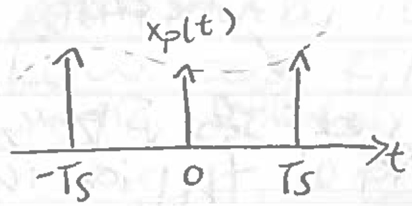
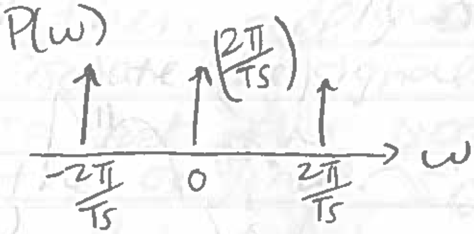
Meg McCauley March 26

# PS08

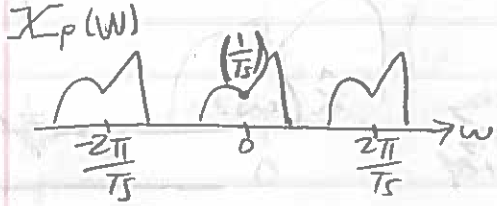
① a.




b.



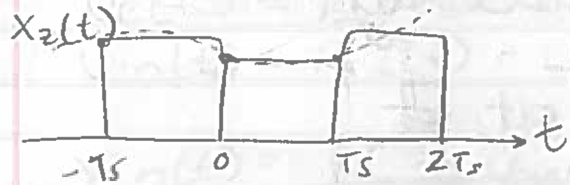

c.



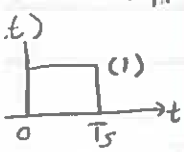

d. $\omega_m = \frac{2\pi}{T_s}$

e. use an ideal low pass filter with a cutoff frequency of $\omega_m$ to get $X(\omega)$ which is $x(t)$ because of the one-to-one Fourier transform property

g.




h.




$$X_z(\omega) = \int_{-\infty}^{\infty} z(t)\, e^{-j\omega t}\, dt$$
$$= \cancel{\int_{-\infty}^{0} z(t) e^{-j\omega t} dt}^{\,0} + \int_0^{T_s} z(t) e^{-j\omega t} dt + \cancel{\int_{T_s}^{\infty} z(t) e^{-j\omega t} dt}^{\,0}$$
$$= \int_0^{T_s} z(t)\, e^{-j\omega t}\, dt$$
$$= \frac{1}{-j\omega}\left[e^{-j\omega t}\right]\Big|_0^{T_s} = \frac{1}{-j\omega}\left[e^{-j\omega T_s} - \cancel{e^{-j\omega \cdot 0}}^{\,1}\right] \cdot \frac{2}{2}$$
$$= \frac{2}{2j\omega}\left[1 - e^{-j\omega T_s}\right] \cdot e^{j\frac{\omega}{2}T_s} \cdot e^{-j\frac{\omega}{2}T_s}$$
$$= \frac{-2}{\omega} \cdot \frac{1}{2j}\left[e^{j\frac{\omega}{2}T_s} - e^{-j\frac{\omega}{2}T_s}\right] \cdot e^{-j\frac{\omega}{2}T_s}$$
$$= \frac{T_s \cdot 2 \cdot 2}{-2\omega T_s} \sin\left(\frac{\omega T_s}{2}\right) e^{-j\frac{\omega}{2}T_s}$$
$$= 4T_s\, \text{sinc}\left(\frac{\omega T_s}{2}\right) e^{-j\frac{\omega}{2}T_s}$$

