LAB 2 REPORT

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Contents

Symbols:

Vl→ Left wheel velocity

Vr→ Right wheel velocity

V_{robot}→ Linear velocity of the robot

L→ Distance between left and right wheel

r_r →Radius of right wheel

r_l →Radius of left wheel

θ→Relative heading of the robot in inertia frame

ω_l→ Angular velocity of right wheel

ω_r→ Angular velocity of right wheel

ω→ Angular velocity of mobile robot

1-Introduction

In the first part of this lab, we will assemble an input-output model for the entire system. The robot will be driving in a rectangle box l= 750mm and w=500mm. The system's inputs are the 2 PWM signals and the output of the system is the readings of the sensors. We get the position of out robot from the 2 laser range sensors, and will consist of the distance to a wall in front and to the right of the robot, respectively. And its orientation from the IMU. The IMU will return an absolute bearing indicating the angle of the robot with respect to magnetic north. We will use the model that we derived in the previous lab to create a state estimator. In the first part we will work on our model of the full system, and also we will drive the robot in a box and collect the data for our hardware output. In the next part we will derive required matrices for our state estimation. We will use this hardware data, the mathematical model and implement out state estimator using MATLAB. Since the system is not linear, as we concluded in lab one, we decided to use Extended Kalman Filter to do our state estimations. What Kalman filter does is to minimioze the difference between the measured values and the predicted results.

2- System Model and Hardware output

2-1 System Model

As discussed in the first lab we modeled our system as follows and derived its state equations.

For our 2-wheeled robot we have:

$$x = \begin{bmatrix} r_x \\ r_y \\ \theta \end{bmatrix}$$

$$z = \begin{bmatrix} d_x \\ d_y \\ \alpha \end{bmatrix}$$

$$u = \begin{bmatrix} T_l \\ T_r \end{bmatrix}$$

where x is the state of the system, z is the measured outputs from the sensors, and u is the input given to the system.

Linear velocity of the robot is expressed in equation 5-1. Linear velocity is related to the angular velocity by the following formula:

$$V=\omega R \qquad \text{Equation 1-1}$$

Where V is the linear velocity. ω is the angular velocity of a wheel with radius of R.

$$V_{robot}=R^2 * (\omega_r+\omega_l) \qquad \text{Equation 1-2}$$

Angular velocity of the mobile robot is expressed in equation 1-2.

$$\omega=RL * (\omega_r-\omega_l) \qquad \text{Equation 1-3}$$

The next state of the robot can be calculated by taking the velocity of the robot and transforming it into generalized a coordinate vector by the following equation:

$$r_{xt+\Delta}= r_{xt}+V_{robot*}\Delta\cos(\theta_t) \qquad \text{Equation 1-4}$$

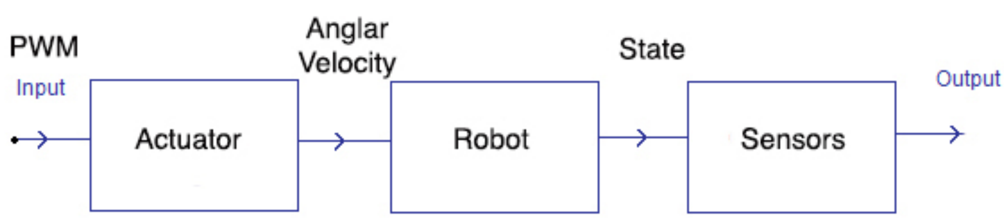
$$r_{yt+\Delta}= r_{yt}+V_{robot*}\Delta\sin(\theta_t) \qquad \text{Equation 1-5}$$

$$\theta_{t+\Delta}= \theta_t+\Delta\omega \qquad \text{Equation 1-6}$$

$$\begin{bmatrix} rx_{t+1} \\ ry_{t+1} \\ \theta_{t+1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} rx_t \\ ry_t \\ \theta_t \end{bmatrix} + \begin{bmatrix} \Delta\cos(\theta_t)\, R^2(\omega_l+\omega_r) \\ \Delta\sin(\theta_t)\, R^2(\omega_l+\omega_r) \\ \Delta \mathrm{RL}(\omega_r-\omega_l) \end{bmatrix}$$

$$\begin{bmatrix} rx_{t+1} \\ ry_{t+1} \\ \theta_{t+1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} rx_t \\ ry_t \\ \theta_t \end{bmatrix} + \begin{bmatrix} \Delta\cos(\theta_t) & \Delta\cos(\theta_t) \\ \Delta\sin(\theta_t)R^2 & \Delta\sin(\theta_t)R^2 \\ -\Delta\mathrm{RL} & \Delta\mathrm{RL} \end{bmatrix} \begin{bmatrix} \omega_l \\ \omega_r \end{bmatrix}$$

