Astronomy & Mathematics in Ancient India

This report is submitted as the fulfilment of the Semester Long Project requirements of B.Tech (IT and Mathematical Innovations) at Cluster Innovation Center, University Of Delhi

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9. **Abstract** : Àryabhata is the first known astronomer to have initiated a continuous counting of solar days, designating each day with a number. The Àryabhatiya is a summary of Hindu mathematics up to his time, including astronomy, spherical trigonometry, arithmetic, algebra and plane trigonometry. The project aims to investigate the historical development of mathematics and astronomy through study of the original texts or of ancient structures and instruments. Study of determining the timing of the festivals and agricultural operations, traditional methods for periodic correction of the astronomical models, study of the simplicity and optimality of the algorithms developed in Indian mathematics and Astronomy and demonstrating them are important aspects of the project.	
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Preface and Acknowledgement

For six months of sixth semester 2023, we did a project at Cluster Innovation Centre, Delhi University under the mentorship of **Dr. Sonam Tanwar**. The main theme of the internship was to review and understand astronomy and mathematics in ancient India. This internship project is a part of our 4-year bachelor's program which is conducted at the Cluster Innovation Centre, University of Delhi.

Under the mentorship of Ma'am, we could understand astronomy and mathematics of ancient India, focusing on the life of *Aryabhata* and his major work *The Aryabhatiya*. We also introduced a new correction constant in the formula that we derived earlier. We also simulated the filter using polycarbonate as the filter material.

We are very appreciative of Dr. Sonam Tanwar, the professor who introduced this project to me and for giving us this wonderful opportunity to work on this real-life problem . They gave us very in-time valuable instructions and put us in contact with experts in the field and also taught us the standard rules and techniques to observe while writing a research paper. We are very grateful for getting the appropriate support to pursue this internship opportunity in which we could learn many new dimensions of 'how to do an efficient review work' .

Certificate of Originality

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Table Of Content

Abstract

India has a rich tradition of mathematics and astronomy. The celestial constants like earth's rotation per solar orbit, days per solar orbit, days per lunar orbit provided by Aryabhatta in 'Àryabhatiya' are the most accurate and precise calculations up to the 18th Century. Àryabhata is the first known astronomer to have initiated a continuous counting of solar days, designating each day with a number. ĀRYABHATĪYA The *Āryabhatīya*, the astronomical text written by Āryabhata (born 476), is one of the landmarks of the history of astronomy. The *Āryabhatīya* is divided into four parts and is the summary of Hindu mathematics up to his time, including astronomy, spherical trigonometry, arithmetic, algebra and plane trigonometry. Some of his formulas are correct, others not. The first appearance of the sine of an angle appears in the work of Àryabhata. Obviously the accuracy of the ancient Indian astronomical data is not just coincidence. Obtained precise constants are supportive of the suggestion that the information derives from an accurate ancient source.

In this semester-long project we aim to aims to investigate the historical development of mathematics and astronomy (not astrology) through study of the original texts or of ancient structures and instruments. Relationship between the astronomical phenomena and the Panchanga for determining the timing of the festivals and agricultural operations, traditional methods for periodic correction of the astronomical models, study of the simplicity and the optimality of the algorithms developed in Indian mathematics and astronomy are important aspects of the projectThe project interprets, reviews and explains the two sections of The Aryabhatiya - The Gitika and Ganita or the Mathematics in simpler and easier language. We have given the detailed section wise explanation of the sections for anyone's understanding towards astronomy and its mathematics. Further, we have designed a website to summarise and explain The Aryabhatiya visually for better interpretations and understanding.

Introduction

Aryabhata - The author

The Aryabhatiya is a **composition** of Aryabhata, the composition was named after him. The Aryabhata is a different person from his namesake of the tenth century A.D., hence is named Aryabhata I.

- Aryabhata I: the author of Aryabhatiya
- Aryabhata II: the author of Maha-Siddhanta

The first Indian satellite was named after him, put into orbit on April 19, 1975.

Aryabhata was an Asmaka who lived in Patliputra in Magadha and wrote his Aryabhatiya there

- ○ Commentators **Paramesvara** and **Raghunatha-raja** interpreted that Aryabhata lived in **Kusumapura** and wrote his Aryabhatiya there. As did the Persian scholar Al-Biruni
- ○ Commentator **Bhaskara**, the earliest commentator, identifies Kusumapura with **Patliputra** in ancient **Magadh,** or modern **Patna** in **Bihar**
- ○ He refers to Aryabhatta as **Asmaka**, indicating that Aryabhata was born in Asmaka

The famous **University of Nalanda**, situated in Patna had a special provision for the study of astronomy in the University. The **astronomical observatory** was a special feature of this University. Aryabhata was designated as **Kulapa** (meaning Kulapati or **Head of a University**). It is quite likely that Aryabhata was the Head of the University of Nalanda, which was a flourishing state in the 5th and 6th A.D. when Aryabhata lived.

In the Kali year 3600, Aryabhata was 23 years old, where, Kali year 3600 corresponds to A.D. 499. Aryabhata was born on **March 21, A.D. 476**

How do we interpret the exact birth year?

- ○ Arybhata himself mentioned, **"When sixty times sixty years and three quarter-*yugas* (of the current *yuga*), twenty-three years had passed since my birth"**.
- ○ The majority of the commentators interpreted that Aryabhata was born at the end of the Kali year 3600 which happened to occur on Sunday, March 21, A.D. 499 at mean noon
- ○ According to the commentators, the object of specifying the end of Kali year 3600 was to show that at that time, the precession of the equinoxes amounted to zero

and the mean positions of the planet obtained from the astronomical parameters given in the *Gitika-pada* did not require any correction.

- However, another commentator, **Haridatta** interpreted the above mentioned verse differently: **‘When sixty times sixty years and three quarter-yugas had elapsed(of the current *yuga*), twenty-three years of my age have passed since then**’.

According to Bhaskara I,
Aryabhata’s profession was teaching and Panduranga-svami, Latadeva and Nisanku learnt astronomy from Aryabhata. Latadeva is the most important one as he earned his name as a great scholar and teacher of astronomy.

From the writings of Varahamihira, we learn that Latadeva was at least the author of two works on astronomy; in one, the day was measured from midnight at Lanka. The book also ascribes to him the authorship of two commentaries- Romaka Siddhanta and Paulisa-Siddhanta.

According to Persian scholar Al-Buiruni, Lataveda was the author of Surya-Siddhanta. Reference to Lataveda has also been made by Brahmagupta and his commentator Prthudaka. Prthudaka has quoted a number of verses from some works of lataveda. These verses are in arya metre and their language and style are similar to those of Aryabhatiya.

Aryabhata’s Works

- Aryabhata I wrote at least two **works on astronomy :**
    1. **Aryabhatiya**
    2. **Aryabhata-Siddhanta**
- The former is well known and the latter is known only through reference to it in later works.
- **Varahamihira** has distinguished the two works by the reckoning of the day adopted in them.
- Other differences in the two works of Aryabhata I have been noted by Bhaskara I in the Maha- Bhaskariya.