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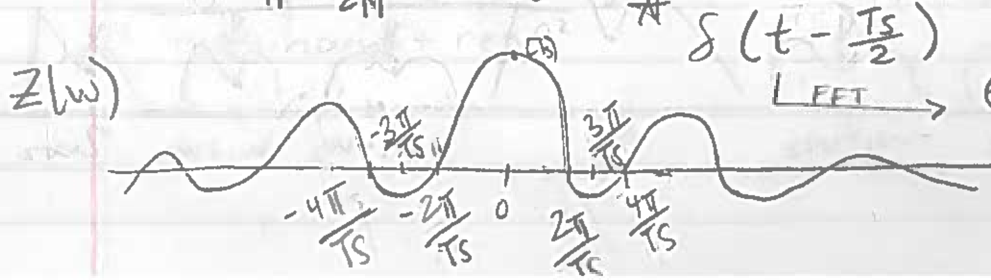
$$\frac{\sin\left(\cancel{2}\pi \frac{T_s}{\cancel{2}} \frac{\omega}{2\pi}\right)}{\pi \cdot \frac{\omega}{2\pi}} \cdot \frac{T_s}{T_s} = T_s \cdot \text{sinc}\left(T_s \cdot \cancel{\pi} \cdot \frac{\omega}{2\pi}\right)$$

$\downarrow \times \longrightarrow \boxed{T_s \cdot \text{sinc}\left(T_s \cdot \frac{\omega}{2\pi}\right) \times e^{-j\omega\frac{T_s}{2}}}$

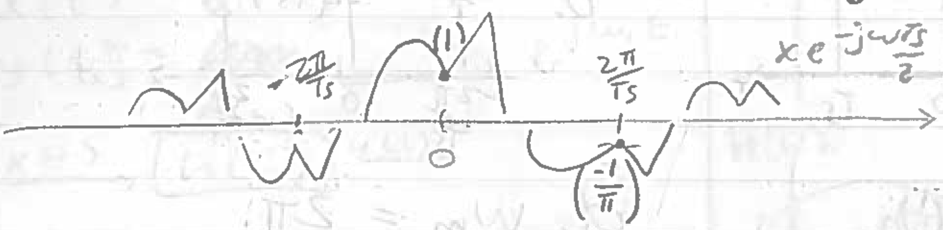
$* \;\; \delta\left(t - \frac{T_s}{2}\right) \xrightarrow{\text{FFT}} e^{-j\omega\frac{T_s}{2}}$




times $e^{-j\omega\frac{T_s}{2}}$

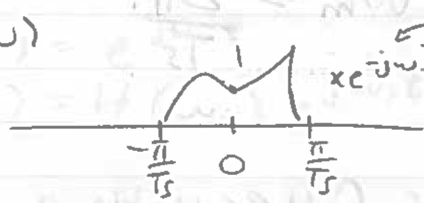
cont.

$X_z(\omega)$



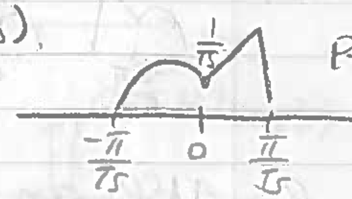

time delay
$xe^{-j\omega\frac{T_s}{2}}$ Slightly distorted on the sides

$\bar{x}(\omega)$




← time delay
$xe^{-j\omega\frac{T_s}{2}}$ slightly distorted

$\hat{x}(\omega)$




perfect

$\bar{x}(\omega)$ is slightly distorted, at a height of 1, and has a time delay of $e^{-j\omega\frac{T_s}{2}}$
$\hat{x}(\omega)$ is a perfect copy, at a height of $\frac{1}{T_s}$, and has no time delay