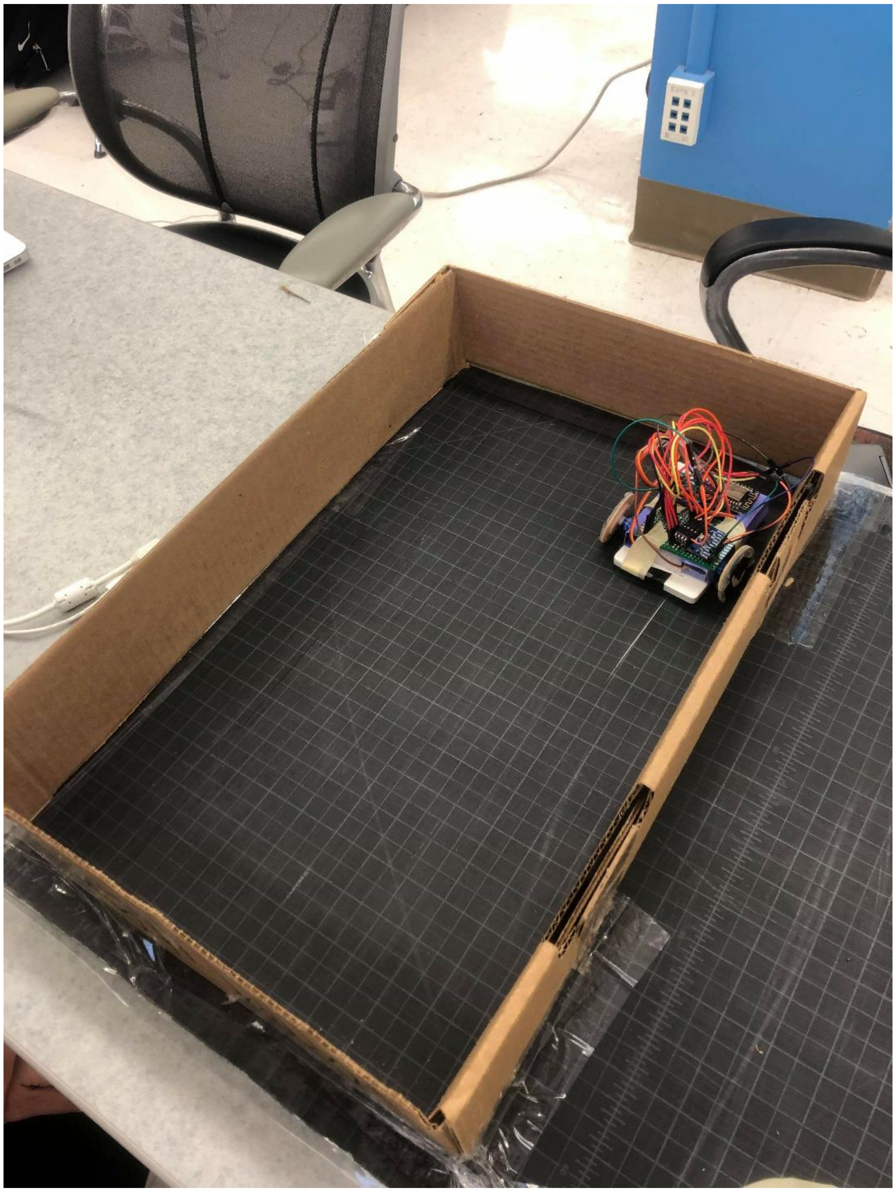
Now we have our state equations.



2-2-Output of Hardware

The goal of this section is to collect data from the actual hardware, and linearize the collected output and create the output function for our state estimator.

2-2-a- Method



To perform this part of our experiment, we put the robot in 500mm X 750mm box, and measured the initial state of the robot from the readings of the sensors, then we gave different inputs to the system for a specific time and then collected the sensor readings for each state from the sensor readings. 2 range sensors give us the position of our robot and the IMU gives us the orientation of our robot.

