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Multi-Tone Intermodulation Distortion Performance of 3rd Order Microwave Circuits

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Abstract - **Analytic expressions are presented to predict the response of a 3rd order memoryless microwave circuit under a relative narrow-band spectrum composed of a general number of equally spaced tones with constant amplitude and correlated or uncorrelated phases. This enabled the study of the relation between two-tone and multi-tone characterization, and showed that normal Noise Power Ratio measurement procedures are optimistic in up to 7dB when evaluating real co-channel distortion.**

I. Introduction

Although two-tone measurements still represent the industry standard in intermodulation distortion (IMD) characterization, nowadays, engineers seek for alternative test procedures closer to the system's final operation regime. Microwave circuits intended for telecommunications applications are expected to handle one or more carriers modulated with non-null information signals, i.e., finite bandwidth excitations. One way to model such inputs is to use a combination of various equally spaced tones with constant power and correlated or uncorrelated phases. Due to the statistics central limit theorem, when the number of tones is made large enough (in practice more than about 10 sinusoids [1]), and their relative phases are uncorrelated (i.e., each tone is synthesized from an independent reference), that kind of input tends to a narrow-band noise excitation and the system's response simulates a noise power ratio (NPR) test [2]. In fact, it can be easily understood that a sufficiently large number of equally spaced, but uncorrelated tones, is not different from another group of randomly located frequencies (uncommensurated tones), or band limited white noise.

From that introduction, it can be concluded that the derivation of a set of analytic expressions capable of describing the system's response under any number of **n** equally spaced tones, is a very useful result as it will allow the integration (and comparison) of the more usual ways of IMD characterization: under two-tone, three-tone, general multi-tone and band-limited noise.

The first aim of the present work is to present such a mathematical result, and to provide comparison relations of practical significance between these alternative IMD measurement procedures.

II. General Multi-Tone Response Calculations

Consider a narrow-band excitation, x(t), composed of **n** constant amplitude tones, and equally separated by a frequency increment $\Delta\omega$:

$$x(t) = \sum_{i=-n}^{n} X_i e^{j\omega_i t} \tag{1}$$

where $\omega_i=\omega_0+(i-1)\Delta\omega$ for $0<i\leq n$ and $\omega_i=-\omega_0+(i+1)\Delta\omega$ if $-n\leq i<0$.

Now imagine this input signal feeds a memoryless 3rd order microwave circuit (a class where a very large group of microwave and RF "linear" narrow band mixers and amplifiers can be included) in which the characteristic functions is: $y(t)=k_1x(t)+k_2[x(t)]^2+k_3[x(t)]^3$ (2)
Due to the system's nonlinearities, new in-band frequency components will be generated, as is represented in Fig. 1, and is predicted by substitution of (1) into (2):

$$y(t) = k_1\sum_{i=-n}^{n} X_i e^{j\omega_i t} + k_2\sum_{i_1=-n}^{n}\sum_{i_2=-n}^{n} X_{i_1}X_{i_2}e^{j(\omega_{i_1}+\omega_{i_2})t} + \\ + k_3\sum_{i_1=-n}^{n}\sum_{i_2=-n}^{n}\sum_{i_3=-n}^{n} X_{i_1}X_{i_2}X_{i_3}e^{j(\omega_{i_1}+\omega_{i_2}+\omega_{i_{23}})t} \tag{3}$$

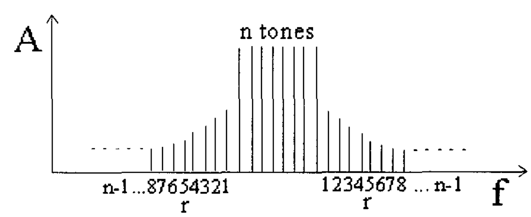

Fig. 1 – Output signal spectrum of a 3rd order nonlinear circuit excited by n equally spaced tones of constant amplitude.

These new mixing products can be classified as adjacent channel distortion (ACP) or co-channel distortion (CCP), whether their frequencies are located to the left or right of the original spectrum, or exactly over it, respectively. Observing the form of (3) and assuming that all tones have equal amplitude, $X_{-n},\ldots,\ldots X_n=X$, the computation of the amplitude of each in-band intermodulation product, ω_r only depends on the number of different arrangements $(\omega_x,\omega_y,\omega_z)$ verifying: $\omega_r=\omega_x+\omega_y+\omega_z$. Thus, the problem is reduced to a simple, although laborious, combinatory analysis.

