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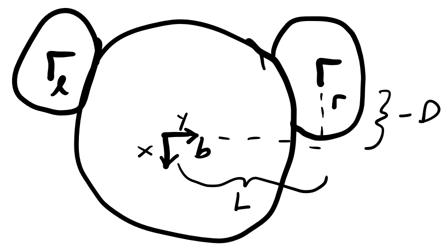
Inverse Kinematics



$$T_{bl} = (0, -D, -L)$$

$$T_{br} = (0, -D, L)$$

$$A_{bl} = \begin{bmatrix} 1 & 0 & 0 \\ -L & 1 & 0 \\ D & 0 & 1 \end{bmatrix} \qquad A_{br} = \begin{bmatrix} 1 & 0 & 0 \\ L & 1 & 0 \\ -D & 0 & 1 \end{bmatrix}$$

$$A_{lb} = \begin{bmatrix} 1 & 0 & 0 \\ L & 1 & 0 \\ -D & 0 & 1 \end{bmatrix} \qquad A_{rb} = \begin{bmatrix} 1 & 0 & 0 \\ -L & 1 & 0 \\ D & 0 & 1 \end{bmatrix}$$

$$V_\ell = A_{\ell b} V_b$$

$$\begin{bmatrix} \dot{\theta} \\ V_{x_\ell} \\ V_{y_\ell} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ L & 1 & 0 \\ -D & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ V_x \\ V_y \end{bmatrix}$$

$$= \begin{bmatrix} \dot{\theta} \\ L\dot{\theta} + V_x \\ -D\dot{\theta} + V_y \end{bmatrix}$$

$$V_r = A_{rb} V_b$$

$$\begin{bmatrix} \dot{\theta} \\ V_{x_r} \\ V_{y_r} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -L & 1 & 0 \\ D & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ V_x \\ V_y \end{bmatrix}$$

$$= \begin{bmatrix} \dot{\theta} \\ -L\dot{\theta} + V_x \\ D\dot{\theta} + V_y \end{bmatrix}$$

$$V_{x_l} = r\dot{\phi}_l \qquad V_{x_r} = r\dot{\phi}_r$$

$$V_{y_l} = 0 \qquad V_{y_l} = 0$$

$$\begin{bmatrix} \dot{\theta} \\ r\dot{\phi}_l \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ L & 1 & 0 \\ -D & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ v_x \\ v_y \end{bmatrix}$$

$$r\dot{\phi}_l = \begin{bmatrix} L & 1 & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ v_x \\ v_y \end{bmatrix}$$

$$0 = \begin{bmatrix} -D & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ v_x \\ v_y \end{bmatrix}$$

$$\begin{bmatrix} \dot{\theta} \\ r\dot{\phi}_r \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -L & 1 & 0 \\ D & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ V_x \\ V_y \end{bmatrix}$$

$$r\dot{\phi}_r = \begin{bmatrix} -L & 1 & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ V_x \\ V_y \end{bmatrix}$$

$$0 = \begin{bmatrix} D & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ V_x \\ V_y \end{bmatrix}$$

$$\begin{bmatrix} \dot{\phi}_l \\ \dot{\phi}_r \end{bmatrix} = \underbrace{\frac{1}{r} \begin{bmatrix} L & 1 & 0 \\ -L & 1 & 0 \end{bmatrix}}_{H} \begin{bmatrix} \dot{\theta} \\ V_x \\ V_y \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \underbrace{\begin{bmatrix} -D & 0 & 1 \\ D & 0 & 1 \end{bmatrix}}_{Q} \begin{bmatrix} \dot{\theta} \\ V_x \\ V_y \end{bmatrix}$$

$$\dot{\phi}_l = \frac{\dot{\theta} L}{r} + \frac{V_x}{r} \quad \ldots\ldots \quad (1)$$

$$\dot{\phi}_r = -\frac{\dot{\theta} L}{r} + \frac{V_x}{r} \quad \ldots\ldots \quad (2)$$

→ note that in my code the right and left are flipped so that "right" is the right wheel in the forward driving direction

Forward Kinematics

$$V_b = H^+ u$$

$$H^+ = H^T (H H^T)^{-1}$$

$$H^+ = \begin{bmatrix} L/r & -L/r \\ 1/r & 1/r \\ 0 & 0 \end{bmatrix} \left(\begin{bmatrix} L/r & 1/r & 0 \\ -L/r & 1/r & 0 \end{bmatrix} \begin{bmatrix} L/r & -L/r \\ 1/r & 1/r \\ 0 & 0 \end{bmatrix} \right)^{-1}$$

$$H^+ = \begin{bmatrix} r/2L & -r/2L \\ 1/2 & 1/2 \\ 0 & 0 \end{bmatrix}$$

$$V_b = \begin{bmatrix} r/2L & -r/2L \\ 1/2 & 1/2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{\phi}_l \\ \dot{\phi}_r \end{bmatrix}$$

$$\dot{\theta} = \left(\frac{r}{2L}\right)\dot{\phi}_l - \left(\frac{r}{2L}\right)\dot{\phi}_r \quad \ldots \; (3)$$

$$v_x = \frac{1}{2}\dot{\phi}_l + \frac{1}{2}\dot{\phi}_r \quad \ldots \; (4)$$

→ note that in my code the right and left are flipped so that "right" is the right wheel in the forward driving direction