2-2-b-Results:

As we can see the sensor readings for each input is in table 1-1:

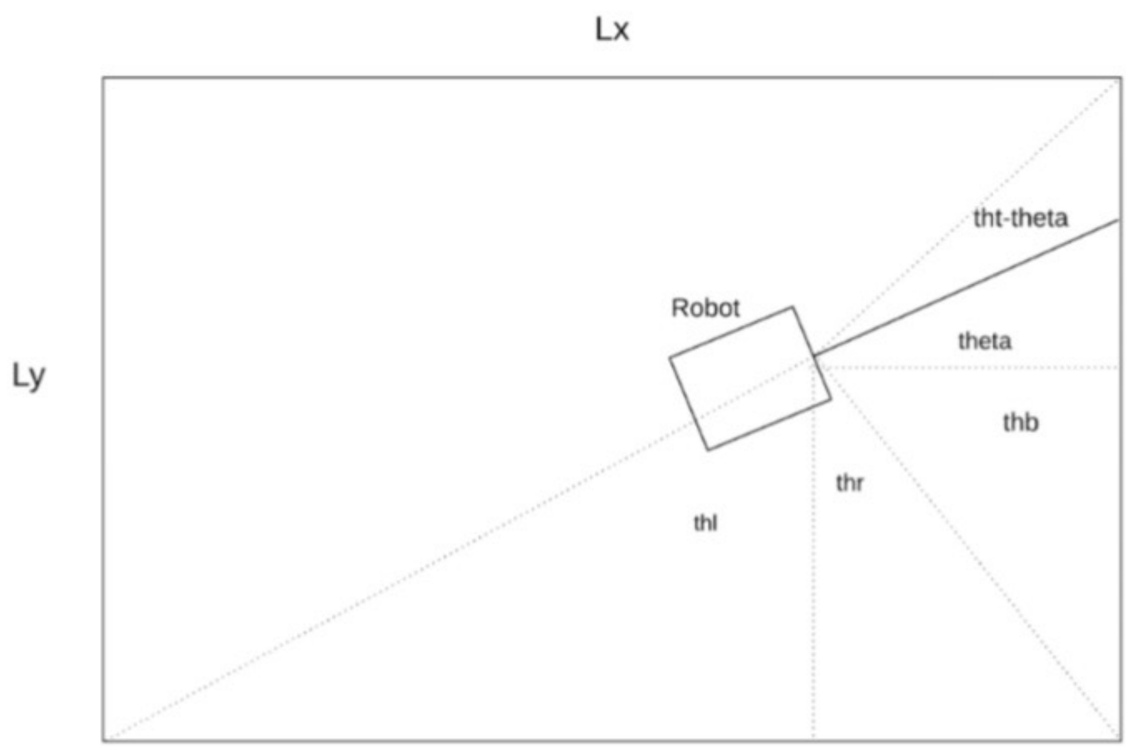
2-3 Deriving the Output Equation

As we can see, we have the position of robot as our state in our state equations; however, what the sensors give us is the distance from the walls of the box. We need to find the relationship between our state variables and our actual hardware outputs, which in our case are the sensor readings. We will get z=h(x) from this results. I used this derivation that I found on github. I will attach the link to the source. To do the following derivation, we made some assumptions:

1)$0<\theta<90$

2) T_r and T_l have the same magnitude, opposite sign if turning, otherwise same sign

3) $\theta=\alpha$ assuming we have the MPU calibrated at the 0 inputs.



$$\theta_{tht} = \tan^{-1}\left(\frac{L_y - r_y}{L_x - r_x}\right)$$

$$\theta_{thb} = \tan^{-1}(\frac{r_y}{L_x - L_x})$$

$$\theta_{thr} = \tan^{-1}(\frac{L_x - r_x}{r_y})$$

$$\theta_{thl} = \tan^{-1}(\frac{r_x}{r_y})$$

d_x is the front range sensor reading which varies based on the orientation of the robot.

$$d_x = \frac{L_x - r_x}{\cos(\theta)} \qquad 0 < \theta < \theta_{tht}$$

$$d_x = \frac{L_y - r_y}{\sin(\theta)} \qquad \theta > \theta_{tht}$$

$$d_x = \frac{L_x - r_x}{\cos(\theta)} \qquad \theta < 0, |\theta| < |\theta_{thb}|$$

$$d_x = \frac{r_y}{\sin\ |\theta|} \qquad \theta < 0, |\theta| > \theta_{thb}$$

d_y is the right range sensor reading which varies based on the orientation of the robot.

$$d_y = \frac{r_y}{\cos|\theta|} \qquad 0 < \theta < \theta_{thr}$$

$$d_y = \frac{L_x - r_x}{\sin\ |\theta|} \qquad \theta > \theta_{thr}$$

$$d_y = \frac{r_y}{\cos|\theta|} \qquad \theta < 0, |\theta| < |\theta_{thl}|$$

$$d_y = \frac{r_x}{\sin|\theta|} \qquad \theta < 0, |\theta| > \theta_{thl}$$

Note that in our derivation, we assumed that $\theta=\alpha$.

