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PS10 April 7

(1) $\dot{y} + y = x$

↓[Laplace transform]

$s \cdot Y(s) + Y(s) = X(s)$

$s \cdot Y(s) + Y(s) = \frac{1}{s}$

$\frac{Y(s)[\cancel{s+1}]}{\cancel{s+1}} = \frac{\frac{1}{s}}{s+1}$

$Y(s) = \frac{1}{s(s+1)}$

$Y(s) = \frac{1}{s} + \frac{-1}{s+1}$

Inverse Laplace transform

$Y(s) = u(t) - e^{-t} \cdot u(t)$

$\boxed{Y(s) = u(t)[1 - e^{-t}]}$

(Shift in time domain is multiplication by complex exponential in frequency domain + vice versa)

$u(t) \leftarrow$ step function

↓[Laplace transform]

$\frac{1}{s}$ } Laplace transform of the impulse

$\frac{1}{s} \cdot 1 = \frac{1}{s}$

$\frac{A}{s} + \frac{B}{s+1} = \frac{1}{s(s+1)}$

$\frac{A(s+1)}{s(s+1)} + \frac{B(s)}{s(s+1)} = \frac{1}{s(s+1)}$

$A(s+1) + B(s) = 1$

$B = \frac{1 - A(s+1)}{s} = \frac{\cancel{1} - s \cancel{+1}}{s} = -1$

if $A = 1$

*Have Oscar explain Laplace transforms better

(2) A.

$Y_{sp}(s)$	sp = set point
$\oplus$	"adder"; add the two things together
$E(s)$	error point; $Y_{sp}(s) - Y(s) = E(s)$
$K(s), H(s)$	transfer functions
$X(s)$	input signal
$Y(s)$	output

from below ↓ ...

$\frac{Y(s)}{Y_{sp}(s)} = \frac{\frac{Y_{sp}(s) \cdot K(s) \cdot H(s)}{1 + K(s) \cdot H(s)}}{Y_{sp}(s)}$

Known:

$Y(s) = X(s) \cdot H(s)$

$Y(s) = (E(s) \cdot K(s)) \cdot H(s)$

$Y(s) = (Y_{sp}(s) - Y(s)) \cdot K(s) \cdot H(s)$

$Y(s) = [Y_{sp}(s) \cdot K(s) \cdot H(s)] - [Y(s) \cdot K(s) \cdot H(s)]$

$Y(s) + [Y(s) \cdot K(s) \cdot H(s)] = [Y_{sp}(s) \cdot K(s) \cdot H(s)]$

$Y(s)[1 + K(s) \cdot H(s)] = Y_{sp}(s) \cdot K(s) \cdot H(s)$

$\boxed{\frac{Y(s)}{Y_{sp}(s)} = \frac{K(s) \cdot H(s)}{1 + K(s) \cdot H(s)}}$

Black's equation

② A cont.

$$\frac{Y(s)}{Y_{sp}(s)} = \frac{K(s)\cdot H(s)}{1+K(s)\cdot H(s)} \Rightarrow K(s) = \frac{K_I}{s}$$

$$\frac{Y(s)}{Y_{sp}(s)} = \frac{\frac{K_I}{s}\cdot H(s)}{1+\frac{K_I}{s}\cdot H(s)} \cdot \frac{s}{s} = \boxed{\frac{K_I \cdot H(s)}{1+K_I\cdot H(s)}}$$

$\lim_{s\to 0}$, then above $\to 1$

DC gain is 1, so no, it does not depend on the value of K_I

② B.

tell you about stability of your system

$H(s) = \frac{1/\tau}{s+1/\tau}$

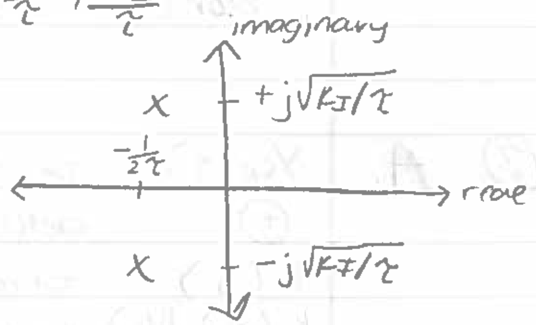
pole occurs when function evaluates to ∞ (denom = 0) ↗ solve for s
zeros occur when functions evaluate to 0 (num = 0) ↘ solve for s

