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Procedia Engineering 48 (2012) 158 – 163
Procedia Engineering
www.elsevier.com/locate/procedia

MMaMS 2012

Inverse and forward dynamic analysis of two link manipulator

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Abstract

In the paper we show the possibilities of physical modeling in Matlab/SimMechanics on a simple mechanical model of two link manipulator. We deal with its direct and inverse dynamics. Considering direct problem we investigate mechanical movement of the robot under the action of given forces and moments. As regards the issue of inverse motion we identify forces which cause a prescribed motion of the effector. Both problems are solved using block diagrams in Simulink and SimMechanics. Results are presented graphically.

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Keywords: Kinematic and dynamic analysis, double pendulum, direct and inverse problems in dynamics, computer simulation, Matlab, SimMechanics..

Nomenclature

m_i	mass of the *i*th link, *i*=1,2 (kg)
l_i	length of the *i*th link, *i*=1,2 (m)
t	time (s)
t_f	end time (s)
φ_i	angle of the *i*th link (rad)
$\dot{\varphi}_i$	angular velocity of the *i*th link (rad/s)
$\ddot{\varphi}_i$	angular acceleration of the *i*th link (rad/s^2)

1. Introduction

Computer modeling is an effective tool to accelerate and improve the design of new mechanical systems. Matlab and SimMechanics are appropriate software tools for creating computer models. In dynamic analysis there are two basic tasks - inverse and direct analysis. In inverse problem we identify forces which cause a prescribed motion of the effector. As far as direct problem is concerned we investigate the movement of the mechanical system under the action of given forces.

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doi:10.1016/j.proeng.2012.09.500

2. Description of the solved model

We investigate the model of double pendulum which is a quite often analysed dynamic system (Fig. 1). Location of the endpoint L of member 2 in the coordinate system O, x, y is given by:

$$\begin{aligned} x_L &= l_1 \cos \varphi_1 + l_2 \cos(\varphi_1 + \varphi_2) \\ y_L &= l_1 \sin \varphi_1 + l_2 \sin(\varphi_1 + \varphi_2) \end{aligned} \tag{1}$$

The task is to investigate the necessary movement in kinematic pairs which would ensure the relocation of the end point L of member 2 from position L_0 (4.5, 0) to position L_1 (0, 3) (Fig. 2). At the beginning and at the end of the movement the velocity and acceleration of point L should be zero [1-6].

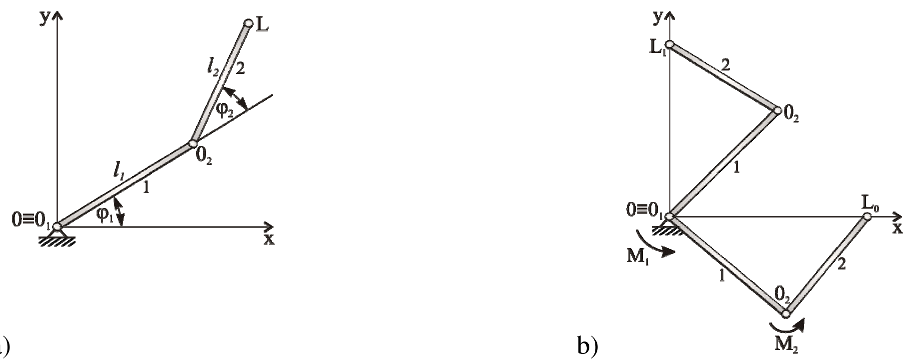

Fig. 1 a) Mechanical system of two link manipulator; b) Starting and final position of the mechanical system.

