Project 2 Report

Engineering Analysis and Computation I

Ahmed Jaheen - 900212943
Mina Yasser - 900214039
Mohamed ElFahla - 900192563

Submitted to Dr. Mohamed Rashed
This report is prepared for ENGR3202, section 1

School of Science and Engineering
The American University in Cairo
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List of contributors

1. **Ahmed Jaheen**
   Contributions: Wrote ***main.m****,* ***newtonRaphson.m****,* ***CalculateF.m****,* ***CalculateJacobian.m****. Also, the latex code for this report and the report.*

2. **Mina Yasser**
   Contributions: Wrote ***gaussElimination.m****,* ***predictPlateSize.m****. Also, the report.*

3. **Mohamed ElFahla**
   Contributions: Wrote ***gaussJordan.m****,* ***plotRelativeErrors.m****,* ***plotPressureVsRadius.m****. Also, the report.*

Problem Definition

The problem involves developing a **MATLAB** code to calculate the average depth of a lake based on discrete depth measurements taken at specific points within the lake. The lake's plan is represented as a set of data points, each comprising 3-dimensional coordinates (x, y, z) representing the location and depth of the lake at that point.

The task is to employ numerical integration methods, specifically the Composite 1/3 **Simpson's rule**, **Single Trapezoidal rule**, and **Single** 3/8 **Simpson's rule**, to approximate the lake's average depth. The depth measurements are treated as discrete values, and the numerical integration methods are applied to compute the lake's total volume.

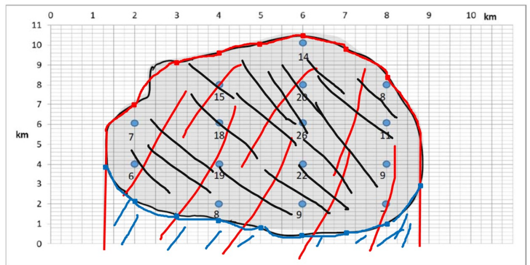
Additionally, **MATLAB** will use numerical triple integration to calculate the volume of each small element within the lake based on discrete depth measurements. Furthermore, numerical double integration will be used to compute the area of the lake's boundary from the discrete coordinate data.

The final output will be the lake's average depth, obtained by dividing the total volume (computed using numerical integration) by the lake's total surface area (computed using numerical double integration). The project requires verifying and validating the developed computer program and comparing its results with known datasets or analytical solutions.

Solution algorithm

1. **Input:**
   (a) The depth measurements and corresponding coordinates (x, y, z) of various points within the lake.
   (b) The coordinates (x, y) of the lake's boundary points for calculating the area.
2. **Average Depth Calculation:**
   (a) Composite 1/3 Simpson's Rule:
      i. Divide the depth measurements into equal intervals (if the number of data points is odd, the last break will use the 3/8 Simpson's rule).
      ii. For each interval, apply the 1/3 Simpson's rule to compute the average depth.
      iii. Sum up the average depths from all intervals to get the total volume.
   (b) Single 3/8 Simpson's Rule:
      i. If the number of data points is odd, use the 3/8 Simpson's rule for the last interval (which is not considered in the Composite 1/3 Simpson's rule).
      ii. Compute the average depth for this interval using the 3/8 Simpson's rule.
      iii. Add this average depth to the total volume obtained from the Composite 1/3 Simpson's rule.
   (c) Total Volume:
      i. The sum of volumes obtained from the Composite 1/3 Simpson's rule and the Single 3/8 Simpson's rule will give the lakes total volume.
3. **Numerical Triple Integration (x, y, z) for the Depth:**
   (a) For each depth measurement (x, y, z), apply numerical triple integration to obtain the volume of the corresponding small element in the lake.
   (b) Sum up the volumes from all the elements to calculate the total volume earlier.
4. **Numerical Double Integration (x, y) for the Area:**
   (a) Use numerical double integration (x, y) to calculate the area of the lake's boundary. This will give the surface area of the lake.
   (b) If the lake is open (no boundary provided), assume a limit based on the data points' extreme coordinates.
5. **Average Depth Calculation and Final Output:**
   (a) Divide the total volume obtained from the numerical triple integration by the total surface area obtained from the numerical double integration to calculate the lakes average depth.
   (b) Display the computed average depth as the final output.