To write the output equations based on state variables, we will have:

$$\theta > 0, |\theta| < \theta_{tht}, |\theta| < \theta_{thr} \qquad z = \begin{bmatrix} \frac{L_x - r_x}{\cos|\theta|} \\ \frac{r_y}{\cos|\theta|} \\ \alpha = \theta \end{bmatrix}$$

$$\theta > 0, |\theta| < \theta_{tht}, |\theta| > \theta_{thr} \qquad z = \begin{bmatrix} \frac{L_x - r_x}{\cos|\theta|} \\ \frac{L_x - r_x}{\sin|\theta|} \\ \alpha = \theta \end{bmatrix}$$

$$\theta > 0, |\theta| > \theta_{tht}, |\theta| < \theta_{thr} \qquad z = \begin{bmatrix} \frac{L_y - r_y}{\sin|\theta|} \\ \frac{r_y}{\cos|\theta|} \\ \alpha = \theta \end{bmatrix}$$

$$\theta > 0, |\theta| > \theta_{tht}, |\theta| > \theta_{thr} \qquad z = \begin{bmatrix} \frac{L_y - r_y}{\sin|\theta|} \\ \frac{L_x - r_x}{\sin|\theta|} \\ \alpha = \theta \end{bmatrix}$$

$$\theta < 0, |\theta| < \theta_{thb}, |\theta| < \theta_{thl} \qquad z = \begin{bmatrix} \frac{L_x - r_x}{\cos|\theta|} \\ \frac{r_y}{\cos|\theta|} \\ \alpha = \theta \end{bmatrix}$$

$$\theta < 0, |\theta| < \theta_{thb}, |\theta| > \theta_{thl} \qquad z = \begin{bmatrix} \frac{L_x - r_x}{\cos|\theta|} \\ \frac{r_x}{\sin|\theta|} \\ \alpha = \theta \end{bmatrix}$$

$$\theta < 0, |\theta| > \theta_{thb}, |\theta| < \theta_{thl} \qquad z = \begin{bmatrix} \frac{r_y}{\sin|\theta|} \\ \frac{r_y}{\cos|\theta|} \\ \alpha = \theta \end{bmatrix}$$

$$\theta < 0, |\theta| > \theta_{thb}, |\theta| > \theta_{thl} \qquad z = \begin{bmatrix} \frac{r_y}{\sin|\theta|} \\ \frac{r_x}{\sin|\theta|} \\ \alpha = \theta \end{bmatrix}$$

To use them in our output model all cases need to be linearized. I did only one of them to clarify the procedure.

$$d_x = \frac{-1}{\cos\theta 0}(r_x - r_{x0}) - \frac{(L_x - r_{x0}) sin\theta 0}{(\cos\theta 0)^2}(\theta - \theta 0) + d_{x0}$$

$$d_y = \frac{1}{\cos\theta 0}(r_y - r_{y0}) - \frac{(r_y) sin\theta 0}{(\cos\theta 0)^2}(\theta - \theta 0) + d_{y0}$$

$$z = \begin{bmatrix} \frac{-1}{\cos\theta 0} & 0 & \frac{(L_x - r_{x0}) sin\theta 0}{(\cos\theta 0)^2} \\ 0 & \frac{1}{\cos\theta 0} & \frac{(r_y) sin\theta 0}{(\cos\theta 0)^2} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_x \\ r_y \\ \theta \end{bmatrix} + \begin{bmatrix} \frac{r_{x0}}{\cos\theta 0} - \frac{(L_x - r_{x0}) sin\theta 0}{(\cos\theta 0)^2}\theta 0 + d_{x0} \\ \frac{-r_{y0}}{\cos\theta 0} - \frac{(r_y) sin\theta 0}{(\cos\theta 0)^2}\theta 0 + d_{y0} \\ 0 \end{bmatrix}$$

