Workshop 1

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1 Principal regression analysis

1.1 Definitions

Stock labelling: $\alpha \in \{1, \ldots, N\}$ Time steps numeration: $t \in \{1, \ldots, T\}$

Return of asset α at time t:

$$\tilde{r}_\alpha(t) = \frac{p(t) - p(t-1)}{\tilde{\sigma}_\alpha}\,, \tag{1}$$

meaning: difference of prices divided by standard deviation of this difference in the last $\tilde{T}$ days.

Normalized return of the asset α:

$$r_\alpha(t) = \frac{\tilde{r}_\alpha(t) - \langle \tilde{r}_\alpha \rangle}{\sigma_\alpha}\,, \tag{2}$$

(mean and std over the whole period T).

Below: examples of indicators

Inverse volatility weighted index return:

$$I(t) = \frac{1}{N} \sum_\alpha r_\alpha(t) \tag{3}$$

Futures market index:

$$I_0(t) = \frac{1}{N} \sum_\alpha s_\alpha r_\alpha(t) \tag{4}$$

s_α +1 if α is a future in bond sector, otherwise -1.

1.2 Parametric approach

Average instantaneous volatility:

$$\sigma^2(t) = \frac{1}{N} \sum_\alpha r_\alpha^2(t) \tag{5}$$

Average instantaneous correlation between pairs of stocks:

$$\rho(t) = \frac{1}{N(N-1)} \sum_{\alpha \neq \beta} \frac{r_\alpha(t) r_\beta(t)}{\sigma^2(t)} \tag{6}$$

Defining three correlation functions:

$$L_I(\tau) = \frac{\langle I(t-\tau) I^2(t) \rangle}{\langle I^2(t) \rangle} \tag{7}$$

$$L_\sigma(\tau) = \frac{\langle I(t-\tau) \sigma^2(t) \rangle}{\langle I^2(t) \rangle} \tag{8}$$

$$L_\rho(\tau) = \frac{\langle I(t-\tau) \rho(t) \rangle}{\langle I^2(t) \rangle} \tag{9}$$

(the two partial leverage effects: correlation and volatility)
Regression:

$$\rho(t) = L_\rho(\tau)\, I(t-\tau) + \langle \rho(t) \rangle + \varepsilon(t, \tau) \tag{10}$$

(the same for $\sigma(t)$).

1.3 Matrix approach

Unconditional correlation matrix: **C**
Correlations between stocks:

$$r_\alpha(t) r_\beta(t) = C_{\alpha\beta} + D_{\alpha\beta}(\tau) I(t-\tau) + \epsilon_{\alpha\beta} \tag{11}$$

$$D_{\alpha\beta}(\tau) = \frac{\langle r_\alpha(t) r_\beta(t) I(t-\tau) \rangle}{\langle I^2(t) \rangle} \tag{12}$$