They're both zero at $\frac{\pi}{T_s}$

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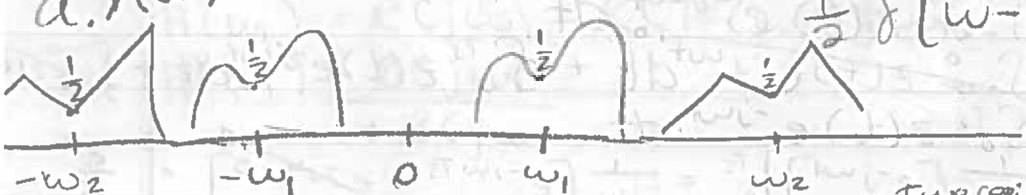
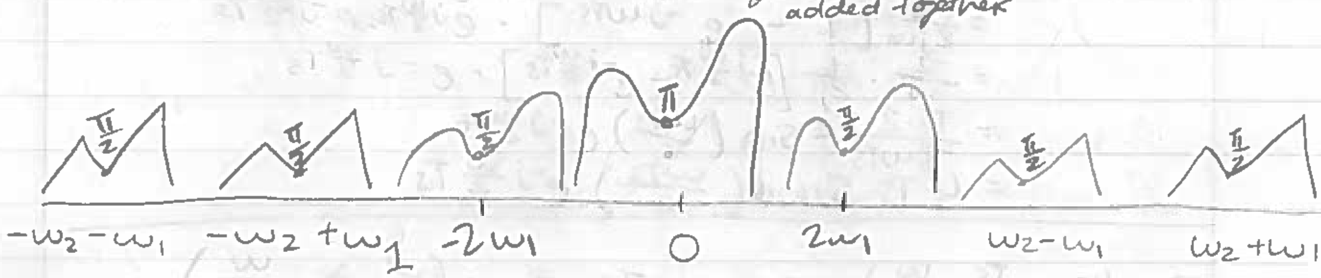
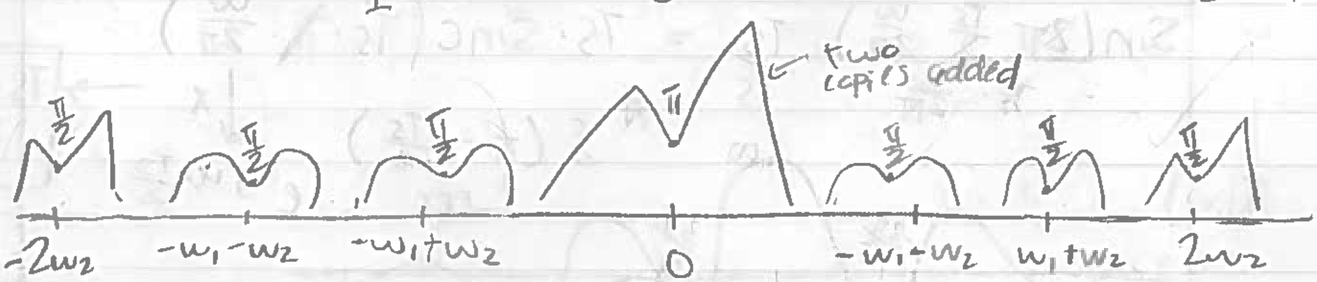
a. $Y(\omega)$

FFT of $\cos(\omega t) = \frac{1}{2}\delta(\omega-\omega_1) + \frac{1}{2}\delta(\omega+\omega_1)$

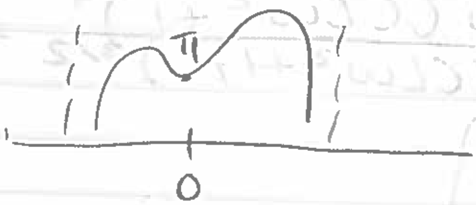


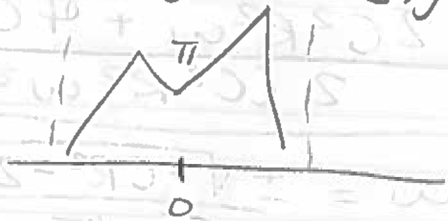

b.



c. Multiply by cos and then apply an ideal Low pass filter to isolate the signal and divide by $\pi$ to get the scaling right to produce the original signal

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a. $V_{in}(t) = V_R(t) + V_L(t) + V_{out}(t)$ ← b/c KVL

$V_L(t) = L \frac{d}{dt} i(t)$

$V_R(t) = R\, i(t)$

$V_{in}(t) = R\, i(t) + L \frac{d}{dt} i(t) + V_{out}(t)$

$V_{in}(t) = RC \frac{d}{dt} V_{out}(t) + CL \frac{d^2}{dt^2} V_{out}(t) + V_{out}(t)$

