FA 22 ISE 201 Probability Homework

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2022-10-19

Question 1

An industrial training company that offers week-long courses to corporations has three instructors on its permanent staff. The company receives requests for its courses from its many corporate clients. The course fee charged by the training company is $20,000 per course. The company also has a pool of qualified instructors in the local area (predominantly retired business school faculty) that it can draw upon whenever demand for their courses exceeds supply of permanent instructors. Under a standardized arrangement, an instructor in the pool receives 55% of the course fee whenever the instructor teaches the course. The weekly demand for courses obeys the probability distribution

num_courses	prob
0	0.05
1	0.15
2	0.25
3	0.25
4	0.15
5	0.10
6	0.05

The company will obviously utilize its own instructors to teach courses whenever possible. Pool instructors will be scheduled to teach only if the demand for the courses exceeds the number of permanent staff.

> Verify that the data in the table above corresponds to a probability distribution

```
# ADD CODE HERE
any(prob>1)
```

```
## [1] FALSE
```

```
any(prob<0)
```

```
## [1] FALSE
```

```
sum(prob)
```

```
## [1] 1
```

ADD YOUR COMMENTS BELOW THIS STATEMENT

The data in table corresponds to probability distribution because

1. All the probabilities are less than 1.
2. All the probabilities are greater than 0.
3. Sum of the probabilities is 1.

> What is the probability that all of the permanent staff are idle in a particular week?

Note: You can write equations within dollar signs, for example, such as $f(x_i) = P(X = x_i)$, or $F(x) = P(X \leq x)$

```
# ADD CODE HERE
data_frame_prob <- as.data.frame(prob_dist_courses)
data_frame_prob$prob[1]
```

```
## [1] 0.05
```

ADD YOUR COMMENTS BELOW THIS STATEMENT

We are choosing the probability from index 1 as data frame follows 1 based indexing.

Here,

P(all permanent staff are idle in a week)

= P(weekly demand of the courses are 0)

= P(0)

= 0.05

> What is the probability that the permanent staff is busy in a particular week?

```
# ADD CODE HERE
prob_dist_courses[4,] + prob_dist_courses[5,] + prob_dist_courses[6,] + prob_dist_courses[7,]
```

```
## num_courses        prob
##       18.00        0.55
```

ADD YOUR COMMENTS BELOW THIS STATEMENT

P(permanent staff is busy in a particular week)

= P(weekly demand is atleast 3 courses)

= P(3) + P(4) + P(5) + P(6)

= 0.55

Note : In code we have index starting from 4 as it follows 1 based indexing

> One of the permanent staff is a star teacher. The company schedules her to teach on every possible occasion to maximize the chances of repeat customers. What is the probability that she will be busy in a given week?

```
# ADD CODE HERE
1 - prob_dist_courses[1,2]
```

```
## prob
## 0.95
```

ADD YOUR COMMENTS BELOW THIS STATEMENT

P(star teacher is busy)
= P(There will be atleast 1 course per week)
= 1 - P(0 courses per week)
= 1 - 0.05
= 0.95

> What is the mean and standard deviation of the weekly revenue after deducting payments to pool instructors?

```
# ADD CODE HERE
#Weekly revenue after deducting payments to pool instructors
revenue <- ifelse(data_frame_prob$num_courses <=3,
       data_frame_prob$num_courses*20000, 20000*3 +
       (data_frame_prob$num_courses - 3)*20000*0.45)

data_frame_prob$revenue <- revenue

# For computing the variance we shall use the formula
# Var(X) = E[X^2] - E[X]^2

mean <- sum(data_frame_prob$prob * data_frame_prob$revenue)
square_of_mean <- mean*mean

mean_of_square <- sum(revenue * revenue * data_frame_prob$prob)
variance = mean_of_square - square_of_mean
variance
```

```
## [1] 510750000
```

```
standard_deviation <- sqrt(variance)
standard_deviation
```

```
## [1] 22599.78
```

ADD YOUR COMMENTS BELOW THIS STATEMENT

For finding weekly revenue after deducting payments to pool instructors
1. When number of courses are less than 3, then the payment will be number of courses multiplied by 20000.
2. If number of courses are more than 3 then 55% of the course fee on subjects is taken by pool instructor goes to the pool instructor. Hence, only 45% is received by the permanent staff members.

We have then added a column “revenue” in the data_frame_prob for easier computation of mean.

For computing the variance we shall use the formula:
Var(X) = E[X^2] - E[X]^2

> Adding more instructors to the staff involves an incremental cost of $2500 per instructor per week. How many instructors, if any, should the company add to its permanent staff in order to maximize expected profit?

```
# ADD CODE HERE
Min_faculty = ceiling(prob[5]*(num_courses[5]-3) + prob[6]*(num_courses[6]-3) + prob[7]*(num_
courses[7]-3))
Min_faculty
```

```
## [1] 1
```

ADD YOUR COMMENTS BELOW THIS STATEMENT
Number of instructos the company should add to its permanent staff in order to maximize expected profit is 1

QUESTION 2

Historical data indicate that the starting salary for a new graduate in a leading tech company can be modeled using a Normal distribution with mean $90,000 and standard deviation $20,000. Suppose that second-year salaries increase by exactly 20%. Also, suppose tht the bonus each year can be modeled using a Normal distribution with mean $25,000 and standard deviation $5,000. Suppose that the bonus is independent of the initial salary (and is also independent of the annual salary increase).

> What is the expected annual compensation (salary plus bonus) for a new hire?

```
# ADD CODE HERE
mean_salary <- 90000
mean_bonus <- 25000
expected_annual_compensation <- mean_salary + mean_bonus
```

ADD YOUR COMMENTS BELOW THIS STATEMENT
The expected annual compensation for a new hire is 115000

> What is the standard deviation of the annual compensation for a new hire?

```
# ADD CODE HERE
standard_deviation_salary <- 20000
standard_deviation_bonus <- 5000
variance <- standard_deviation_salary^2 + standard_deviation_bonus^2
SD <- sqrt(variance)
SD
```

```
## [1] 20615.53
```

ADD YOUR COMMENTS BELOW THIS STATEMENT
The standard deviation of the annual compensation for a new hire is 20615.53

> What is the expected annual compensation for an employee after completing one year at the firm, i.e., just after the salary increase is announced?

```
# ADD CODE HERE
salary_increase <- 1.2*90000
compensation <- salary_increase + mean_bonus
compensation
```

```
## [1] 133000
```

ADD YOUR COMMENTS BELOW THIS STATEMENT

The expected annual compensation for an employee after completing one year at the firm is 133000.

> What is the standard deviation of an employee's annual compensation for an employee after completing one year at the firm, i.e., just after the salary increase is announced?

```
# ADD CODE HERE
standard_deviation_salary <- 20000*1.2
standard_deviation_bonus <- 5000
variance <- standard_deviation_salary^2 + standard_deviation_bonus^2
SD <- sqrt(variance)
SD
```

```
## [1] 24515.3
```

ADD YOUR COMMENTS BELOW THIS STATEMENT

The standard deviation of an employee's annual compensation for an employee after completing one year at the firm is 24515.2

> What is the probability that an employee's annual compensation after completing one year in the firm, i.e., just after the salary increase is announced, will exceed $140,000?

```
# ADD CODE HERE
probability <- pnorm(140000,compensation,SD)
probability_annual_comp_after_completing_one_year <- 1-probability
probability_annual_comp_after_completing_one_year
```

```
## [1] 0.3876168
```

ADD YOUR COMMENTS BELOW THIS STATEMENT

The probability that an employee's annual compensation after completing one year in the firm is 0.3876168