Machine Learning Assignment 1 - EE 871

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① Reddy - Mikks Problem:

$$\max z = C_E x_E + C_I x_I \quad (C_E, C_I = 3, 2)$$ - Taken from class example

$$z = 3x_E + 2x_I$$

subjected to the constrains:

$x_E + 2x_I \leq 6$ — ①

$2x_E + x_I \leq 8$ — ②

$-x_E + x_I \leq 1$ — ③

$x_I \leq 2$ — ④

$x_E \geq 0$ — ⑤

$x_I \geq 0$ — ⑥

a) By using graphical method and Libre office solver, the optimal point obtained is:

$x_E = \frac{10}{3}$	$z = \frac{38}{3}$
$x_I = 4/3$	

b) $z = \frac{38}{3}$ (optimal value) - Using graphical method and Libre office solver

$$z = 3 \times \frac{10}{3} + \frac{2 \times 4}{3} = \frac{38}{3}$$

c) Lagrange multipliers for each of the constrains

Lagrangian :

$$L(\underline{x}, \underline{\mu}) = f(\underline{x}) + \sum_{j=1}^{6} \mu_j g_j \quad | \quad \mu_j \le 6$$

$$\nabla_x z(\underline{x}) + \nabla_x (\mu_1 g_1) + \nabla_x (\mu_2 g_2) = 0$$

$$\begin{bmatrix} \frac{\partial z}{\partial x_I} \\ \frac{\partial z}{\partial x_E} \end{bmatrix} + \mu_1 \begin{bmatrix} \frac{\partial g_1}{\partial x_I} \\ \frac{\partial g_1}{\partial x_E} \end{bmatrix} + \mu_2 \begin{bmatrix} \frac{\partial g_2}{\partial x_I} \\ \frac{\partial g_2}{\partial x_E} \end{bmatrix} = 0$$

$$\begin{bmatrix} 2 \\ 3 \end{bmatrix} + \mu_1 \begin{bmatrix} 2 \\ 1 \end{bmatrix} + \mu_2 \begin{bmatrix} 1 \\ 2 \end{bmatrix} = 0$$

$$2\mu_1 + \mu_2 = -2$$

$$\mu_1 + 2\mu_2 = -3$$

$$-3\mu_2 = 4 \Rightarrow \boxed{\mu_2 = -\frac{4}{3}, \quad \mu_1 = -1/3}$$

d) ① and ② → These constrains' intersection solution is obtained. These are binding/active constrains

③, ④, ⑤, ⑥ → The solution is not obtained by the intersection of any of these points. Hence, there are called non-binding, constrains.

