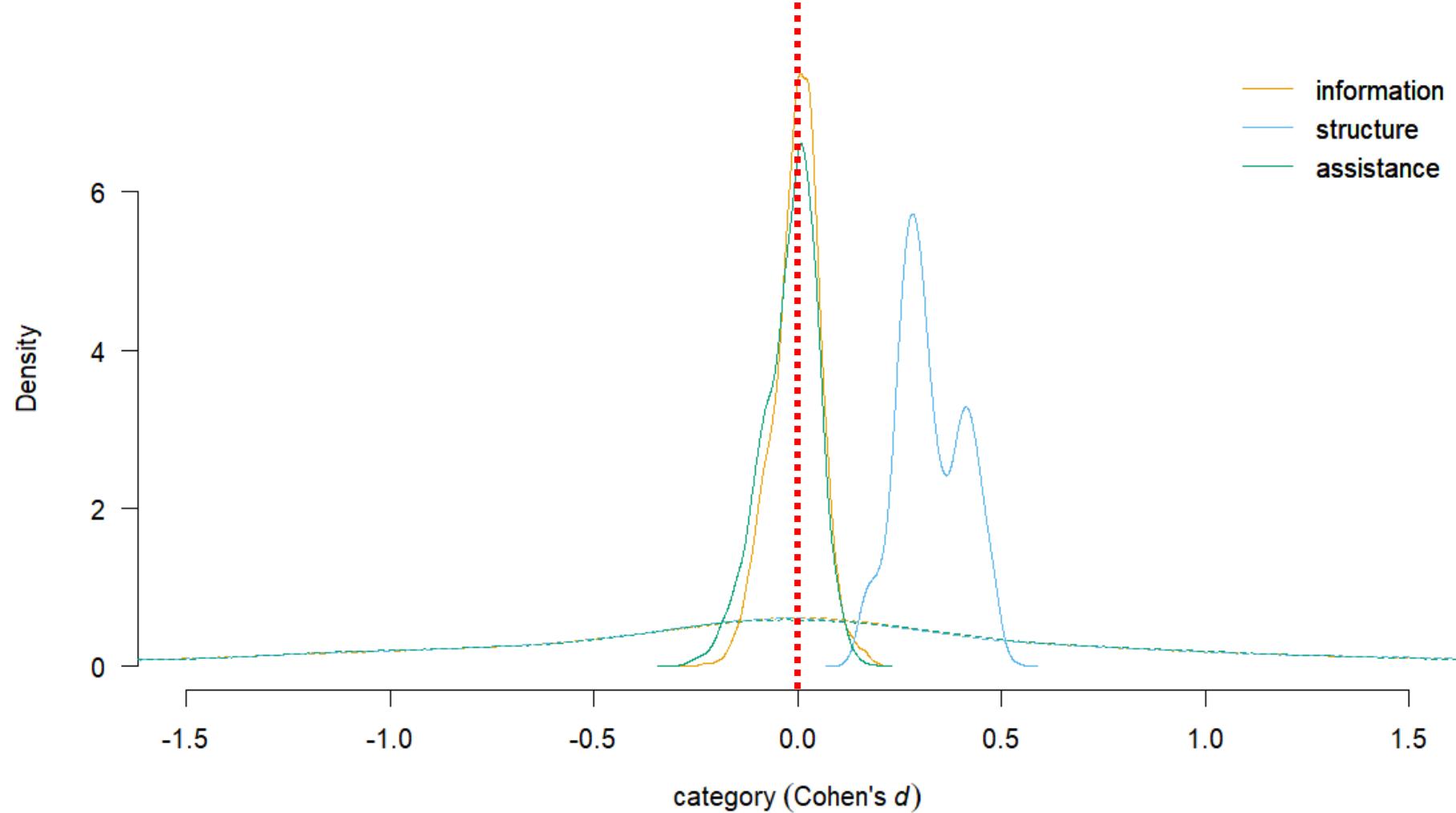


Mertens, S., Herberz, M., Hahnel, U. J. J., Brosch, T. (2022). The effectiveness of nudging: A meta-analysis of choice architecture interventions across behavioral domains. *Proceedings of the National Academy of Sciences*

Maier, M., Bartoš, F.\* T.D. Stanley, David R. Shanks, Adam, J.L. Harris & Wagenmakers, E.-J. (2022). No evidence for nudging after adjusting for publication bias. *Proceedings of the National Academy of Sciences*

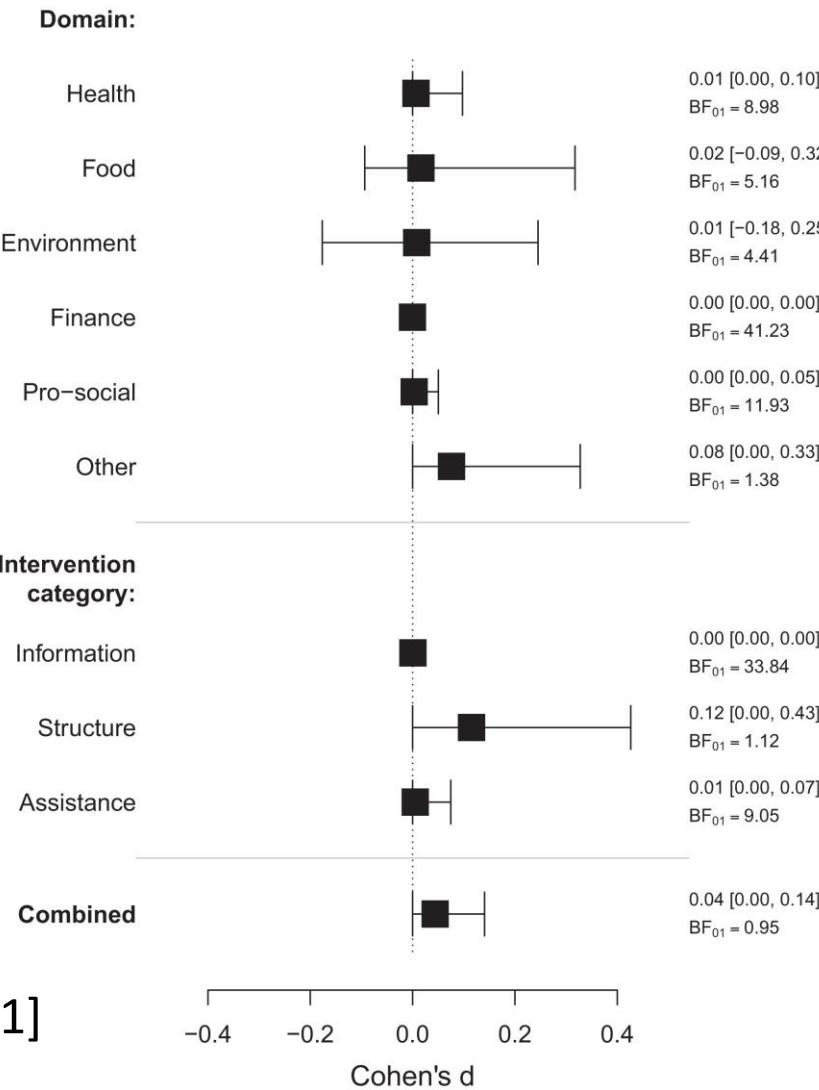
# Example: RoBMA

- Effect:  $BF_{10} = 1.20$ ,  $\mu = 0.06$ , 95% CI [0.00, 0.17]
- Heterogeneity:  $BF_{rf} = \text{Inf}$ ,  $\tau = 0.36$ , 95% CI [0.27, 0.45]
- Publication Bias:  $BF_{pb} = 1.02 \times 10^{13}$
- Moderation
  - Domain:  $BF_{10} = 2.33$
  - Category:  $BF_{10} = 1.60 \times 10^{11}$
- Subgroups by *Category*
  - Information:  $BF_{10} = 0.08$ ,  $\mu_{\text{information}} = 0.00$ , 95% CI [-0.13, 0.11]
  - Structure:  $BF_{10} = \text{Inf}$ ,  $\mu_{\text{structure}} = 0.32$ , 95% CI [0.17, 0.48]
  - Assistance:  $BF_{10} = 0.09$ ,  $\mu_{\text{assistance}} = 0.02$ , 95% CI [-0.18, 0.10]



# Example: RoBMA

- Effect:  $BF_{10} = 1.20$ ,  $\mu = 0.06$ , 95% CI [0.00, 0.17]
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# Simulation Study

1.  $\mu = (0, 0.2, 0.5)$
2.  $\beta = (0, 0.2, 0.5)$
3.  $\tau = (0, 0.2, 0.4)$
4.  $K = (30, 100)$
5. Publication bias
  - a. No bias:  $\omega_1 = 1, \omega_2 = 1, \omega_3 = 1$
  - b. Moderate bias:  $\omega_1 = 0.2, \omega_2 = 0.5, \omega_3 = 1$
  - c. Strong bias:  $\omega_1 = 0, \omega_2 = 0, \omega_3 = 1$

$\omega_1$ = Nonsignificant studies

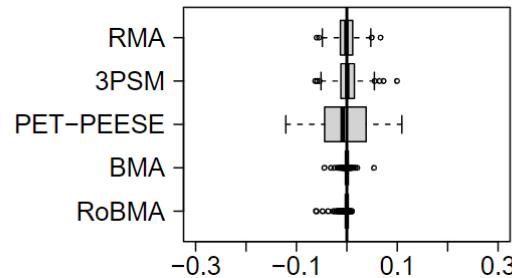
$\omega_2$ = Marginally significant studies

