EE311 - Digital System Design - Assignment 3

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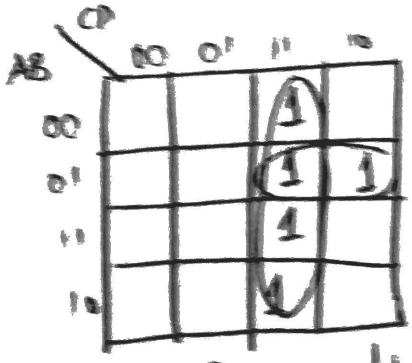
1) PLA Implementation

$f(A, B, C, D) = \Sigma min(3, 6, 7, 11, 15)$

$g(A, B, C, D) = \Sigma min(1, 3, 4, 7, 9, 13)$

$h(A, B, C, D) = \Sigma min(4, 6, 8, 10, 11, 12, 14, 15)$

KMAP - f

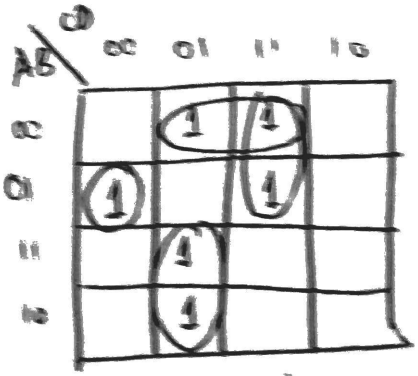


2-groups

Simplified expression

$f = \bar{A}BC + CD$

KMap - g

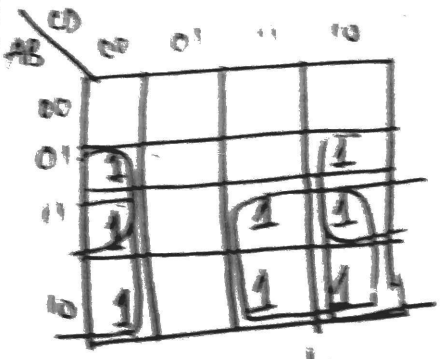


4-groups

Simplified exp:

$g = \bar{A}\bar{B}D + \bar{A}CD + A\bar{C}D + \bar{A}B\bar{C}\bar{D}$

KMap - h

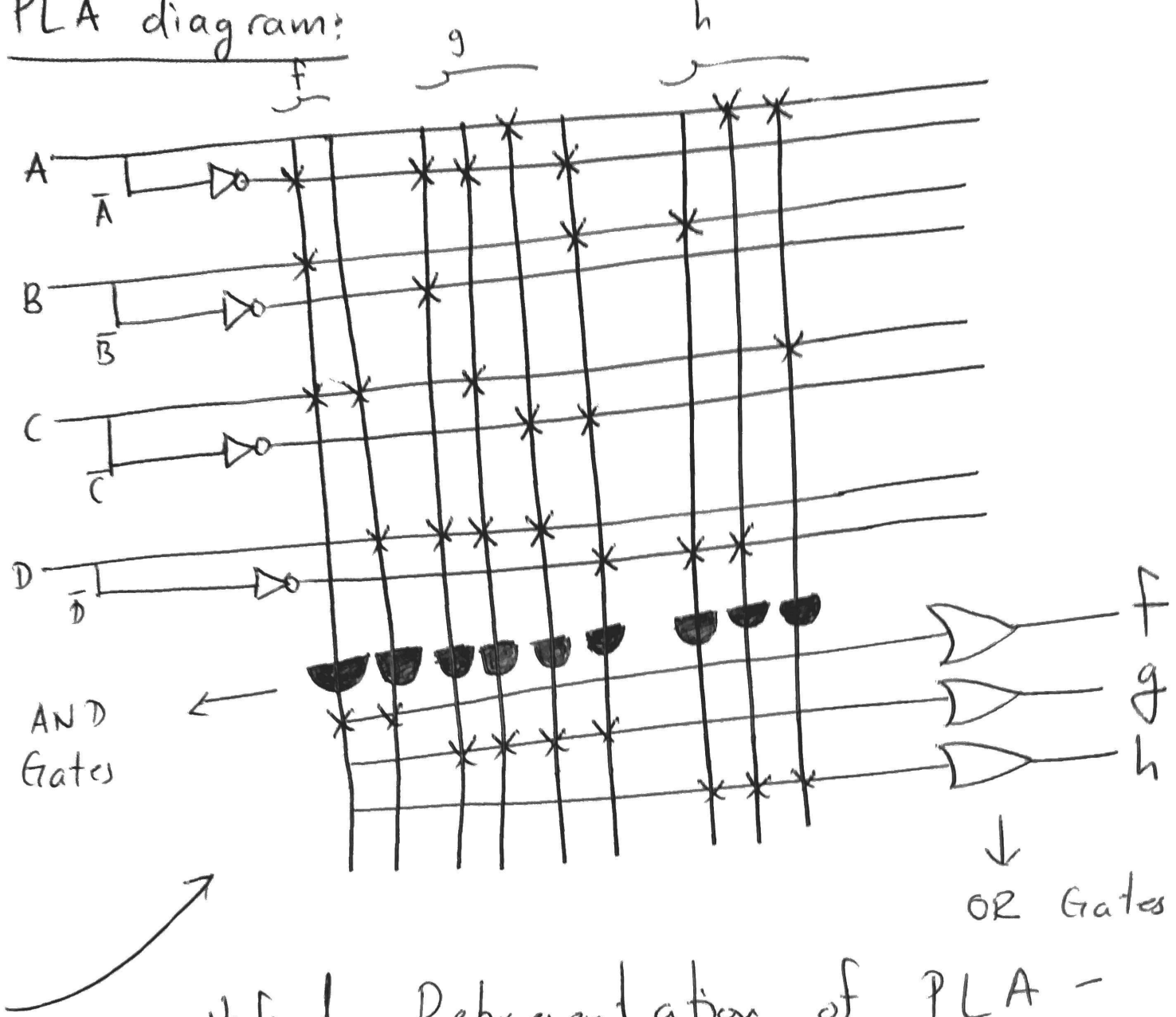


3-Groups

Simplified exp:

$h = B\bar{D} + A\bar{D} + AC$

PLA diagram:



Non-Simplified Representation of PLA -

Uses 9 AND Gate → S

Using Simplified Version:

$f = \bar{A}BC + CD$

$g = \bar{A}\bar{B}D + \bar{A}CD + A\bar{C}D + \bar{A}B\bar{C}\bar{D}$

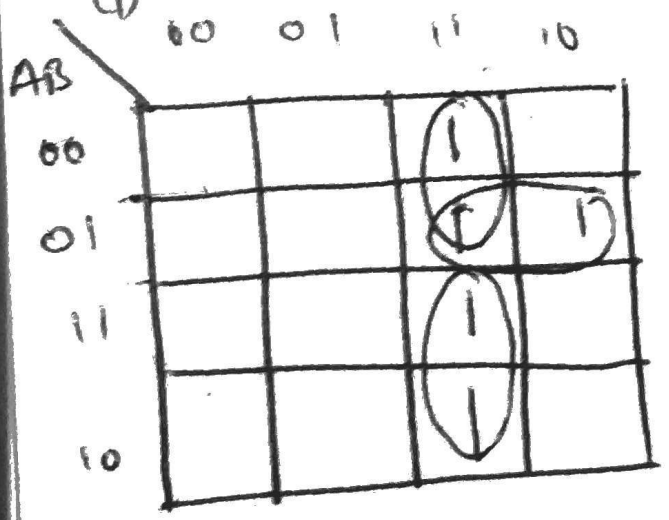
$h = B\bar{D} + A\bar{D} + AC$

Simplifying PLA for Q1

Draw K-Map again:

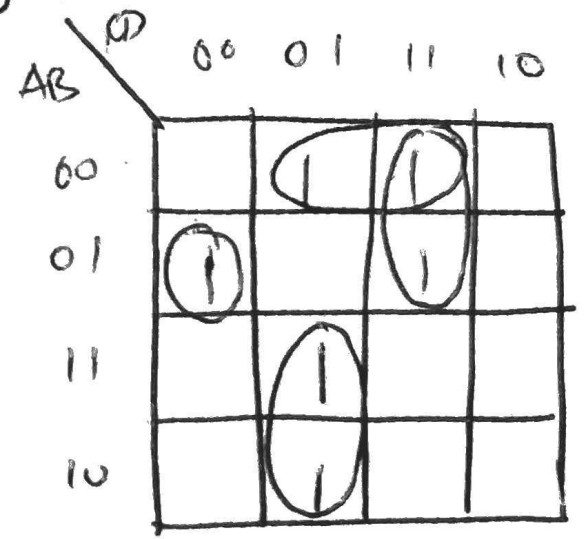
Has 8 terms

f



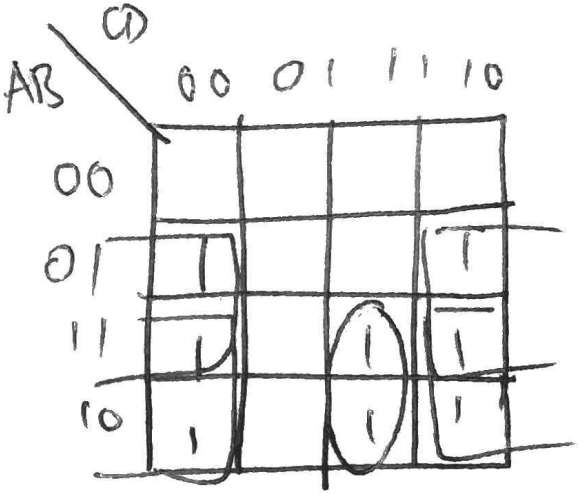
$$f = \underline{\bar{A}CD}^{0} + \bar{A}BC + \underline{ACD}^{2}$$

g



$$g = \bar{A}B\bar{C}\bar{D} + \bar{A}\bar{B}D + \underline{\bar{A}CD}^{1} + \underline{A\bar{C}D}$$

h



$$h = \underline{ACD}^{2} + B\bar{D} + A\bar{D}$$

PLA Table is (Simplified to 8 - terms)

