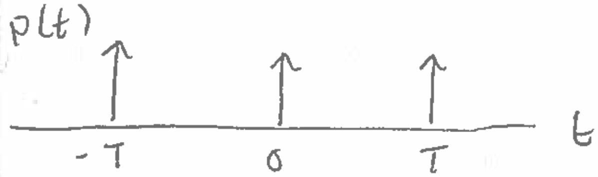
Meg McCauley

PS07 March 12

① a.



b. series $= \sum_{-\infty}^{\infty} C_k e^{j\frac{2\pi}{T}kt}$ where $\omega_0 = \frac{2\pi}{T}$

$C_k = \frac{1}{T}\int_{-T/2}^{T/2} p(t)\, e^{-j\omega_0 k t}\, dt$

$C_k = \frac{1}{T}\int_{-T/2}^{T/2} \delta(t)\, dt = \frac{1}{T}$

$\boxed{\text{series} = \sum_{-\infty}^{\infty} \frac{1}{T} e^{j\omega_0 k t}}$

$e^{j\omega_0 k t} = 2\pi\delta(\omega - k\omega_0)$

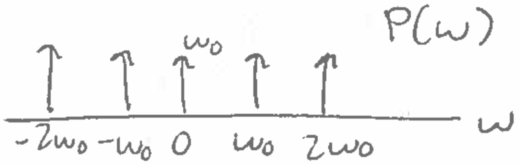
c. $X(\omega) = \sum_{k=-\infty}^{\infty} C_k\, 2\pi\, \delta(\omega - \omega_0)$

$\boxed{X(\omega) = 2\pi \sum_{k=-\infty}^{\infty} C_k\, \delta(\omega - k\omega_0)}$

d. $P(\omega) = \sum_{k=-\infty}^{\infty} \left(\frac{1}{T}\right) 2\pi\, \delta(\omega - \omega_0) = \frac{2\pi}{T} \sum_{k=-\infty}^{\infty} \delta(\omega - k\omega_0)$

$= \boxed{\omega_0\, p(\omega)}$

e.



② a. TA (total area) $= \sum_{k=-\infty}^{\infty} \frac{\omega_0}{2\pi} Q(k\omega_0)$

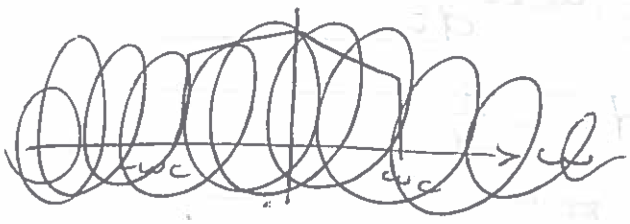
$h(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega) e^{j\omega t}\, d\omega = \frac{1}{2\pi}\left(\int_{-\infty}^{-\omega_c} 0 e^{j\omega t} d\omega \cdot \int_{\omega_c}^{\infty} 0 e^{j\omega t} d\omega \right.$

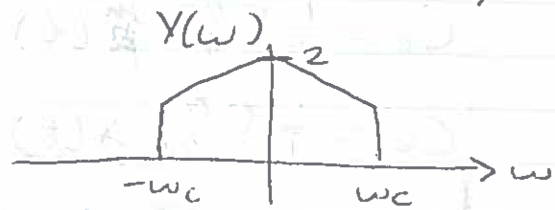
$\left. + \int_{-\omega_c}^{\omega_c} 1 e^{j\omega t} d\omega \right)$

$= \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} e^{j\omega t}\, d\omega = \frac{1}{2\pi} + \frac{1}{jt} e^{j\omega_c t} - \frac{1}{jt} e^{-j\omega_c t}$

$= \frac{1}{\pi t}\left(\frac{1}{2j} e^{j\omega_c t} - \frac{1}{2j} e^{-j\omega_c t}\right) = \frac{1}{\pi t}\sin(\omega_c t) = \boxed{\frac{\sin(\omega_c t)}{\pi t}}$

(2) b. To get $Y(\omega)$ you multiply $X(\omega)$ with the LTI $H(\omega)$. $H(\omega)$ has an amplitude of 0 on the left of $-\omega_c$ and the right of ω_c, so these areas are cut off in $Y(\omega)$. $H(\omega)$ between $-\omega_c$ and ω_c has an amplitude of 1 so $X(\omega)$ stays constant at those points.