Commentaries on the Aryabhatiya

- Commentaries in Sanskrit
    - Bhaskara I's Commentary
        - Earliest commentary on the *Aryabhatiya*
        - Written in *Valabhi* in *Saurastra* (modern Kathiawar) in the year 629 A.D.
        - It sets forth a comprehensive exposition of the contents of the *Aryabhatiya*.

    - Prabhakara's Commentary
        - From two passages in Bhaskara I's commentary on the *Aryabhatiya,* it appears that Prabhakara was an earlier commentator of *Aryabhatiya.*
        - Prabhakara's Commentary has not survived the ravages of time, nor has it been mentioned by any later writer.

    - Somesvara's Commentary
        - A manuscript of Somesvara's Commentary on the *Aryabhatiya* exists in the Bombay University Library.
        - The contents of this commentary show that, as acknowledged by the author himself, it is a summary of Bhaskara I's commentary.
        - In the commentary on the *Ganita-pada*, Somesvara has set some new examples besides those taken from the commentary of Bhaskara I.
        - Somesvara commentary does not throw any light on the life and works of its author.

    - Suryadeva Yajva's Commentary
        - Suryadeva's commentary is usually known by the following names: *Aryabhata-prakasa, Bhata-prakasa, Prakasa, Aryabhata-prakasika, Bhata-prakasika* and *Prakasika.*
        - This commentary has been elucidated further by notes and examples given by Yallaya and has been used as a source book by Paramesvara in writing his own commentary on *Aryabhatia.*
        - About Suryadeva:
            - Suryadeva was born on Monday, 3rd *tithi* of the dark half of Magha, Saka 1113 (A.D 1191).
            - He was a Brahmana of Nidhruva *gotra.*
            - He belonged to the Cola country (roughly comprising Tanjore and Trichinopoly districts of Tamil Nadu) and was a resident of the town Gangapura.
            - He is the author of at least five commentaries:

        - An exposition of Govinda-svami's *bhasya* on the *Maha-Bhaskariya* of Bhaskara I (A.D. 629)
        - Commentary on the *Aryabhatiya*
        - Commentary on the *Maha-yatra* of Varahamihira
        - Commentary on the *Laghu-manasa* of Manjula (A.D. 932)
        - Commentary on the *Jataka-paddhati* of Sripati (A.D. 1039)

- Paramesvara's Commentary
    - Paramesvara's Commentary on the was edited by H. Kern and printed in Leiden (Holland) in A.D. 1874. It was reprinted in A.D. 1906 by UdayaNarain Singh along with his Hindi translation of *Aryabhatiya*.
    - In writing his commentary, the author has utilised Suryadeva's commentary, and has quoted from the *Surya-siddhanta,* the *Brahma-sphuta-siddhanta* of Brahmagupta, the *Brhatsamhita* of Varahamihira, the *Sisya-dhi-vrddhida* of Lalla, the *Trisatika* of Sridhara, and the *Lilavati* of Bhaskara II. he has also referred to his *Mahabhaskariya-bhasya-vyakhya Siddhantadipika,* which was written sometime after A.D. 1431.
    - Paramesvara hails from Kerala. He lived in the village Asvattha (modern Alattur). His first composition was his commentary on the *Laghu-bhaskariya* which he wrote in A.D. 1408. His *Drgganita* was written in A.D. 1431 and his *Goladipika* in A.D. 1443.
    - Paramesvara wrote a number of books on astronomy, astrology and allied subjects.

- Yallaya's notes on Suryadeva's Commentary
    - Yallaya has written notes on Suryadeva's commentary dealing with the second, third and fourth *Padas* of the *Aryabhatiya.* His commentary on each verse of the *Aryabhatiya* consists of Suryadeva's commentary followed by Yallaya's notes where necessary.
    - A manuscript of this commentary exists in the Lucknow University Library.
    - Yallaya was the son of Sridhararya and a pupil of Suryacarya.
    - Skandasomesvara was not only the place where this commentary was written, but also the place to which Yallaya actually belonged. It was situated 4º 5' to the east of the Hindu prime meridian.
    - This commentary was written in 1480 A.D., which corresponds to the Kali year 4581.
    - In his commentary, Yallaya has given the following tables:
        - Table of linear measures

8 paramanus	=	*1 trasarenu*
8 trasarenus	=	*1 ratharenu*
8 ratharenus	=	*1 kosa*
8 kosas	=	*1 tilabija*
8 tilabijas	=	*1 sarsapa*
8 sarsapas	=	*1 yava*
8 yavas	=	*1 angula*
12 angulas	=	*1 hasta*
4 hastas	=	*1 danda*
2000 dandas	=	*1 krosa*
4 krosas	=	*1 yojana*

- *Table of grain measures*

4 kudubas	=	*1 prastha*
4 prasthas	=	*1 adha*
4 adhas	=	*1 drona*
5 dronas	=	*1 khari*

- *Table of gold or silver measures*

4 vrihis	=	*1 gunja*
2 gunjas	=	*1 masaka*
2 masakas	=	*1 gumarta*
10 gumartas	=	*1 suvarna*
1.5 suvarnas	=	*1 karsa*
4 karsas	=	*1 pala*

- *Names of 29 notational places*
  (1) eka, (2) dasa, (3) sata, (4) sahasra, (5) ayuta, (6) laksa, (7) prayuta, (8) koti, (9) dasakoti, (10) satakoti, (11) arbuda, (12) nyarbuda, (13) kharva, (14) maha-kharva, (15) padma, (16) maha-padma, (17) sankha, (18) maha-sankha, (19) ksoni, (20) maha-ksoni, (21) ksiti, (22) maha-ksiti, (23) ksobha, (24) maha-ksobha, (25) parardha, (26) sagara, (27) ananta, (28) cintya and (29) bhuri.

- Nilakantha Somayaji's Commentary
  - This commentary bears the name ***Maha-bhasya*** and has been published in the ***Trivandrum Sanskrit Series***, Nos. 101, 112, 185.
  - Nilakantha like Yallaya has commented only upon the *Ganita, Kala-kriya* and *Gola Padas* of the *Aryabhatiya.*

- About the commentator:
    - Nilakantha's father was called Jataveda, the same being the name of his maternal uncle.
    - His younger brother was named Sankara.
    - He was a Brahmana, the follower of the Asvalayana-sutra, and belonged to the Gargya-gotra.
    - His teacher in astronomy was Damodra, son of the commentator Paramesvara (A.D. 1431); and his teached in Vedanta was Ravi.
    - He was a native of the village Kunda, which has been identified with Trkkantiyur in South Malabar, Kerala.
- This commentary was written **after A.D. 1502**.
- Nilakantha was born in December A.D. 1444. Hence at the time of writing the present commentary he was above 60 years of age.
- Nilakantha's commentary on *Aryabhatia* is a valuable work as it incorporates the advances made in astronomy up to his time and contains a good deal of matters of historical interest.
- At one place in the commentary, Magadha and Baudhayana are reported to have stated in their works the amount of precision of the equinoxes for their times. The following hemistich is ascribed to the *Garga-samhita:*
    - If b, k and h be the base, the upright and the hypotenuse of a right angled triangle, then $\mathbf{b^2 + k^2 = h^2}$.