$\mathbf{V}_{in}(\omega) = RC\, j\omega \mathbf{V}_{out}(\omega) + CL(j\omega)^2 \mathbf{V}_{out}(\omega) + \mathbf{V}_{out}(\omega)$

$\mathbf{V}_{in}(\omega) = \mathbf{V}_{out}(\omega)\left[RC\, j\omega - CL\omega^2 + 1\right]$

$$\boxed{\frac{\mathbf{V}_{out}(\omega)}{\mathbf{V}_{in}(\omega)} = \frac{1}{RCj\omega - CL\omega^2 + 1}} \leftarrow H(\omega)$$

$$\frac{1}{\sqrt{(RC\omega)^2 + (CL\omega^2 + 1)^2}} = |H(\omega)|$$

↑ magnitude of $H(\omega)$

$$\frac{1}{\sqrt{\text{imaginary}^2 + \text{real}^2}}$$

d. $$\frac{d}{d\omega}\left[\frac{1}{\sqrt{(RC\omega)^2+(CL\omega^2+1)^2}}\right]=0$$

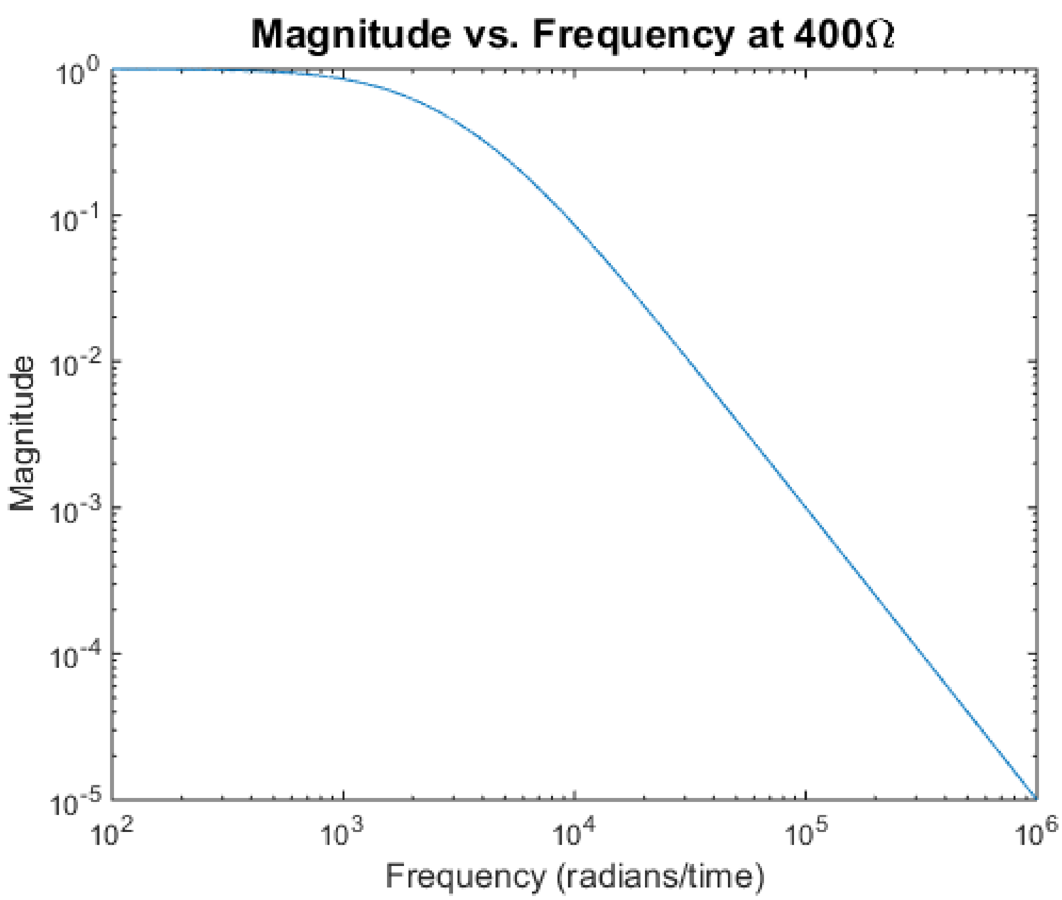
$$\frac{-2C^2R^2\omega+4CL\omega(CL\omega^2+1)}{2(C^2R^2\omega^2+(CL\omega^2+1)^2)^{3/2}}=0$$