$\theta \geq 0, |\theta| < \theta_{tht} |\theta| < \theta_{thr}$

$$H=\begin{bmatrix}\frac{-1}{\cos\theta 0} & 0 & \frac{(L_x-r_{x0})sin\theta 0}{(\cos\theta 0)^2}\\ 0 & \frac{1}{\cos\theta 0} & \frac{(r_y)sin\theta 0}{(\cos\theta 0)^2}\\ 0 & 0 & 1\end{bmatrix}\qquad C=\begin{bmatrix}\frac{r_{x0}}{\cos\theta 0}-\frac{(L_x-r_{x0})sin\theta 0}{(\cos\theta 0)^2}\theta 0+d_{x0}\\ \frac{-r_{y0}}{\cos\theta 0}-\frac{(r_y)sin\theta 0}{(\cos\theta 0)^2}\theta 0+d_{y0}\\ 0\end{bmatrix}$$

$\theta \geq 0, |\theta| < \theta_{tht}|\theta| > \theta_{thr}$

$$H=\begin{bmatrix}\frac{-1}{\cos\theta 0} & 0 & \frac{(L_x-r_{x0})sin\theta 0}{(\cos\ \theta 0)^2}\\ \frac{-1}{\sin\theta 0} & 0 & \frac{\theta 0}{|\theta 0|}\frac{(L_x-r_{x0})cos\theta 0}{(\sin\ \theta 0)^2}\\ 0 & 0 & 1\end{bmatrix}\qquad C=\begin{bmatrix}\frac{r_{x0}}{\cos\theta 0}-\frac{(L_x-r_{x0})sin\theta 0}{(\cos\theta 0)^2}\theta 0+d_{x0}\\ \frac{r_{y0}}{\sin\theta 0}-\frac{(L_x-r_{x0})cos\theta 0}{(\sin\theta 0)^2}|\theta 0|+d_{y0}\\ 0\end{bmatrix}$$

$\theta > 0, |\theta| > \theta_{tht}|\theta| < \theta_{thr}$

$$H=\begin{bmatrix}0 & \frac{-1}{\sin|\theta 0|} & \frac{\theta 0}{|\theta 0|}\frac{(L_y-r_{y0})cos\theta 0}{(\sin|\theta 0|)^2}\\ 0 & \frac{1}{\cos\theta 0} & \frac{(r_{y0})sin\theta 0}{(\cos\ \theta 0)^2}\\ 0 & 0 & 1\end{bmatrix}\qquad C=\begin{bmatrix}\frac{r_{x0}}{\sin|\theta 0|}-\frac{(L_y-r_{y0})cos\theta 0}{(\sin|\theta 0|)^2}|\theta 0|+d_{x0}\\ \frac{-r_{y0}}{\cos\theta 0}-\frac{(r_{y0})sin\theta 0}{(\cos\theta 0)^2}\theta 0+d_{y0}\\ 0\end{bmatrix}$$

$\theta > 0, |\theta| > \theta_{tht}|\theta| > \theta_{thr}$

$$H=\begin{bmatrix}0 & \frac{-1}{\sin|\theta 0|} & -\frac{\theta 0}{|\theta 0|}\frac{(L_y-r_{y0})cos\theta 0}{(\sin\ |\theta 0|)^2}\\ \frac{-1}{\sin|\theta 0|} & 0 & -\frac{\theta 0}{|\theta 0|}\frac{(L_x-r_{x0})cos\theta 0}{(\sin|\theta 0|)^2}\\ 0 & 0 & 1\end{bmatrix}\quad C=\begin{bmatrix}\frac{r_{x0}}{\sin|\theta 0|}+\frac{(L_y-r_{y0})cos\theta 0}{(\sin|\theta 0|)^2}\theta 0+d_{x0}\\ \frac{r_{y0}}{\sin|\theta 0|}+\frac{(L_x-r_{x0})cos\theta 0}{(\sin|\theta 0|)^2}\theta 0+d_{y0}\\ 0\end{bmatrix}$$