$\omega_3$ = Significant studies

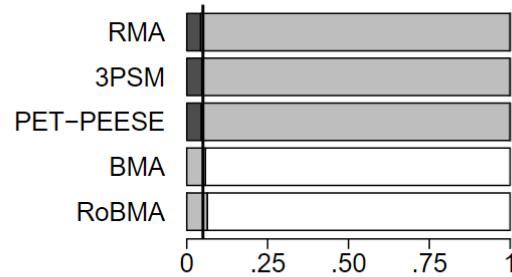
# Simulation Study Results (Select Cases)

$\mu$  (estimate)

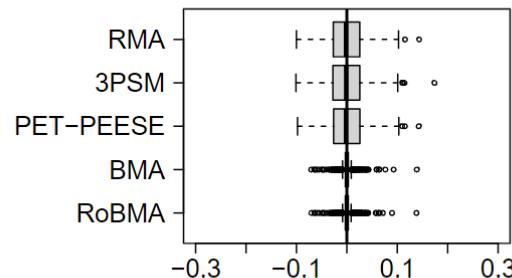
No effect ( $\mu = 0$ )  
No moderation ( $\beta = 0$ )  
No heterogeneity ( $\tau = 0$ )  
No pub. bias



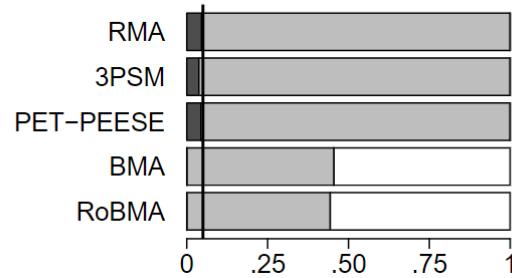
$\mu$  (test)



$\beta$  (estimate)



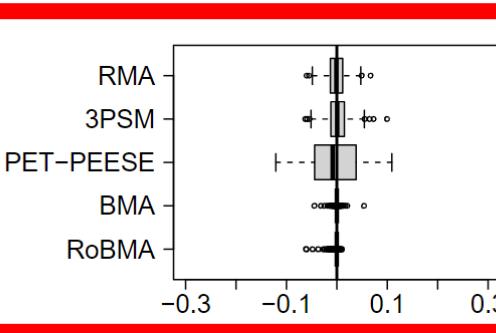
$\beta$  (test)



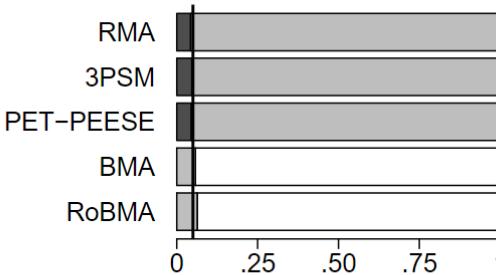
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No effect ( $\mu = 0$ )  
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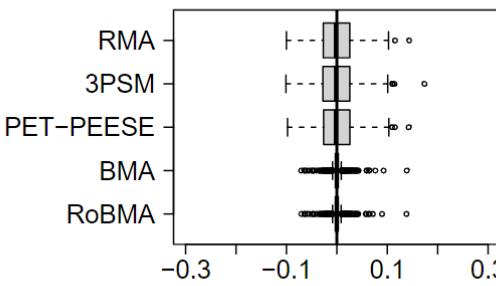
$\mu$  (estimate)



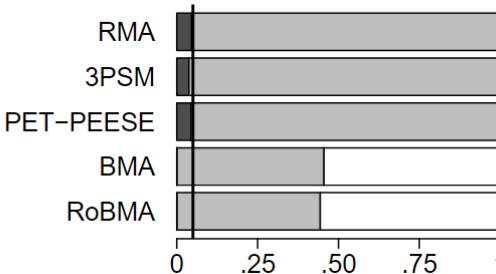
$\mu$  (test)



$\beta$  (estimate)



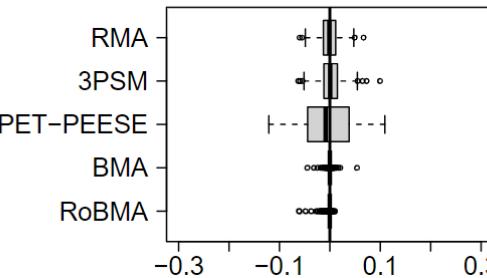
$\beta$  (test)



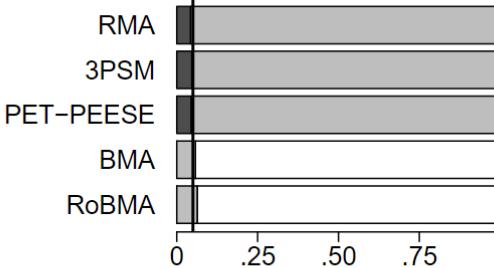
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$\mu$  (estimate)

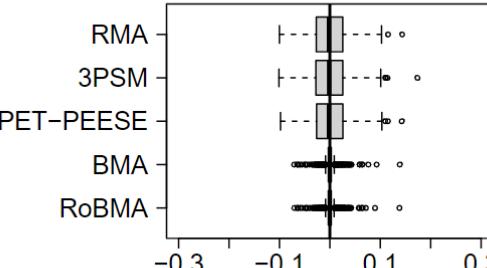
No effect ( $\mu = 0$ )  
No moderation ( $\beta = 0$ )  
No heterogeneity ( $\tau = 0$ )  
No pub. bias



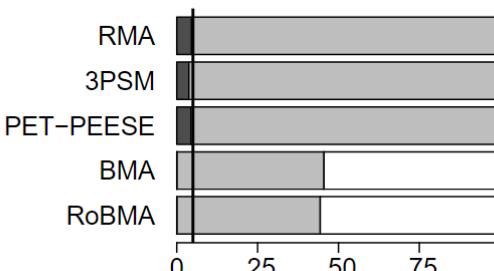
$\mu$  (test)



$\beta$  (estimate)



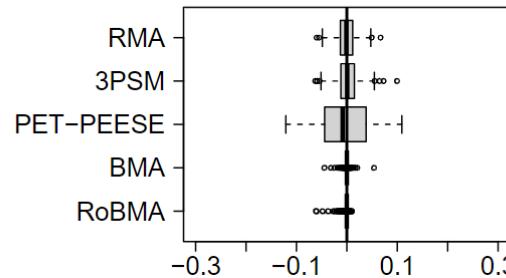
$\beta$  (test)



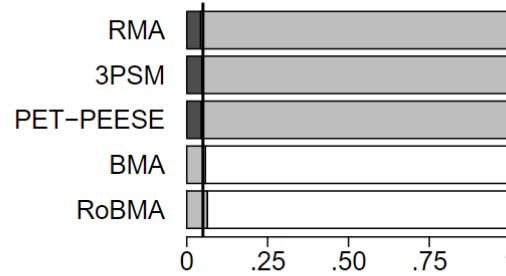
# Simulation Study Results (Select Cases)

$\mu$  (estimate)

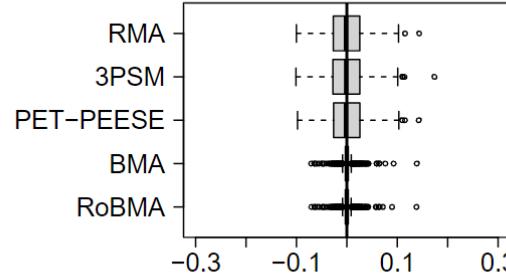
No effect ( $\mu = 0$ )  
No moderation ( $\beta = 0$ )  
No heterogeneity ( $\tau = 0$ )  
No pub. bias



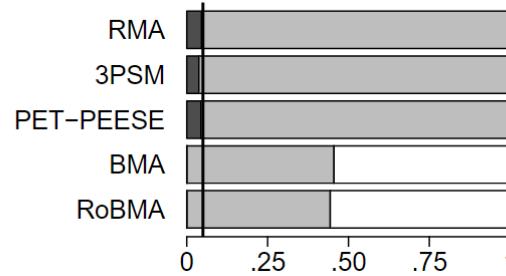
$\mu$  (test)



$\beta$  (estimate)



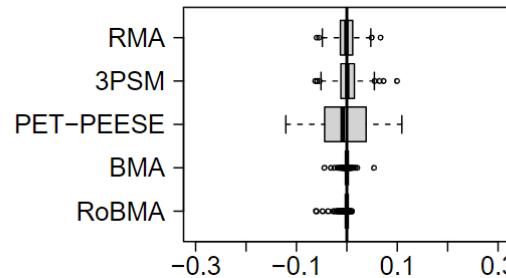
$\beta$  (test)



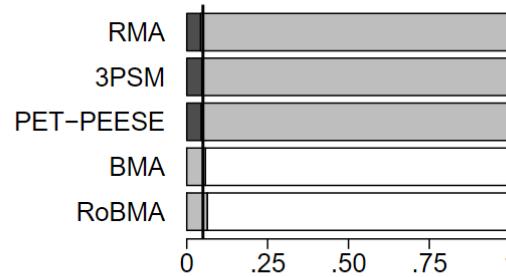
# Simulation Study Results (Select Cases)

$\mu$  (estimate)

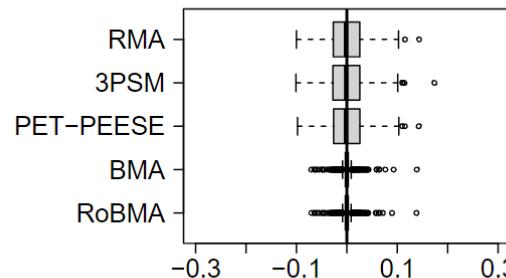
No effect ( $\mu = 0$ )  
No moderation ( $\beta = 0$ )  
No heterogeneity ( $\tau = 0$ )  
No pub. bias