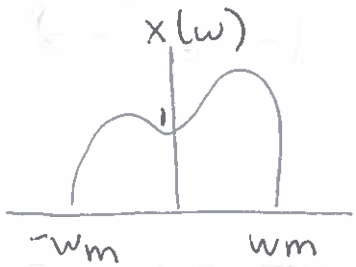




c. As explained above, the only remants of the signal exist between $-\omega_c$ and ω_c. Because $H(\omega)$ is zero below $-\omega_c$ and above ω_c, it destroys these areas, equivalently creating a low pass filter with frequency of ω_c.

d. See graphs at end

(3) a.



$y(t) = x(t)\cos(\omega_c t)$

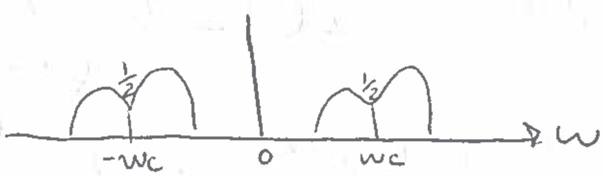
$\cos(x) = \frac{1}{2}(e^{jx} + e^{-jx})$

where $x = \omega_c t$

$y(t) = \frac{1}{2}(e^{j\omega_c t} + e^{-j\omega_c t})\,x(t)$

$e^{j\omega_0 t}x(t) \longrightarrow X(\omega-\omega_0)$

$Y(\omega) = \frac{1}{2}[X(\omega-\omega_0) + X(\omega+\omega_0)]$



Halves the amplitude and duplicates the frequencies in both the negative and positive sides

