02-08-25

(Lecture-1)

Probility:

terms:

Experiment	Random variable
Event	contineous variable
sample point	Discreate variable.
Sample space	

Random Experiment:

A random experiment is an experiment that can be repeated any number of times. under some identical conditions.

Example:- Tossing a fair coin or through a die and observe that the top show

The numbers of road accident per day in a dhaka city.

Event: Any possible outcome or a set of possible outcome of random experiment is called an event. Generally, events are denoted by capital letters A, B, C,....

Example: If the sample space of drown an unbiasied die is {1, 2, 3, 4, 5, 6} and the set of odd number is denoted by $A = \{1, 3, 5\}$. Then A is an event of the obtained odd number is sample. sample space.

Complementary events :-

The complements of an event implies the non-occurrence of the event. Therefore the complement of an event E contains those points of the sample space which are not in E. The complement of event E is denoted by $\bar{E}$. Both B and $\bar{E}$ are complement of each other.

$$P(E) + P(\bar{E}) = 1$$

or $$P(\bar{E}) = 1 - P(E)$$

Different approch of probability

- Clasical or prior approch
- Relative or emperical approch
- Axiomatic approch.

Clasoical or prior approch

The probability of an event

$$P = \frac{\text{Number of favorable outcome}}{\text{Total number of possible outcome}}$$

Considers that in an experiment the event A contain n (A) of these (that is favourable) outcome, then the probality of A is given by $P(A) = \frac{n(A)}{n(S)}$ where n (S) is the total number of outcome.

Example :- If we want to know the probility of getting a king in a draw from a pack of 52 cards then the total number of cases (or outcome)

$n(S) = 52$

Total number of ~~kai~~ king $n(A) = 4$

where A is the event of getting a king

$$P(A) = \frac{n(A)}{n(S)} = \frac{4}{52} = \frac{1}{13}$$

Question:-

A fair coin is tossed two times. Contrast the sample space of the experiment. What is the probability getting

a) All head

b) At least one head

c) At best one head

d) A head and a tail

Solution:- A fair coin is tossed two times. The sample space of the experiment

$S = \{HH, HT, TH, TT\}$

The number of sample point is $n(S) = 4$

a) Let the event $A: \{HH\}$

$\therefore n(A) = 1$

So, the required probality is

$$P(A) = \frac{n(A)}{n(S)} = \frac{1}{4}$$

b/ Let. the event B : At lest one head

$B : \{HH, HT, TH\}$

$n(B) = 3$

$\therefore P(B) = \frac{n(B)}{n(S)} = \frac{3}{4}$

c/ Let the event c : At best one head

$c : \{HT, TH, TT\}$

$n(c) = 3$

$P(c) = \frac{n(c)}{n(S)} = \frac{3}{4}$

d/ Let the event d : At head and a tail

$d : \{HT, TH\}$

$n(d) = 2$

$P(d) = \frac{n(d)}{n(S)} = \frac{2}{4} = \frac{1}{2}$

Lecture - 2

03-08-25

Question 2:

An unbaised coin is tossed four times. what is the possiballity of getting:

① At least 3 heads

② At best 1 heads.

Solution:- A unbaised coin is tossed four time. the sample space for the experiment

	HH	HT	TH	TT
HH	HHHH	HHHT	HHTH	HHTT
HT	HTHH	HTHT	HTTH	HTTT
TH	THHH	THHT	THTH	THTT
TT	TTHH	TTHT	TTTH	TTTT

The total number of possible outcome of sample space $n(S) = 16$

1. Let the event A: At lest 3 heads the set of favouriable cases of event A is.

$A = \{HHHH, HHHT, HHTH, HTHH, HTHH, THHH\}$

$n(A) = 5$

The requried ~~po~~ probability is $P(A) = \frac{n(A)}{n(S)} = \frac{5}{16}$

2/ let event B : at best 1 head. The set of favorable cases of event

$B = \{ HTTH, TH\cancel{H}TT, TTHT, TTTT \}$

$\cancel{B}$ $n(B) = 5$

So, The required probability is

$$P(B) = \frac{n(B)}{n(S)} = \frac{5}{16}$$

Question 3: An unbaised coin is tossed 10 times. Find the probability of getting

1. Just 3 heads
2. At lest one head
3. At lest two heads
4. At lest three heads
5. At best one head
6. At best two heads
7. At best three heads.

$^{n}C_{r}$

Solution:-

If an unbaised coin is tossed 10 times then the total numbers of sample point is $2^{10} = 1029 = n(S)$

1. The ~~possible~~ probability of getting just 3 heads

$$P(\text{Just 3 heads}) = \frac{n(A)}{n(S)} = \frac{^{10}C_3}{1024} = \frac{120}{1024} = 0.1172$$

Extra notes:-

Rough

at lest one	$P(1) + P(2) + P(3) + \ldots$ $[1 - P(0)]$
at best one	$P(0) + P(1)$
at best two	$P(0) + P(1) + P(2)$
at best three	$P(0) + P(1) + P(2) + P(3)$
at lest 2	$1 - P(0) - P(1)$