$$\frac{Y(s)}{Y_{sp}(s)} = \frac{\left(K_I \cdot \frac{1/\tau}{s+1/\tau}\right)}{s + \left(K_I\cdot\frac{1/\tau}{s+1/\tau}\right)} \cdot \frac{s+1/\tau}{s+1/\tau} = \frac{\frac{K_I}{\tau}}{s(s+1/\tau)+\frac{K_I}{\tau}}$$

$$= \frac{\frac{K_I}{\tau}}{s^2+\frac{s}{\tau}+\frac{K_I}{\tau}}$$

No zeros
Poles $\rightarrow s^2 + \frac{s}{\tau} + \frac{K_I}{\tau} = 0$

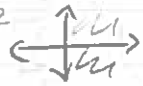
$$\boxed{s = \frac{-1}{2\tau} \pm j\sqrt{K_I/\tau}}$$



zero: Your function is zero.
Pole: $\frac{1}{s-\text{pole}} \Rightarrow \mathcal{L} \Rightarrow e^{\text{pole}\,t}\cdot u(t)$

- imaginary makes things oscillate
- real is $+ \rightarrow$ growth or $- \rightarrow$ decay

oscillations that grow which is instability; if you have a pole on the right side

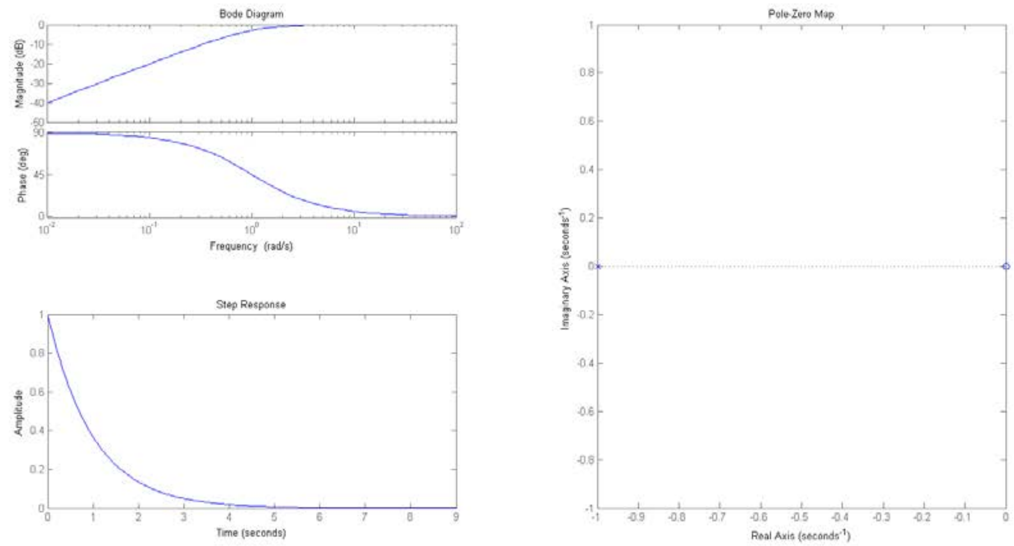


you have an unstable system

(ex. w/ swinging pendulum that swings harder + harder and breaks)

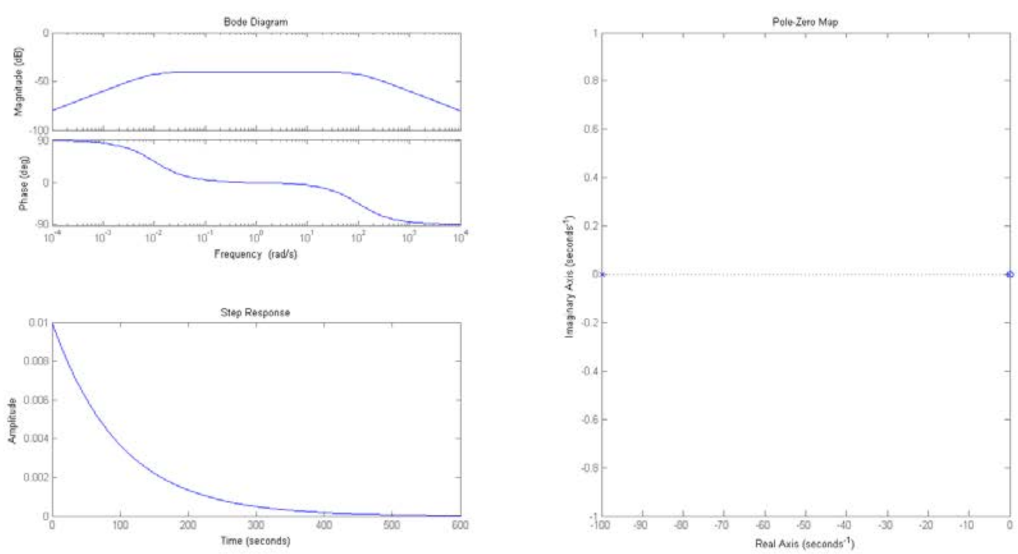
oscillations that shrink down to zero (ex. w/ swinging pendulum that slows and stops)