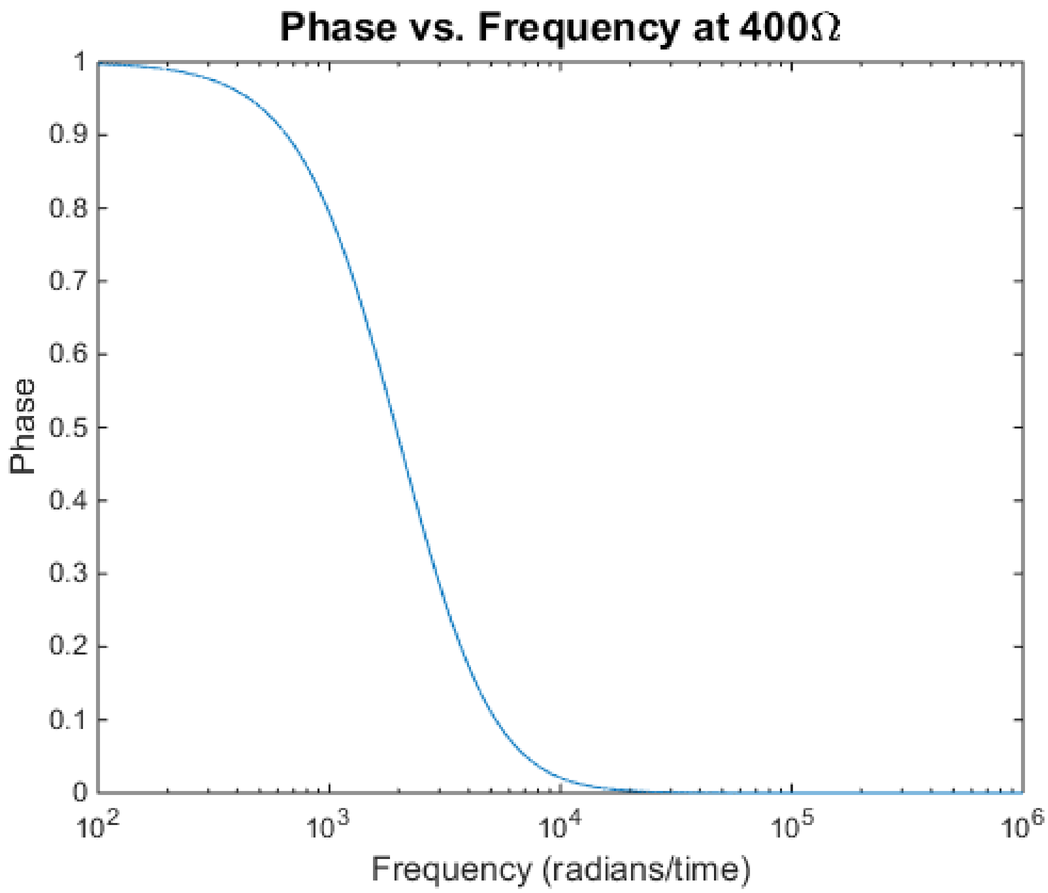
$$\boxed{\omega=\frac{\pm\sqrt{-CR^2-2L}}{\sqrt{2}\sqrt{C}\;L}}$$

e. See MATLAB code + graphs at end of assignment
(colloboration credit to Jacob Riedel)

```
R = 400;
L = 10e-2;
C = 10e-7;

w = logspace (2,6,10000);

mag = 1./(sqrt((R.*C.*w).^2+(w.^2.*C.*L+1).^2));
phase = mag.*exp(-1i.*atan(w.*R.*C));

loglog (w,mag)
title('Magnitude vs. Frequency at 400\Omega','FontSize',14)
xlabel('Frequency (radians/time)','FontSize',12)
ylabel('Magnitude','FontSize',12)

figure;
semilogx(w,phase)
title('Phase vs. Frequency at 400\Omega','FontSize',14)
xlabel('Frequency (radians/time)','FontSize',12)
ylabel('Phase','FontSize',12)
```