The *formulae* next presented were derived from a such rigorous combinatory analysis, for each class of mixing products that verify $\omega_x+\omega_y-\omega_z=\omega_r$. Because there are many arrangements involving same three frequencies that fall on ω_r, the number of mixing components should be multiplied by the correspondent multinomial coefficient [3] which is 6 when we are dealing with $\omega_x+\omega_y-\omega_z$, 3 for $\omega_x+\omega_x-\omega_y$ and also 3 in case $\omega_x+\omega_x-\omega_x$. The expressions were then proved by formal mathematical induction.

II.1 – Adjacent Channel Distortion

By using Fig. 1 as a reference, we define the ACP tone under study as $\omega_x+\omega_y-\omega_z=\omega_r$, where $1\le r\le n-1$. The number of mixing products for each type of input frequency combination is:

- $\omega_x+\omega_y-\omega_z$ mixing type: $N_1(n,r)=\left(\frac{n-r}{2}\right)^2-\frac{\varepsilon}{4}$ (4)
- $2\omega_x-\omega_y$ mixing type: $M_1(n,r)=\left(\frac{n-r}{2}\right)+\frac{\varepsilon}{2}$ (5)

where $\varepsilon=\mathrm{mod}\left(\frac{n+r}{2}\right)$; (6)

and mod(a/b) stands for the remainder of a/b.

It can be proved that this expressions gives the same result as the recursive series presented on [4] for r=1. However, it also provides the amplitude of any other ACP tone ($1\le r\le n-1$).

II.2 – Co-Channel Distortion

To derive the number of co-channel mixing products in a certain position inside the input spectrum, we divided the set of **n** input tones in two blocks, one to the right of the test point of **b** tones ($0\le b<\mathrm{round}(n/2)$), and another to the left composed of (n-b-1) input tones, as is depicted in Fig. 2.

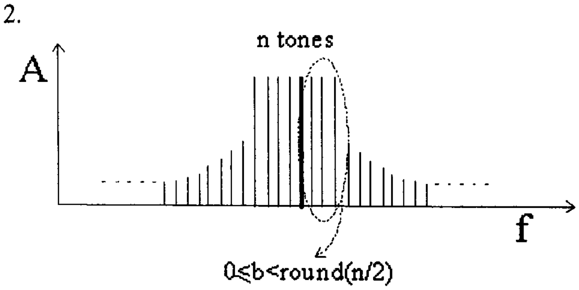

Fig. 2 – Co-channel mixing products generated by a 3rd order nonlinear circuit excited by n input tones.

The number of in-band beat products that fall on $\omega_b=\omega_x+\omega_y-\omega_z$ for each type of mixing arrangement is:

- $\omega_x+\omega_y-\omega_z$ mixing type :

$$\begin{aligned} N_2(n,b) &= \left(\frac{n-b-2}{2}\right)^2-\frac{\varepsilon}{4}+ \\ &+\left(\frac{b-1}{2}\right)^2-\frac{\varepsilon_1}{4}+b(n-b-2)+b \end{aligned} \quad (7)$$

- $2\omega_x-\omega_y$ mixing type :

$$M_2(n,b)=\left(\frac{n-b-2}{2}\right)+\frac{\varepsilon}{2}+\left(\frac{b-1}{2}\right)+\frac{\varepsilon_1}{2} \quad (8)$$

- $\omega_x+\omega_y-\omega_y$ mixing type : $S_1(n)=(n-1)$ (9)
- $\omega_x+\omega_x-\omega_x$ mixing type : $S_2=1$ (10)

where $\varepsilon=\mathrm{mod}\left(\frac{n+b}{2}\right)$ and $\varepsilon_1=\mathrm{mod}\left(\frac{b+1}{2}\right)$. (11-12)

II.3 – Co-Channel Distortion With One Tone Shut Down

This type of distortion corresponds to the measurement results obtained with a conventional NPR test. In fact, since CCP will fall exactly on the original spectrum, and is really indistinguishable from the circuit's much larger linear response, the natural way of measuring these distortion components is to observe them directly within an input spectrum notch. This notch corresponds to the tone that we considered shut down. Again, the number of co-channel beat products that fall on $\omega_b=\omega_x+\omega_y-\omega_z$ for each type of mixing arrangement is:

- $\omega_x+\omega_y-\omega_z$ mixing type :

$$\begin{aligned} N_3(n,b) &= \left(\frac{n-b-2}{2}\right)^2-\frac{\varepsilon}{4}+ \\ &\left(\frac{b-1}{2}\right)^2-\frac{\varepsilon_1}{4}+b(n-b-2) \end{aligned} \quad (13)$$

- $2\omega_x-\omega_y$ mixing type :

$$M_3(n,b)=\left(\frac{n-b-2}{2}\right)+\frac{\varepsilon}{2}+\left(\frac{b-1}{2}\right)+\frac{\varepsilon_1}{2} \quad (14)$$

where ε and ε_1 follow their definition given for (11-12).