Problem 3A:



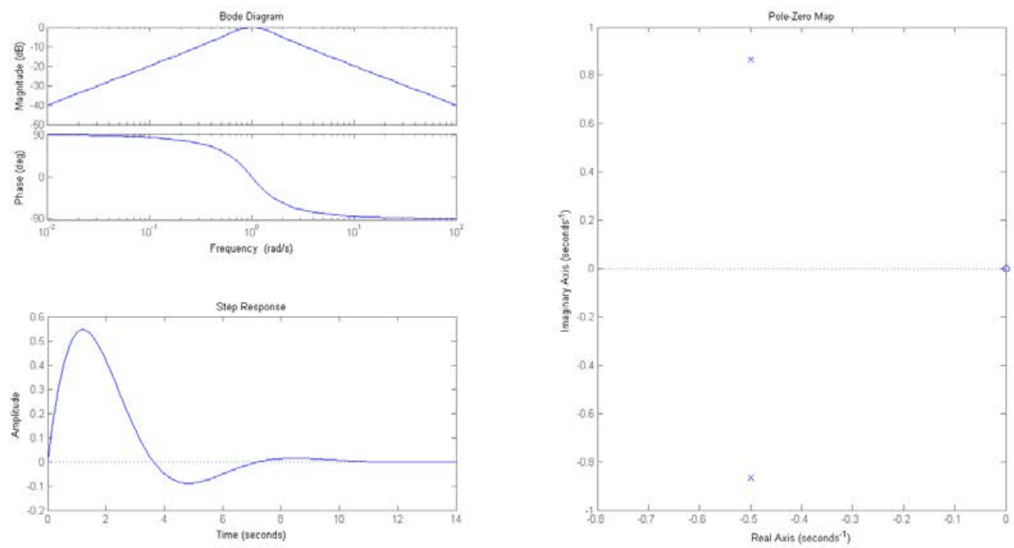
This is a first order system because the denominator, *s + 1*, is first degree. There is one real pole at (-1,0) and one real zero at (0,0). There are no oscillations in the step response because the poles don't have any imaginary parts. The bode plot is a high pass filter. There is one turn at 10^0 because there is one pole on the x-axis. All the poles are negative so the system is stable and the step response is decaying.

Problem 3B:



This is a second order system because the denominator has s^2 which is a second degree. There are two negative, purely real poles, and one real zero at (0,0). The bode plot has two turns: one very close to zero (for the pole on the far right) and at one 100 (for the pole on the far left). The bode plot is a band pass filter. All poles are negative so the system is stable and the step response is decaying. The pole located at -100 is having a rapid decay effect compared to the pole at 0.01. At 0.01, the change is happening slowly so there is a jump on the time scale of the y-axis of the step response, but you can only see it if you zoom in on the beginning of the graph.

Problem 3C:



This is a second order system because s^2 is in the denominator. There are two poles, one with a positive imaginary part, and one with a negative imaginary part, both with the real part of -0.5. There is also one completely real zero on the far right at (0,0). The bode plot is a band pass. The oscillations in the step response are caused by the imaginary parts in the complex poles. The slope of the bode plot depends on the number of zeros, in this case, one. The graph corresponds in such a way that as you pass the location of each pole, the bode plot has a turn. There are two turns because there are two imaginary poles. The bode plot turns twice, creating an upside down V, at the same place because the imaginary poles were at the same place. The slope of the bode plot appears to be 1, but since this is on a log-log scale, the slope actually has an exponent of 1.

