Submodular Function Calibration for Fast and Effective Data Subset Selection

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Let us understand the title word-by-word ....

What is a Submodular Function?

Given any set of elements,

$$V = \left\{ \right\}$$

What is a Submodular Function?

- It is a set function, more generally, $f : 2^V \rightarrow \mathbb{R}$
- Possess the diminishing returns property - if for every $A \subseteq B \subseteq V$ and every $x \in V \setminus B$, the following inequality holds:

$$f(A \cup \{x\}) - f(A) \geq f(B \cup \{x\}) - f(B) \tag{1}$$

- Naturally suitable for modeling set diversity.

There are many examples of Submodular Functions like ...

- Entropy function: $H(X) = -\sum p(x) \log p(x)$
- Log determinant of a positive semidefinite matrix: $\log\det(X + \epsilon I)$ where $\epsilon > 0$
- Facility location: $f(S) = \sum_{j \in \mathcal{C}} \max_{i \in S} u_{ij}$ where $\mathcal{C}$ is the set of clients
- Set cover function: $f(S) = \sum_{u \in \mathcal{U}} \min(1, \sum_{s \in S} w_{su})$ where $\mathcal{U}$ is the universe of elements
- Graph cut capacity: $f(S) = \sum_{i \in S, j \notin S} c_{ij}$ where c_{ij} is the capacity of the edge between i and j
- Mutual information: $I(X;Y) = H(X) + H(Y) - H(X,Y)$
- Rank function of a matroid: $f(S) = \text{rank}(S)$
- Coverage function: $f(S) = |\cup_{s \in S} \mathcal{C}_s|$ where $\mathcal{C}_s$ is the set covered by s

Which Submodular Function do we use in this work?

- We use the Facility Location (FL) Function.
- Definition of FL -

$$f_V(X) = \sum_{j \in V} \max_{i \in X} s(i, j) \tag{2}$$

where:

- V is the ground set.
- X is the set on which we want to evaluate the FL function.
- i, j - index elements in V and X respectively.
- $s(i, j)$ is the similarity function that measures the affinity or similarity from between elements i and j. $s(i, j)$, may not necessarily equal $s(j, i)$. In fact, we sometimes prefer to have asymmetric similarities.

Facility Location (FL) Function

How do we use the FL function?

We try to find a subset $X^* \in V$ which maximizes it.

Why do we want to maximize FL?

- Intuitively (and loosely) speaking, a set X^* which maximizes FL, is a set whose elements are most similar to all elements of the ground set V.
- Hence, this set can act as a good representative of the ground set V, potentially useful for downstream applications.

How do we maximize FL?

- Submodular Function Maximization is NP hard, in general.
- But, greedy algorithm on function gains has a $1 - 1/e$ guarantee, and works well in practice.

Where does our work fit in the literature?

- To the best of our knowledge, our work is the first one to present a detailed *empirical* evaluation of applying submodular functions for the task of data subset selection in a supervised learning setting.
- We show that a well *calibrated (or tuned)* (will be explained later) submodular function can beat popular state-of-the-art data subset selection baselines.
- We mainly focus on computer vision datasets (CIFAR-10/100, TinyImagenet, Imagenet) in this work, but our approach can be extended to other modalities, too.
- We scale submodular selection to ground set sizes of $\approx$ 1M, i.e. to the Imagenet dataset.

What our work does not show?

- We do not claim to introduce any new kind of submodular function, but rather show that submodular functions can be tuned to get the right set of parameters, and such a function can be a strong data subset selector.