2: The probability of at least one head

$$= 1 - P(0)$$

$$= 1 - \frac{^{10}C_0}{1029} = 1 - \frac{1}{1029} = 0.999$$

3. The probability of at least two heads

$$= 1 - P(0) - P(1)$$

$$= 1 - \frac{{}^{10}C_0}{1024} - \frac{{}^{10}C_1}{1024}$$

$$= 0.989$$

4. The probability of at least three heads

$$= 1 - P(0) - P(1) - P(2)$$

$$= 1 - \frac{{}^{10}C_0}{1024} - \frac{{}^{10}C_1}{1024} - \frac{{}^{10}C_2}{1024}$$

$$= 0.945$$

5. The probability of at best one head

$$= P(0) + P(1)$$

$$= \frac{{}^{10}C_0}{1024} + \frac{{}^{10}C_1}{1024} = 0.011$$

6. The probability of of best ~~two~~ three head

$$= P(0) + (P(1) + P(2) + P(3))$$

$$= \frac{{}^{10}C_0}{1024} + \frac{{}^{10}C_1}{1024} + \frac{{}^{10}C_2}{1024} + \frac{{}^{10}C_3}{1024}$$

$$= 0.171$$

Question:- Two unbaised coin and an unbaised die are tossed once, write down the sample space and find the probability of the following events:-

1. Opposite face of the coin and odd numbers on the die.

2. even number of the die.

Solution:- If two unbaised coin and unbaised die are tossed once then the sample space is given by:-

S	1	2	3	4	5	6
HH	HH1	HH2	HH3	HH4	HH5	HH6
HT	HT1	HT2	HT3	HT4	HT5	HT6
TH	TH1	TH2	TH3	TH4	TH5	TH6
TT	TT1	TT2	TT3	TT4	TT5	TT6

1. The probality of opposite face of two coin and odd numpers on die. The favourible case of event

$A = \{HT1, HT3, HT5, TH1, TH3, TH5\}$

$\therefore n(A) = 6$

So, the reauried probability is

$$P(A) = \frac{n(A)}{n(S)} = \frac{6}{24} = \frac{1}{4}$$

2. The probability of even numbers of the die, the favorable case of event is

$B : \{HH2, HH4, HH6, HT2, HT4, HT6, TH2, TH4, TH6, TT2, TT4, TT6\}$

$\therefore n(B) = 12$

So, the reauried probability is

$$P(B) = \frac{n(B)}{n(S)} = \frac{12}{24} = \frac{1}{2}$$

Some important formulas

Let us consider two events A and B.

1. ~~A a~~ If A and B are mutually exclusive then

$P(A \cup B) = P(A) + P(B)$ [mutually exclusive events : there are no common elements]

or $P(A \cap B) = \emptyset$

2. If A and B are non mutually exclusive then

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

3. A If A and B are independent then

$P(A \cap B) = P(A) \cdot P(B)$

4. If A and B are dependent then

$P(A \cap B) = P(A) \cdot P(B \setminus A)$ [$B \setminus A$ means B is dependent on A]

or $P(A \cap B) = P(B) \cdot P(A \setminus B)$

5. $P(A) + A(\bar{A}) = 1$ [$P(A)$ is the happening and $A(\bar{A})$ is the non-happening event]

6. $P(A \cap \bar{B}) = P(A) - P(A \cap B)$

$P(\bar{A} \cap B) = P(B) - P(A \cap B)$

7. $P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B}) = 1 - P(A \cup B)$

8. $P(\bar{A} \setminus \bar{B}) = \dfrac{P(\bar{A} \cap \bar{B})}{P(\bar{B})}$

8. $P(A\bar{B} \setminus \bar{A}) = \dfrac{P(\bar{A} \cap \bar{B})}{P(\bar{A})}$

9. At least one / any one / A or B / either A or B

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

10. Both A and B $\quad P(A \cap B) = P(A) \cdot P(B)$

11. None of two / neither A nor B

$P(\overline{A \cup B}) = 1 - P(A \cup B)$

12. Only A / Just A / A but not B

$P(A \cup \bar{B}) = P(A) - P(A \cap B)$

13. Only B / Just B / B but not A

$P(\bar{A} \cup B) = P(B) - P(A \cap B)$

14. $P(\bar{A} \cap \bar{B} \cap \bar{C}) = P(\bar{A}) - P(A \cap B) - P(A \cap C) + P(\ldots)$
$+ P(A \cap B \cap C)$

15. $P(\bar{A} \cap \bar{B} \cap C) = P(C) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$

Lecture 3 $\quad$ 4 $\quad$ 09/8/25

Question:- A coin and a die are thrown ~~simulate~~ simultaneously. In event A: head of the coin and event B: even number on the die then.

i.) show that the events A and B are independent

ii/ Find the value of $P(A \cup B)$.

Solution:- If a coin and die are thrown simultaneously then the sample space is given below.