○ Raghunatha-raja's Commentary

- A manuscript of this commentary is available in Lucknow University Library.
- Raghunatha-raja belonged to Karnata (Karnataka or Mysore) and was a king. His mother's name was Lakshmi and his genealogy was as follows:
    - Venkata → Nagaraja → Kondabhupa → Raghunatha-raja
- In his commentary, Raghunatha-raja gives the Rsine of the local latitude as equal to 962' 38".
- At another place in the commentary, he gives the times of rising of the signs for his local place as follows:

Sign	Time of rising in *vinadikas*
Aries	243
Taurus	271
Gemini	311
Cancer	335
Leo	327
Virgo	313

        - Ahobila, the native place of the commentator, was situated approximately in latitude 15º 50’ N and longitude 1º 40’ E of the Hindu prime meridian. It is the same Ahbila as is situated in Kurnool district, Anshra State.
        - It is said that Ahobila was the capital of the demon king Hiranyakasipu, whose son Prahlada was saved from the wrath of his father by god Nrsimha at this very place.
        - This commentary was written in A.D. 1597.
        - The commentary contains a large number of solved examples. 41 of these examples have been taken from the commentary of Bhaskara I, 15 from the commentary of Suryadeva and some from the works of Bhaskara II.

    - Madhava’s (son of Virupaksa) Commentary
        - Madhava, the commentator, was a Brahmana of Atreya Gotra, and belonged to the family (*anvaya)* of Vantula.
        - His commentary on *Aryabhatiya* was his earliest work.
        - His other commentaries were on the *Narada-samhita* and on the *Laghu-jataka* and the *Brhajjataka* of Varahamihira.

    - Bhutivisnu’s Commentary
        - A manuscript of Bhutivisnu’s commentary on the *Aryabhatiya*, entitled *Bhatapradipa,* exists in the Royal Library at Berlin.
        - Bhutivisnu is the author of a commentary on the *Surya-siddhanta* also, of which an incomplete manuscript exists in Lucknow University Library.
        - Bhutivisnu belonged to the lineage of Garga and was the son of Devaraja and the grandson of his own namesake Bhutivisnu.
        - Bhutivisnu’s commentary on *Aryabhatiya* was written earlier than his commentary on *Surya-siddhanta.*
    - Ghatigopa’s Commentary
        - Two manuscripts of this commentary exist in Kerala University Oriental Research Institute and Manuscripts Library, Trivandrum.
        - Ghatigopa was a devotee of God Padmanabha and a pupil of Paramesvara (different from his namesake, not the author of *Drgganita*).
        - K.V Sharma believes that Ghatigopa is Prince Godavarma Koyittampuran (A.D. 1810-60), a member of the scholarly family of Kilamanoor and a resident of Trivandrum, who bore the appellation ‘Manikkaran’ (=clockman) in Malayalam and ‘Ghatigopa’ in Sanskrit.

    - Kodandarama’s Commentary

        - A complete manuscript of Kodandarama's commentary on the *kalakriya-pada* of the *Aryabhatiya,* in Sanskrit versus along with Telugu meaning, exists in the Government Oriental Manuscripts Library, Madras. It is called *Aryabhatatantra-ganita.*
        - Kodandarama is also the author of a work called *Aryabhatavani,* which was meant to be a sequel to *Aryabhatiya.*

    - Kodandarama's Commentary
        - Kodandarama wrote a commentary in Telugu also. It is on the first three *padas* only, and bears the name *Sudhataranga.*
        - This commentary has been edited by V. Lakshminarayana Sastri and published in *Madras Government Oriental Series* in 1956.

    - Virpaksa's Commentary
        - A manuscript of this commentary exists in the Oriental Manuscripts Library, Mysore.

    - Krsnadasa's Commentary
        - A manuscript of Krsnadasa's commentary covering the *Gitika-pada* occurs in the collection of K.V. Sarma.
        - Krsnadasa is identified with Koccu-Krsnan Asan (A.D. 1756-1812), of the family of Netumpayil in the Tiruvalla taluk of South Kerala, well known in Malayalam literary circles as the author of several poetical works. He is also the author of a number of astrological works in Malayalam.

    - Krsna's Commentary
        - A manuscript of *Aryabhatiya-vyakhya,* a commentary in Malayalam, entitled *Bhasayam Krsna-tika,* exists in the library of the Indian Office, London.
        - It is difficult to say whether the author of this commentary was the same persona as Krsnadasa or different from him.

    - Two commentaries by Ghatigopa
        - In addition to his commentary in Sanskrit, Ghatigopa wrote tow commentaries in Malayalam, both on the *Ganita, Kalakriya* and *Gola Padas* only. The larger commentary extends to 1850 *granthas* (1 *grantha* = 32 letters), and the smaller one extends to 1200 *granthas.*
        - Of the larger commentary, there exist two manuscripts in the Kerala University Oriental Research Institute and Manuscripts Library, Trivandrum.

        - Of the smaller commentary, there are three manuscripts in the Kerala University Oriental Research Institute and Manuscripts Library, Trivandrum and one in the Government Sanskrit College Library, Trivandrum.

    - Anonymous commentary in Marathi
        - A commentary in Marathi exists in the Bombay University Library, Bombay. The name of the author is not mentioned.

THE ARYABHATIYA

The contents

It deals with both mathematics and astronomy and is written in **121 stanzas.**
The subject matter of Aryabhatiya is divided into 4 sections/ chapters/ or *Padas.* The word *pada* means quarter or one-fourth.

- *Pada 1: Gitika Pada*
- *Pada 2: Ganita pada*
- *Pada 3 : Kalakriya pada*
- *Pada 4: Gola pada*

The two compositions

- Aryabhatiya is supposed to be a collection of two compositions:
    - *Dasagitika sutra*
        - Aphorisms in 10 *gitika* stanzas
        - Consists of *Pada 1*, stating the astronomical parameters in *gitika* metre
    - *Aryastasata* or *Aryabhata-tantra*
        - 108 stanzas in *arya* metre
        - Consists of the second, third and fourth *padas*
    - *Dasagitika sutra* and *Aryastasata* both begin with an invocatory stanza and end with a concluding stanza in praise of the work and look like two different works.
    - The commentator Bhaskara I regards the two as two different works and designates his commentaries on them by names of *Dasagitika-sutra-vyakhya* and *Aryabhata-tantra-bhasya*, respectively.
    - Other commentators of the *Aryabhatiya*, Suryadeva, Raghunatha-raja, Yallaya and Nilakantha too hold the same opinion.
    - The north Indian astronomer Brahmagupta too believes the same.
    - The *Dasagitika sutra* was issued as a separate tract and was meant for the freshers who were expected to learn the astronomical parameters before embarking upon the study of proper astronomy. The *Aryastasata* was meant for those who had mastered the *Dasagitika sutra* and were qualified for the study of proper astronomy.