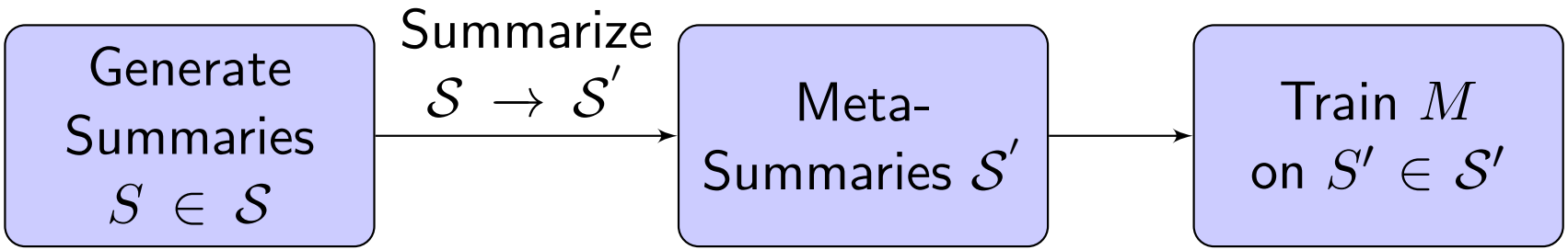
How do we tune the Submodular Function?

To answer this question, we have to first ask what is the objective that we are trying to achieve?

Our Objective

Given a machine learning model M, training algorithm $\mathcal{A}$, a training-dataset V_{tr}, test-dataset V_{te}, we want to find $S \subset V_{tr}$ such that $\mathcal{A}(M, S)$, achieves a highest test-accuracy (a_{te}), i.e. accuracy on V_{te}, where $\mathcal{A}(M, S)$ denotes training a model M on dataset S.

How do we tune the Submodular Function?



Summary Generation Procedure

Input: Dataset $V_{tr} = \{x_i, y_i\}_{i=1}^{i=n}$, Subset size m, Model M, Feature Extractor $\mathcal{F}$, Training Algorithm $\mathcal{A}$, Facility Location Function Parameter Space $\mathcal{X}$. ▷ Calibration Procedure Inputs.

Output: Set of summaries $\mathcal{S}$ ▷ The Output.

procedure GRIDSEARCH
  for $y \in \mathcal{X}$ **do** ▷ Loop over set of FL hyperparameters
    $F \leftarrow \mathcal{F}(V_{tr})$ ▷ Make Design Matrix F
    $P \leftarrow \text{SIMMAT}(F)$ ▷ Make Similarity Matrix P
    $f_{fl}(X; V_{tr}, y) = \sum_{v \in V_{tr}} \max_{x \in X} P_y(v, x)$ ▷ Instantiate FL function
    $I \leftarrow \text{GREEDYMAX}(f_{fl}, m)$
    $\mathcal{S} \leftarrow \mathcal{S} \cup I$
  end for
end procedure

Meta Summarization

Make Jaccard Matrix

Input: Set of summaries $\mathcal{S}$

Output: Jaccard Matrix J ▷ The Output.

procedure MakeJaccardMatrix
    Initialize empty Jaccard Matrix J
    for $S_1 \in \mathcal{S}$ **do** ▷ Outer Loop.
        for $S_2 \in \mathcal{S}$ **do** ▷ Inner Loop.
            $J(i,j) = \frac{|S_1 \cap S_2|}{|S_1 \cup S_2|}$ ▷ Assign Jaccard Matrix Elements
        end for
    end for
end procedure

Meta Summarization

Meta Summarize

Input: Jaccard Matrix J, Set of summaries $\mathcal{S}$, Meta Summary Size mm.

▷ Meta Summarization Inputs.

Output: Summary of summaries (we call them meta-summaries) $\mathcal{Q}$, Meta-summary indices I_{ms} in greedy-max order. ▷ The Output

1: **procedure** MetaSummarize
2: $f_{fl}(X;\mathcal{S}) = \sum_{t\in\mathcal{S}} \max_{x\in X} J(t,x)$ ▷ Index FL via Jaccard Matrix J
3: $I_{ms} \leftarrow$ GreedyMax(f_{fl}, mm) ▷ Meta-summary indices via greedy max.
4: $\mathcal{Q} = S(I_{ms})$ ▷ Index actual summaries with I_{ms}
5: **end procedure**