We try to find the time diagram of angular rotations φ_1 and φ_2 in the form of a 5th degree polynomials:

$$\begin{aligned} \varphi_1(t) &= a_1 t^5 + a_2 t^4 + a_3 t^3 + a_4 t^2 + a_5 t + a_6 \\ \varphi_2(t) &= b_1 t^5 + b_2 t^4 + b_3 t^3 + b_4 t^2 + b_5 t + b_6 \end{aligned} \tag{2}$$

The constants in the polynomials are determined by the initial and end conditions of the endpoint L for l_1 = 2 [m], l_2= 3 [m], m_1=2 [kg]; m_2=3 [kg]. The degree of these polynomials is chosen so that they are able to express all the desired conditions [7-11]. After substituting the initial and final conditions we obtain the following values of their respective constants:

a_1 = 0.1687	b_1 = 0.1854
a_2 = -0.8437	b_2 = -0.9268
a_3 = 1.1249	b_3 = 1.2357
a_4 = 0	b_4 = 0
a_5 = 0	b_5 = 0
a_6 = -0.5601	b_6 = 0.9221

We obtain constants a_6 , a_5 , a_4 for φ_1 and b_6 , b_5 , b_4 for φ_2 motion:
φ_1(t=0)= = -32,08918386°;
a_6=φ_1(0)= -32,08918386*pi/180=-0,5601;
φ_2(t=0)= = 52,83110034°;
b_6= φ_2(0)= 52,83110034*pi/180= 0,9221;
$\dot{\varphi}_1(t=0) = a_5 = 0;$
$\ddot{\varphi}_1(t=0) = 2a_4 = 0;$ a_4=0;
$\dot{\varphi}_2(t=0) = b_5 = 0;$
$\ddot{\varphi}_2(t=0) = 2b_4 = 0;$ b_4=0;

Constants a_1 , a_2 , a_3 and b_1 , b_2 , b_3 from equations:

$$\varphi_1(t_f) - \varphi_1(t=0) = a_1 t_f^5 + a_2 t_f^4 + a_3 t_f^3 \tag{3}$$

for a_4=0 and a_5=0,

$$0 = 5a_1 t_f^4 + 4a_2 t_f^3 + 3a_3 t_f^2 \tag{4}$$

for $\dot{\varphi}_1(t_f) = 0$,

$$0 = 20a_1 t_f^3 + 12a_2 t_f^2 + 6a_3 t_f \tag{5}$$

for $\ddot{\varphi}_1(t_f) = 0$;

Constants a_1, a_2, a_3 and b_1, b_2, b_3 from equations:

$$\varphi_2(t_f) - \varphi_2(t=0) = b_1 t_f^5 + b_2 t_f^4 + b_3 t_f^3 \tag{6}$$

for b_4=0 and b_5=0,

$$0 = 5b_1 t_f^4 + 4b_2 t_f^3 + 3b_3 t_f^2 \tag{7}$$

for $\dot{\varphi}_2(t_f) = 0$,

$$0 = 20b_1 t_f^3 + 12b_2 t_f^2 + 6b_3 t_f \tag{8}$$

for $\ddot{\varphi}_2(t_f) = 0$;

in matrix form:

$$\begin{bmatrix} t_f^5 & t_f^4 & t_f^3 \\ 5t_f^4 & 4t_f^3 & 3f_f^2 \\ 20t_f^3 & 12t_f^2 & 6t_f \end{bmatrix} . \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} \varphi_1(t_f) - \varphi_1(0) \\ 0 \\ 0 \end{bmatrix} \tag{9}$$

and

$$\begin{bmatrix} t_f^5 & t_f^4 & t_f^3 \\ 5t_f^4 & 4t_f^3 & 3f_f^2 \\ 20t_f^3 & 12t_f^2 & 6t_f \end{bmatrix} . \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} \varphi_2(t_f) - \varphi_2(0) \\ 0 \\ 0 \end{bmatrix} \tag{10}$$

Trajectory of the endpoint L is shown in Fig.2. Time diagram of angular rotation, angular velocity and angular acceleration of the pendulum members is shown in Fig.3.