These expressions validate the empirical ones obtained by trial and error and previously published in [4]. A brief comparison between these and the previous *II.2* results, clearly indicates that now there are less mixing components of type $\omega_x+\omega_y-\omega_z$, and none of $\omega_x+\omega_y-\omega_y$ or $\omega_x+\omega_x-\omega_x$ falling on the position where the input tone has zero amplitude. This is a primary indication that the notch produces a non negligible impact in the co-channel distortion under measurement. And also shows that the intuitive reasoning (which supports the conventional NPR measurement procedure) which assumes that the elimination of only one out of a large number of **n** tones, would not perturb the measurement, is, indeed, false.

III. Two-Tone versus Multi-Tone Distortion Characterization

Although it may sound counter intuitive, Volterra-Wiener theories [5] state that any 3rd order system may be completely characterized by only a set of three-tone tests. This means that no additional essential information should be expected by simply increasing the number of input tones. In fact, many of the strange results found in multi-tone testing of practical circuits can be explained by higher order contributions.

Since a general 3[rd] order system is completely characterized by its first three Volterra kernels or frequency domain nonlinear transfer functions, a minimum of three tones is required in theory to evaluate its distortion performance. A two-tone excitation is able to only generate products of the type $\omega_x+\omega_y-\omega_x$ or $\omega_x+\omega_x-\omega_x$ and can never produce terms of the form $\omega_x+\omega_y-\omega_z$. However, if the system can be approximately considered as memoryless, (at least for the signal it should handle), its nonlinear transfer function will not depend on frequency, and thus the contribution to distortion of the $\omega_x+\omega_y-\omega_z$ terms equals the one of $\omega_x+\omega_y-\omega_y$ or $\omega_x+\omega_y-\omega_x$, except for the number of different combinations of each type. In this case, a noise, multi-tone or even CW test may be sufficient to identify the system's distortion origin. Therefore, we should conclude that if the memoryless restriction applies, it is possible to derive relations between multi-tone and the usual two-tone test results.

The following expressions present these comparison results by showing integrated adjacent channel power ratio (ACPR), multi-tone IMD to carrier ratio (M-IMR), middle band usual noise power ratio (NPR) and corrected noise power ratio (CNPR) (for a total constant input power, $P_{tone}=\frac{P_{in}}{n}$), relative to the measured IMR obtained from a conventional two-tone test (IMR_2).

III.1 – ACPR versus IMR_2

ACPR is herein defined as the ratio between total linear output power to integrated distortion power contained in the adjacent channel bandwidth. Thus, according to (3) through (6), ACPR can be given by:

$$ACPR_{dBc} = IMR_{2dBc} + \\ +10\log\left(\frac{n^3}{2*\left(8\sum_{r=1}^{n-1}N_1(n,r)+2\sum_{r=1}^{n-1}M_1(n,r)\right)}\right) \quad (15)$$

where

$$\sum_{r=1}^{n-1}N_1(n,r)=\frac{2n^3-3n^2-2n}{24}+\frac{\varepsilon}{8}$$
$$\sum_{r=1}^{n-1}M_1(n,r)=\frac{n^2-\varepsilon}{4}$$
and $\varepsilon=\mathrm{mod}\left(\frac{n}{2}\right)$.

III.2 – M-IMR versus IMR_2

M-IMR is the ratio of the linear output power per tone to the distortion power of one certain tone in the adjacent channel. It is therefore:

$$\mathrm{M-IMR_{dBc}} = IMR_{2dBc} - 10\log(4) + \\ +10\log\left(\frac{n^2}{4N_1(n,r)+M_1(n,r)}\right) \quad (16)$$