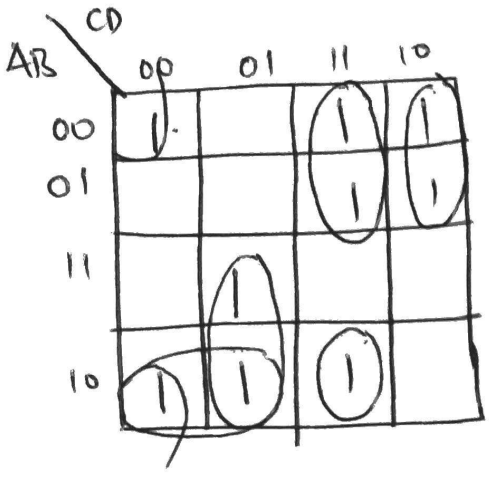
Term	Inputs A	B	C	D	f	g	h
$\bar{A}CD$	0	–	1	1	1	1	0
$\bar{A}BC$	0	1	1	–	1	0	0
$AC\bar{D}$	1	–	1	1	1	0	3
$\bar{A}B\bar{C}\bar{D}$	0	1	0	0	0	1	0
$\bar{A}\bar{B}D$	0	0	–	1	0	1	0
$\bar{A}\bar{C}D$	0	–	0	1	0	1	0
$A\bar{C}D$	1	–	0	1	0	1	0
$B\bar{D}$	–	1	–	0	0	0	1
$A\bar{D}$	1	–	–	0	0	0	1

2) a) $f_1(A,B,C,D) = \Sigma m(0,2,3,6,7,8,9,11,13)$

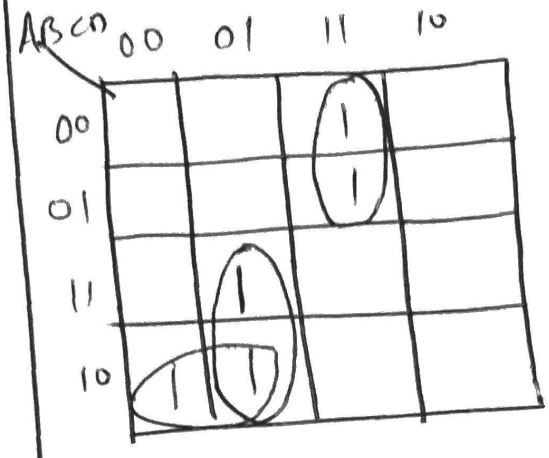
$f_2(A,B,C,D) = \Sigma m(3,7,8,9,13)$

$f_3(A,B,C,D) = \Sigma m(0,2,4,6,8,12,13)$

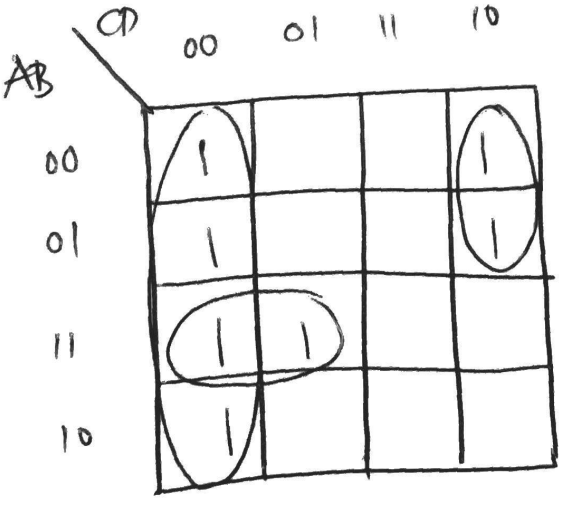
KMap f_1:



KMap - f_2



K-Map f_3

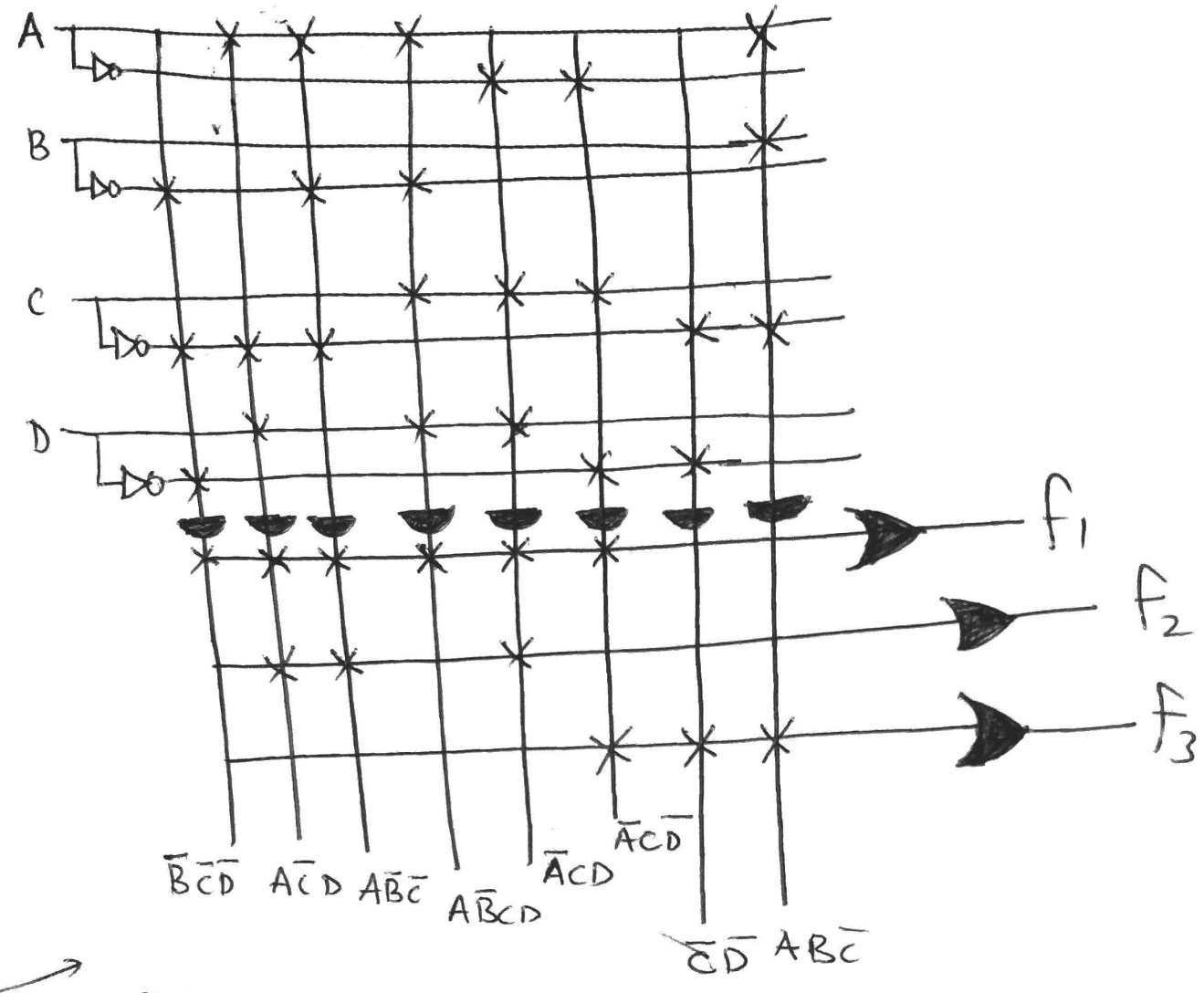


Expressions Simplified for each function (Such that combined # is least)

$f_1 = \bar{B}\bar{C}\bar{D} + A\bar{C}D^{(3)} + A\bar{B}\bar{C}^{(2)} + A\bar{B}CD + \bar{A}CD^{(4)} + \bar{A}C\bar{D}^{(1)}$

$f_2 = A\bar{B}\bar{C}^{(2)} + \bar{A}CD^{(3)} + \bar{A}C\bar{D}^{(4)}$

$f_3 = \bar{C}\bar{D} + AB\bar{C} + \bar{A}C\bar{D}^{(1)}$



PLA Diagram (Almost minimized) — 8 terms

Product term	Inputs A	B	C	D	Outputs f_1	f_2	f_3
$\bar{B}\bar{C}\bar{D}$	–	0	0	0	1	0	0
$A\bar{C}D$	1	–	0	1	1	1	0
$A\bar{B}\bar{C}$	1	0	0	–	1	1	0
$A\bar{B}CD$	1	0	1	1	1	0	0
$\bar{A}CD$	0	–	1	1	1	1	0
$\bar{A}C\bar{D}$	0	–	1	0	1	0	1
$\bar{C}\bar{D}$	–	–	0	0	0	0	1
$AB\bar{C}$	1	1	0	–	0	0	1

2b) $f_1(A, B, C, D) = cd + ad + \bar{a}b\bar{c}\bar{d}$

$f_2(A, B, C, D) = b\bar{c}\bar{d} + a\bar{c} + ad$

Make KMap for each of the expression and then find minimum row POA.

$f_1 = (a+\bar{a})(b+\bar{b})cd + a(b+\bar{b})(c+\bar{c})d + \bar{a}b\bar{c}\bar{d}$

$f_1 = abcd + ab\bar{c}d + \bar{a}bcd + \bar{a}\bar{b}cd + a\bar{b}\bar{c}d + ab\bar{c}d + \bar{a}b\bar{c}\bar{d}$

ab \ cd	$\bar{c}\bar{d}$	$\bar{c}d$	cd	$c\bar{d}$
$\bar{a}\bar{b}$	0 (0)	0 (1)	1 (3)	0 (2)
$\bar{a}b$	1 (4)	0 (5)	1 (7)	0 (6)
ab	0 (12)	1 (13)	1 (15)	0 (14)
$a\bar{b}$	0 (8)	1 (9)	1 (11)	0 (10)

$f_2 = ab\bar{c}\bar{d} + \bar{a}b\bar{c}d + a(b+\bar{b})\bar{c}(d+\bar{d}) + a(b+\bar{b})(c+\bar{c})\bar{d}$