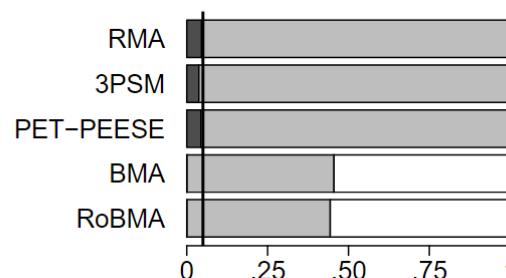
$\mu$  (test)



$\beta$  (estimate)

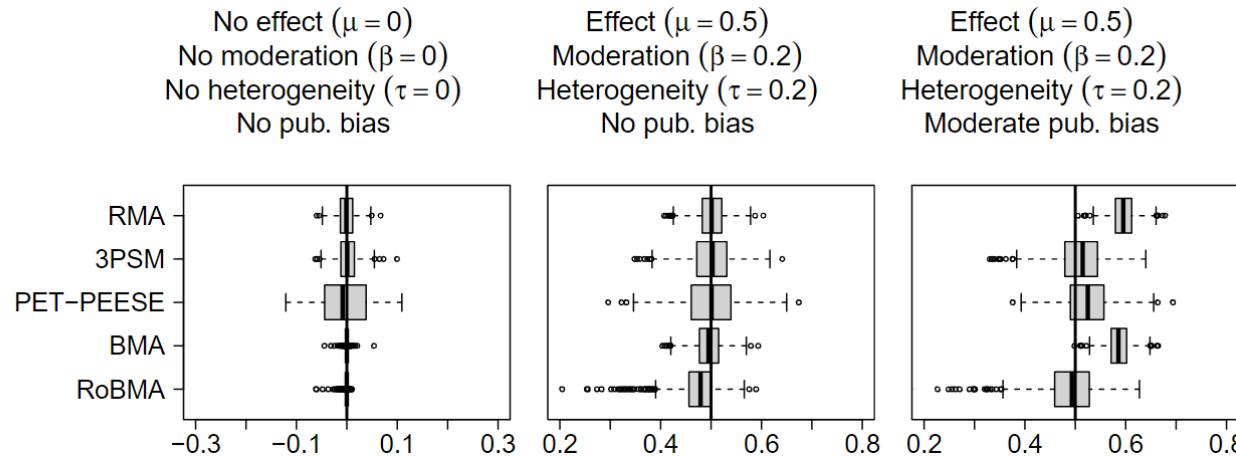


$\beta$  (test)

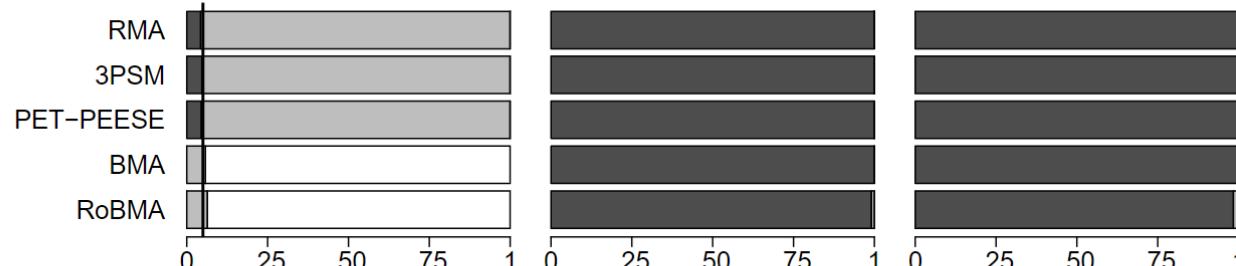


# Simulation Study Results (Select Cases)

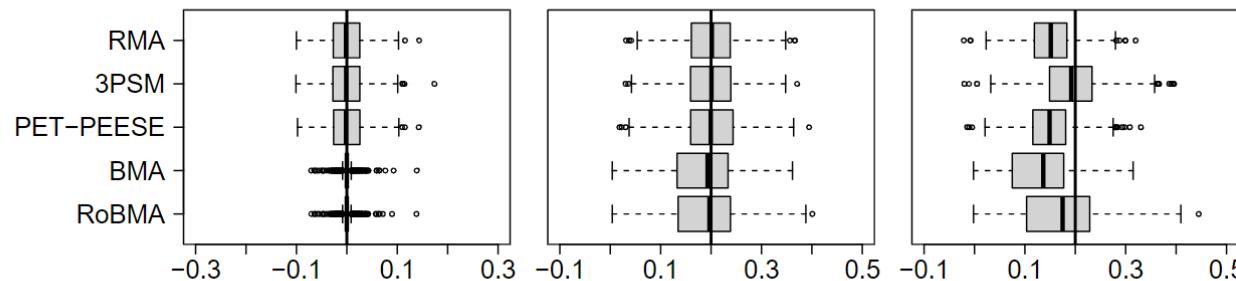
$\mu$  (estimate)



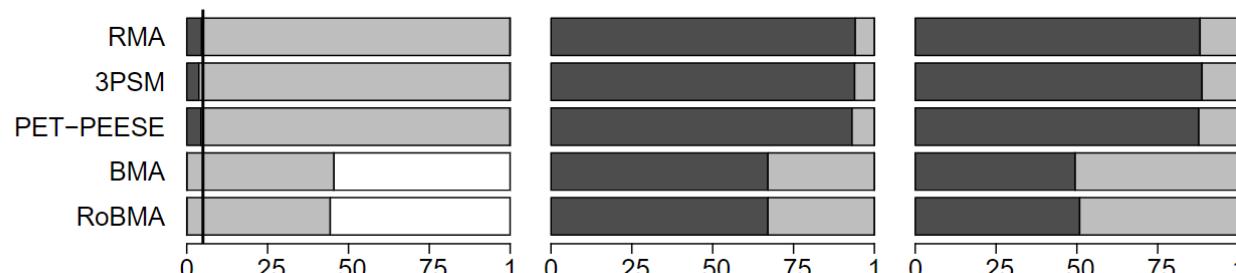
$\mu$  (test)



$\beta$  (estimate)



$\beta$  (test)



# Simulation Study Results (Select Cases)

$\mu$  (estimate)

$\mu$  (test)

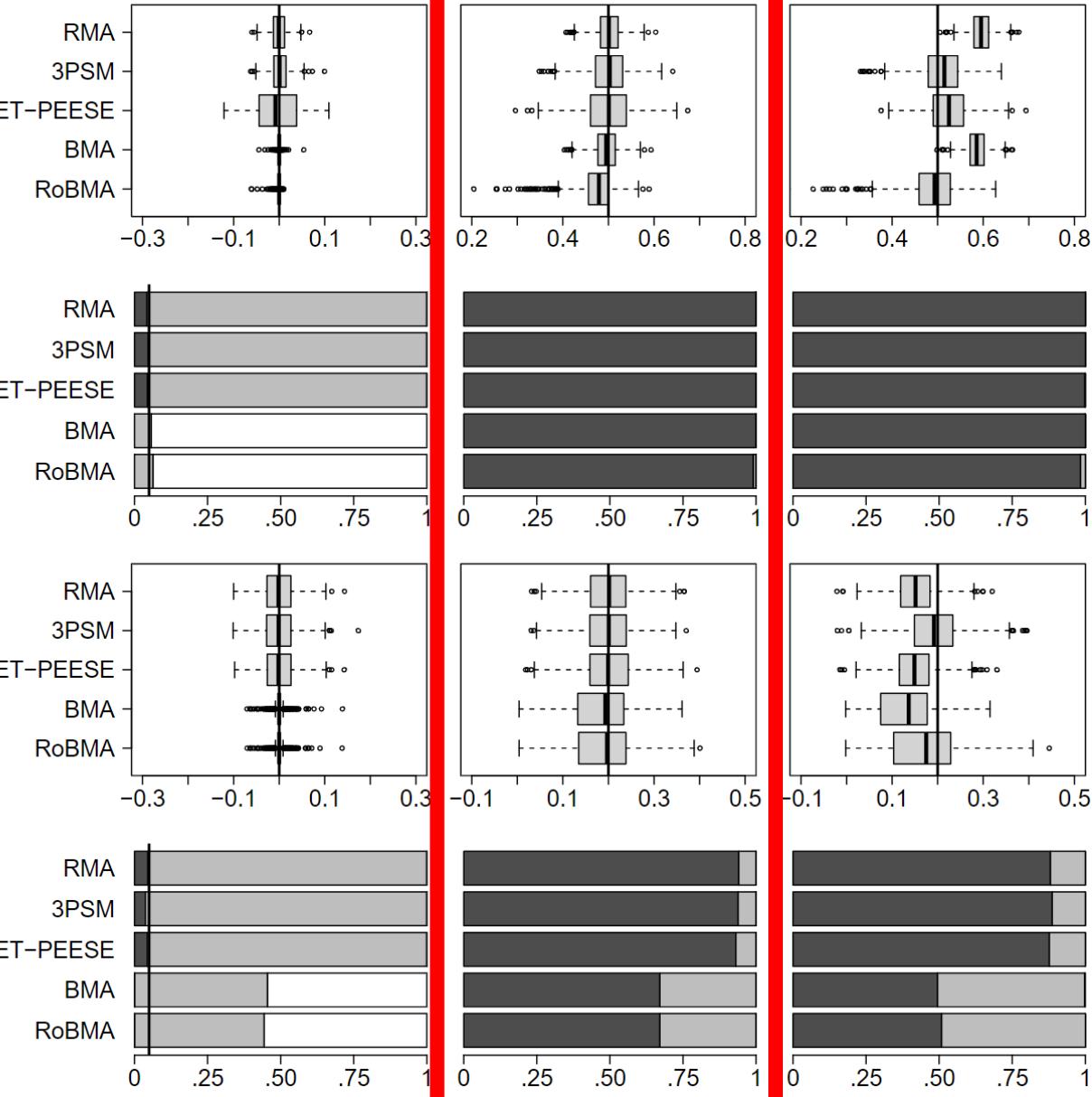
$\beta$  (estimate)