S	1	2	3	4	5	6
H	H1	H2	H3	H4	H5	H6
T	T1	T2	T3	T4	T5	T6

Total number of possible outcome of the sample space is $n(S) = 12$

The set of favorable cases of the event A

A: {H1, H2, H3, H4, H5, H6}

$n(A) = 6$

and the set of favourable cases of the event B

B: {H2, H4, H6, T2, T4, T6}

$n(B) = 6$

$P \cap$

The set of common element of event A and B

$\{H_2, H_4, H_6\}$

$n(A \cap B) = 3$

We know, $P(A \cap B) = P(A) \cdot P(B)$ for independent

$$\Rightarrow \frac{n(A \cap B)}{n(S)} = \frac{n(A)}{n(S)} \cdot \frac{n(B)}{n(S)}$$

$$\Rightarrow \frac{3}{12} = \frac{6}{12} \cdot \frac{6}{12}$$

$$\Rightarrow \frac{1}{4} = \frac{1}{2} \cdot \frac{1}{2}$$

$$\Rightarrow \frac{1}{4} = \frac{1}{4}$$ So (the event) are independent

ii/ $A \cup B = P(A) + P(B) - P(A \cap B)$

$$= \frac{1}{2} + \frac{1}{2} - \frac{1}{4}$$

$$= \frac{3}{4}$$

Question:- Consider two events A and B such as

$P(A) = \frac{1}{8}$, $P(A \setminus B) = \frac{1}{4}$ and $P(B \setminus A) = \frac{1}{6}$.

Examin the following statement and comment on valid reason of each of these

i) A and B are independent

ii) A and B are mutually exclusive

iii) $P(\bar{A} \setminus \bar{B}) = 0.5$

Solution:-

$P(A \cap B) = P(A) \cdot P(B \setminus A)$

$= \frac{1}{8} \cdot \frac{1}{6}$

$= \frac{1}{48}$

Again $\frac{P(A \cap B)}{P(A \setminus B)} = P(B)$

$\Rightarrow P(B) = \frac{\frac{1}{48}}{\frac{1}{4}} = \frac{1}{48} \times 4 = \frac{1}{6}$

$\Rightarrow P(B) = \frac{\frac{1}{48}}{\frac{1}{64}} \Rightarrow \frac{1}{48} \times 4 =$

H.W: Given that $P(A) = \frac{3}{8}$, $P(B) = \frac{5}{8}$ and $P(A \cup B) = \frac{3}{4}$, find $P(A \setminus B)$, $P(B \mid A)$ are A an B independent?

— x — x — x —

Question:

The probability that a ~~he~~ husband will alive 12 more years is 0.25 and the probability that his wife will allive 12 more years is 0.33. find the probability that

i. both will be alive in 12 years.

ii. At least one will be alive in 12 years

iii. Neighter will be alive in 12 years

iv/ only the wife will be alive in 12 years.

v/ only the husband will be alive in 12 years.

Soln:- Let the event

H = The husband is alive in 12 years

W = The wife " " " "

Here $P(H) = 0.25$

$P(W) = 0.33$

i) $P(H \cap W)$ ii) $P(H \cup W)$ iii) $P(\overline{H \cup W})$

iv) $P(\bar{H} \cup W)$ v) $P(H \cup \bar{W})$

Soln:-

i. $A(H \cap B) = P(H) \cdot P(B)$

$= 0.25 \times 0.33$

$= 0.0825$

ii. $P(H \cup W) = P(H) + P(B) - P(H \cap B)$

$= 0.25 + 0.33 - 0.0825$

$= 0.4975$

iii) $P(\overline{H \cup W}) = 1 - P(H \cup W)$

$= 1 - 0.4975$

$= 0.5025$

iv. $P(\bar{H} \cup W) = P(W) - P(\cancel{AAB}\ H' \cap W)$

$= 0.33 - 0.0825$

$= 0.2475$

v. $P(H \cup \bar{W}) = P(H) - P(H \cap W)$

$= 0.25 - 0.0925$

$= 0.1675$

Question:

A problem of statistis is given to three students A, B, and C. whose chances of solving it are $\frac{2}{5}$, $\frac{3}{10}$, $\frac{7}{15}$ respectively. what is the probability that problem will be solved?

Soln:-

let the event A, B, C

A: solved by first student

B: " " second "

C: " " third "

The probability that the problem will be solved is

(0.776) Ans

$P(A \cup B \cup C) = ?$

Question:- In a survey on the shampoo use of women, it is found that ~~25~~ 50% women use sunsilk 45% use clear, 40% meril, 25% sunsilk and clear, 10% clear and meril, 16% maril and sunsilk and 8% all the above.

A women is selected at random find the probability that

i. use at least one.

ii. use non.

iii. use only Sun silk

Soln:- $P(A) = 5\% = 0.5,\quad P(B) = 45\% = 0.45$

$P(C) = 40\% = 0.4,\quad P(A \cap B) = 25\% = 0.25$

$P(B \cap C) = 10\% = 0.1\quad P(C \cap A) = 16\% = 0.16$

$P(A \cap B \cap C) = 8\% = 0.08$

we have to find:

i. $P(A \cup B \cup C) = 0.92$

ii. $P(\overline{A \cup B \cup C}) = 0.08$

iii) $P(A \cap \overline{B} \cap \overline{C} = 0.17$.