Meta-training

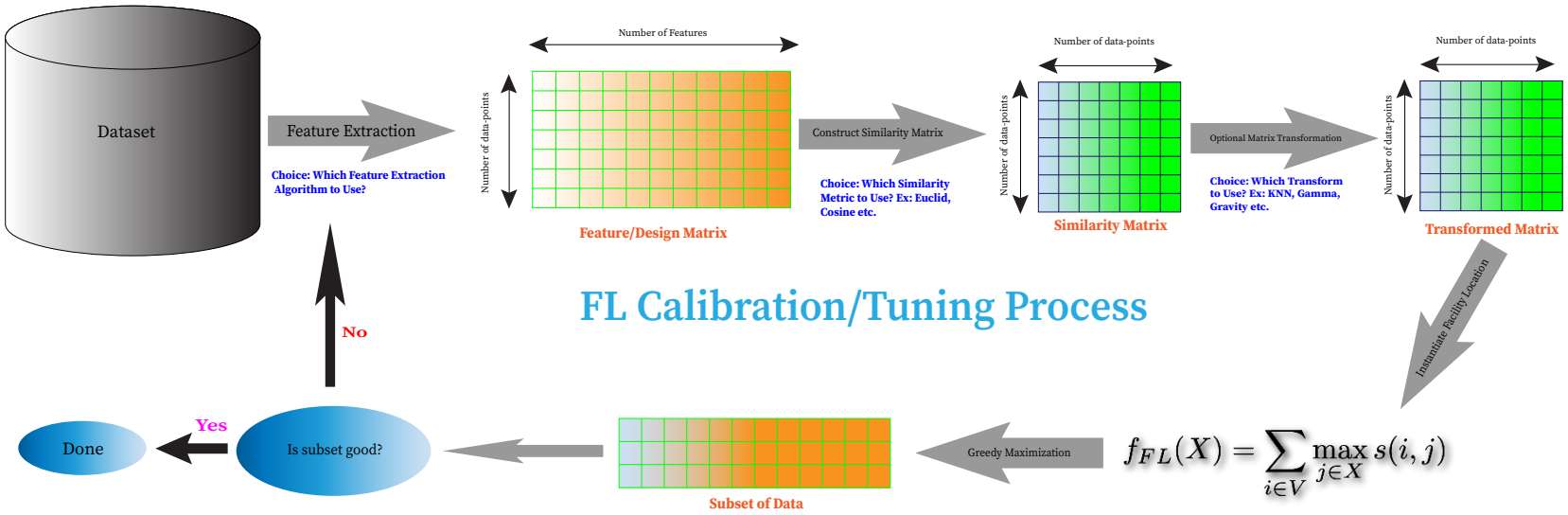
Train model on Meta-summaries

Input: Model M, Set of Meta-summaries $\mathcal{Q}$, Training Algorithm $\mathcal{A}$, Test Dataset V_{te} ▷ Meta-training Inputs.

Output: Best performing summary Q^*, and respective best trained model M^*

1: **procedure** MetaTrain
2: $m_{te} = 0$ ▷ Initialize Max Test Accuracy
3: **for** $Q \in \mathcal{Q}$ **do** ▷ Loop over meta-summaries.
4: $a_{te} = \mathcal{A}(M, Q)$ ▷ Train model to get test accuracy.
5: $m_{te} = \max(a_{te}, m_{te})$ ▷ Get max test accuracy.
6: **end for**
7: $M^*, Q^* = \arg\max(m_{te})$ ▷ Get best summary and respective trained model.
8: **end procedure**