$\beta$  (test)

No effect ( $\mu = 0$ )  
No moderation ( $\beta = 0$ )  
No heterogeneity ( $\tau = 0$ )  
No pub. bias

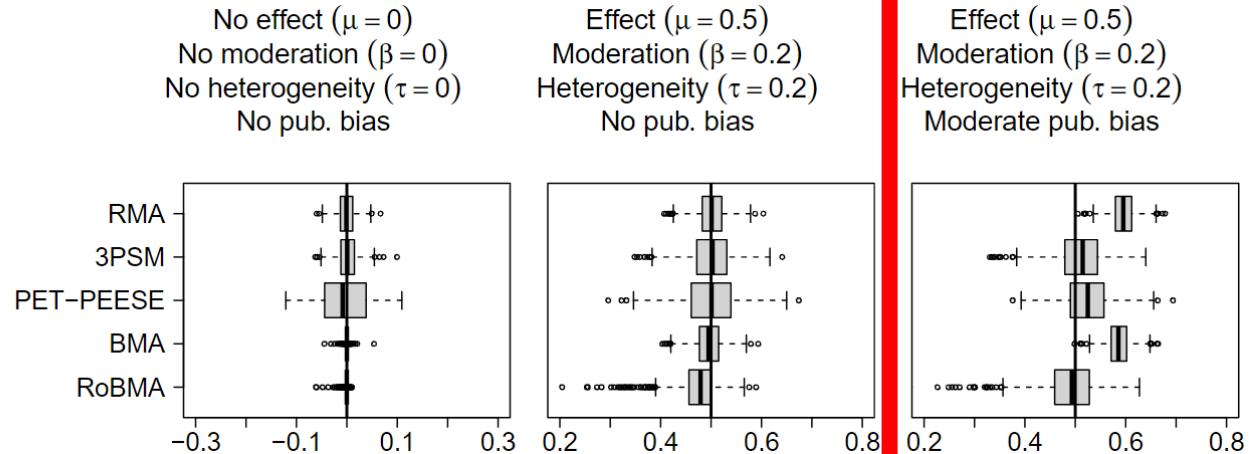
Effect ( $\mu = 0.5$ )  
Moderation ( $\beta = 0.2$ )  
Heterogeneity ( $\tau = 0.2$ )  
No pub. bias

Effect ( $\mu = 0.5$ )  
Moderation ( $\beta = 0.2$ )  
Heterogeneity ( $\tau = 0.2$ )  
Moderate pub. bias

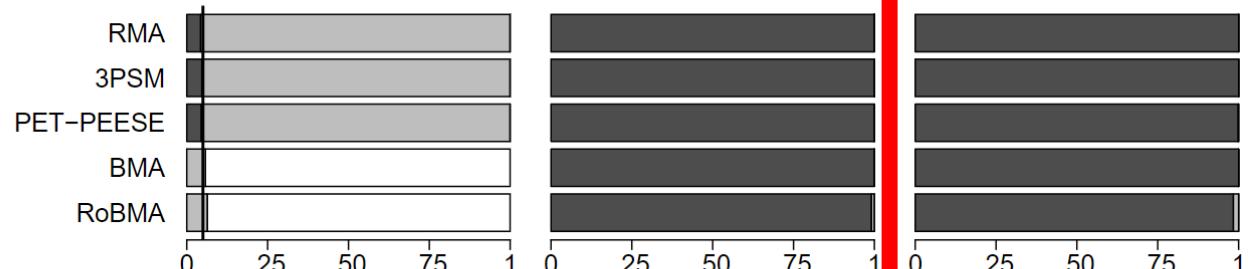


# Simulation Study Results (Select Cases)

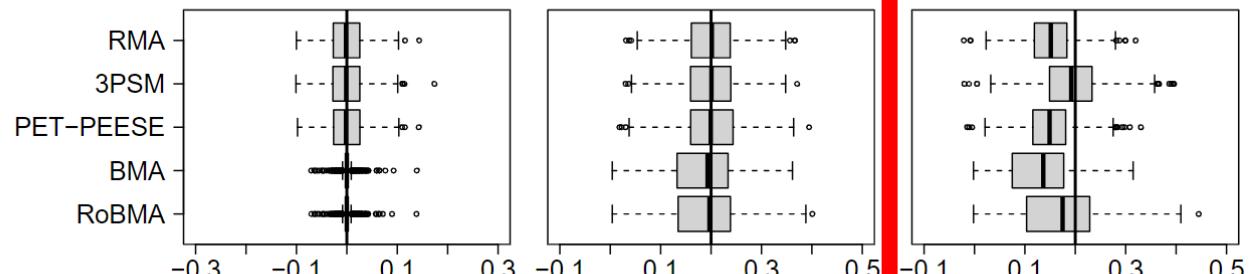
$\mu$  (estimate)



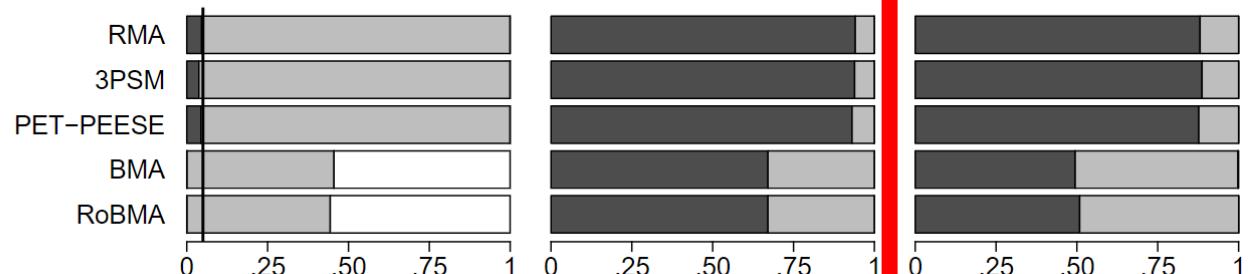
$\mu$  (test)



$\beta$  (estimate)



$\beta$  (test)



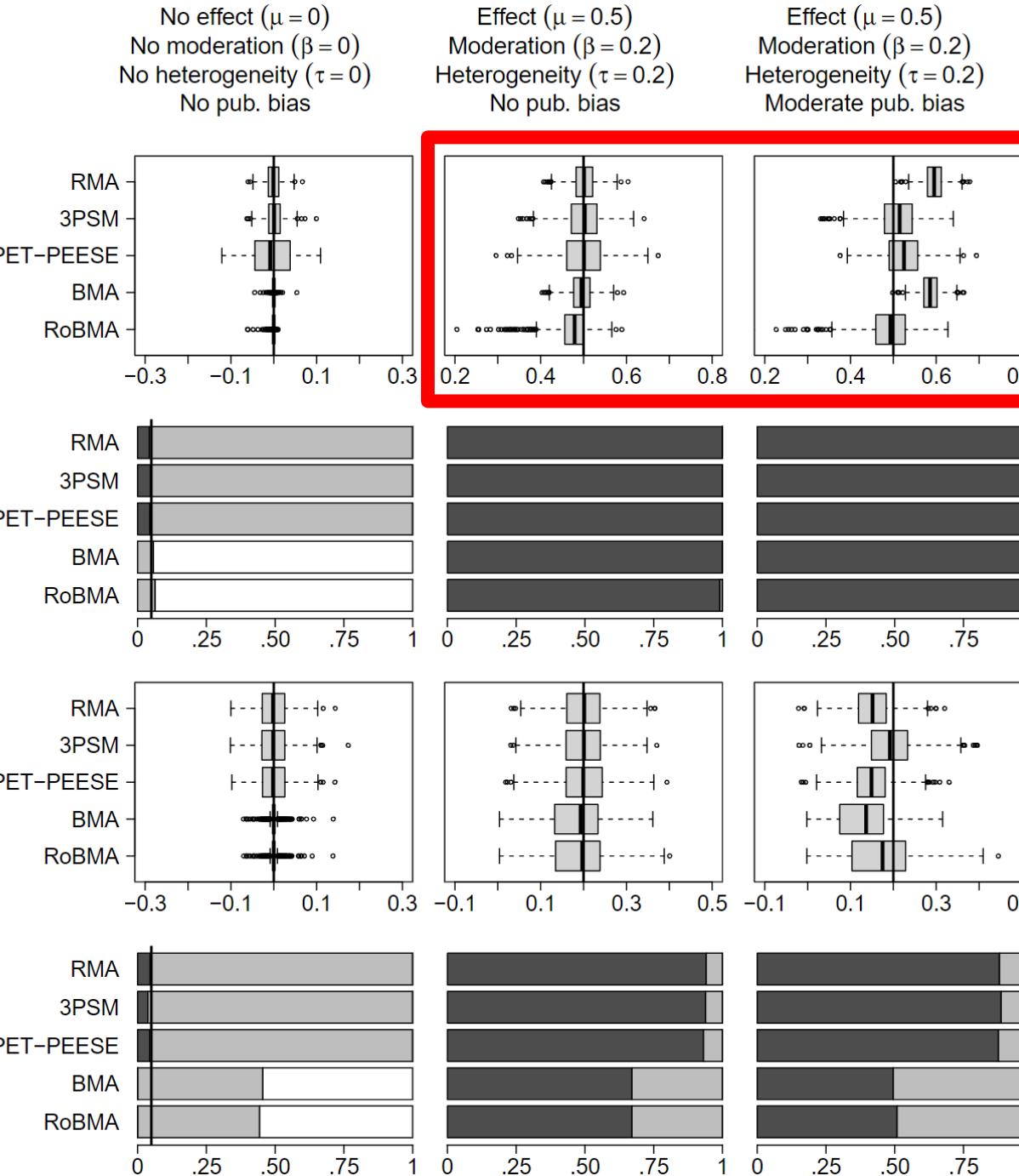
# Simulation Study Results (Select Cases)