Lecture - 4: 10-8-25

Random variable and Probability function

Formula 1. In case of discrete random variable

i. Condition of the probability function: $\sum P(x) = 1$

ii. ~~Discrete~~ Distribution function $F(x) = \sum P(x)$

1. Problem:- If $P(x) = \frac{x+1}{14}$, $x = 1, 2, 3, 4$ then prove $P(x)$ is a probability function and find the value of $P(2 \le x < 4)$

Question 2: The probability distribution of discrete random variable x is given below:

$$P(x) = \frac{2x+k}{56}, \quad x = -3, -2, -1, 0, 1, 2, 3$$

i. Find the value of k

ii. Show that $P(x)$ is a probability function

iii) Find distribution function and show that $P(0 < x \le 2) = F(2) - F(0)$

iv) Find the value of $P(-2 \le x \le 2)$

$\underline{\text{Sol}^n:-1:}$

i) Given that, $P(x) = \frac{x+2}{14}$, $x = 1, 2, 3, 4$

We know the function is probability function if

$\sum P(x) = 1$

Now,

$$\sum_{x=1}^{4} P(x) = P(x=1) + P(x=2) + P(x=3) + P(x=4)$$

$$= \frac{1+1}{14} + \frac{2+1}{14} + \frac{3+1}{14} + \frac{4+1}{14}$$

$$= \frac{2}{14} + \frac{3}{14} + \frac{4}{14} + \frac{5}{14}$$

$$= \frac{2+3+4+5}{14} = \frac{14}{14} = 1$$

So, $P(x)$ is probability function.

ii) $P(2 \leq x < 4) = P(x=2) + P(x+3)$

$$= \frac{1+2}{14} + \frac{1+3}{14}$$

$$= \frac{3}{14} + \frac{4}{14}$$

$$= \frac{7}{14} = \frac{1}{2} = 0.5$$

Soln:- 2

If $P(x)$ be the probability function then

$$\sum P(x) = 1$$

$$\Rightarrow \sum_{x=-3}^{3} P(x) = 1$$

$$\Rightarrow \frac{2(-3)+k}{56} + \frac{2(-2)+k}{56} + \frac{2(-1)+k}{56} + \frac{2(0)+k}{56}$$

$$+ \frac{2(1)+k}{56} + \frac{2(2)+k}{56} + \frac{2(3)+k}{56} = 1$$

$$\Rightarrow \frac{-6+k}{56} + \frac{-4+k}{56} + \frac{-2+k}{56} + \frac{0k}{56} + \frac{2+k}{56} + \frac{4+k}{56} + \frac{6+k}{56} = 1$$

$$\Rightarrow \frac{\cancel{-6}+k \cancel{-4}+k + \cancel{-2}+k + k + \cancel{2}+k + \cancel{4}+k + \cancel{6}+k}{56} = 56$$

$$\Rightarrow 7k = 56$$

$$\Rightarrow k = 8$$

ii/ Now, $P(x) = \frac{2x+8}{56}$, $x = -3, -2, -1, 0, 1, 2, 3$

if $P(x)$ is probability function then

$\sum P(x) = 1$,

Now,

$$\sum_{x=-3}^{3} P(x) = \frac{2(-3)+8}{56} + \frac{2(-2)+8}{56} + \frac{2(-1)+8}{56} + \frac{2(0)+8}{56}$$

$$+ \frac{2(1)+8}{56} + \frac{2(2)+8}{56} + \frac{2(3)+8}{56}$$

$$= \frac{-6+8}{56} + \frac{-4+8}{56} + \frac{-2+8}{56} + \frac{8}{56}$$

$$+ \frac{2+8}{56} + \frac{4+8}{56} + \frac{6+8}{56}$$

$$= \frac{2}{56} + \frac{4}{56} + \frac{6}{56} + \frac{8}{56} + \frac{10}{56} + \frac{12}{56} + \frac{14}{56}$$

$$= \frac{2+4+6+8+10+12+14}{56}$$

$$= \frac{56}{56} = 1$$ so $(P(x))$ is probability function

iii/ We have $P(x) = \frac{x+4}{28}$ | $\frac{2x+8}{56} = \frac{2(x+4)}{56}$

where, $x = -3, -2, -1, 0, 1, 2, 3$. | $P(x) = \frac{x+4}{28}$

The probability distribution and distribution function of x are given below

x	-3	-2	-1	0	1	2	3
$P(x)$	$\frac{1}{28}$	$\frac{2}{28}$	$\frac{3}{28}$	$\frac{4}{28}$	$\frac{5}{28}$	$\frac{6}{28}$	$\frac{7}{28}$
$F(x)$	$\frac{1}{28}$	$\frac{3}{28}$	$\frac{6}{28}$	$\frac{10}{28}$	$\frac{15}{28}$	$\frac{21}{28}$	$\frac{28}{28}$

L.H.S

$P(0 < x \leq 2) = P(x=1) + P(x=2)$

$= \frac{5}{28} + \frac{6}{28} = \frac{11}{28}$

R.H.S.