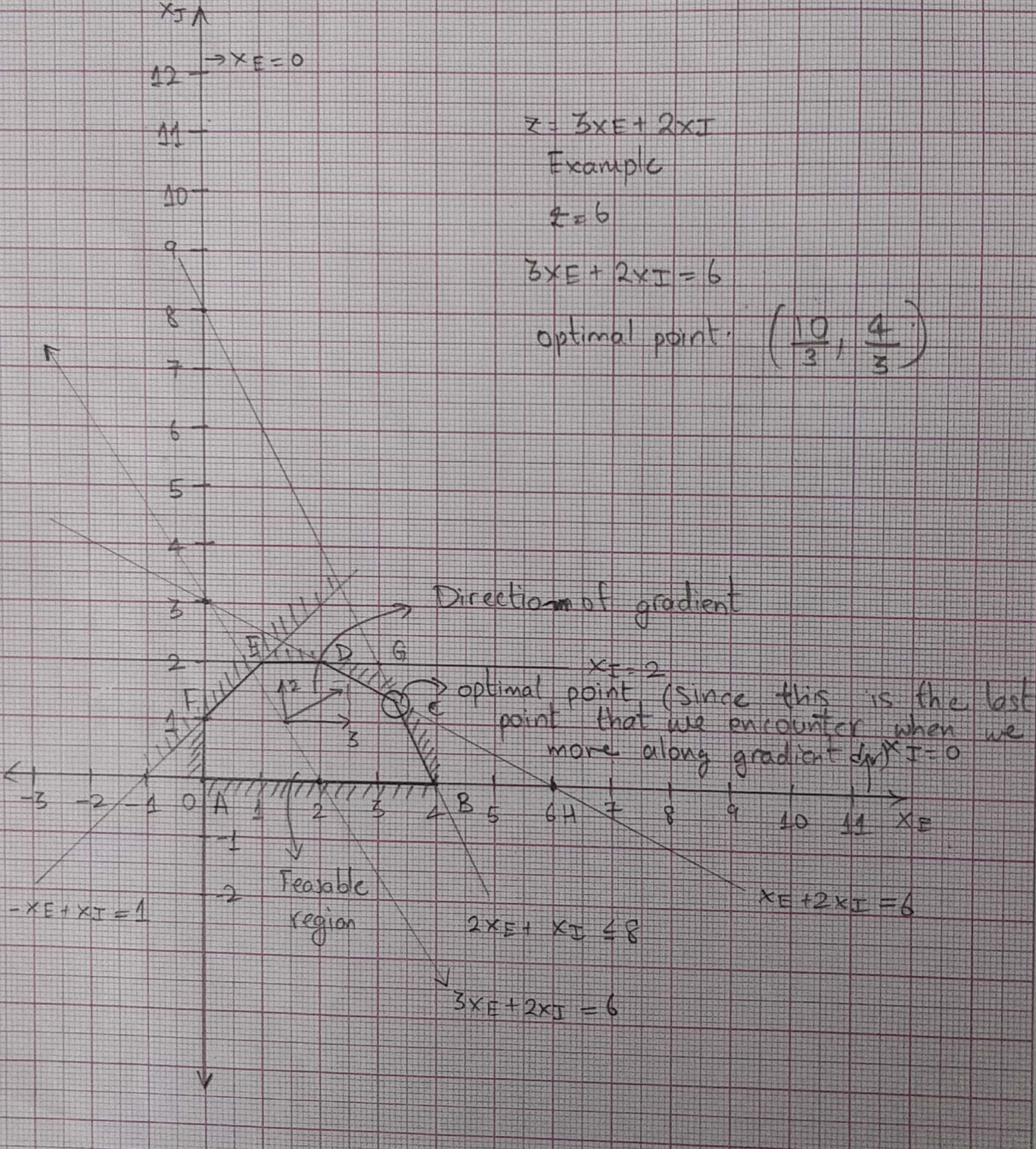
e) For constrains ① and ② surplus value = 0, since the solution lies on these lines:-

Optimal point $(x_E, x_I) = \left(\frac{10}{3}, \frac{4}{3}\right)$

Scale:

X-axis: 1cm = 1 unit

Y-axis: 1cm = 1 unit



For the other constrains

3 $-x_E + x_I \le 1$

$-\frac{10}{3} + \frac{4}{3} + s_3^2 = 1$

$s_3^2 = 1 + 10/3 - 4/3$

$\boxed{s_3^2 = 3}$

4 $x_I \le 2$

$x_I - 2 \le 0$

$x_I - 2 + s_4^2 = 0$

$s_4^2 = 2 - \frac{4}{3}$

$\boxed{s_4^2 = \frac{2}{3}}$

5 $-x_E \le 0$

$-x_E + s_5^2 = 0$

$\boxed{s_5^2 = x_E = \frac{10}{3}}$

6 $-x_I + s_6^2 = 0$

$s_6^2 = x_I$

$x_I = s_6^2$

$\boxed{s_6^2 = \frac{4}{3}}$

f) → Costs of A and B are same

→ Invest more money in resource B → Since lagrange multiplier corresponding to resource B has a higher magnitude compared to that of resource A.

$|\mu_2| = 4/3$ and $|\mu_1| = 1/3$

$|\mu_2| > |\mu_1|$

→ For same change in the quantity of resources, increase in income will be higher for resource B

2) a) (i) Cost function →

$z = C_E x_E + C_I x_I = 3x_E + 2x_I$

Binding constrains are:

① $x_E + 2x_I \le 6$ → Slope $= -\frac{1}{2}$

② $2x_E + x_I \le 8$ → Slope $= -2$

→ Slope must lie in between slopes of the 2 - binding constrain

$\frac{1}{2} \le \frac{C_E}{C_I} \le 2$ → for solution to remain the same

$\therefore \frac{1}{2} \le \frac{C_E}{C_I} \le 2$

Keeping $C_E = 3$ (constant).