$\mu$  (estimate)

$\mu$  (test)

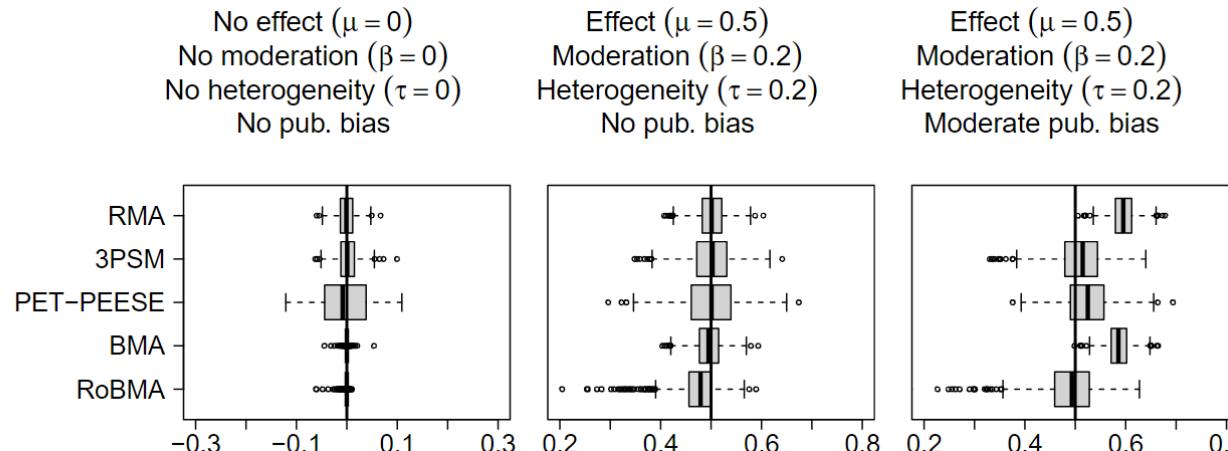
$\beta$  (estimate)

$\beta$  (test)

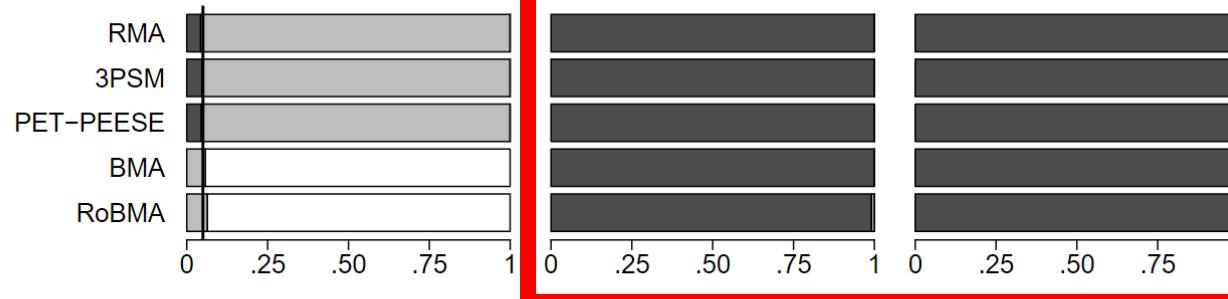


# Simulation Study Results (Select Cases)

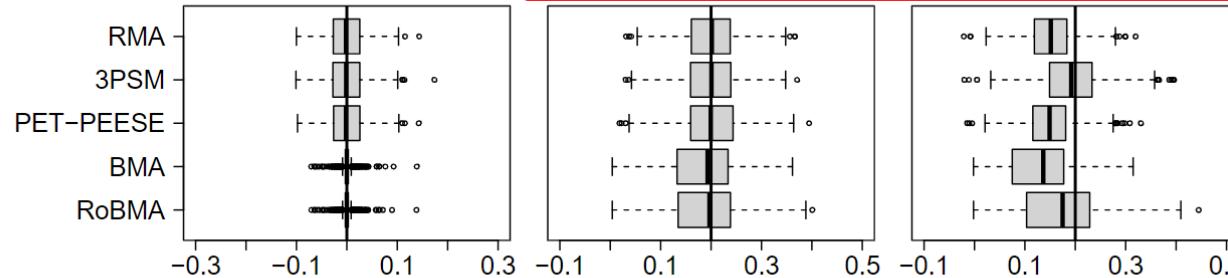
$\mu$  (estimate)



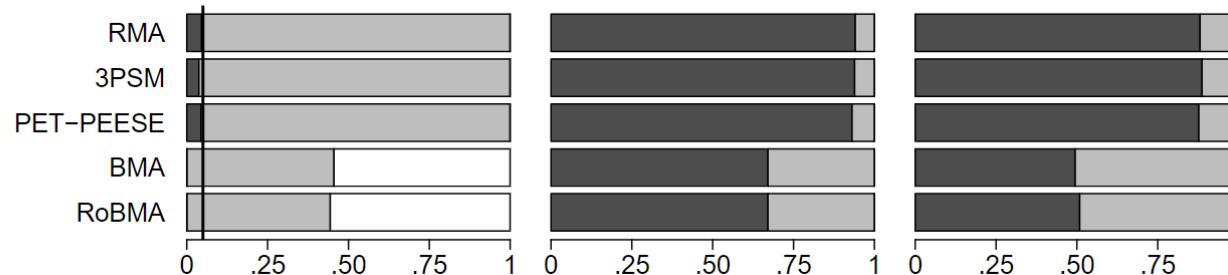
$\mu$  (test)



$\beta$  (estimate)

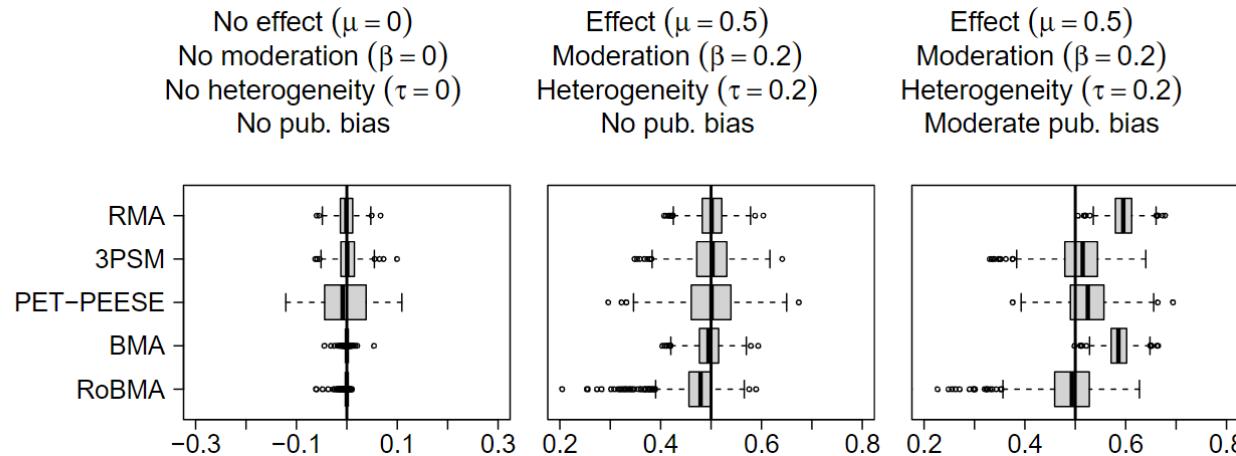


$\beta$  (test)