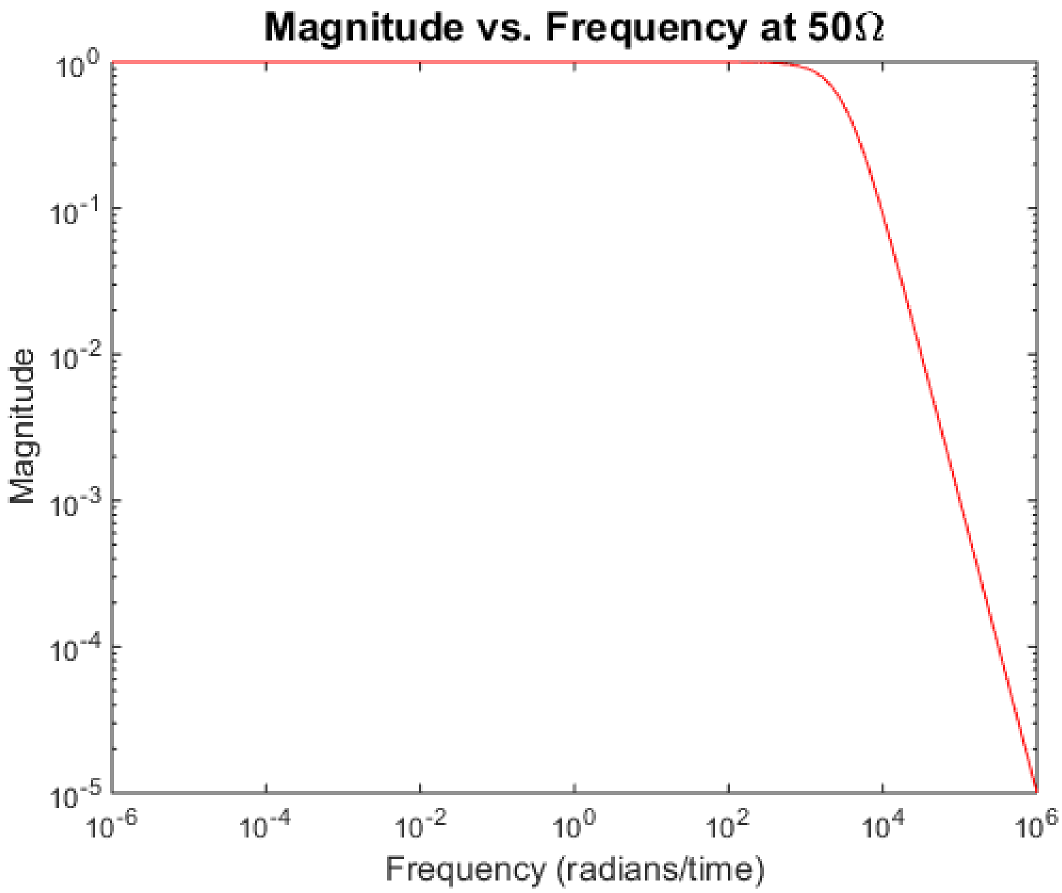
```
Warning: Imaginary parts of complex X and/or Y arguments ignored
```