$F(2) - F(0) = \frac{21}{28} - \frac{10}{28} = \frac{11}{28}$

$\therefore$ L.H.S = R.H.S

iv) We have, $P(x) = \frac{x+4}{28}$

$P(-2 \leq x \leq 2) = P(x=-2) + P(x=-1) + P(x=0) + P(x=1) + P(x=2)$

$= \frac{2}{28} + \frac{3}{28} + \frac{4}{28} + \frac{5}{28} + \frac{6}{28}$

$= \frac{2+3+4+5+6}{28} = \frac{20}{28} = \frac{5}{7} = 0.714$

In case of two discrete random variables

i. In the case of finding the value of the constant ($k/a/b$ etc)

we know $\sum_x \sum_y P(x,y) = 1$ or $\sum_y \sum_x (P(x,y)) = 1$

ii. The marginal probability function of x is ~~P(x)~~

$P(x) = \sum_y P(x,y)$

The marginal probability function of y is

$P(\cancel{y}) = \sum_x P(x,y)$

iii) the ~~Contid~~ Conditional probability of x given y is $P(x|y) = \frac{P(x,y)}{P(y)}$

The Conditional probability of y given x is

$$P(y|x) = \frac{P(x,y)}{P(x)}$$

Question:-

The Joint probability function of two random variables x and y given below:

$$P(x,y) = \frac{x+2y}{16},$$

$x = 0, 1$ and $y = 0, 1, 2, 3$

i) Find the marginal probability function x and y

ii) Find the conditional Probability function of x and y.

Ans:-

i) $x \rightarrow \frac{x+3}{4}$, $y \rightarrow \frac{4y+1}{16}$

ii) $x|y \rightarrow \frac{x+2y}{4y+1}$

13-08-25

lecture-5

For contineous random variable

In case of one ~~cont~~ contineous random variable.

i. In order to find the value of constant (k/a/b etc) or condition of the probability density function, we know,

$$\int_{-\infty}^{\infty} f(x)\, dx = 1$$

ii.) Distribution function $F(x) = \int_{-\infty}^{x} f(x)\, dx$

iii) $P(a<x<b) = \int_{a}^{b} f(x)\, dx$

iv) $P(x \geq a) = \int_{a}^{\infty} f(x)\, dx$

v) $P(x<b) = \int_{-\infty}^{b} f(x)\, dx$

Question 1: The probability density function of a contineous random variable is given by

$$f(x) = k(x+1) \;;\; 0 \leq x \leq 1$$
$$= 0 \;;\; \text{otherwise}$$

i. Find the value of k

ii. find the distribution function and show that

$$F(1) - F(0) = P(x>0)$$

Question 2: show that $f(x) = \frac{1}{30}(3+2x)$; $2 < x < 5$ is a probability density function and find the value of $P(x \geq 4)$

H.W

Question 3: The probability density function of a contineous random variable is given by

$f(x) = ax^2$; $0 < x < 4$

$= 0$ otherwise

Find the value of a ~~no~~ and $P(1 \leq x < 3)$

Question 4: The probability density function of a contineous random variable is given below:

$f(x) = kx^2 + kx + \frac{1}{8}$; $0 < x < 2$

$= 0$; otherwise

i) Find the value of ~~k~~ k

ii) Find the value of $P(1 < x < 2)$

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Solution - 1:

We know the function is probability function if

$$\int_{-\infty}^{\infty} f(x)\, dx = 1$$

Now,

$$\int_0^1 k(x+1)\, dx = 1$$

$$\Rightarrow k \left[\int_0^1 (x+1)\, dx \right] = 1$$

$$\Rightarrow k \left[\frac{x^2}{2} + x \right]_0^1 = 1$$

$$\Rightarrow k \cdot \left[\frac{1}{2} + 1 \right] = 1$$

$$\Rightarrow k \left[\frac{3}{2} \right] = 1$$

$$\Rightarrow k = \frac{2}{3}$$

So, the function is $f(x) = \frac{2}{3}(x+1)$

ii) Distribution function is

$$F(x) = \int_{-\infty}^{x} f(x)\, dx$$

$$= \int_0^x \frac{2}{3}(x+1)\, dx$$

$$= \frac{2}{3} \int_0^x (x+1)\, dx$$

$$\Rightarrow \frac{2}{3}\int_0^x (x+1)\,dx$$

$$\Rightarrow \frac{2}{3}\left[\frac{x^2}{2}+1\right]_0^x$$

$$\Rightarrow \frac{2}{3}\left[\frac{x^2}{2}+x\right]$$

$$\Rightarrow \cancel{\frac{2x}{3}} \quad \frac{x^2}{3}+\frac{2x}{3} \Rightarrow \frac{x^2+2x}{3}$$

$$\therefore f(x) = \frac{x^2+2x}{3}$$

$$L.H.S = F(1) - F(0)$$

$$= \frac{1+2}{3} + \frac{0+2\cdot 0}{3} = \frac{3}{3} + 0 = 1$$

$$R.H.S \Rightarrow P(x>0)$$

$$= \int_0^1 \frac{2}{3}(x+1)\,dx$$

$$= \frac{2}{3}\left[\frac{x^2}{2}+1\right]_0^1 = \frac{2}{3}\left(\frac{1}{2}+1\right)$$