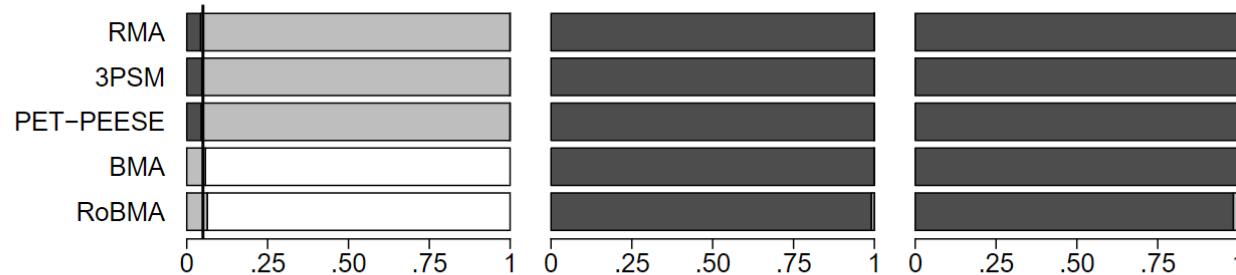


# Simulation Study Results (Select Cases)

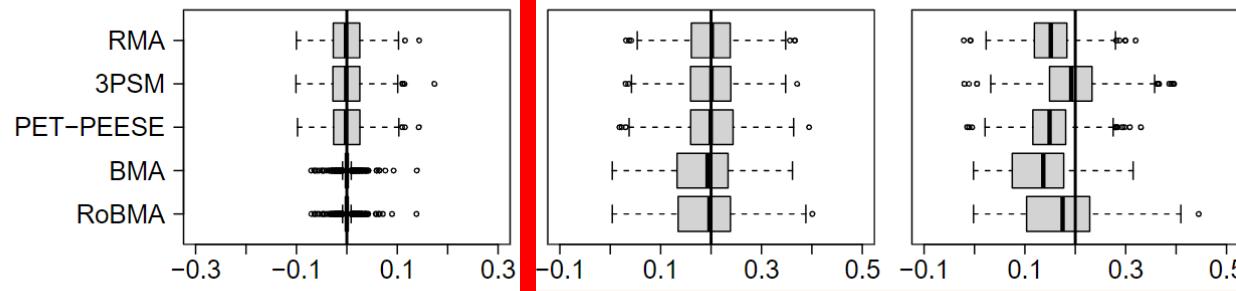
$\mu$  (estimate)



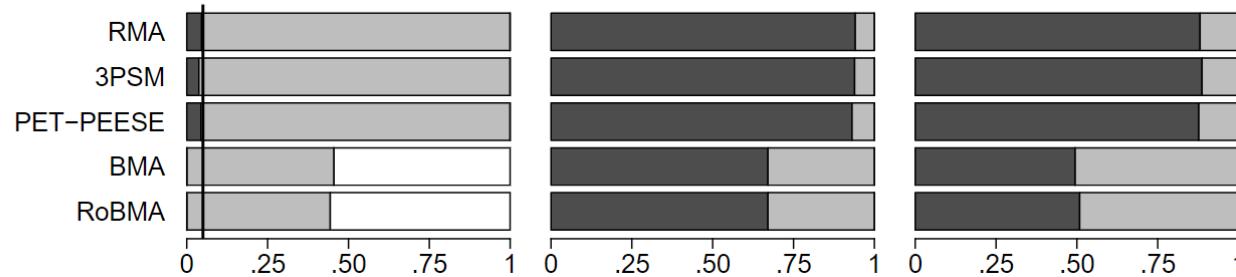
$\mu$  (test)



$\beta$  (estimate)

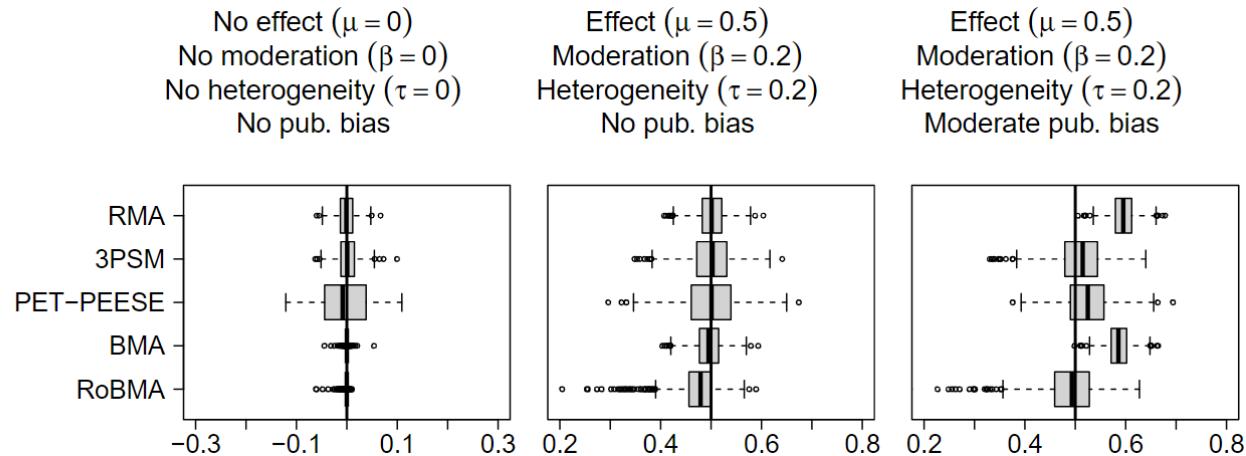


$\beta$  (test)

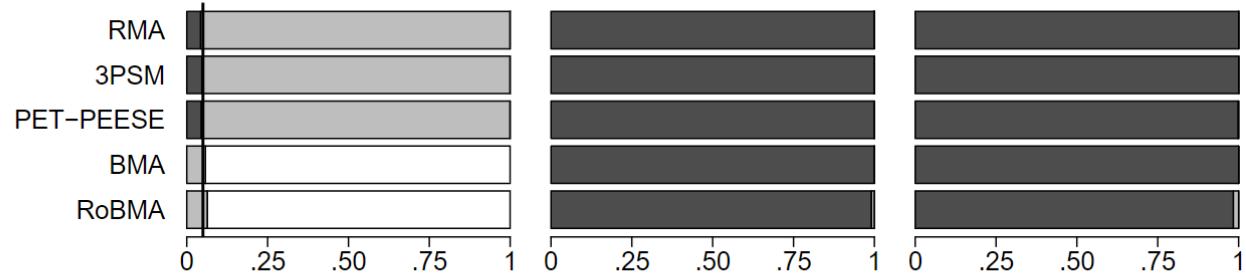


# Simulation Study Results (Select Cases)

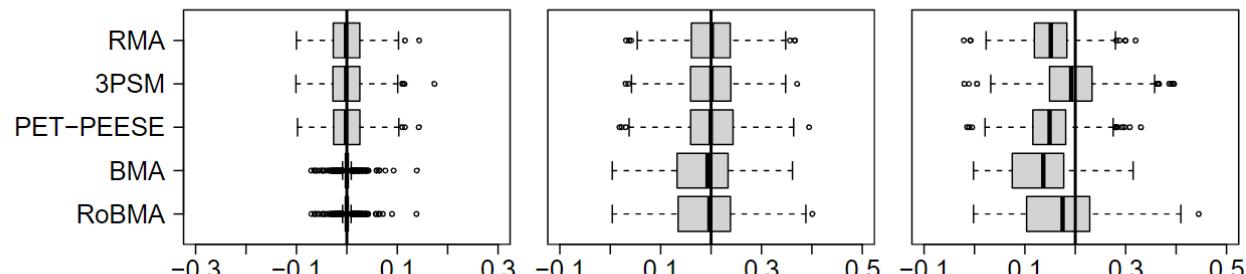
$\mu$  (estimate)



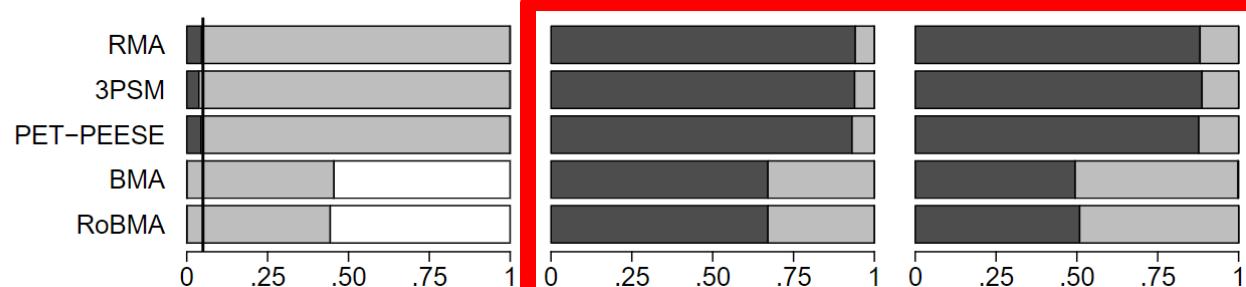
$\mu$  (test)