Flowchat of Tuning Process

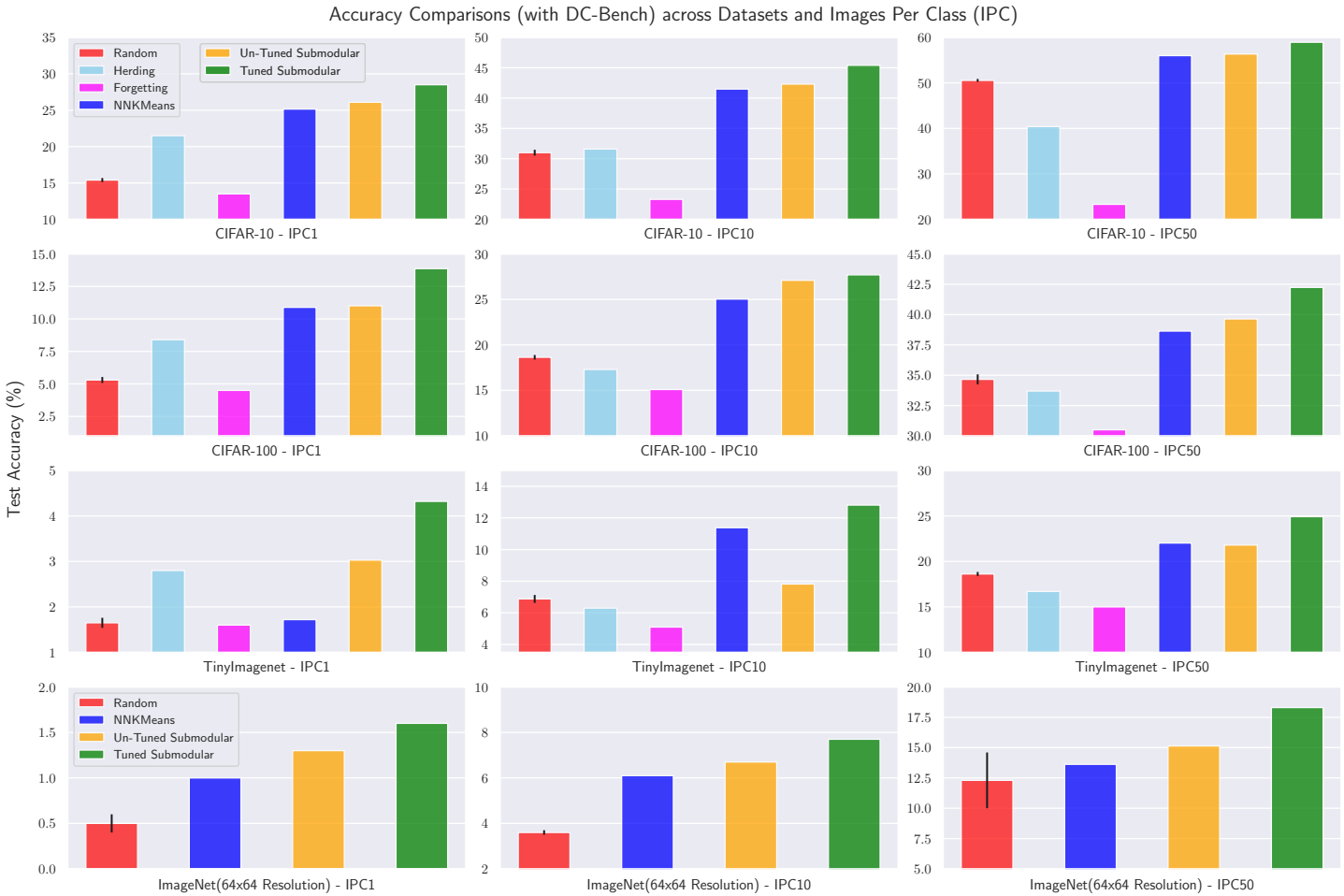


Evaluation Framework

We evaluate our method on the following subset selection benchmarks (mainly on computer vision benchamarks) -