$= ab\bar{c}\bar{d} + \bar{a}b\bar{c}\bar{d} + ab\bar{c}d + a\bar{b}\bar{c}d + a\bar{b}\bar{c}\bar{d} + ab\bar{c}\bar{d} + a\bar{b}c\bar{d} + abc\bar{d}$

ab \ cd	$\bar{c}\bar{d}$	$\bar{c}d$	cd	$c\bar{d}$
$\bar{a}\bar{b}$	0 (0)	0 (1)	0 (3)	0 (2)
$\bar{a}b$	1 (4)	0 (5)	0 (7)	0 (6)
ab	1 (12)	1 (13)	0 (15)	1 (14)
$a\bar{b}$	1 (8)	1 (9)	0 (11)	1 (10)

Simplified Table using Espresso Algorithm is:

(Minimum Row PLA Table)

Expression/Term	Input A	B	C	D	f_1	f_2
✓ $\bar{A}B\bar{C}\bar{D}$	0	1	0	0	1	0
~~$B\bar{C}\bar{D}$~~	~~-~~	~~1~~	~~0~~	~~0~~	~~0~~	~~1~~
✓ CD	-	-	1	1	1	0
✓ $A\bar{D}$	1	-	-	0	0	1
✓ $A\bar{C}D$	1	-	0	1	1	1

Expression can be written as:

$f_1 = \bar{A}B\bar{C}\bar{D} + CD + A\bar{C}D$

$f_2 = \sout{B\bar{C}\bar{D}} + A\bar{D} + A\bar{C}D$

Common term

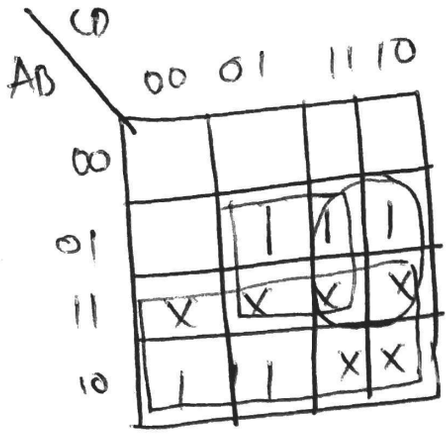
3) BCD to XS3 code converter:

Truth table for BCD to XS3

BCD	XS3
A B C D	W X Y Z
0 0 0 0	0 0 1 1
0 0 0 1	0 1 0 0
0 0 1 0	0 1 0 1
0 0 1 1	0 1 1 0
0 1 0 0	0 1 1 1
0 1 0 1	1 0 0 0
0 1 1 0	1 0 0 1
0 1 1 1	1 0 1 0
1 0 0 0	1 0 1 1
1 0 0 1	1 1 0 0
1 0 1 0	X X X X
1 0 1 1	X X X X
1 1 0 0	X X X X
1 1 0 1	X X X X
1 1 1 0	X X X X
1 1 1 1	X X X X

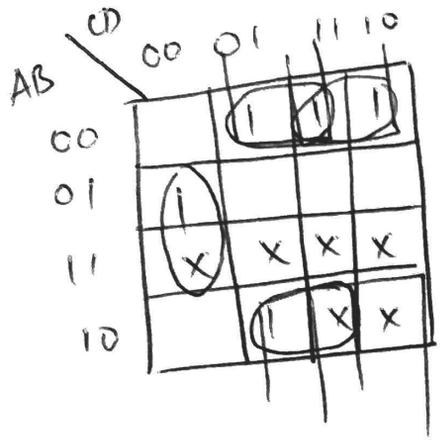
K Map for BCD to XS3

W →



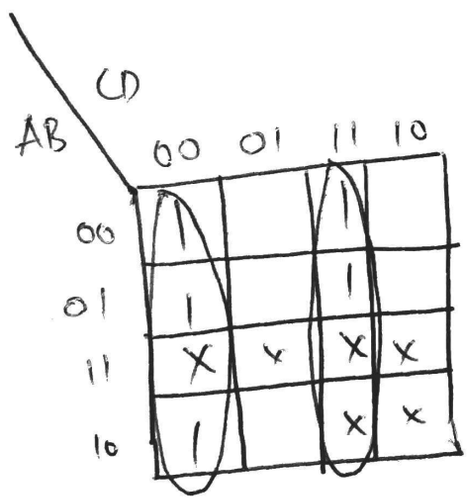
$W = A + BC + BD$

X →



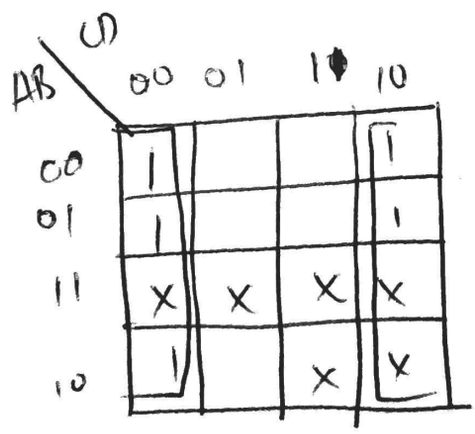
$X = \bar{B}C + \bar{B}D + B\bar{C}\bar{D}$