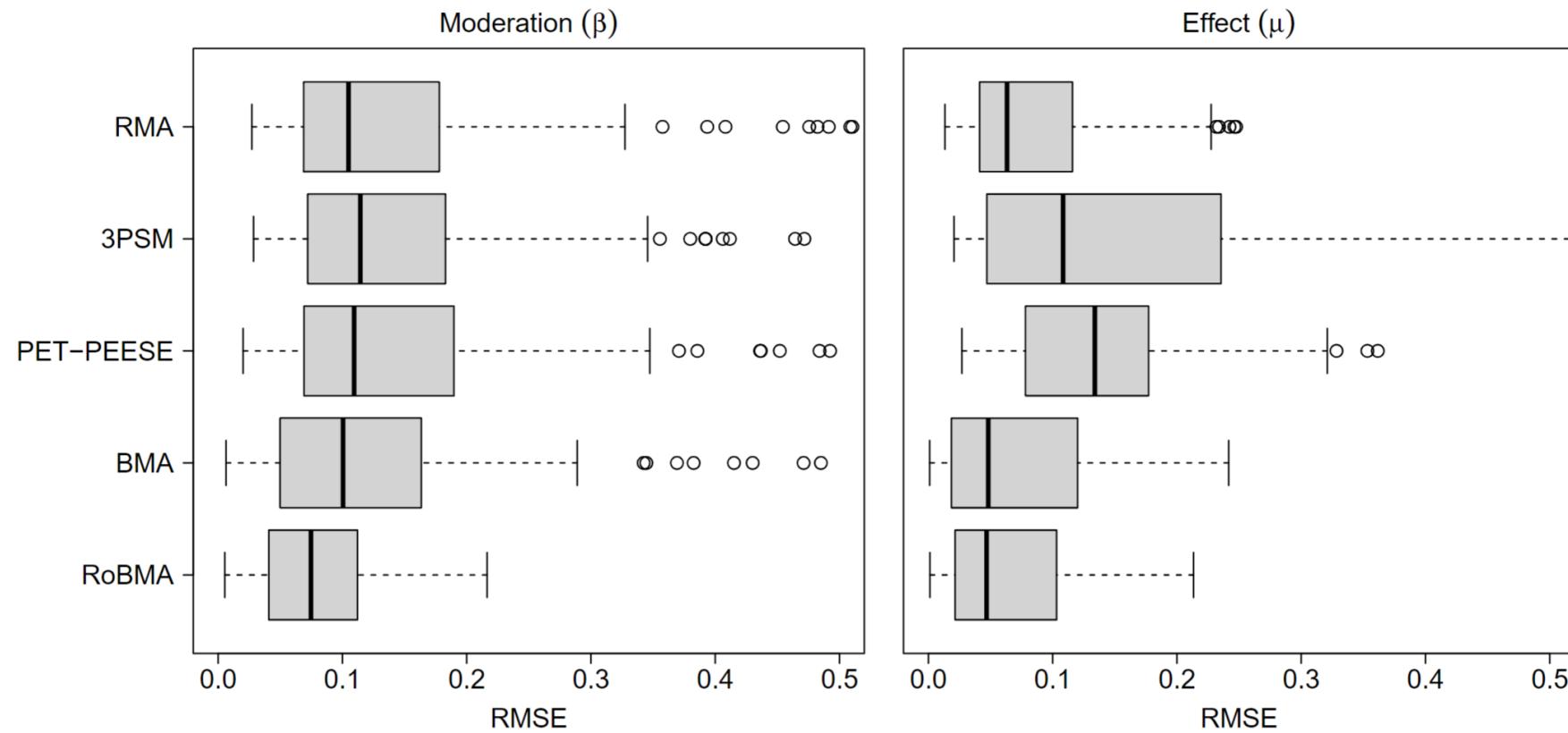
$\beta$  (estimate)



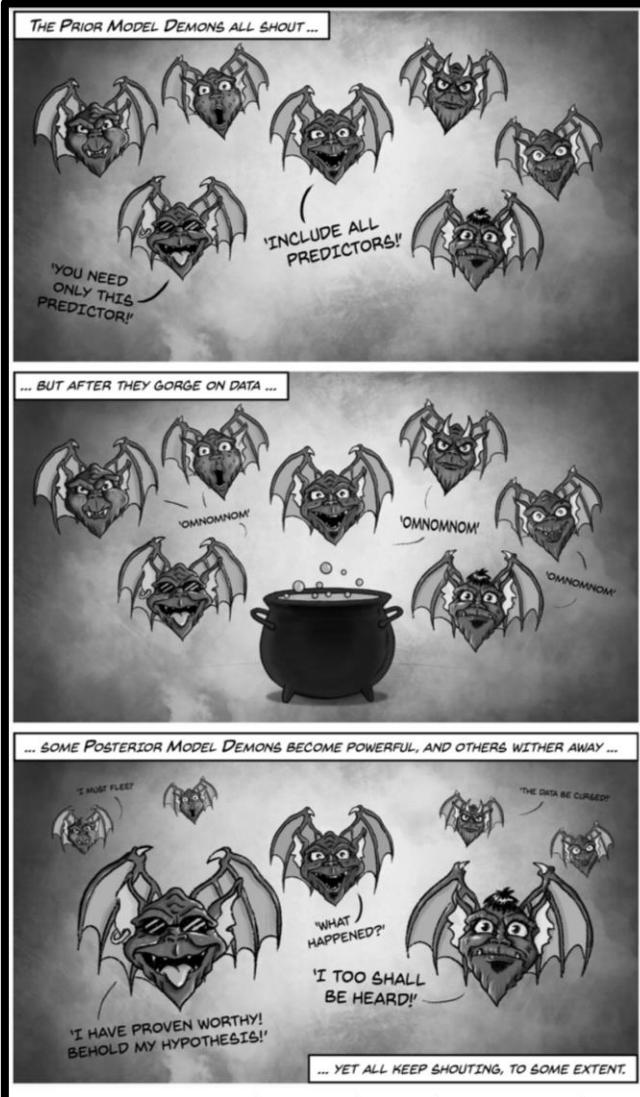
$\beta$  (test)



# Simulation Study Results (Across Conditions)



# How to Run RoBMA



prior parameter distributions

$$p(\Theta_0|\mathcal{H}_0^f) : \mu = 0$$

$$p(\text{data}|\Theta_0, \mathcal{H}_0^f) : y \sim \text{Normal}(\mu, \text{se})$$

Specify the

posterior parameter distributions

$$p(\Theta_0|\mathcal{H}_0^f, \text{data}) = \frac{p(\text{data}|\Theta_0, \mathcal{H}_0^f) p(\Theta_0|\mathcal{H}_0^f)}{p(\text{data}|\mathcal{H}_0^f)}$$

$$p(\Theta_1|\mathcal{H}_1^f, \text{data}) = \frac{p(\text{data}|\Theta_1, \mathcal{H}_1^f) p(\Theta_1|\mathcal{H}_1^f)}{p(\text{data}|\mathcal{H}_1^f)}$$

marginal likelihoods

$$p(\text{data}|\mathcal{H}_0^f) = \int p(\text{data}|\Theta_0, \mathcal{H}_0^f) p(\Theta_0|\mathcal{H}_0^f) d\Theta_0$$

$$p(\text{data}|\mathcal{H}_1^f) = \int p(\text{data}|\Theta_1, \mathcal{H}_1^f) p(\Theta_1|\mathcal{H}_1^f) d\Theta_1$$

Update the

Draw inference

Bayes factors

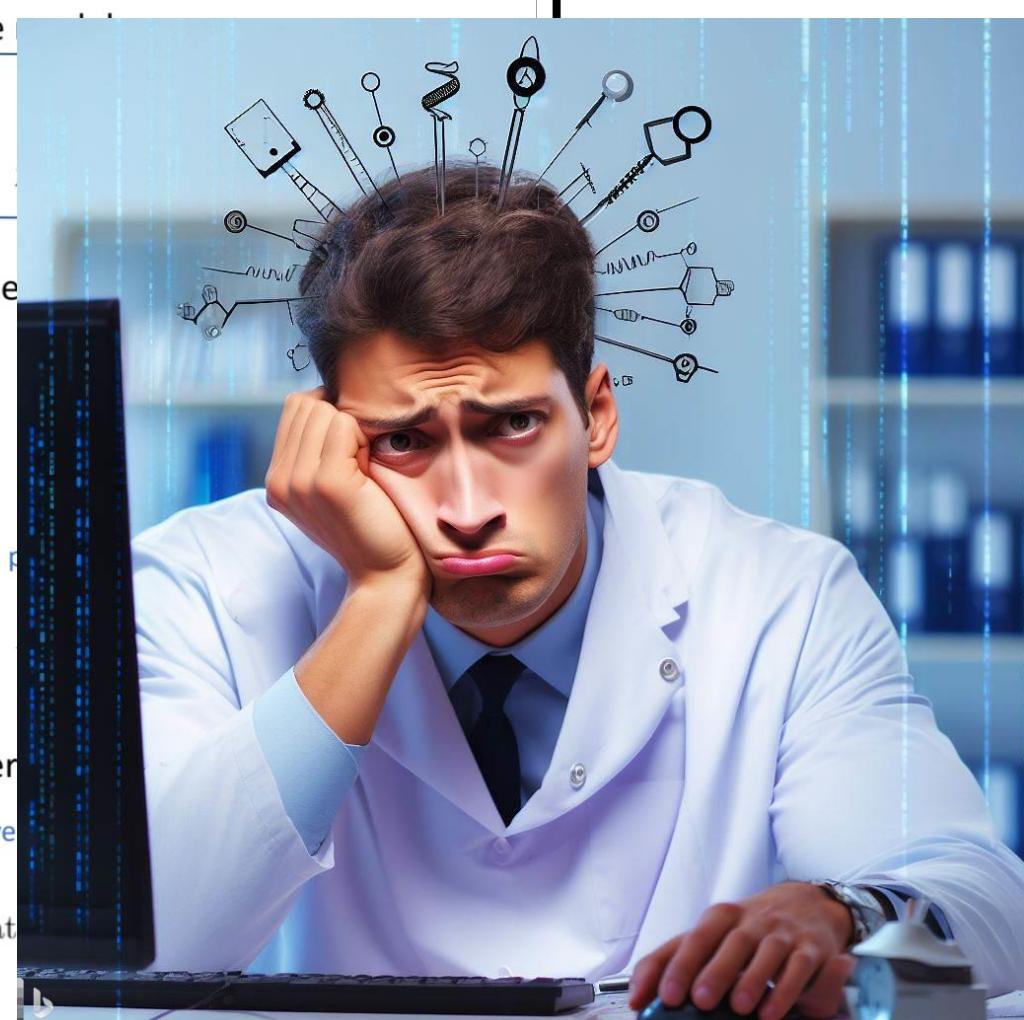
$$\text{BF}_{10} = \frac{p(\text{data}|\mathcal{H}_1^f)}{p(\text{data}|\mathcal{H}_0^f)}$$

$$\frac{p(\text{data} | \mathcal{H}_1^f)}{p(\text{data} | \mathcal{H}_0^f)} = \frac{p(\mathcal{H}_1^f | \text{data})}{p(\mathcal{H}_0^f | \text{data})} \Bigg/ \frac{p(\mathcal{H}_1^f)}{p(\mathcal{H}_0^f)}$$