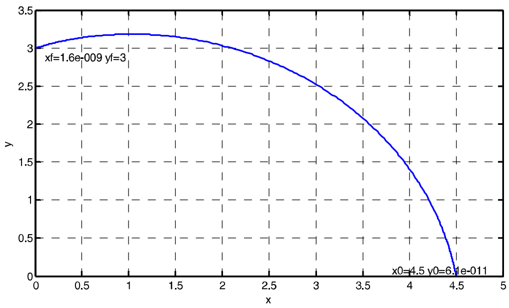

Fig. 2 Trajectory of the endpoint L of the mechanical system.

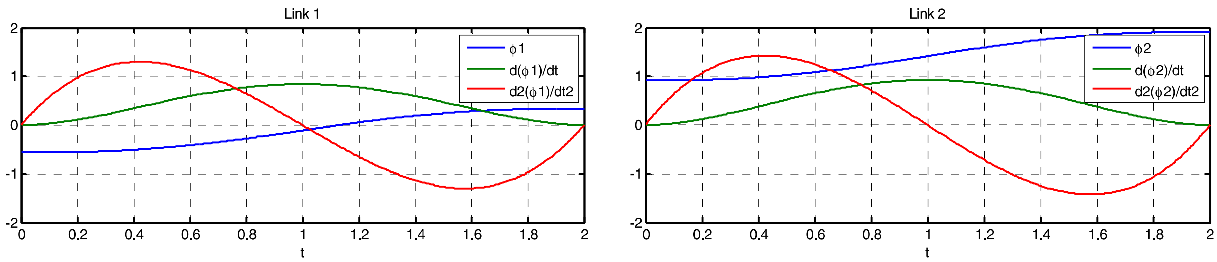

Fig. 3 Time diagram of the mechanical system.

3. Inverse problem

While solving the inverse problem we determine the moments M_1 and M_2 required to obtain the prescribed movement of the mechanical system. Inputs are the prescribed movements of its individual members [12]. Member 2 is to be moved from position L_0 to desired position L_1 along trajectory determined in the preceding paragraph. Block diagram of the mechanical system for solving the inverse problem is in Fig. 4. Blocks of inverse dynamics problem are described in Fig. 5, Fig. 6 and Fig. 7.

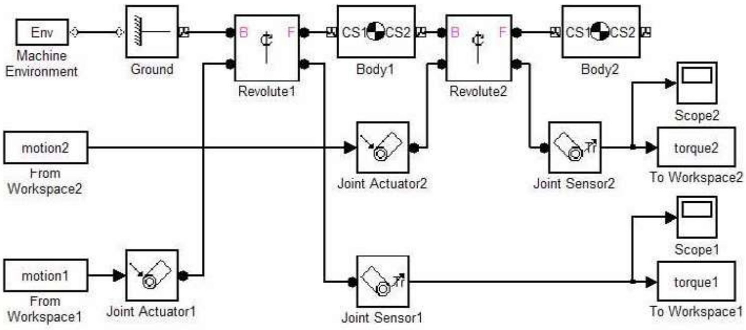

Fig. 4 Block diagram of the mechanical system in SimMechanics (inverse dynamics).

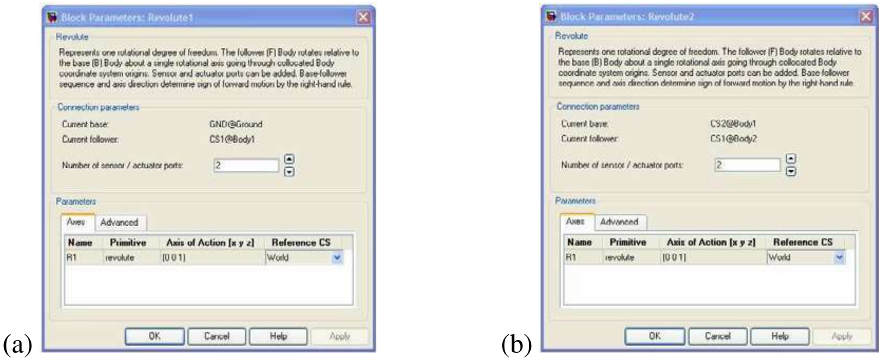

Fig. 5 Parameters of the block Revolute 1 (a) parameters of the block Revolute 2 (b).