Aryabhatiya the work of the Brahma School

- Aryabhata was a follower of the Brahma school of Hindu astronomy.
- His devotion to God Brahma has led people to suppose that Aryabhata acquired his knowledge of astronomy by performing penance in propitiation of God Brahma.
- Aryabhata's devotion to Brahma was indeed of a high order. For, in his view, the end of learning was the attainment of the Supreme Brahman and this could be easily achieved by the study of astronomy.
- Aryabhata's prediction for the Brahma school of astronomy may have been inspired by two main considerations:
    - Firstly, the Brahma school was the most ancient school of Hindu astronomy promulgated by God Brahma himself.
    - Secondly, the astronomers of *Kusumapura*, where Aryabhata lived and wrote his *Aryabhatiya*, were the followers of that school.

Notable Features

- *The alphabetical system of numeral notation*
    - This is different from the *katapayadi* system but much **more effective** in expressing numbers briefly in verse.
- *Circumference-diameter ratio,* $\pi = 3.1416$
    - Aryabhata states that
        - circumference : diameter = 62832 : 20000
        - Which is equivalent to saying that $\pi = 3.1416$
        - This **value of π is correct** to four decimal places and is **better** than the value 3.141666 given by the Greek astronomer Ptolemy.
        - Aryabhata has called this value 'approximate'.
- *The table of sine differences*
    - Aryabhata is probably the earliest astronomer to have given a table of sine-differences.
    - He has also stated geometrical and theoretical methods for **constructing sine-tables**.
- *Formulae for sin θ, when* $\theta > \pi/2$
    - Aryabhata uses the following formulae:
        - $\sin(\pi/2 + \theta) = \sin \pi/2 - \text{versin}\ \theta$
        - $\sin(\pi/2 + \theta) = \sin \pi/2 - \text{versin}\ \pi/2 - \sin \theta$
        - $\sin(3\pi/2 + \theta) = \sin \pi/2 - \text{versin}\ \pi/2 - \sin \pi/2 + \text{versin}\ \theta$
    - These formulae were later used by Brahmagupta also.
- *Solution of indeterminate equations of the following types:*

    - $N = ax + b = cy + d = ez + f = \ldots..$
    - $(ax \neq c) / b =$ a whole number
    - Aryabhata is the earliest to have given the **general solution of problems** of the following types which reduce to the solution of above equations:
        - Find the number which yields 5 as the remainder when divided by 8, 4 as the remainder when divided by 9, and 1 as the remainder when divided by 7.
        - 16 is multiplied by a certain number, the product is diminished by 138 and the difference thus obtained being divided by 487 is found to be exactly divisible. Find the m multiplier and the quotient.
- *Theory of the Earth's Rotation*
    - It was believed that the Earth was stationary and lay at the centre of the universe and all heavenly bodies revolve around the Earth.
    - Aryabhata differed from other astronomers.
    - He held the view that **Earth rotates about its axis** and the **stars are fixed** in space.
    - According to Aryabhata, the period of one sidereal rotation of the Earth is **$23^h56^m4^s.1$**[1]. The corresponding modern value is **$23^h56^m4^s.091$**[2].
- *The astronomical parameters*
    - The astronomical parameters given by Aryabhata are much better than those given by earlier astronomers.
    - The method used for their determination has been indicated by him in the *Gola-pada*.
- *Time and divisions of time*
    - Aryabhata does not believe in the theory of creation and annihilation of the world.
    - For him, **time is a continuous process**, without beginning and the end.
    - The beginnings of the *yuga* and *kalpa*, according to him, have nothing to do with any terrestrial occurrence. They are purely based on astronomical phenomena depending on the positions of planets in the sky.
    - In the *Smrtis* as also in the *Surya-siddhanta*, we have the following pattern of time division:
        - 1 *Kalpa* = 14 *Manus*
        - 1 *Manu* = 71 *yugas*
        - 1 *Yuga* = 43,20,000 years
    - In order to make the *Kalpa* equivalent to 1000 *Yugas* (approx.), every *Manu* is supposed to be preceded and followed by a period of $\frac{2}{5}$ of a *yuga*, called twilight.
    - A period of 3.95 *yugas* is further earmarked for the time spent in the creation of the world, so that when the world order starts all planets occupy the same place.

- A *Kalpa* is defined as a day of Brahma, 2 *Kalpas* as a nychthemeron (day and night) of Brahma, 720 *Kalpas* as a year of Brahma, and 100 years (or 72,000 *Kalpas*) as the lifespan of Brahma.
- The age of Brahma, according to *Surya-siddhanta*, at the beginning of the current *Kalpa*, was 50 years.
- The current *Kalpa* is the first day of the 51st year of Brahma's life, and 6 with their twilights and $27\frac{9}{10}$ *yugas* had elapsed at the beginning of the Kaliyuga since the beginning of the current *Kalpa*.
- A *Yuga* is taken to be composed of 4 smaller *yugas* bearing the names Krta, Treta, Dvapara and Kali. the lengths of these smaller *yugas* are supposed to be
  - Krta : 17,28,000 years
  - Treta : 12,96,000 years
  - Dvapara : 8,64,000 years
  - Kali : 4,32,000 years
- Aryabhata rejects this scheme of time division and replaces it with the following:
  - 1 day of Brahma or *Kalpa* = 14 *Manus* or 1008 *yugas*
  - 1 *Manu* = 72 *yugas*
  - 1 *yuga* = 43,20,000 years
- He has dispensed with the periods of twilight and the time spent in creation, and has simplified the scheme enormously.
- Since $1008 \equiv 0 \pmod 7$, every *Kalpa* under this scheme begins on the same day, which is an additional advantage.
- Under this scheme, 6 *Manus* and $27\frac{3}{4}$ *yugas* had elapsed at the beginning of Kaliyuga since the beginning of the current *Kalpa*.
- Aryabhat too divided a *yuga* into 4 smaller *yugas*, but he takes them to be of equal durations : **quarter-*yugas***, the duration of each being **10,80,000 years**. This is a more scientific division, as in every quarter-*yuga* the planets make an integral number of revolutions around the Earth.
- Although the time-divisions given in the *Surya-siddhanta* and by Aryabhata differ so much, they have been so adjusted that the beginning of the current Kaliyuga according to both falls on the **same day**, viz, Friday, February 18, 3102 B.C.
- Aryabhata has also divided his *yugas* into 2 divisions :
  - *Utsarpini :* further divided into *Dussama & Susama*
  - *Apasarpini :* further divided into *Susama & Dussama*

* *Theory of the planetary motion*
  - The computation of the planetary positions in the *Aryabhatiya* is based on the following hypothesis:
    - **Hypothesis 1 :** In the beginning of the current yuga, which occurred ion Wednesday, 32,40,000 years before the commencement of the current quarter-yuga, all the planets together with the Moon's apogee and the

Moon's ascending node were in conjunction at the first point of the asterism *Asvini*,.

- **Hypothesis 2 :** The mean planets revolve in geocentric circular orbits. The mean motions of the planets are given in terms of revolutions performed by the planets round the Earth in a period of 43,20,000 years. These revolutions are based on Aryabhata's own observations, and constitute the main distinguishing feature of Aryabhata's astronomy.
- **Hypothesis 3 :** The true planets move in eccentric circles or in epicycles.
  - The *manda* epicycles are not the actual epicycles but the mean epicycles corresponding to the mean distances of the planets.
  - The radius of the *sighra* concentric and *sighra* eccentric is equal to the planet's distance called *mandakarna*.
- **Hypothesis 4 :** All planets have equal linear motion in their respective orbits.