$$= \frac{2}{3}\cdot\frac{3}{2} = 1$$

$$\therefore L.H.S = R.H.S$$

$\int x = 0$
1

Solution 2:-

We know, probability ~~fo~~ function is

$$\int_{-\infty}^{\infty} F(x)\, dx = 1$$

$$\therefore \int_{2}^{5} \frac{1}{30}(3+2x)\, dx$$

$$\Rightarrow \frac{1}{30}\int_{2}^{5} (3+2x)\, dx$$

$$\Rightarrow \frac{1}{30}\left[3x + x^2\right]_{2}^{5}$$

$$\Rightarrow \frac{1}{30}\left[(15+25) - (6+4)\right]$$

$$\Rightarrow \frac{1}{30}\left[40 - 10\right]$$

$$\Rightarrow \frac{1}{30} \times 30 = 1$$

So, it is a probability function.

~~The density function is:~~

$$F(x) = \int_{a}^{x} f(x)\, dx$$

$$\therefore F(x) = \int_{4}^{5} \frac{1}{30}(3+2x)$$

$$= \frac{1}{30}\int_{4}^{5} (3+2x)$$

$$= \frac{1}{30}\left[3x + x^2\right]_{4}^{5}$$

$= \frac{1}{30}\left[3x+x^2\right]_4^5$

$\Rightarrow \frac{1}{30}\left[(15+25)-(12+16)\right]$

$\Rightarrow \frac{1}{30}(40-28)$

$\Rightarrow \frac{1}{30}\times 12$

$\Rightarrow \frac{2}{5}$

i) In case of the joint probability density function $\int_y \int_x f(x,y)\,dx\,dy = 1$

or $\int_x \int_y f(x,y)\,dy\,dx = 1$

ii)

(a) The marginal probability function of x is

$f(x)$ or $g(x) = \int_y f(x,y)\,dy$

(b) ~~T~~The marginal probability function of y is

$f(y)$ or $h(y) = \int_x f(x,y)\,dx$

iii) The conditional probability density function of x given y is

$$f(x|y) = \frac{f(x,y)}{h(y)} \text{ or } f(x|y) = \frac{f(x,y)}{f(y)}$$

The conditional probability density function of y given x is

$$f(y|x) = \frac{f(x,y)}{g(x)} \text{ or } f(y|x) = \frac{f(x,y)}{f(x)}$$

iv) x and y are independent if

$$f(x,y) = f(x) \cdot f(y)$$

Question:

The joint probability density function of the two contineous random variable x, y are given below:

$$f(x,y) = k(8-x-y); \quad 0 \le x \le 2, \; 0 \le y \le 2$$
$$= 0 \; ; \text{ otherwise}$$

i) find the value of k

ii) find the marginal probability density function of x and y.

iii) Find the conditional density function of x, y.

Solution:-

We know the condition for probability density function is $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\, dy\, dx = 1$

Now, $\int_0^2\int_0^2 k\,(8-x-y)\, dy\, dx = 1$

$\Rightarrow k\int_0^{2} \left[8y - xy - \frac{y^2}{2}\right]_0^2 dx = 1$

$\Rightarrow k\int_0^{2} [16 - 2x - 2]\, dx = 1$

$\Rightarrow k\int_0^{1} (14 - 2x)\, dx = 1$

$\Rightarrow k\,[14x - x^2]_0^2 = 1$

$\Rightarrow k\,[28 - 4] = 1$

$\Rightarrow k \cdot 24 = 1$

$\Rightarrow k = \frac{1}{24}$

So the function is

$$f(x,y) = \frac{1}{24}(8-x-y)$$

ii) The marginal probability function of x is

$$g(x) = \int_y f(x,y)\,dy = \int_0^2 \frac{1}{24}(8-x-y)\,dy$$

$$= \frac{1}{24}\left[8y - xy - \frac{y^2}{2}\right]_0^2 = \frac{1}{24}[16-2x-2]$$

$$= \frac{1}{24}[14-2x] = \frac{1}{12}(7-x)$$

The marginal probability function of y is

$$h(y) = \int_x f(x,y)\,dx = \int_0^2 \frac{1}{24}(8-x-y)\,dx$$

$$= \frac{1}{24}\left[8x - \frac{x^2}{2} - yx\right]_0^2$$

$$= \frac{1}{24}[16-2-2y) = \frac{1}{12}(7-\cancel{x}\,y)$$

iii) The conditional density function of x is

$$f(x|y) = \frac{f(x,y)}{h(y)} = \frac{\frac{1}{24}(8-x-y)}{\frac{1}{12}(7-y)} \ldots$$

for y:
~~Sing~~ :- $$f(y|x) = \frac{f(x,y)}{g(x)} = \frac{\frac{1}{24}(8-x-y)}{\frac{1}{12}(7-x)}$$

iv) we know $f(x,y) = f(x)\cdot f(y)$

$$\frac{1}{24}(8-x-y) \neq \frac{1}{12}(7-x)\cdot\frac{1}{12}(7-y)$$

14-08-25 Lecture 6

Binomial Distribution:-

Let, P be the probability of occurence and Q be the probability of non-occurence of a perticular event in a single trial so that $p+q=1$. If the experiment is repeated for n independent trials, then the probability of an event may be expressed as

$$P(x) = \binom{n}{x} p^x q^{n-x}, \quad x = 0, 1, 2 \cdots, n$$

where, $q = 1-p$ and $\binom{n}{x} = {}^nC_x$;

n = Number of trials;

p = probability of success and $0 \le p \le 1$

Special case: If $n=1$; the distribution is known as unit (or point) ~~biono~~ binomial distribution or Bernauli distribution.