$\theta < 0, |\theta| < \theta_{thb,}|\theta| < \theta_{thl}$

$$H=\begin{bmatrix}\frac{-1}{\cos\theta 0} & 0 & \frac{(L_x-r_{x0})sin\theta 0}{(\cos\ \theta 0)^2}\\ 0 & \frac{1}{\cos\theta 0} & \frac{(r_y)sin\theta 0}{(\cos\ \theta 0)^2}\\ 0 & 0 & 1\end{bmatrix}\quad C=\begin{bmatrix}\frac{r_{x0}}{\cos\theta 0}-\frac{(L_x-r_{x0})sin\theta 0}{(\cos\theta 0)^2}\theta 0+d_{x0}\\ \frac{-r_{y0}}{\cos\theta 0}-\frac{(r_y)sin\theta 0}{(\cos\theta 0)^2}\theta 0+d_{y0}\\ 0\end{bmatrix}$$

$\theta < 0, |\theta| < \theta_{thb,} |\theta| > \theta_{thl}$

$$H = \begin{bmatrix} \frac{-1}{\cos\theta 0} & 0 & \frac{(L_x - r_{x0})sin\theta 0}{(\cos\ \theta 0)^2} \\ \frac{1}{\sin|\theta 0|} & 0 & -\frac{\theta 0}{|\theta 0|}\frac{(r_{x0})cos\theta 0}{(\sin\quad \theta 0)^2} \\ 0 & 0 & 1 \end{bmatrix} \quad \text{C=}\begin{bmatrix} \frac{r_{x0}}{\cos\theta 0} - \frac{(L_x - r_{x0})sin\theta 0}{(\cos\theta 0)^2}\theta 0 + d_{x0} \\ \frac{-r_{y0}}{\sin|\theta 0|} + \frac{(r_{x0})cos\theta 0}{(\sin\quad \theta 0)^2}|\theta 0| + d_{x0} \\ 0 \end{bmatrix}$$

$\theta < 0, |\theta| > \theta_{thb,} |\theta| < \theta_{thl}$

$$H = \begin{bmatrix} 0 & \frac{1}{\sin|\theta 0|} & -\frac{\theta 0}{|\theta 0|}\frac{r_{y0}cos\theta 0}{(\sin\theta 0)^2} \\ 0 & \frac{1}{\cos\theta 0} & \frac{(r_y)sin\theta 0}{(\cos\ \theta 0)^2} \\ 0 & 0 & 1 \end{bmatrix} \quad \text{C=}\begin{bmatrix} \frac{-r_{x0}}{\sin|\theta 0|} + \frac{r_{y0}cos\theta 0}{(\sin\theta 0)^2}\theta 0 + d_{x0} \\ \frac{-r_{y0}}{\cos\theta 0} - \frac{(r_y)sin\theta 0}{(\cos\ \theta 0)^2}\theta 0 + d_{x0} \\ 0 \end{bmatrix}$$

$\theta < 0, |\theta| > \theta_{thb,} |\theta| > \theta_{thl}$

$$H = \begin{bmatrix} 0 & \frac{1}{\sin|\theta 0|} & -\frac{\theta 0}{|\theta 0|}\frac{r_{y0}cos\theta 0}{(\sin\theta 0)^2} \\ \frac{1}{\sin|\theta 0|} & 0 & -\frac{\theta 0}{|\theta 0|}\frac{(r_{x0})sin\theta 0}{(\cos\ \theta 0)^2} \\ 0 & 0 & 1 \end{bmatrix} \quad \text{C=}\begin{bmatrix} \frac{-r_{x0}}{\sin|\theta 0|} + \frac{r_{y0}cos\theta 0}{(\sin\theta 0)^2}|\theta 0| + d_{x0} \\ \frac{-r_{y0}}{\sin|\theta 0|} + \frac{(r_{x0})sin\theta 0}{(\cos\ \theta 0)^2}|\theta 0| + d_{x0} \\ 0 \end{bmatrix}$$

3-Extended Kalman Filter:

Now that we have the Extended Kalman Filter given part meaning, robot's controls and observations we can use Extended Kalman Filter to do our state estimation. The goal is to estimate the path of the robot.

Extended Kalman Filter 's algoythm of operation consists of two major parts:

1)Prediction:

$$\hat{\mu}_t = g(u_t, \mu_{t-1})$$

$$\hat{\Sigma}_t = G_t \Sigma_{t-1} G_t^T + R_t$$

.