- *Innovations in planetary computation*
  - Earlier astronomers performed four corrections in the case of superior planets (Mars, Jupiter and Saturn) and as many as five corrections in the case of inferior planets (Mercury and venus) in order to obtain their true positions.
  - Aryabhata changed the old pattern of corrections and in the case of inferior planets, reduced the number of corrections from **five to three**.
  - In the case of superior planets, a **pre-correction** (= to half the equation of centre) was also predicted.
  - In the case of finding planetary distances, the *Surya-siddhanta* prescribed the following formula :
    - $\frac{manadakarna + sighrakarna}{2}$
  - But Aryabhata changed it to :
    - $\frac{manadakarna \times sighrakarna}{R}$
- *Celestial latitudes of the planets*
  - It is Aryabhata who for the first time gave the **correct method** for finding the **celestial latitude of planets**, both superior and inferior.
- *Use of radian measure in minutes*
  - Aryabhata is the earliest astronomer to use the **radian measure of 3438'** for the **radius** of his circle. His table of **Rsine-differences** is also given in the same measure.

Importance and popularity

- Conciseness of expressions, superiority of astronomical constants and innovations in astronomical methods made Aryabhatiya an excellent book on economy.
- Aryabhata School was made, where the exponents called themselves 'disciples of Aryabhata'
- These disciples stated Aryabhata as the God and his teachings of the highest esteem.
- Bhaskara 1(an exponent of the Aryabhata School) stated that no one except Aryabhata has been able to know the motion of heavenly bodies.
- Later, Bhaskara 1 earned a great name as an astronomical teacher and was a well known 'guru'.
- His commentary on the Aryabhatiya, was recognised as a work of great scholarship and he was designated as an 'all knowing commentator'
- The popularity of the Aryabhatiya can be determined by the following facts-
    - There is hardly any work that deals with hindu astronomy without referring to the Aryabhatiya
    - There are a lot of commentaries on Aryabhatiya by people from far flung places as South India
    - There are ample of works that are solely based on the Aryabhatiya
    - Calendrical texts and tables used in South India are based on the Aryabhatiya works of astronomy.
- Aryabhatiya was even studied in northern india at least up to the end of tenth century A.D.
- Brahmagupta, who lived in Rajasthan, in the seventh century, did an intensive study of the work.

Works that are based on the Aryabhatiya

1. The works of Bhaskara I

2. The *Karana-ratna* of Deva (A.D. 689), son of Gojanma
    - A manuscript of this work exists in the Kerala University Oriental Research Institute and Manuscripts Library, Trivandrum.
    - It is a calendrical work in eight chapters, containing 183 verses.
    - This is the earliest work of Aryabhata school that states the precession of the equinoxes and the so-called *Sakabda, Manuyuga* and *Kalpa* corrections.

3. The *Graha-cara-nibandhana* of Haridatta (or Haradatta)
    - This calendrical work was edited by K. V. Sharma and published by the Kuppuswami Sastri Research Institute, Mylapore, Madras in 1954.

    - The work is in three chapters and states the simplified rules and tables for finding the true longitudes of the planets and therefore the *naksatra* and *tithi*, two of the five elements of the Hindu calendar.

4. The *Sisya-dhi-vrddhida* of Lalla (or Ralla)
    - The text of this work was published by Sudhakara Dvivedi at Banaras in A.D. 1886.
    - The author explains the scope of his work as follows:
        - That science of astronomy, as told by Aryabhatta, is difficult to comprehend and therefore is being set forth by Lalla in such a way as to be easily understood by students.
        - Although having mastered the *sastra* composed by Aryabhatta, his pupils have written astronomical *tantras*, but they have not been able to describe the methods properly. Hence, Lalla aims at stating the procedures in a proper sequence.
    - Lalla was the son of Samba, popularly known as Bhatta Trivikrama, and the grandson of the learned scholar Taladhvaja.
    - Two features of the *Sisya-dhi-vrddhida* stand out more than others:
        - Arrangement of subject matter under two distinct heads -
            - *Grahaganita* (dealing with astronomical calculations
            - *Goladhyaya* (dealing with the celestial sphere, cosmogony, astronomical instruments, etc.)
        - Language. The language used by Lalla is, at places, highly poetic and appealing. Some of his expressions and similes are so nice that posterior writers could not resist copying them.

5. The *Karana-prakasa* of Brahmadeva (A.D. 1092)
    - This calendrical work was edited by Sudhakara Dvivedi together with his own commentary.
    - It makes use of the *bija* correction prescribed by Lalla, and *tithis* calculated from this work differ by about 2 to 3 *ghatis*, being in excess, from those calculated from the parameters of the *Aryabhatiya.*
    - This work was used in South India, particularly in Maharashtra, amongst Vaisnavas, who preferred the 11th *tithi* calculated from this work.

6. The *Bhatatulya* of Damodra
    - The epoch used in this work is A.D. 1417.
    - Damodra was a son of Padmanabha and grandson of Narmada.
    - Use of Lalla's *bija* correction is made in this work also.
    - A manuscript of this work exists in the Deccan College Library, Puna.

7. The *Karana-paddhati* of Putumana Somayaji (A.D. 1732)
    - This work has been published in the *Trivandrum Sanskrit Series* (No. 126) and the *Madras Government Oriental Series* (No. 98).

8. The *Aryabhata-siddhanta-tulya-karana* by Virasimhaganaka, son of Kasiraja
    - Three manuscripts of this work occur in Anup Sanskrit Library, Bikaner.

SUMMARY OF ARYABHATIYA

The Gitika Section

- In the 13 stanzas composed in the *gitika* metre, comprising 10 aphorisms (*sutra)* of this chapter, Aryabhata sets forth the basic definitions and important astronomical parameters and tables and gives definitions of :
  - Larger units of time: *kalpa, manu, yuga*
  - The circular units: sign, degree, minute
  - The linear units: *yojana, hasta, angula*
- The section also states the number of rotations of the Earth and the revolutions of the Sun, Moon, planets etc. in 43,20,000 years, the time and place from which the planets are supposed to have started motion at the beginning of the current *yuga* as well as the time elapsed since the beginning of the current *Kalpa* up to the beginning of *Kaliyuga*, the positions of the apogees and the ascending nodes of the planets in the time of the author and the orbits of the Sun, Moon and the planets, the obliquity of the ecliptic and the inclinations (to the ecliptic) of the orbits of the Moon and the planets, the epicycles of the Sun, Moon and the planets and a table of sine - differences.
- A beginner in astronomy is supposed to learn them by heart so that he might not feel any difficulty while making calculations later on.
- For the convenience of the beginner, this chapter was written as an independent tract and issued under the name *Dasaditika-sutra*.

Invocation and Introduction

Stanza 1:
"Having paid obeisance to God Brahma - who is one and many, the real God, the Supreme Brahman - Aryabhata sets forth the three, viz, mathematics (*ganita*), reckoning of time (*kalakriya*) and celestial sphere (*gola*)."