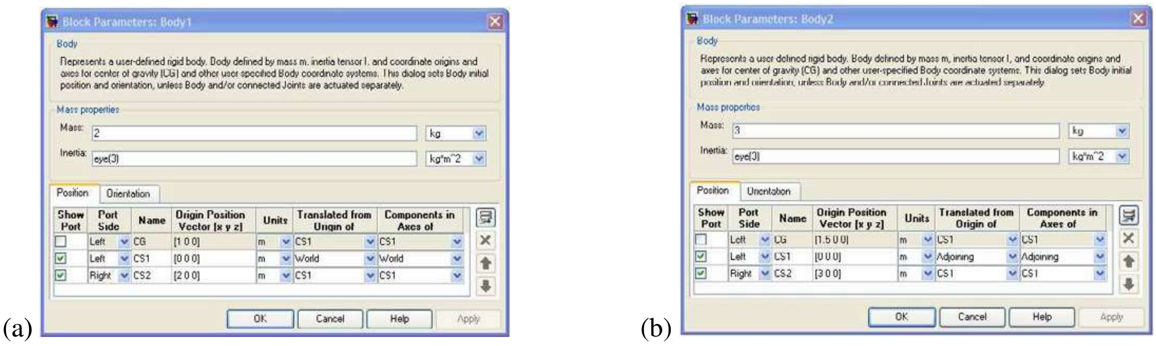

Fig. 6 Parameters of the block Body 1 (a) parameters of the block Body 2 (b).

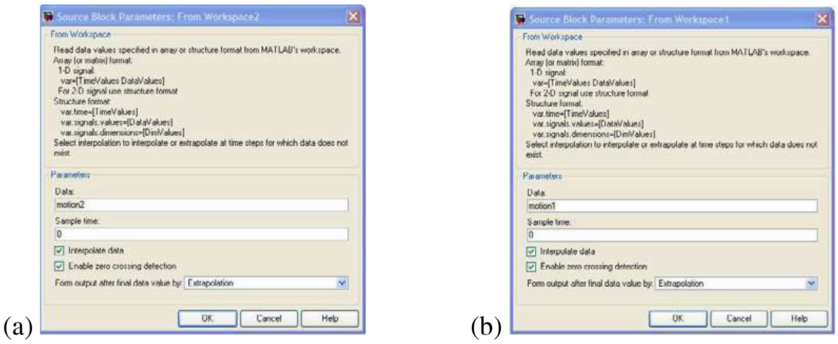

Fig. 7 Parameters of the block From Workspace 2 (a) parameters of the block From Workspace 1 (b).

Time diagrams of moments in the kinematic pairs M_1 and M_2 obtained by solving the model in the above block diagram are shown in Figure 8.

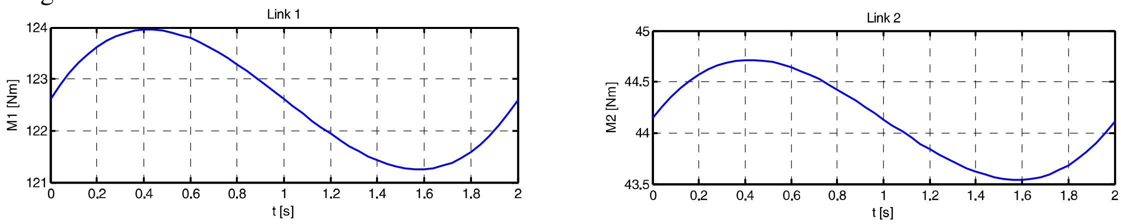

Fig. 8 Time diagram of actuator forces of the mechanical system.

4. Direct problem

Within direct problem we investigate the movement of the mechanical system under the load of forces obtained in the previous paragraph. Aplication of the results from the inverse problem on the model in Fig. 9 should result in motion as prescribed at the begining of the analysis.