6. **Error Analysis and Validation:**
   (a) Perform error analysis to evaluate the accuracy of the integration methods used.
   (b) Validate the computational tool's results by comparing them with analytical solutions or known datasets for test cases.
7. **Area Calculation:**

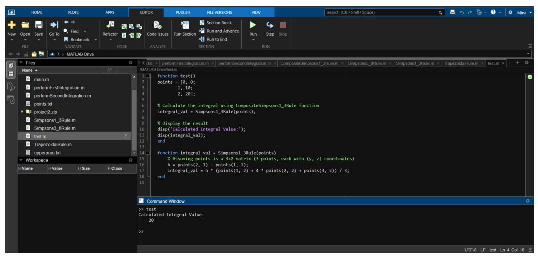


The area of the lake is the difference between the the upper and the lower integral and each one of them evaluated numerically with the most accurate numerical methods

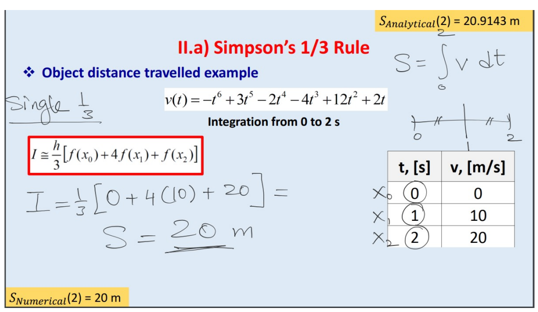
Validation of the computer program

Firstly, we will test our function for Single Simpson's 1/3 rule in compare with the solved example in the lecture notes:

1. **Single Simpson's 1/3 rule function:**

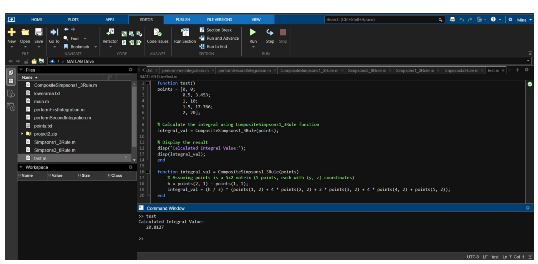


2. **Solved example from lecture notes:**

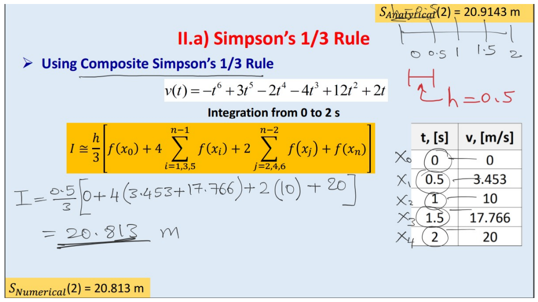


Next, we will test our function for Composite Simpson's 1/3 rule in compare with the solved example in the lecture notes:

1. **Composite Simpson's 1/3 rule function:**

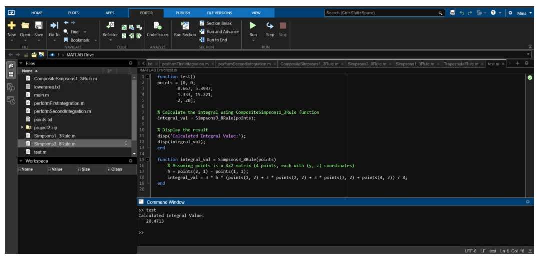


2. **Solved example from lecture notes:**

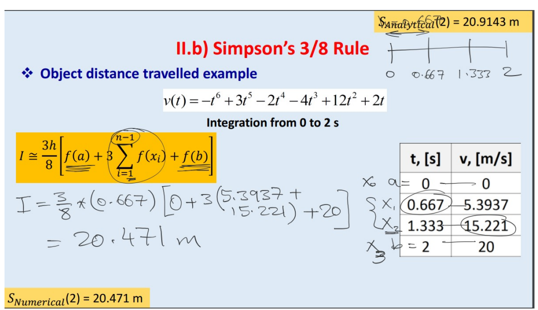


Then, we will test our function for Single Simpson's 3/8 rule in compare with the solved example in the lecture notes:

1. **Single Simpson's 3/8 rule function:**

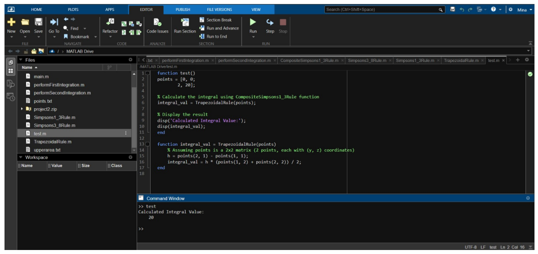


2. **Solved example from lecture notes:**

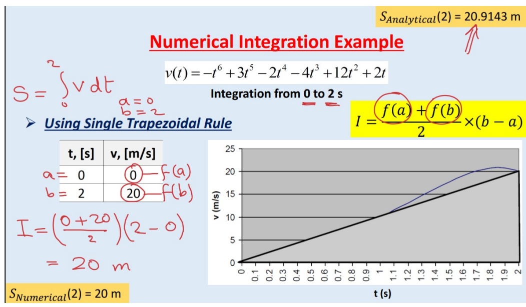


After that, we will test our function for Single Trapezoidal rule in compare with the solved example in the lecture notes:

1. **Single Trapezoidal rule function:**

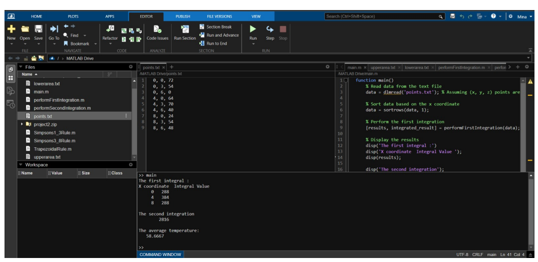


2. **Solved example from lecture notes:**

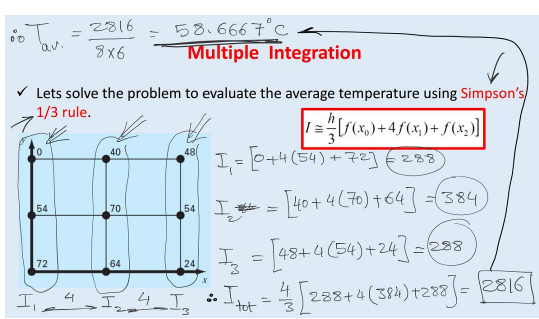


After that, we will test our function for Multiple integration in compare with the solved example in the lecture notes:

1. **Multiple integration function:**



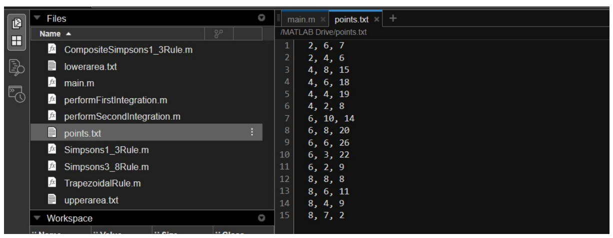
2. **Solved example from lecture notes:**



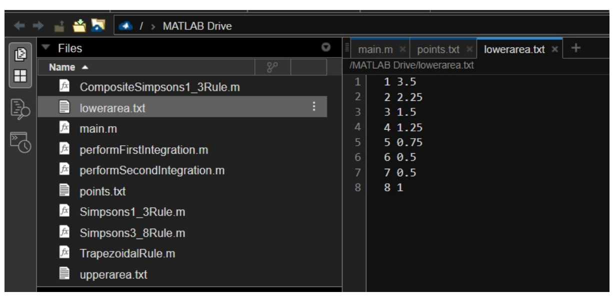
User Guide

The project is written in **MATLAB**, allowing you to compile it using **MATHWORKS** online or offline platform. Once the compilation process is complete, the program will be initiated as following:

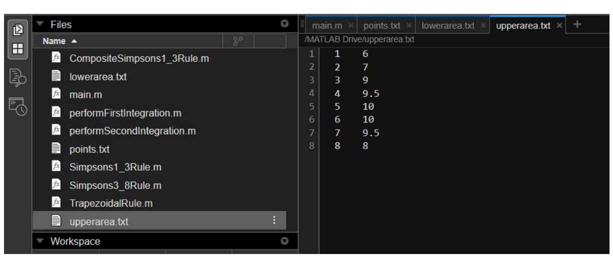
1. Firstly, the user has to write the points in the following format (x, y, z) in a text file as following:



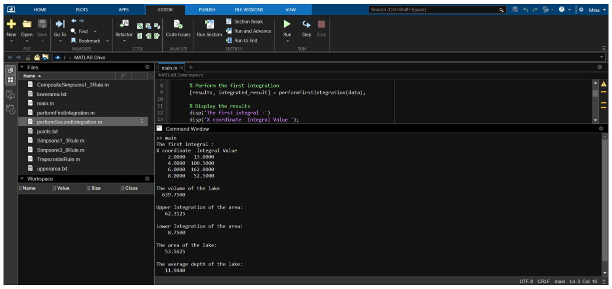
2. Secondly, the user has to write the points in the lower surface of the irregular shape he wants the average in in the following format (x, y) in a text file as following:



3. Thirdly, the user has to write the points in the upper surface of the irregular shape he wants the average in in the following format (x, y) in a text file as following:



4. Finally, the user will receive the volume and the area of the lake in addition to the average depth, and then the program will terminate.



Bibliography

[1] Dr. Mohamed Rashed. (2023). *Lecture Notes*.

[2] Dr. Mostafa Youssef. (2023). *Lecture Notes*.