- Obeisance to Brahma points to the school to which the author Aryabhata belongs.
- Brahma is spoken as one and many, as writes the commentator Bhaskara I:
  - When viewed as the unchangeable and unsustained God, He is one, but when taken to reside in the bodies of so many living beings, He is many.
  - Or, in the beginning He was only one, but later He became twofold - man and woman - and created all living beings and became many.

    - Or, viewed as the omnipresent God (*visvarnpa*), He is unquestionably one and many.
- He is called the real God, because the other gods having been created by Him are not real gods. He is called the 'Supreme Brahman' (*param brahma*), because He is the root cause of the world.
- Bhaskara I thinks that the first half of the stanza may be interpreted also as
    - Obeisance to the two *Brahmans* - the *Sabda-Brahman (satya devata)* and the *Para-Brahman*.
    - Or else, as obeisance to the triad, Hiranyagarbha (the supreme body, consisting of the subtle bodies of all the living beings taken collectively), the Causative Power of the Supreme Body *(satya devata)*, and the Master of that Power (*Para-brahma,* the supreme brahman)
- According to Bibhuibhushan Datta, *kam* in the text may be interpreted as *anandakam* (meaning 'supreme bliss'), *satyam* as *satsvarupam* (meaning 'really existent truth') and *devatam* as *cit-svarupam* (meaning 'pure intelligence'). The text should then be translated as:

    > "Having paid obeisance to the Supreme Brahma - who is one and also many, who is supreme bliss, really existent truth, and pure intelligence - Aryabhata sets forth the three, viz, mathematics (*ganita*), reckoning of time (*kalakriya*) and celestial sphere (*gola*)."

Method of Writing Numbers

Stanza 2:
"The *varga* letters (*k* to *m*) should be written in the *varga* places and the *avarga* letters (*y* to *h*) in the *avarga* places. The *varga* letters take the numerical values 1, 2, 3, etc. from *k* onwards; the numerical value of the initial *avarga* letter *y* is equal to *n* plus *m* (i.e. 5+25). In the places of the two nines of zeros which are written to denote the notational places, the nine vowels should be written (one vowel in each pair of the *varga* and *avarga* places). In the *varga (*and *avarga),* places beyond the places denoted by the nine vowels too assumed vowels or other symbols should be written, if necessary."

- In the Sanskrit alphabet, the letters k to m have been classified into five *vargas* (classes) - *ka-varga, ca-varga, ṭa-varga, ta-varga* and *pa-varga*. These letters are therefore referred to above as *varga* letters. These are supposed to bear the numerical values 1 - 25 as shown in the following table:

Varga	Letters and their numerical values				
ka-varga	*k* = 1	*kh* = 2	*g* = 3	*gh* = 4	*ṅ* = 5

ca-varga	*c* = 6	*ch* = 7	*j* = 8	*jh* = 9	*ñ* = 10
ṭa-varga	*ṭ* = 11	*ṭh* = 12	*ḍ* = 13	*ḍh* = 14	*ṇ* = 15
ta-varga	*t* = 16	*th* = 17	*d* = 18	*dh* = 19	*n* = 20
pa-varga	*p* = 21	*ph* = 22	*b* = 23	*bh* = 24	*m* = 25

- The letters y to h are called *avarga* letters because they are not classified into *vargas*. These letters bear the following numerical values:

  y = 30, *r* = 40, *l* = 50, *v* = 60,
  ś = 70, *ṣ* = 80, *s* = 90, *h* = 100

- The values of the above letters are taken to increase by 10 because the *avarga* letters are written in the *avarga* places, and increase by 1 in the *avarga* place, means increase by 10 in the *varga* place.
- On the analogy of *varga* and *avarga* classification of the letters, the notational places are also divided into the *varga* and *avarga* places.
  - The odd places denoting the units' place, the hundreds' places, the ten thousands' place and so on, are called the *varga* places (because 1, 100, 10,000 etc are perfect squares).
  - The even places denoting the tens' place, the thousands' place and so on are called the *avarga* places (because 10, 1000 etc are non-square numbers).
- The text says that the *varga* letters (*k* to *m*) should be written in the *varga* places and the *avarga* letters (*y* to *h*) in the *avarga* places. How?
  - The notational places are written first. The usual practice in India is to denote them by ciphers:

    0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

    Instead it is suggested that they should be denoted by nine vowels (*a, i, u, ṛ, ḷ, e, o, ai, au*) in the following manner:

    au au ai ai o o e e ḷ ḷ ṛ ṛ u u i i a a

  - When a letter is joined with a vowel (for example in gṛ, the letter g is joined with the vowel ṛ), the letter denotes a number and the vowel the place where that number is to be written down. Thus, gṛ stands for the number g=3 written in the *varga* occupied by the vowel ṛ in the *varga* place as below: (A=avarga, V=varga)

A	V	A	V	A	V	A	V
ṛ	ṛ	u	u	i	i	a	a

	g	0	0	0	0	0	0
=	3	0	0	0	0	0	0

Thus gṛ = 3000000

- Similarly, *ṅiśibuṇḷṣkhṛ* (=*ṅ+i, ś+i, b+u, ṇ+ḷ, ṣ+kh+ṛ*) denotes the number which is obtained by writing the *ṅ* in the *varga* place and *ś* in the *avarga* place occupied by the vowel *i; b* in the *varga* place occupied by the vowel *u; ṇ* in the *avarga* place occupied by the vowel *ḷ;* and *ṣ* in the *avarga* place and *kh* in the *varga* place occupied by the vowel *ṛ* as follows:

ḷ	ḷ	ṛ	ṛ	u	u	i	i	a	a
	ṇ	*ṣ*	*kh*	0	*b*	*ś*	*ṅ*	0	0
=1	5	8	2	2	3	7	5	0	0

Thus *ṅiśibuṇḷṣkhṛ* = 1582237500

- The rule stated in the above stanza is meant to provide a key deciphering the numerical values borne by the letter chronograms.
- The instruction *'ṅmau yah'* serves two purposes.
  - Firstly, it gives the value of the letter *y* as equal to *ṅ* plus *m* (5+25 = 30)
  - Secondly, it suggests that the conjoint letter *ṅm* means *ṅ+m.*

- The statement of "two nines of zeros" in the text refers to the Indian method of writing the notational places by mens of zeros. In the present primary schools in India, when a student is taught to write large numbers, he is first made to write the notational places by means of zeros arranged horizontally as follows:

0 0 0 0 0 0 0 0 0

The teacher then points to the first zero and says "units' place", then to the next zero and says "tens' place", then to the next zero and says "hundreds' place" and so on.

Revolution-numbers and Zero Point

Stanza 3-4

"In a *yuga,* the eastward revolutions of the Sun are 43,20,000; if the Moon, 5,77,53,336; of the Earth, 1,58,22,37,500; of Saturn, 1,46,564; of Jupiter, 3,64,224; of Mars, 22,96,824; of Mercury and Venus, 43,20,000; of the Moon's apogee, 4,88,219; of (the *sighrocca* of) Mercury, 1,79,37,020; of (the *sighrocca* of) Venus, 50,22,388; of (the *sighroccas* of) the other planets, 43,20,000; of the moon's ascending node in the opposite direction (i.e. westward),

2,32,226. These revolutions commenced at the beginning of the sign Aries on Wednesday at Sunrise at Lanka (when it was commencement of the current *yuga*).”