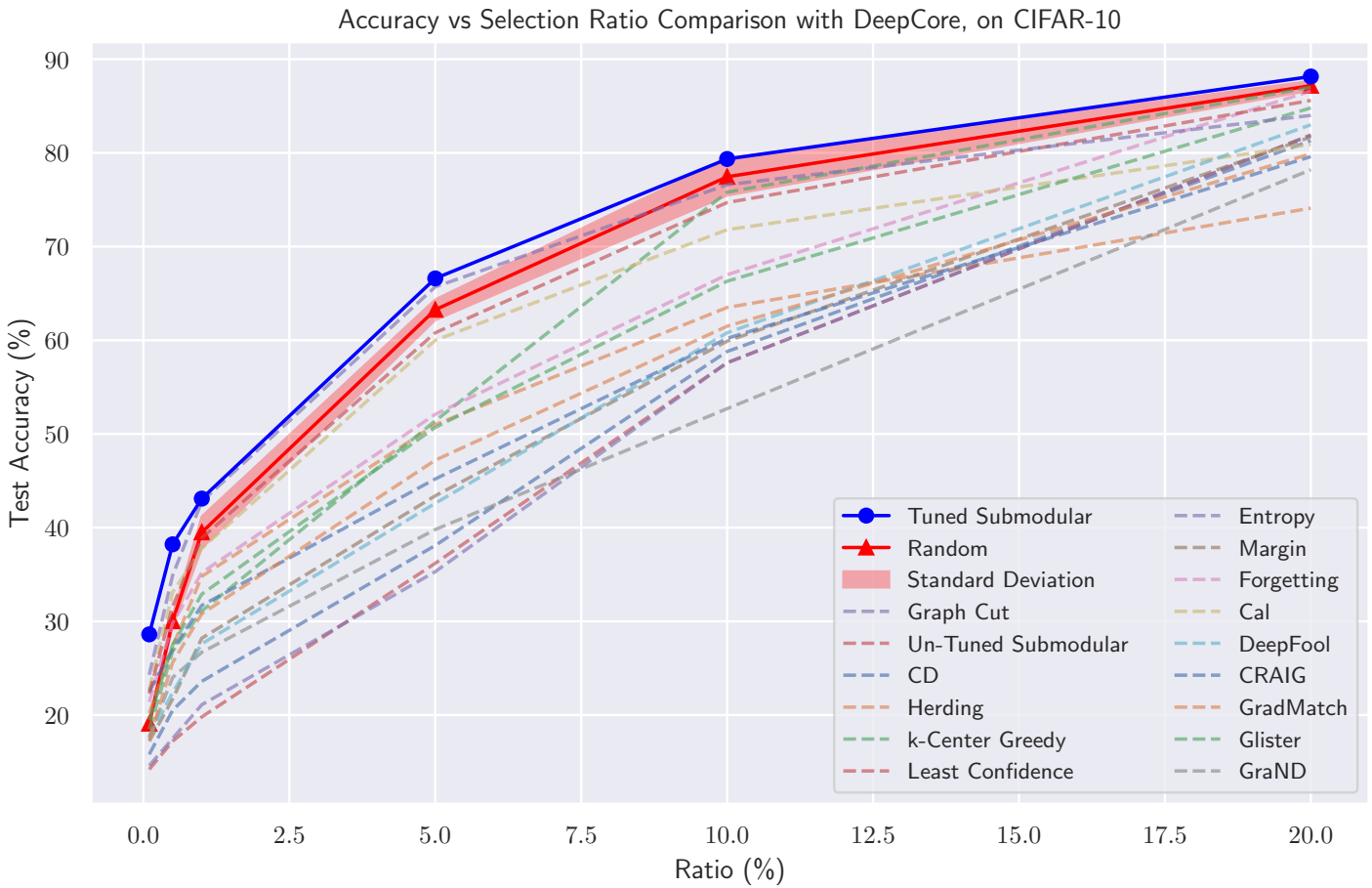
- DC-Bench (https://dc-bench.github.io/)
  - Mainly for Dataset Distillation, but includes some subset selection baselines too.
  - Datasets - CIFAR10, CIFAR100, TinyImagenet, ImageNet (64×64 Resolution)
  - Models M used are ConvNet (Depth $= 3$ & 4)
- DeepCore (https://github.com/PatrickZH/DeepCore)
  - Consists of different subset selection benchmarks into a unified setup.
  - Datasets - CIFAR10, CIFAR100
  - Model used here is ResNet18.