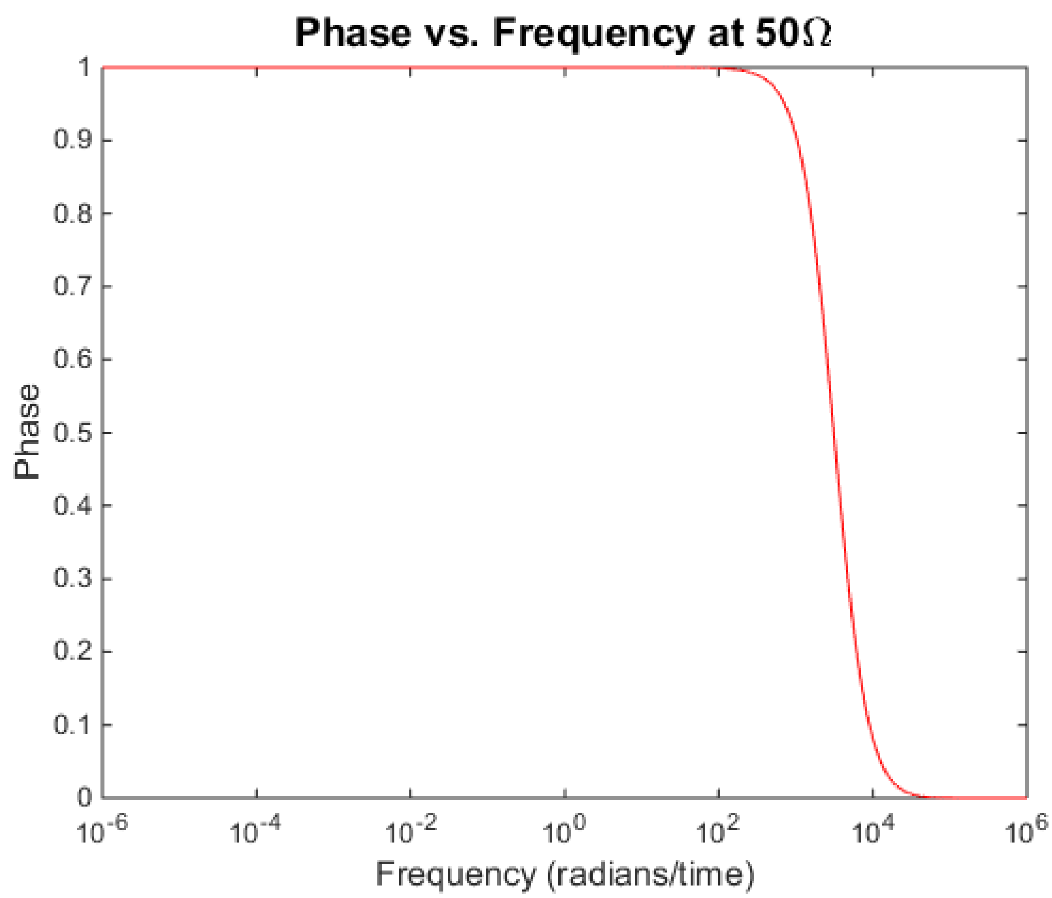





```
R = 50;
L = 10e-2;
C = 10e-7;

w = logspace (-6,6,10000);
mag = 1./(sqrt((R.*C.*w).^2+(w.^2.*C.*L+1).^2));
phase = mag.*exp(-1i.*atan(w.*R.*C));


loglog(w,mag,'r')
title('Magnitude vs. Frequency at 50\Omega','FontSize',14)
xlabel('Frequency (radians/time)','FontSize',12)
ylabel('Magnitude','FontSize',12)
figure;
semilogx(w,phase,'r')
title('Phase vs. Frequency at 50\Omega','FontSize',14)
xlabel('Frequency (radians/time)','FontSize',12)
ylabel('Phase','FontSize',12)
```

```
Warning: Imaginary parts of complex X and/or Y arguments ignored
```


Magnitude vs. Frequency at 50Ω
Magnitude
10^0
10^-1
10^-2
10^-3
10^-4
10^-5
10^-6
10^-4
10^-2
10^0
10^2
10^4
10^6
Frequency (radians/time)


Phase vs. Frequency at 50Ω
Phase
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
10^-6
10^-4
10^-2
10^0
10^2
10^4
10^6
Frequency (radians/time)