Underlining condition of Bionomial Distribution

1. There is a fixed number of trials.
2. The trials are independent
3. There are only two outcomes for each trial such as success & and failure
4. Probability of success remains constant from trial to trial.

5. The numbers of success x ($x = 0, 1, 2 \ldots n$) in ~~an~~ n trials is a discrete random variable

Mean, variance and standard deviation of a binomial distribution:

Mean:
$$\mu' = \sum_{x=0}^{n} x\,P(x)$$

$$= 0 \cdot P(0) + 1 \cdot P(1) + 2 \cdot P(2) + \cdots + n\,P(n)$$

$$= 0 + {}^{n}C_1 p^1 q^{n-1} + 2 \cdot {}^{n}C_2 p^2 q^{n-2} + \cdots + n\,{}^{n}C_n p^n q^{n-n}$$

$$= \frac{n!}{1!\,(n-1)!} p \cdot q^{n-1} + 2\,\frac{n!}{2!\,(n-2)!} p^2 q^{n-2} + \cdots + n\,\frac{n!}{n!\,(n-n)!} p^n$$

$$= \frac{n \cdot (n-1)!}{(n-1)!} p \cdot q^{n-1} + 2\,\frac{n \times (n-1) \times \cancel{(n-2)!}}{2!\,\cancel{(n-2)!}} p^2 q^{n-2} + \cdots + n p^n$$

$$= n p q^{n-1} + \cancel{2}\,\frac{n(n-1)}{\cancel{2}} p^2 q^{n-2} + \cancel{np^n}\; n p^n$$

$$= n p q^{n-1} + n(n-1) p^2 q^{n-2} + \cdots + n p^n$$

$$= np\left[q^{n-1} + (n-1) p q^{n-2} + \cdots + p^{n-1}\right]$$

$$= np\,(q+p)^{n-1} \qquad \left[(q+p)^n = q^n + {}^{n}C_1 p q^{n-1} + {}^{n}C_2 p^2 q^{n-2} + \cdots + p^n\right]$$

$$= np\,(1)^{n-1} \qquad \text{and } q + p = 1$$

$$= np$$

So the mean of a ~~biom~~ biomial distribution is np.

* The variance of a ~~bion.~~ binomial distribution $= npq$
* the standard deviation $= \sqrt{npq}$
* Mean of the binomial distribution is grater than its variance

proff: variance $= npq$

$$= np(1-p)$$

$$= np - np^2$$

$$\text{variance} = \text{mean} - np^2$$

$$\text{or mean} = \text{variance} + np^2$$

$$= \text{variance} + (\text{+ve quantity})$$

So, mean > variance.

Problem:- The mean and variance of a binomial distribution are 4 and 4/3 respectively. find (i) probability function

(ii) $P(x=0)$

(iii) $P(x \geq 1)$ or at least one success.

Soln:- n and p are two parameter of a binomial distribution.

we know,

$$mean = np \quad ; \quad variance = npq$$

∴ According to the question

$$np = 4 \quad \text{--- (i)}$$

$$npq = \frac{4}{3}$$

$$\text{or } 4q = \frac{4}{3} \Rightarrow \text{or } q = \frac{1}{3}$$

$$p = 1 - q = 1 - \frac{1}{3} = \frac{2}{3}$$

putting the value of p in (1) we get

$$n \times \frac{2}{3} = 4 \Rightarrow n = 6$$

we know the probability function of a binomial function x is $p(x) = \binom{n}{x} p^x q^{n-x}$; $x = 0, 1, 2 \ldots n$

$\therefore P(x) = {}^nC_x\, p^x q^{n-x}$

$P(x) = {}^6C_x\, p^x q^{6-x}$

ii) $P(x=0) = {}^6C_0\, p^0 q^{6-0}$

$= 1 \cdot 1 q^6 = q^6 = \left(\frac{1}{3}\right)^6 = 0.00137$

iii) $P(x \geq 1) = 1 - P(x=0)$

$= 1 - 0.00137 = \cancel{0.9984}\ 0.9986$

~~Al~~ Alternatively :-

$P(x \geq 1) = P(6) + P(5) + P(4) + P(3) + P(2) + P(1)$

$= {}^6C_6\, p^6 q^0 + {}^6C_5\, p^5 q^1 + {}^6C_4\, p^4 q^2 + {}^6C_3\, p^3 q^3 + {}^6C_2\, p^2 q^4$