DC-Bench Comparison

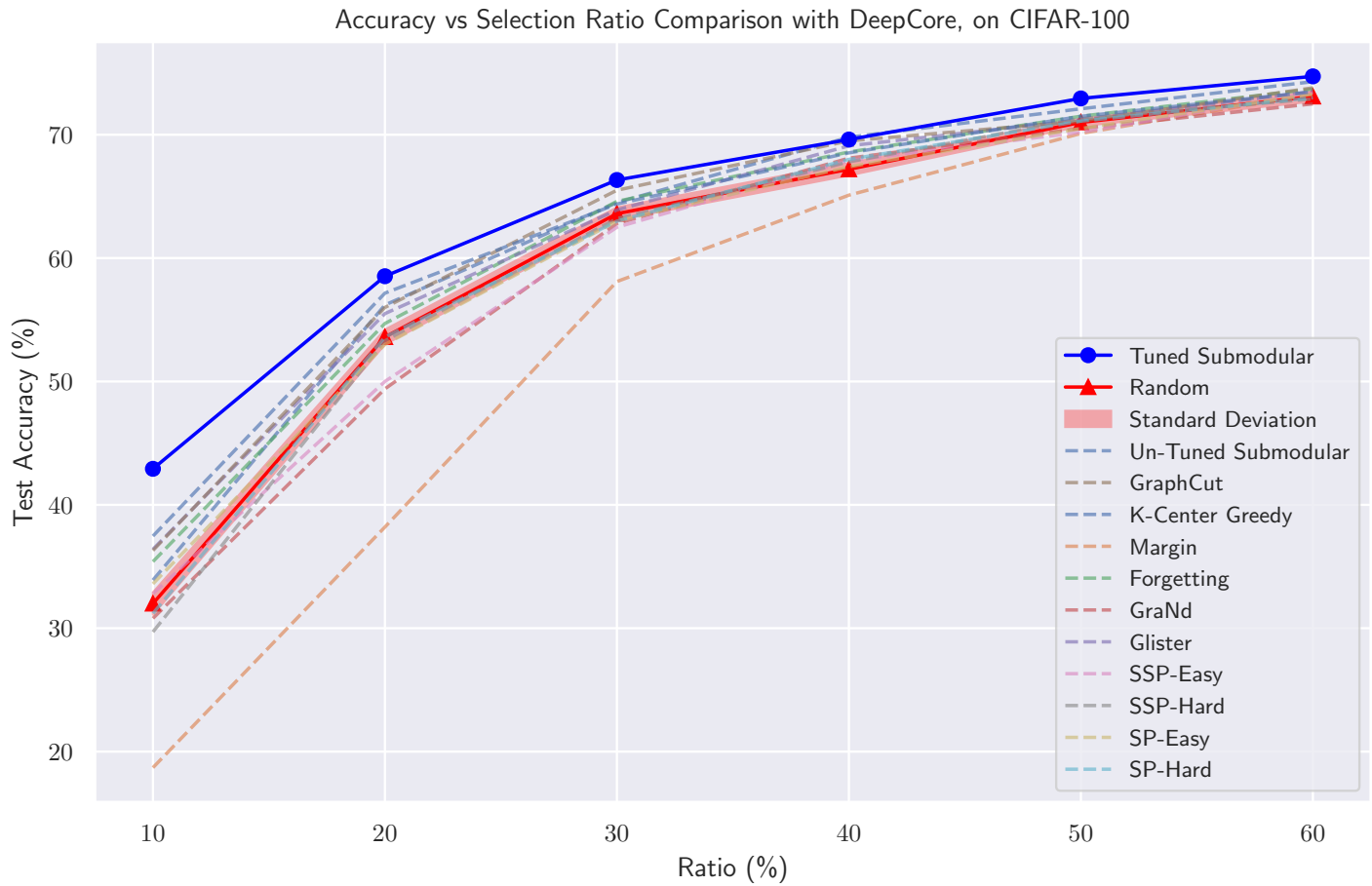


DC-Bench Comparison

DeepCore Comparison



DeepCore Comparison



Conclusion

- We show that a well tuned submodular function can act as an effective data subset selector.
- We hope that this can act as a stepping stone for -
  - Using more different types of submodular functions such as GraphCut, DPP etc, for data subset selection.
  - Scaling submodular selection to modern large scale vision and language datasets, and other data modalities.

Thank You!

Questions?