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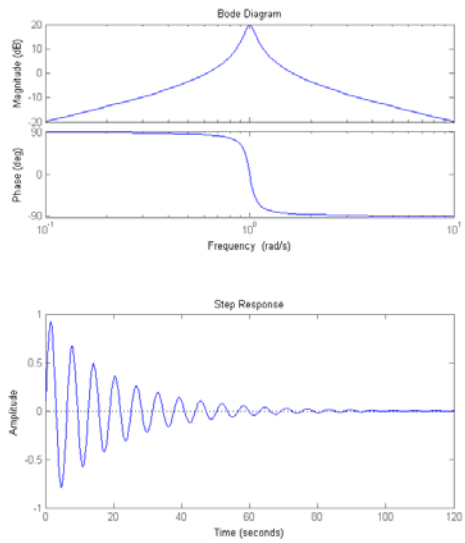
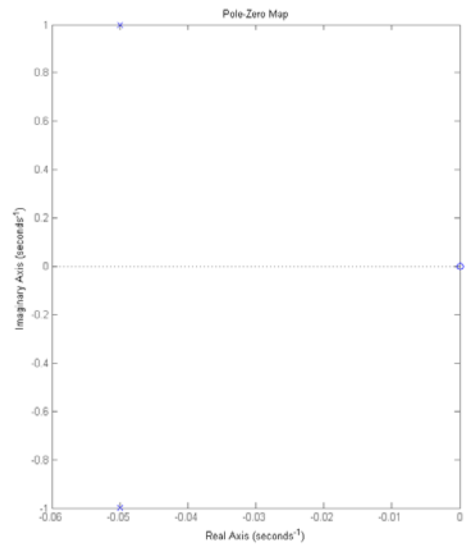
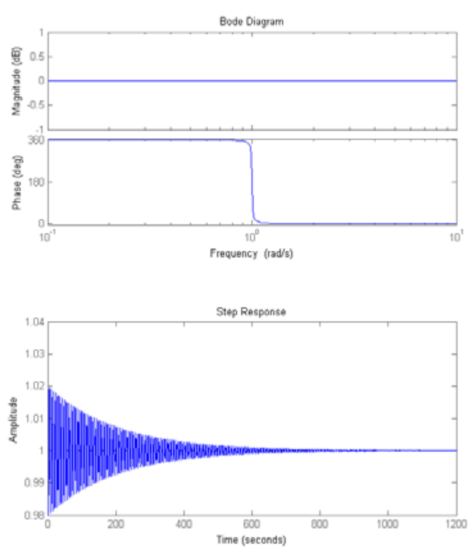
Problem 3D:

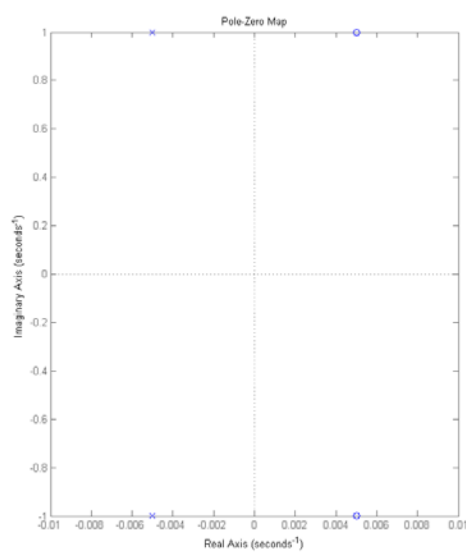




This is a second order system, again, because s^2 is present. There are two poles, one at (-0.05,-1) and the other at (-0.05, 1). There is one zero at (0,0). The bode plot is again a band pass filter, but one that lets only a very small bit of frequency through. The spike that occurs above 0 on the bode plot is due to the resonance. The step response has oscillations due to the imaginary part of the complex poles. The negative real part of the poles causes the step response to exhibit decay.