$+ {}^6C_1\, p^1 q^5$

$= \cancel{1p^6 q} = p^6 + 6p^5 q^1 + 15p^4 q^2 + 20p^3 q^3 + 15p^2 q^4$

$+ 6pq^5$

$= \left(\frac{2}{3}\right)^6 + 6 \cdot \left(\frac{2}{3}\right)^5 \cdot \left(\frac{1}{3}\right) + 15 \cdot \left(\frac{2}{3}\right)^4 \cdot \left(\frac{1}{3}\right)^2 + 20 \cdot \left(\frac{2}{3}\right)^3 \cdot \left(\frac{1}{3}\right)^3$

$+ 15\left(\frac{2}{3}\right)^2 \cdot \left(\frac{1}{3}\right)^4 + 6\left(\frac{2}{3}\right) \cdot \left(\frac{1}{3}\right)^5$

$= \frac{496}{729} + \frac{232}{729} = 0.9986$

Question:- The mean of a binomial distribution is 40 and standard deviation is 6. Calculate n, p, q

Solution:-

Let, x ~~is~~ be a binomial variable with parameters n, ~~&~~ p.

we know,

$$\text{mean} = np \qquad S.D = \sqrt{npq}$$

$$\therefore np = 40 \longleftarrow ①$$

$$\sqrt{npq} = 6 \qquad \therefore npq = 36$$

$$\therefore q = \frac{36}{40} = 0.9$$

$$\therefore p = 1 - 0.9 = 0.1$$

$$\therefore n = \frac{40}{0.1} = 400$$

$$\therefore n = 400, \quad p = 0.1, \quad q = 0.9$$

3/ an unbaised coin is tossed 6 times. Find the probabilities of getting

i) exactly 3 heads

(2) At least 5 heads

(3) At best 3 heads

Solution :- Numbers of trials = 6 $\therefore n = 6$

let x be the number of head,

we have, probability of getting head in a single toss

$$p = \frac{1}{2}$$

$\therefore$ the probability function $p(x) = \binom{n}{x} p^x q^{n-x}$

$$= {}^6C_x \left(\tfrac{1}{2}\right)^x \left(\tfrac{1}{2}\right)^{n-x}$$

$\therefore$ i. $p(x=3) = {}^6C_3 \left(\tfrac{1}{2}\right)^3 \cdot \left(\tfrac{1}{2}\right)^3 = 0.3125 = \frac{5}{16}$

(2) At least 5 heads $= P(6) + P(5)$

$$= {}^6C_6 \left(\tfrac{1}{2}\right)^6 \cdot \left(\tfrac{1}{2}\right)^0 + {}^6C_5 \left(\tfrac{1}{2}\right)^5 \cdot \left(\tfrac{1}{2}\right)^1$$

$$= \frac{1}{64} + \frac{3}{32}$$

$$= \frac{7}{64} = 0.1094$$

3) Probability of getting at least 3 heads

$P(x \leq 3) = P(x=0) + P(x=1) + P(x=2) + P(x=3)$

$= {}^6C_0\, p^0 q^{6-0} + {}^6C_1\, p^1 q^5 + {}^6C_2\, p^2 q^4 + {}^6C_3\, p^3 q^3$

$= \frac{21}{32}$

Problem:- The probability of a new born baby will be boy or girl is equal, then among 5 new born babies, what is the probability that

i) at least one child is boy

ii) at least two children are boy.

Soln:- since the probability of girl or boy are equal, so, $p = q = \frac{1}{2}$, $n = 5$

Let x denoted the number of new born boy

$\therefore P(x) = \binom{5}{x} p^x q^{n-x}$

$= {}^5C_x\, p^x q^{5-x} = {}^5C_x (1/2)^x . (1/2)^{5-x}$

i) Now, $P(0) = {}^5C_0 (\frac{1}{2})^0 . (\frac{1}{2})^5 = \frac{1}{32}$

$\therefore$ probabilities of at least one baby is boy $= 1 - \frac{1}{32}$

$= \frac{31}{32}$

Problem:- Probability that Bangladesh win a cricket ~~match~~ test match against pakistan is given to be $\frac{1}{3}$. If Bangladesh and pakistan play three test match use ~~bio~~ binomial distribution to find the probability that

i) Bangladesh will loss all the ~~the~~ three match.

ii) Bangladesh will win ~~at~~ at lest one test match.

Solⁿ:-

Given that, the probability that bangladesh win is $\frac{1}{3}$, $\therefore p = \frac{1}{3}$

$\therefore q = 1 - \frac{1}{3} = \frac{2}{3}$; $n = 3$

$\therefore$ ~~$P(x) =$~~ Let, x denoted the number of match win

$\therefore P(x) = \binom{3}{x} p^x q^{n-x} = {}^3C_x \left(\frac{1}{3}\right)^x \cdot \left(\frac{2}{3}\right)^{3-x}$

i) Bangladesh loss all the match ; so $x = 0$

$\therefore P(x=0) = {}^3C_0 \left(\frac{1}{3}\right)^0 \cdot \left(\frac{2}{3}\right)^{3-0}$

$= \frac{8}{27} =$ ~~0.07047~~ 0.2963

ii) Bangladesh will ~~at~~ at least one match

$\therefore$ ~~P~~ $P(x \geq 1) = 1 - P(0)$

$= 1 - \frac{8}{27} = \frac{19}{27}$ ~~= 0.8~~

$= 0.7037$