- The ‘Moon’s apogee’ is that point of the Moon’s orbit which is at the remotest or the farthest distance from the Earth, and the ‘Moon’s ascending node’ is that point of the ecliptic where the Moon crosses it in its northward motion.
- The *sighroccas* of Mercury and venus are the imaginary bodies which are supposed to revolve around the Earth with the heliocentric mean angular velocities of Mercury and Venus, respectively, their directions from the Earth being always the same as those of the mean positions of Mercury and venus from the Sun. It will thus mean that the revolutions of Mars, the *sighrocca* of Mercury, Jupiter, the *sighrocca* of Venus, and Saturn, given above, are equal to the revolutions of Mars, Mercury, Jupiter, Venus and Saturn, respectively around the Sun.
- The following table gives the revolutions of the Sun, the Moon and the planets along with their periods of one sidereal revolution.

Table 2. Mean motion of the planets

Planet	**Revolutions in 43,20,000 years**	**Sidereal period in terms if days**		
		Aryabhata I	**Ptolemy**	**Moderns**
Sun	43,20,000	365.25868	365.24666	365.25636
Moon	5,77,53,336	27.32167	27.32167	27.32166
Moon’s apogee	4,88,219	3231.98708	3231.61655	3231.37543
Moon’s asc. node	2,32,226	6794.74951	6794.45587	6794.39108
Mars	22,96,824	686.99974	686.94461	686.9797
Sighrocca of Mercury	1,79,37,020	87.96988	87.969935	87.9693
Jupiter	3,64,224	4332.27217	4330.96064	4332.5887

Sighrocca of Venus	70,22,388	224.69814	224.69890	224.7008
Saturn	1,46,564	10766.06465	10749.49640	10759.201

- The epoch of the planetary motion mentioned in the text marks the beginning of the current *yuga* and not the beginning of the current *Kalpa* as was supposed by P.C. Sengupta.
- The current *Kalpa*, according to Aryabhata I, started on Thursday 1,98,28,80,000 years or 7,24,26,41,32,500 days before the beginning of the current *yuga*; and 1,98,61,20,000 years or 7,25,447,570,625 days before the beginning of the current *Kaliyuga*. The current *Kaliyuga* began on Friday 18, 3102 BC at Sunrise at Lanka (a hypothetical place on the equator where the meridian of Ujjain intersects it, which synchronised with the beginning of the light half of Luna (synodic) month of *Caitra*.

- One thing that deserved special notice is the statement of the Earth's rotations. Aryabhata I was, perhaps, the earliest astronomer in India who advanced the theory of the Earth's rotation and gave the number of rotations that the Earth performs in a period of 43,20,000 years.
- The period of one sidereal rotation of the Earth according of Aryabhata I is 23h 56 min 41 sec. The corresponding modern value is 23h 56min 4s.091. The accuracy of Aryabhata I's value is remarkable.
- Of the other Indian astronomers who upheld the theory of the Earth's rotation, mention may be made of Prthudaka (A.D. 860) and Makkibhatta (A.D. 1377). In the Skanda-purana (1. 1. 31. 71), too, the Earth is described as revolving like a bhramarika (spinning top, potter's wheel or whirlpool).
- The commentators of the *Aryabhatiya*, who hold the opinion that the Earth is stationary, thought that Aryabhata I states the rotations of the Earth because the asterisms, which revolve westward around the earth by the force of the provector wind, see that the Earth rotates eastward.
- These commentators were indeed helpless because Aryabhata I's theory of the Earth's rotation received a severe blow at the hands of Varahamihira (d. A.D. 587) and Brahmagupta (A.D. 628) whose arguments against this theory could not be refuted by any Indian astronomer.
- It is noteworthy that the Greek astronomer Ptolemy, following Aristotle (B.C. 384-322), believed that the Earth was stationary and adduced arguments in support of his view.

Kalpa, Manu And Beginning Of Kali

Stanza 5:
"A day of Brahma (or a Kalpa) is equal to (a period of) 14 Manas, and (the period of one) Manu is equal to 72 *yugas.* Since Thursday, the beginning of the current Kalpa, 6 Manus, 27 *yugas* and 3 quarter yugas had elapsed before the beginning of the current kaliyuga (lit, before bharata)."

1 Kalpa = 14 Manus

1 Manu = 72 *yugas,*

1 Kalpa = 1008 *yugas* or 4,35,45,60,000 years

- Likewise, the time elapsed since the beginning of the current Kalpa up to the beginning of the current *Kaliyuga*

  $= \; 6\,Manus \; + \; 27\,\frac{3}{4} yugas$

  $= \; (6x72 \; + \; 27\,\frac{3}{4}) yugas$

  $= \; (432 \;\; + \; 27\frac{3}{4}) \; x \; 4320000 \; years$

  $= \; 1986120000 \; years \; or \; 725447570625 \; days$

- It is interesting to note that Aryabhata I prefers to say "before Bharata" (*bharatat parvam*) instead of saying "before the beginning of *Kaliyuga*" which is the sense actually intended here.
- Regarding the interpretation of *bharatat parvam* there is difference of opinion amongst the commentators. The commentator Some vara interprets it as meaning 'before the occurrence of the Bharata (battle)". P. C. Sengupta (A.D. 1927) and W.E. Clark (A.D. 1930), too, have interpreted the word bharata as meaning 'the Bharata battle'. the *Mahabharata* we are told that the Bharata battle occurred at the end of the *Dvopara yuga* and before the beginning of the *Kali yuga*:
  - "The battle between the armies of the Kurus and the Pandavas occurred at Syamantapañcaka (Kuruksetra) when it was the junction of Kali and Dvapara."

  So this interpretation of *bharatat purvam* ("before Bharata") is equivalent to *kaliyugat purvam* ('before Kaliyuga'), as it ought to be.

- The commentators Bhaskara I (A.D. 629), Suryadeva (b. A.D. 1191) and others have interpreted *bharatat parvam* as

    - meaning 'before Yudhisthira', i.e., 'before the time when Yudhisthira of the Bharata dynasty relinquished kingship and proceeded on the last journey (*maha-prasthana*)".
- According to these commentators, this event took place on Thursday, the last day of the past Dvapara. But the basis of this assumption is not specified. The commentators simply say: "This is what is well known." (*Iti prasiddhiḥ*).
- According to these commentators, too, *bharatat parvam* ultimately means 'before the beginning of the current *Kaliyuga*'.
- Brahmagupta criticised Aryabhata I for his teaching in the above stanza. He wrote :
    - "Since the measures of a *Manu*, a (quarter) *yuga and a Kalpa* and the periods of time elapsed since the beginnings of *Kalpa and Krtayuga* (as taught by Aryabhata) are not in conformity with those taught in the Smrtis, it follows that Aryabhata is not aware of the mean motions (of the planets)."**
    - "Since Aryabhata states that three quarter yugas had elapsed at the beginning of *Kaliyuga*, the beginning of the current *yuga* and the end of the past *yuga* (according to him) occurred in the midst of *Krtayuga*; so his *yuga* is not the true one."
    - "Since the initial day on which the *Kalpa* started according to (Aryabhata's) sunrise system of astronomy is Thursday and not Sunday (as it ought to be), the very basis has become discordant."
- In reply to Bramagupta's criticism, astronomer Vatesvara(A.D. 904) said:
    - "If the yuga stated by Brahmagupta conforms to the teachings of the Smrtis, how is it that the Moon (according to him) is not beyond the Sun (as stated in the Smrtis). If that is unacceptable because that statement of the Smrtis is false, then, alas, the yuga-hypothesis of the Smrtis, too, is false."
    - "Since a planet does not make complete revolutions during the quarter yugas acceptable to Brahmagupta, son of Jisnu, (whereas it does during the quarter yugas according to Aryabhata), it follows that the quarter yugas of Śrīmad Aryabhata (and not those of Brahmagupta) are the correct ones."
    - "If a Kalpa should begin with a Sunday, how is it that Brahmagupta's Kalpa does not end with Saturday. Brahmagupta's Kalpa being thus contradictory to his own state- ment, it is a fabrication of his own mind (and is by no means authoritative).