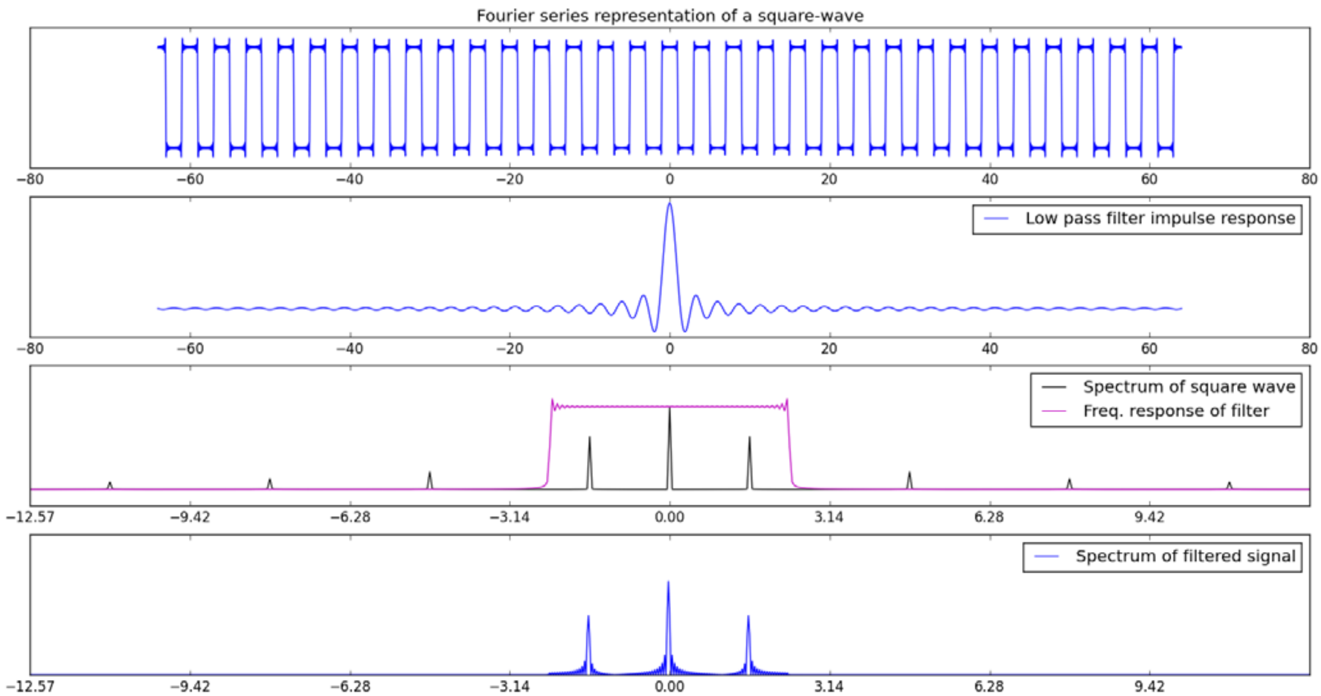


Figure 1: $W_c = 0.75\pi$

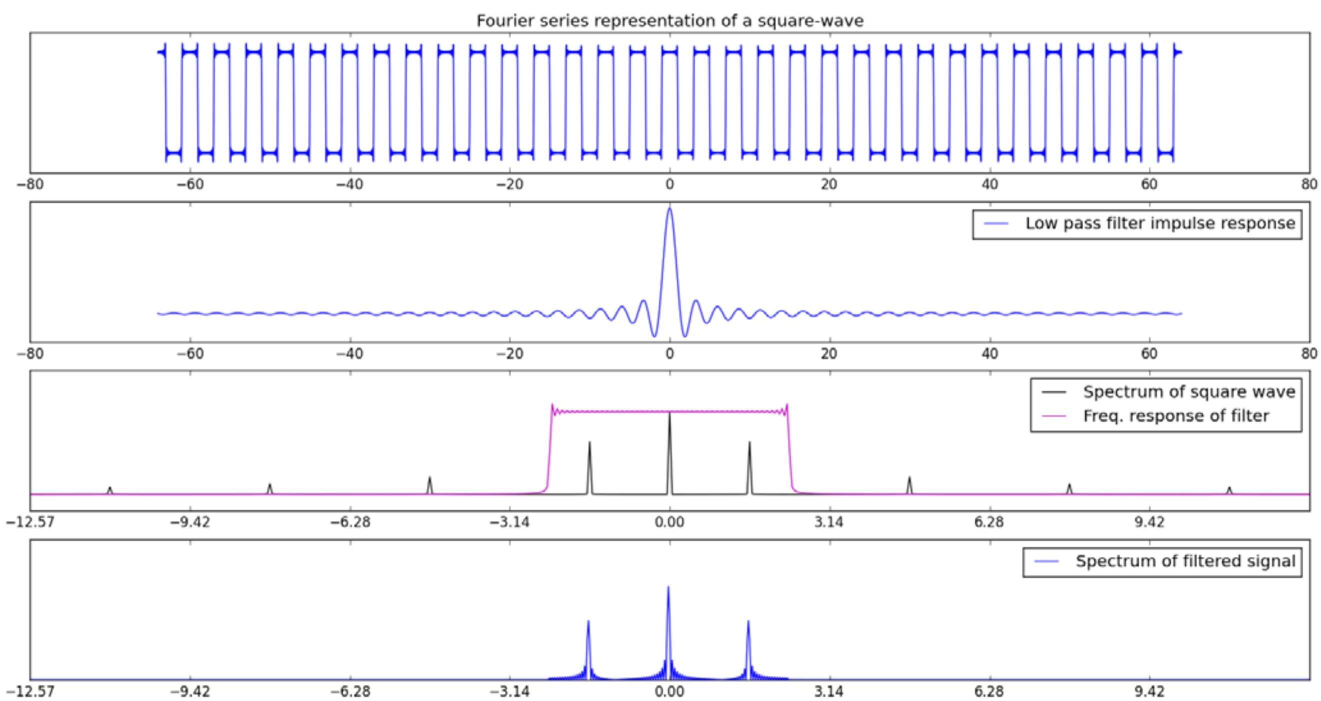


Figure 2: $W_c = 1.75\pi$