III.3 – NPR versus IMR_2

The definition in this work to NPR, follows the one normally used when the test is based in a reasonably large set of discrete but uncorrelated tones. NPR stands for the ration between the linear output power per tone and the output power measured in a co-channel position where the input tone was previously shut down. NPR, this way defined, is therefore distinct from the corrected noise power ratio, CNPR, that would be measured as NPR if it was possible to observe distortion in the same position and under the same excitation condition, except that now no tone were turned-off. So, usual NPR relates to IMR_2 by:

$$NPR_{dBc} = IMR_{2dBc} - 10\log(4) + \\ +10\log\left(\frac{n^2}{4N_3(n,b)+M_3(n,b)}\right) \quad (17)$$

while $CNPR_{dBc}$ can be expressed as:

$$CNPR_{dBc} = IMR_{2dBc} - 10\log(4) + \\ +10\log\left(\frac{n^2}{4N_2(n,b)+M_2(n,b)+\frac{(6S_1(n)+3S_2)^2}{9}}\right) \quad (18)$$

Fig. 3 illustrates these comparison values versus number of input tones **n**.

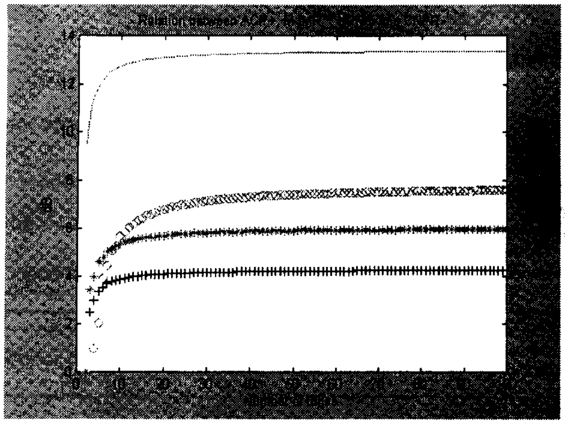

Fig. 3 – Relation between multi-tone ACPR, M-IMR, NPR, CNPR and IMR_2 versus number of input tones.

In order to estimate relations of noise excitation results to the reference two-tone test, we simply need to calculate the limit of (15) through (18) when the number of tones **n** increases indefinitely but total input power remains constant. Thus, for a random gaussian excitation ACPR turns to:

$$ACPR_{N_{dBc}} = \lim_{n\to\infty}(ACPR_{dBc}) = IMR_{2dBc} - 4.25dBc \qquad (19)$$

IMR_N comes:

$$IMR_{N_{dBc}} = \lim_{n\to\infty}(M - IMR_{dBc}) = IMR_{2dBc} - 6dB + \\ + \lim_{n\to\infty} 10\log\left(\frac{n^2}{n^2 - 2nr + r^2}\right) \qquad (20)$$

Which presents its minimum value when the edge of the adjacent channel band is considered (r=1):

$IMR_{NdBc}=IMR_{2dBc}-6dB$

For NPR this limit is:

$$NPR_{N_{dBc}} = \lim_{n\to\infty}(NPR_{dBc}) = IMR_{2dBc} - 6dB + \\ + \lim_{n\to\infty} 10\log\left(\frac{n^2}{n^2 - 2b^2 + 2nb}\right) \qquad (21)$$

Which reaches its minimum value of IMR_{2dBc}-7.7dB in the middle of the input bandwidth ($b = \frac{n}{2}$), and its maximum in the bandwidth extremes (b=0); IMR_{2dBc}-6dB.
Finally, if the same procedure was followed for CNPR, we should obtain:

$$CNPR_{N_{dBc}} = \lim_{n\to\infty}(CNPR_{dBc}) = IMR_{2dBc} - 6dB + \\ + \lim_{n\to\infty} 10\log\left(\frac{n^2}{5n^2 - 2b^2 + 2nb}\right) \qquad (22)$$

Which in the middle of the bandwidth ($b = \frac{n}{2}$) is IMR_{2dBc}-13.4dB, and IMR_{2dBc}-12.99dB in the extremes (b=0).

These latter results prove that the usual NPR test procedure provides optimistic co-channel distortion values of 5.64dB in the middle of the bandwidth, to 7dB in its extremes, compared to the ones that will be really obtained in normal operation when the circuit is driven by a dense spectrum, and all input tones are considered.

IV. Conclusions

In conclusion the derivation of the number of different combinations of three frequencies allowed us the study of the in-band distortion response of a 3rd order memoryless system when subject to **n** equally spaced tones of uniform amplitude. Those closed form results induced the conversion between IMD figures of merit obtained under two-tone, multi-tone and noise tests. The particular discussion on Noise Power Ratio showed that usual multi-tone or noise NPR tests lead to co-channel distortion evaluations that can be as optimistic as 5.64dB to 7 dB.

V. Acknowledges

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