Y →



$Y = CD + \bar{C}\bar{D}$

Z →



$Z = D$

Using the Espresso algorithm for getting min Row PLA

Expression / Term	A	B	C	D	W	X	Y	Z
$B\bar{C}\bar{D}$	-	1	0	0	0	1	0	0
$\bar{B}C\bar{D}$	-	0	1	0	0	1	0	1
$\bar{B}D$	-	0	-	1	0	1	0	0
CD	-	-	1	1	0	0	1	0
$BC\bar{D}$	-	1	1	0	1	0	0	1
BD	-	1	-	1	1	0	0	0
$\bar{C}\bar{D}$	-	-	0	0	0	0	1	1
A	1	-	-	-	1	0	0	0

Minimum PLA:

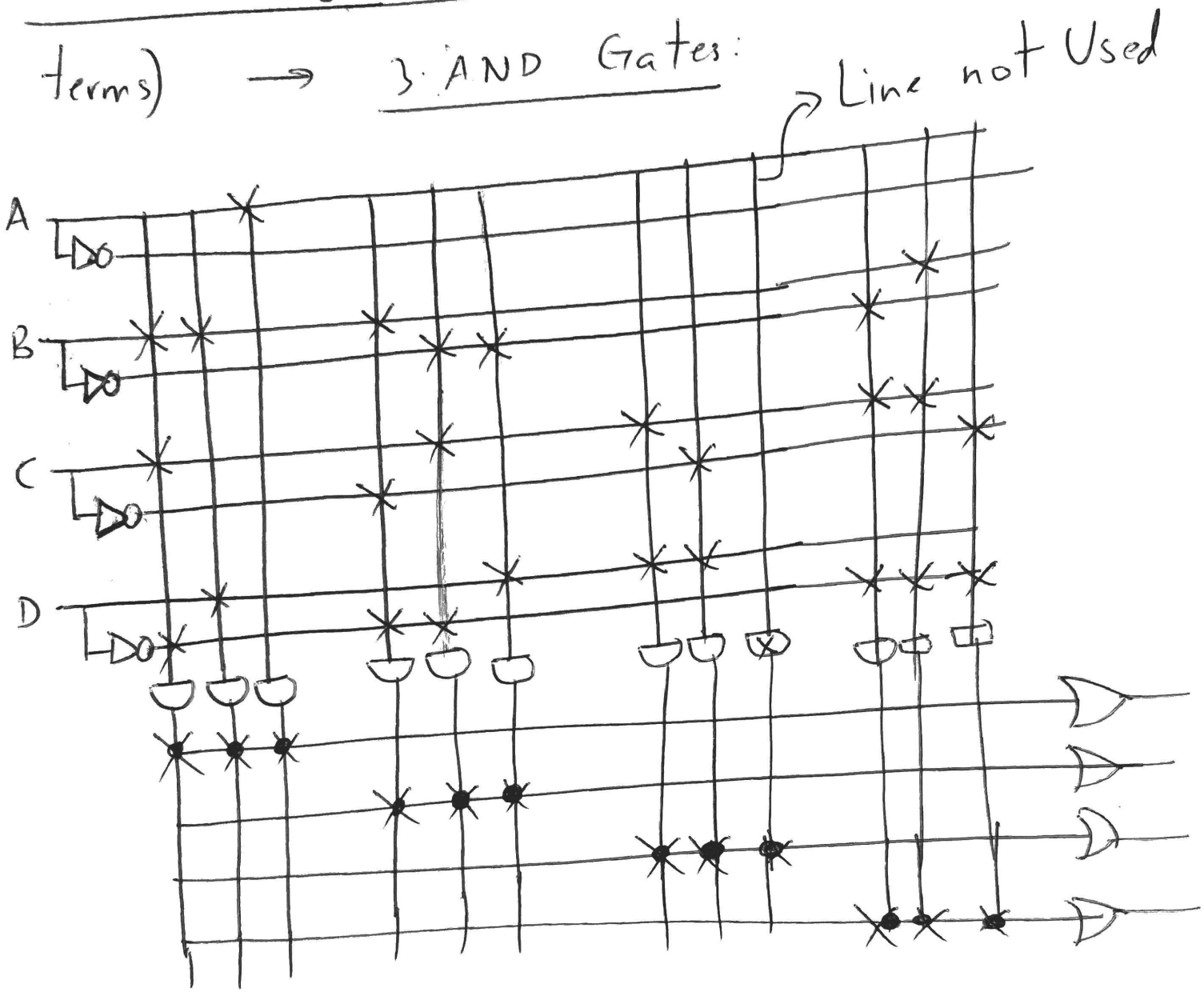
$W = B\bar{C}\bar{D} + BD + A$

$X = B\bar{C}\bar{D} + \bar{B}C\bar{D} + \bar{B}D$

$Y = CD + \bar{C}\bar{D}$

$Z = \bar{B}C\bar{D} + BC\bar{D} + \bar{C}\bar{D}$

PAL Logic Diagram (Take max # of terms) → 3 AND Gates:



OR Gates Fixed Not programmable

4) Truth Table:

	Input			Output			
	X	Y	Z	A	B	C	D
0	0	0	0	0	1	0	0
1	0	0	1	1	1	1	1
2	0	1	0	1	0	1	1
3	0	1	1	0	1	0	1
4	1	0	0	1	1	1	0
5	1	0	1	0	0	0	1
6	1	1	0	1	0	1	0
7	1	1	1	0	1	1	1

Simplifying the K-Maps for each:

A →

X \ YZ	00	01	11	10
0	0 (0)	1 (1)	0 (3)	1 (2)
1	1 (4)	0 (5)	0 (7)	1 (6)

B →

X \ YZ	00	01	11	10
0	(0)	1	1	1 (2)
1	1 (4)	1 (5)	1 (7)	1 (6)

C →

X \ YZ	00	01	11	10
0	0 (0)	1 (1)	0 (3)	1 (2)
1	1 (4)	0 (5)	1 (7)	1 (6)

D →

X \ YZ	00	01	11	10
0	0 (0)	1 (1)	1 (3)	1 (2)
1	0 (4)	1 (5)	1 (7)	0 (6)

Simplified Expression using KMap for each o/p is:

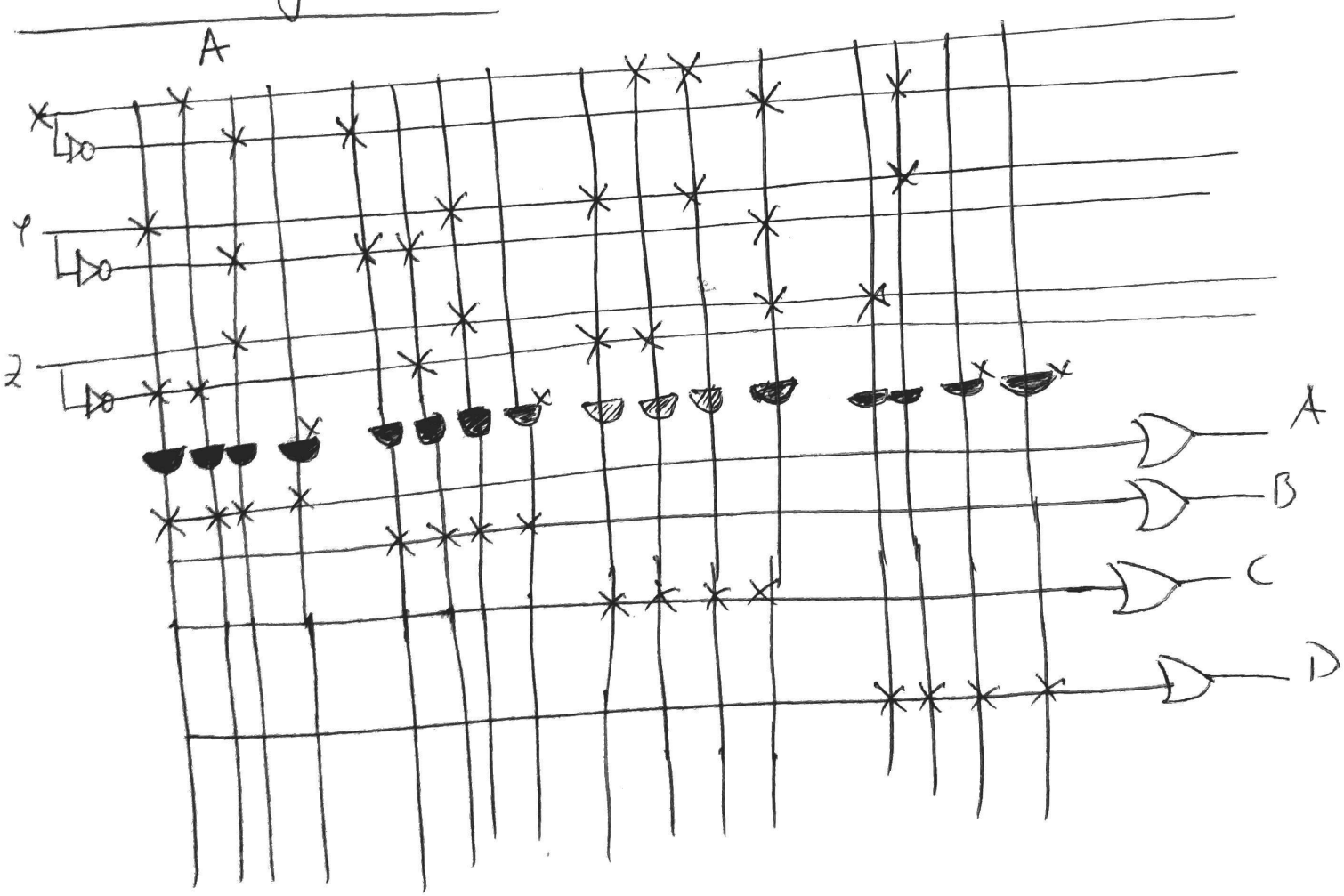
$A = X\bar{Z} + X\bar{Z} + \bar{X}\bar{Y}Z$

$B = \bar{X}\bar{Y} + \bar{Y}\bar{Z} + YZ$

$C = Y\bar{Z} + X\bar{Z} + XY + \bar{X}\bar{Y}Z$ → Max # of AND Gates.