$\frac{1}{2} \le \frac{3}{C_I} \le 2$

$2 \ge \frac{C_I}{3} \ge \frac{1}{2} \Rightarrow \frac{1}{2} \le \frac{C_I}{3} \le 2 \Rightarrow \boxed{\frac{3}{2} \le C_I \le 6}$

Keeping $C_I = 2$ → constant

$\frac{1}{2} \le \frac{C_E}{2} \le 2 \Rightarrow \boxed{1 \le C_E \le 4}$

(ii) ① objective function slope = slope of constrain 1

objective function of the form:

$z = \alpha x_E + 2\alpha x_I$

$\therefore$ The optimal solution of this is along the line segment DE

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The optimal value = 6 (value of ineq constrain)

∴ The range can be described as →

$\left(\frac{10}{3}, \frac{4}{3}\right)$ to $(2, 2)$

line segment

② Objective function of slope = slope of constrain②

Objective function of the form:

$z = 2\alpha X_E + \alpha X_I$

The optimal solution is along the line segment BC

The optimal value = 8 (value of ineq constrain)

The range can be described as →

$(4, 0)$ to $\left(\frac{10}{3}, \frac{4}{3}\right)$ → line segment

b) (i) For constrain 1 to be redundant; it must pass through point $G(3, 2)$ and beyond that it will be redundant.

∴ $X_E + 2X_I \Rightarrow 3 + 2 \times 2 = 7$

∴ $\boxed{X_E + 2X_I \leq 7}$ ← Constrain redundant

For constrain ② to be redundant; must pass through point $H(6, 0)$ and beyond that it will be redundant

$2x_E + x_I \Rightarrow 2 \times 6 + 0 = 12$

$\boxed{2x_E + x_I \leq 12}$ → constrain redundant.

For constrain 3 to be redundant must pass through (0,3) and beyond that will be redundant

$-x_I + x_I \Rightarrow -0 + 3$

$\boxed{-x_E + x_I \leq 3}$

For constrain 4 to be redundant, must pass through →

$-x_E + x_I = 1$

$x_E + 2x_I = 6$

$3x_I = 7$

$\boxed{x_I = 7/3 \quad x_E = 4/3}$ → $\left(\frac{4}{3}, \frac{7}{3}\right)$

$x_I \leq 2$

$\boxed{x_I \leq 7/3}$ → above this redundant

For constrain 5 to be redundant, must pass through (-1,0)

→ $\boxed{x_E \geq -1}$ → and left → redundant

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constrain (6) → can never be redundant → No point of intersection below that

(ii) Resource B was chosen in problem 1F, for maximum reform, limit on resource B should be increased to 12 from 8. At these times, the gross income i.e \$ 18k \$

(iii) The new $\underline{x}^*$ obtained from graph and Libre office

$$x^* = \begin{bmatrix} x_E \\ x_I \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \end{bmatrix}$$

$$x^* = 3x_E^* + 2x_I^* = 3 \times 6 + 2 \times 0 = 18 \text{ k\$}$$

3) 2) a) (i) If the slope of the contour of the objective function is less than the slope of constrain 1 or is greater than the slope of constrain 2, then point C will remain the optimal solution. This is because when the slope value is between these 2 values (said range), the point C is the farthest point in the feasible region when moved along the gradient direction of the objective function. If the slope of the contours of the objective function is less than constrain 1, then point D will be the optimal soln. If more than that of constrain 2, point B becomes optimal solution.

2) a) (ii) When objective function aligns with the constrain 1 the line segment CD is the optimal solution. The objective function value remains same along that line segment. And the similar case happens for constrain 2 along line segment BC.

2) b) (i) Resource/constrain can be rendered redundant only if the intersection of atleast 2 other constrains lie in the infeasable region of the space made by the constrains. The max. constrain limit tells us the max. amount of a particular resource that needs to be stocked.

2) b) (ii) As calculated in 2b) ii) the resource limit of Resource B, can be changed upto the value 12. ~~If~~ You do not need to stock more than 12 tons of resource B, to maximize income.

2b) (iii) Once resource limit is increased to 12 tons, the new solutions are $x^* = (6,0)$.
At this value the gross income is 18k \$

2c) Changing the coefficients of the constrains will change the shape and area of the feasable region, as a result, the optimal solution will also change
It may so happen that the feasable region might become unbounded which ~~might~~ ~~will~~ might ~~it~~ push optima to $-\infty, +\infty$

	A	B	C	D	E
1	Variable →	Xe	Xi		z
2	Example value	3.333333333	1.333333333		12.66666667
3	Constrain Values when example values are substituted above		Less than =	Value	
4	Constrain 1	6		6	
5	Constrain 2	8		8	
6	Constrain 3	-2		1	
7	Constrain 4	1.333333333		2	
8	Constrain 5	-1.333333333		0	
9	Constrain 6	-3.333333333		0	