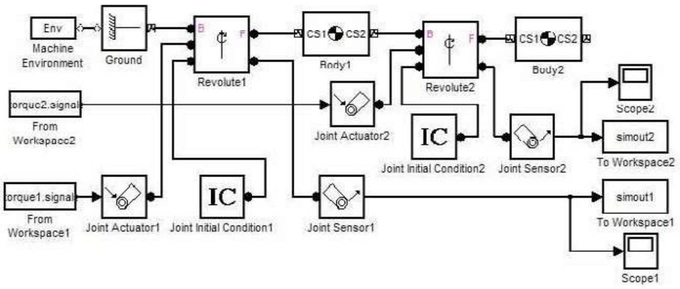

Fig. 9 Block diagram of the mechanical system in SimMechanics (forward dynamics).

Fig. 10 describes parameters of the block from Workspace of fordward dynamics problem.

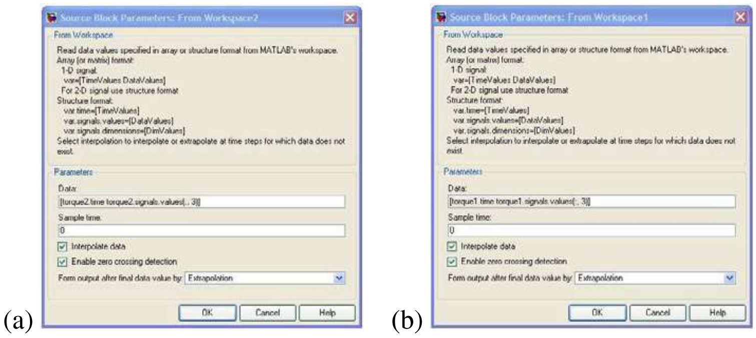

Fig. 10 Parameters of the block From Workspace 2 (a) parameters of the block From Workspace 1 (b).

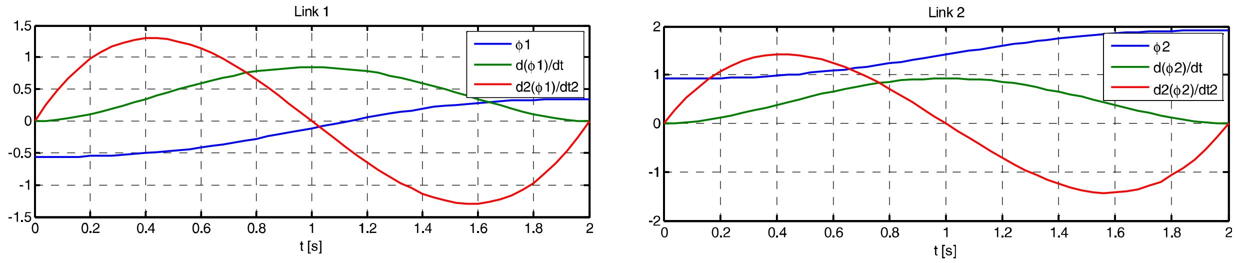

Fig. 11 Time diagram of SimMechanics simulation.

Results obtained from the simulation in Fig. 11 correspond with the prescribed motion we determined from the desired location of the endpoint L that are in Fig.3.

5. Conclusion

The paper presented a procedure for solving direct and inverse dynamics problem of the mechanism of double pendulum by means of SimMechanics modeling. SimMechanics allows us to simulate such mechanical systems which are a common problem in engineering practice. Results obtained are depicted in form of time diagram of the desired variables. Tasks are solved numerically and the model is compiled by using block diagrams. Mastering this methodology provides a suitable tool for solving problems in teaching and practice [13-16].

Acknowledgement

This contribution is the result of the project implementation: Center for research of control of technical, environmental and human risks for permanent development of production and products in mechanical engineering (ITMS:26220120060) supported by the Research & Development Operational Programme funded by the ERDF. This work was also supported in part by the Ministry of Education of the Slovakia Foundation under Grant VEGA no. 1/1205/12, VEGA no. 1/0289/11, VEGA no. 1/0937/12 and VEGA no. 1/0102/11.

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