$D = Z + \bar{X}Y$

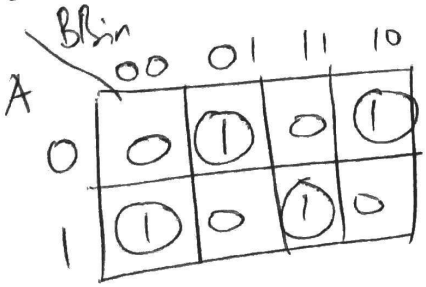
PAL Diagram is:



5) Full Subtractor using PAL:

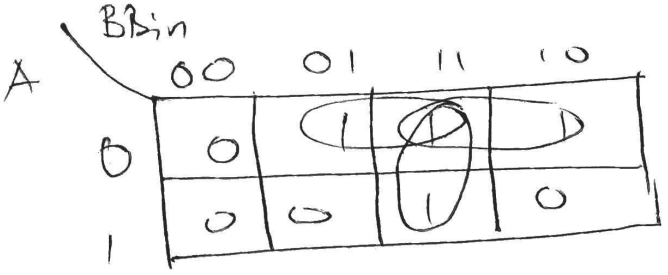
Input			Output	
A	B	Bin	D	Bout
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	0	1
1	0	0	1	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

Difference:



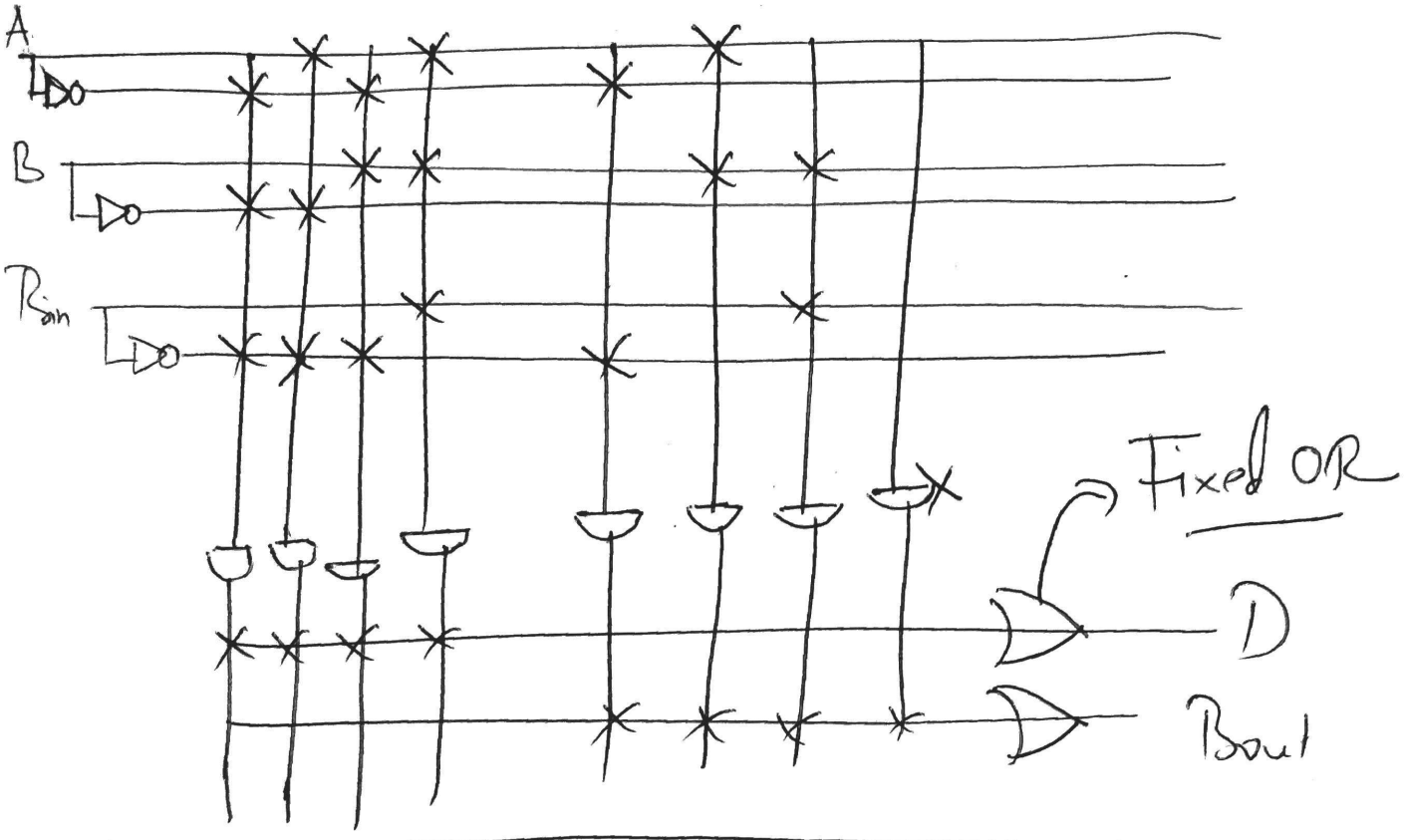
$$D = \bar{A}\bar{B}B_{in} + A\bar{B}\bar{B}_{in} + \bar{A}B\bar{B}_{in} + ABB_{in}$$

Borrow Out (Bout)



$$B_{out} = \bar{A}\bar{B}_{in} + \bar{A}B + BB_{in}$$

Implementation using ~~PLA~~: PAL:



Programmable AND, Fixed OR