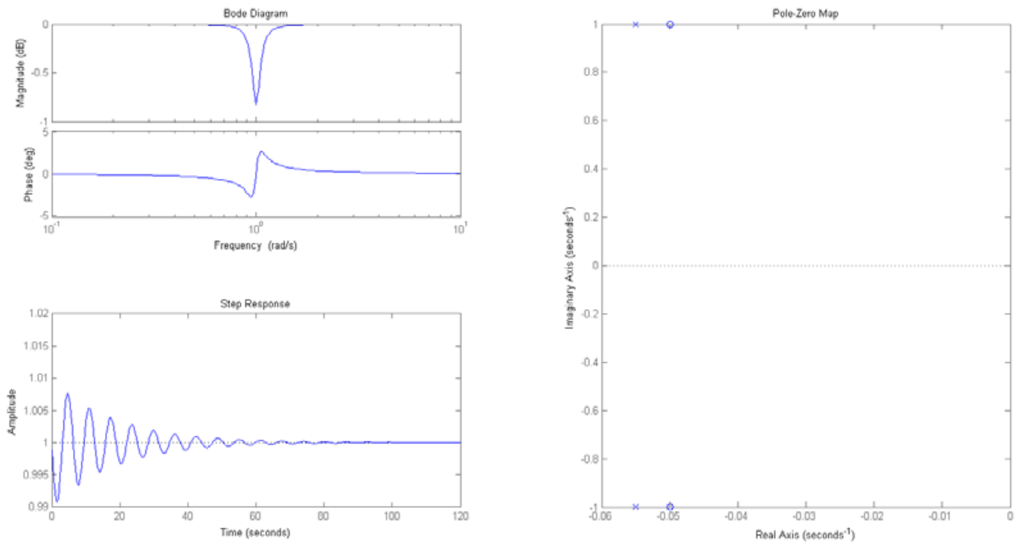
Problem 3E:





This is a second order system. There are two poles: one with a negative real part and a positive imaginary part and the other with a negative real part and a negative imaginary part. There are two zeros reflected over the y-axis. They both have positive, real parts and one has a positive imaginary part while the other has a negative imaginary part. In In the bode plot the two poles and two zeros cancel each other out, because they occur at the same time. This causes a flat bode plot. The power of each frequency is the same, but the phases are being shifted differently, so the frequencies cancel each other out and exponentially decay the step response to zero.

Problem 3F:

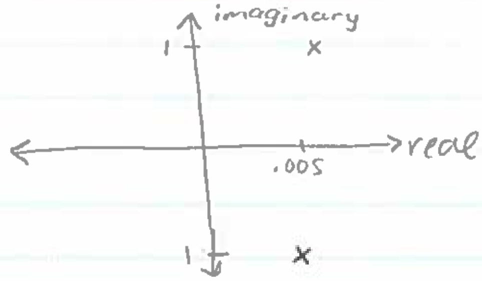


This is a second order system. There are two poles and two zeros. All poles have negative real parts and one of each is above imaginary 0 (positive imaginary) while the other of each is below imaginary 0 (negative imaginary). The zeros are exactly the same, but shifted to the right by ~0.005. The bode plot is a band pass. The oscillations in the step response are due to the imaginary part of the complex poles.

④ a. $H(s) = \dfrac{-1}{s^2 - 0.01s + 1}$

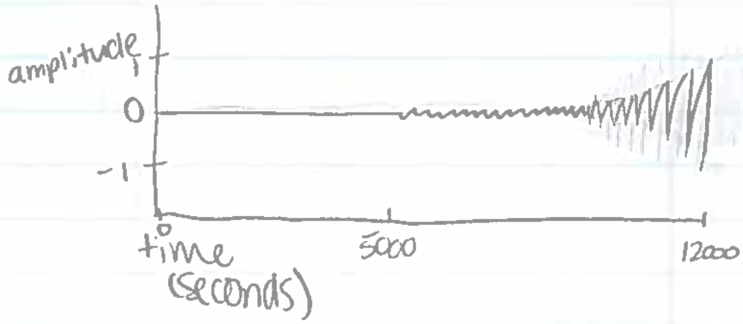
no zeros

pole: $s^2 - 0.01s + 1 = 0$



$$S = \frac{+0.01 \pm \sqrt{.01^2 - 4(1)(1)}}{2(1)}$$

$$S = .005 \pm 0.99987i$$



b. Using proportional control $K = K_p$. Going back to the answer in 2 A, we can use the expression $\frac{KH}{1+KH}$ where $K = K_p$ and $H = H(s)$. Using this information and the pole zero map above, it can be determined that no, this system cannot be stabilized, even with any positive value of K_p big or small because there will always be some negative real part that causes a pole on the left side of the imaginary y-axis.

c. With integral control, $K = \frac{K_I}{s}$. After solving the cubic equation with the variable still in it, it is seen that no matter what, one root will still have a positive part which means there are no ways to stabilize the system.

d. With differential control, $K = sK_D$. Because s is being multiplied, this system can be stabilized.