2) Update:

$$K_t = \hat{\Sigma}_t H_t^T \left(H_t \hat{\Sigma}_t H_t^T + Q_t\right)^{-1}$$

$$\mu_t = \hat{\mu}_t + K_t(z_t - h(\hat{\mu}_t))$$

$$\Sigma_t = (I - K_t H_t)\hat{\Sigma}_t$$

return Σ_t and μ_t

where R_t $and Q_t$ $are\ the\ covarience$ matrices:

$$R_t = E[v_t v_t^T]$$

$$Q_t = E[\omega_t \omega_t^T]$$

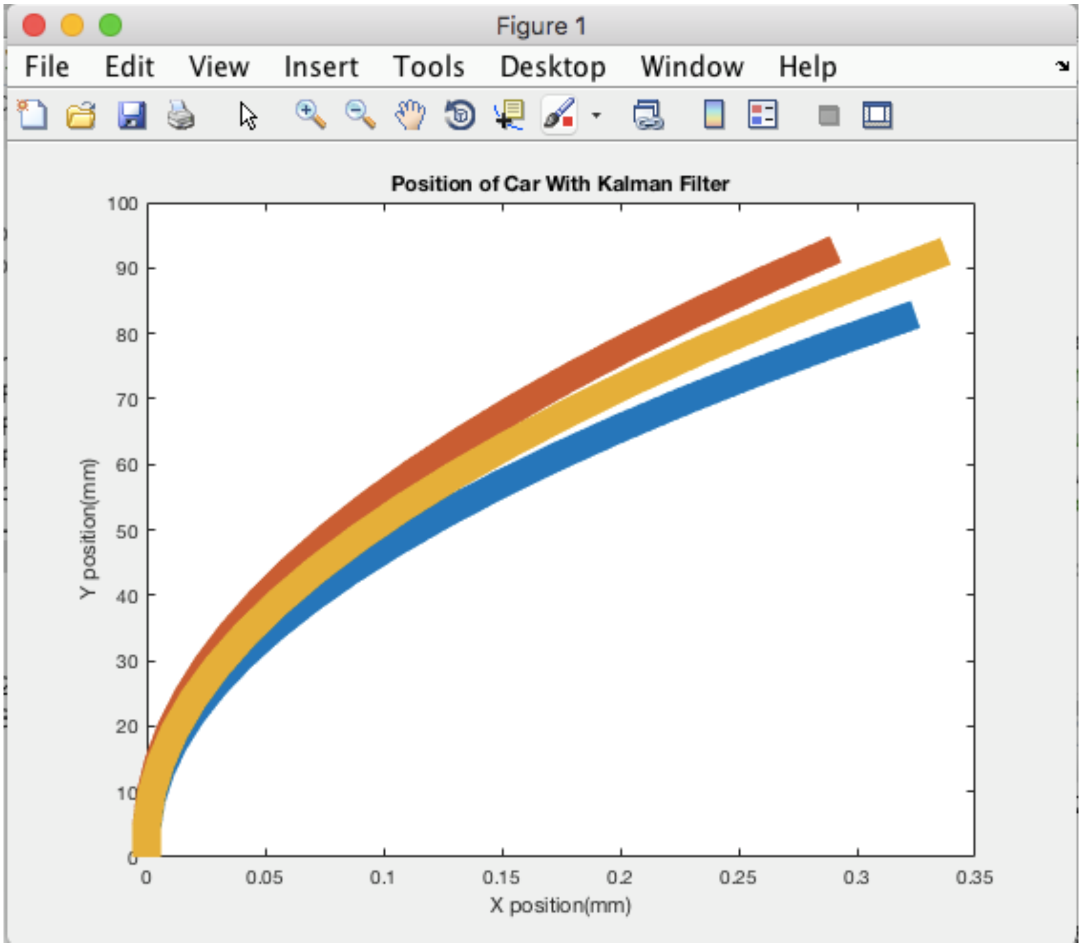
For unknown initial condition we have:

$$\mu_0 = [\,0 \quad 0 \quad 0\,]^T$$

$$\Sigma_0 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

4- Evaluation:

Now that we have the equations for states and hardware output and we have the Extended Kalman Filter algorithm, it is time to evaluate how good is our estimation. To do so we implemented the EKF with matlab and compared the results for actual hardware outputs, theoretical calculations using state equations, and the estimation using EKF. The result is provided in the following graph



5-Conclusion:

In this lab we demonstrated methods that can be used for state estimation. We learned to implement EKF in MATLAB. The code takes a state transition function and a sensor model and computes a corrected state along with the corresponding covariance matrices.

Github link:

https://github.com/meghedib/EE183DA-Lab2