Bayes factor                          Posterior odds                          Prior odds

Model-averaged

$$p(\Theta|\text{data}) = \frac{p(\text{data}|\Theta) p(\Theta)}{\int p(\text{data}|\Theta') p(\Theta') d\Theta'}$$



# RoBMA Implementation (R)

```
library(RoBMA)

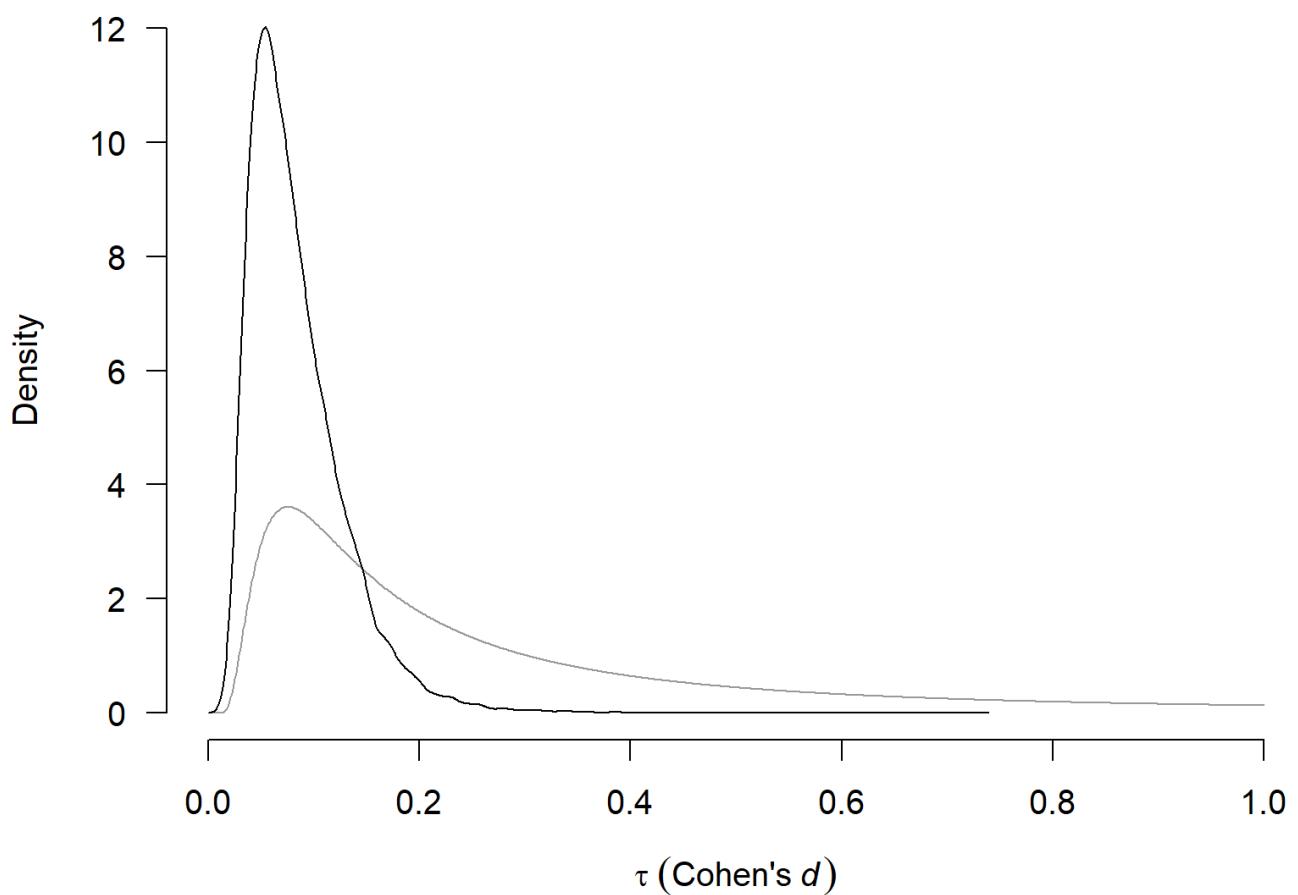
fit <- RoBMA(d = Bem2011$d, se = Bem2011$se)

summary(fit)

> Robust Bayesian Meta-Analysis
>
>          Models Prior prob. Post. prob. Incl. BF
> Effect      18/36     0.500     0.324    0.480
> Heterogeneity 18/36     0.500     0.125    0.143
> Pub. bias    32/36     0.500     0.942   16.297

> Model-averaged estimates
>          Mean Median  0.025  0.975
> mu        0.037  0.000 -0.051  0.218
> tau       0.010  0.000  0.000  0.113
> omega[0,0.025] 1.000  1.000  1.000  1.000
> omega[0.025,0.05] 0.934  1.000  0.332  1.000
> omega[0.05,0.5]  0.784  1.000  0.009  1.000
> omega[0.5,0.95]  0.771  1.000  0.007  1.000
> omega[0.95,0.975] 0.787  1.000  0.007  1.000
> omega[0.975,1]   0.803  1.000  0.007  1.000
> PET         0.758  0.000  0.000  2.790
> PEESE       6.222  0.000  0.000 25.597
```

```
fit <- RoBMA(d = Bem2011$d, se = Bem2011$se)  
plot(fit)  
plot(fit, plot_type = "ggplot")  
plot(fit, parameter = "tau", conditional = TRUE,  
     prior = TRUE, xlim = c(0, 1))
```



```
# specifying an informed one-sided hypothesis test

fit <- RoBMA(
  d = Bem2011$d, se = Bem2011$se,
  priors_effect = prior("normal", parameters = list(mean = 0, sd = 0.30), truncation = list(0, Inf))
)

# specifying only a PET-PEESE style publication bias adjustment

fit <- RoBMA(
  d = Bem2011$d, se = Bem2011$se,
  priors_bias = list(
    prior_PET("Cauchy", parameters = list(0,1), truncation = list(0, Inf), prior_weights = 1/2),
    prior_PEESE("Cauchy", parameters = list(0,5), truncation = list(0, Inf), prior_weights = 1/2)
  )
)
```

```

fit <- RoBMA.reg(~ measure + age, data = df_reg)

summary(fit)

> Robust Bayesian meta-regression

> Components summary:
>          Models Prior prob. Post. prob. Inclusion BF
> Effect      72/144      0.500      0.340 5.150000e-01
> Heterogeneity 72/144      0.500      1.000 1.043068e+23
> Bias        128/144      0.500      0.965 2.797600e+01

> Meta-regression components summary:
>          Models Prior prob. Post. prob. Inclusion BF
> measure    72/144      0.500      0.950      18.940
> age        72/144      0.500      0.154      0.182

> Model-averaged estimates:
>          Mean Median 0.025 0.975
> mu          0.063  0.000 0.000 0.330
> tau         0.213  0.209 0.149 0.301
> omega[0,0.025] 1.000  1.000 1.000 1.000
> omega[0.025,0.05] 1.000  1.000 1.000 1.000
> omega[0.05,0.5]  0.998  1.000 1.000 1.000
> omega[0.5,0.95]  0.997  1.000 1.000 1.000
> omega[0.95,0.975] 0.997  1.000 1.000 1.000
> omega[0.975,1]   0.997  1.000 1.000 1.000
> PET          2.043  2.484 0.000 3.277
> PEESE        1.012  0.000 0.000 9.811
> The estimates are summarized on the Cohen's d scale (priors were specified on the Cohen's d scale).
> (Estimated publication weights omega correspond to one-sided p-values.)

> Model-averaged meta-regression estimates:
>          Mean Median 0.025 0.975
> intercept     0.063  0.000 0.000 0.330
> measure [dif: direct] -0.126 -0.129 -0.216 0.000
> measure [dif: informat] 0.126  0.129 0.000 0.216
> age           0.000  0.000 -0.047 0.047
> The estimates are summarized on the Cohen's d scale (priors were specified on the Cohen's d scale).

fit <- RoBMA.reg(~ measure + age, data = df_reg, priors = list(
  measure = prior_factor("normal", parameters = list(mean = 0, sd = 0.25), contrast = "treatment"),
  age     = prior("gamma", parameters = list(shape = 2, rate = 10))))

```

# Advantages of RoBMA

- Can incorporate uncertainty about the selected model with BMA
- Can provide evidence for either the null or the alternative hypothesis
- Has better performance with small sample sizes
- Has the capacity to incorporate expert knowledge
- Has the potential for sequential updating of evidence

# Disadvantages of RoBMA

- Slow - requires MCMC sampling ( $2^p \times 36$  models)
- Can fail under strong  $p$ -hacking

# Thank you for your Attention

R package: <https://cran.r-project.org/package=RoBMA>

JASP: <https://jasp-stats.org/>

For more about RoBMA:

Maier, M., Bartoš, F., & Wagenmakers, E. J. (2022). Robust Bayesian meta-analysis: Addressing publication bias with model-averaging. *Psychological Methods*.

Bartoš, F., Maier, M., Wagenmakers, E. J., Doucouliagos, H., & Stanley, T. D. (2023). Robust Bayesian meta-analysis: Model-averaging across complementary publication bias adjustment methods. *Research Synthesis Methods*

Bartoš, F., Maier, M., Stanley, T. D., & Wagenmakers, E. J. (2023). Robust Bayesian Meta-Regression—Model-Averaged Moderation Analysis in the Presence of Publication Bias. *PsyArXiv* <https://doi.org/10.31234/osf.io/98xb5>