Planetary Orbits, Earth's Rotation

Stanza 6:

"Reduce the Moon's revolutions (in a *yuga*) to signs, multiplying them by 12 (lit. using the fact that there are 12 signs in a circle or revolution). Those signs mutiplied successively by 30, 60 and 10 yield degrees, minutes and *yojanas*, respectively. (These yojanas give the length of the circumference of the sky). The Earth rotates through (an angle of) one minute of arc in one respiration (=4 sidereal seconds). The circumference of the sky divided by the revolutions of a planet in a yuga gives (the length of) the orbit on which the planet moves. The orbit of the asterisms divided by 60 gives the orbit of the Sun."

$$\text{Orbit of the sky} = 57753336 \times 12 \times 30 \times 60 \times 10 \text{ } yojanas$$

$$= 12474720576000 \text{ } yojanas$$

$$\text{Orbit of the asterisms} = 173260008 \text{ } yojanas$$

$$\text{Orbit of the Sun} = 2887666\tfrac{4}{5} \text{ } yojanas$$

$$\text{Orbit of the Moon} = 216000 \text{ } yojanas$$

$$\text{Orbit of Mars} = 5431291\tfrac{373277}{896851} \text{ } yojanas$$

$$\text{Orbit of (}Sighrocca\text{ of) Mercury} = 695473\tfrac{373277}{896851} \text{ } yojanas$$

$$\text{Orbit of Jupiter} = 34250133\tfrac{699}{1897} \text{ } yojanas$$

$$\text{Orbit of (}Sighrocca\text{ of) Venus} = 1776421\tfrac{255221}{585199} \text{ } yojanas$$

$$\text{Orbit of Saturn} = 85114493\tfrac{5987}{36641} \text{ } yojanas$$

- These orbits are hypothetical and are based on the following two assumptions:
    - That all the planets have equal linear motion in their respective orbits.
    - That one minute of arc (1') of the Moon's orbit is equal to 10 *yojanas* in length.
- From the second assumption, the length of the Moon's orbit comes out to be 216000 *yojanas*. Multiplying this by the Moon's revolution-number (viz. 57753336), we get 12474720576000 *yojanas*. This is the distance described by the Moon in a *yuga*. From the first assumption, this is also the distance described by any other planet in a *yuga*. Hence

$$Orbit\ of\ a\ planet\ = \frac{distance\ described\ by\ a\ planet\ in\ a\ yuga}{revolution-\ number\ of\ that\ planet}$$

This is how the lengths of the orbits of the various planets stated above have been obtained.

- In the case of the asterisms, it is assumed that their orbit is 60 times the orbit of the Sun. By saying that "the orbit of the asterisms divided by 60 gives the orbit of the Sun", Aryabhata I really means to say that "the orbit of the asterisms is 60 times the orbit of the Sun."
- Indian astronomers, particularly the followers of Aryabhata I, believe that the distance described by a planet in a *yuga* denotes the circumference of the space, supposed to be spherical, which is illuminated by the Sun's rays. This space, they call 'the sky' and its circumference "the orbit of the sky'. Bhaskara I said:
    - "(The outer boundary of) that much of the sky as the Sun's rays illumine on all sides is called the clrcumference or orbit of the sky. Otherwise, the sky is beyond limit; it is impossible to state its measure."
    - "For us the sky extends to as far as it is illuminated by the rays of the Sun. Beyond that, the sky is immeasurable."
- According to the Indian astronomers, therefore,
    - $Orbit\ of\ a\ planet\ = \frac{Orbit\ of\ the\ sky}{Planet's\ revolution{-}number}$
- The statement of the Earth's rotation through 1' in one respiration, stated in the text, has been criticised by Brahmagupta, who said:
    - "If the Earth moves (revolves) through one minute of arc in one respiration, from where does it start its motion and where does it go? And, if it rotates (at the same place), why do tall lofty objects not fall down?"
- The reading *bham* (in place of *bhuh*) adopted by the commentators is evidently incorrect. The correct reading is *bhah*, which has been mentioned by Brahmagupta (A.D. 628), Prthudaka (A. D. 860) and Udayadiväkara (A.D. 1073).

Linear Diameters

Stanza 7:

"8000 *ny* make a *yojana*. The diameter of the Earth is 1050 *yojanas*; of the Sun and the Moon, 4410 and 315 *yojanas*, (respectively); of Meru, 1 *yojana*; of Venus, Jupiter, Mercury, Saturn and Mars (at the Moon's mean distance), one-fifth, -one-tenth, one-fifteenth, one-twentieth, and one-twentyfifth, (respectively), of the Moon's diameter. The years (used in this work) are solar years."

- Nr is a unit of length whose measure is equal to the height of a man. Nr is also known as *nara, puruşa, dhanu and danda*, "*Puruşa, dhanu, danda and nara* are synonyms", says Bhaskara 1.

- The diameters of the Earth, the Sun, the Moon, and the Planets stated above may be exhibited in the tabular form as follows:

Table 3. Linear diameters of the Earth etc.

	Linear diameter in *yojanas*	Linear diameter in *yojanas* (at the moon’s mean distance)
Earth	1050	
Sun	4410	
Moon	315	
Mars		12.60
Mercury		21.00
Jupiter		31.50
Venus		63.00
Saturn		15.75

- The following is a comparative table of the mean angular diameters of the planets:

Table 4. Mean angular diameters of the planets

Planet	Mean angular diameter according to		
	Aryabhata I	Greek astronomers	Modern
Moon	31’ 20”	35’ 20” (Ptolemy) Tycho Brahe (15-46-1631)	31’ 8”
Mars	1’ 15”.6	1’ 40”	
Mercury	2’ 6”	2’ 10”	
Jupiter	3’ 9”	2’ 45”	
Venus	6’ 18”	3’ 15”	
Saturn	1’ 34”.5	1’ 50”	

- P. C. Sengupta translates the second half of the stanza as follows:
    - "The diameters of Venus, Jupiter, Mercury, Saturn and Mars are, respectively, 1/5, 1/10, 1/15, 1/20 and 1/25 of the diameter of the Moon, when taken at the mean distance of the Sun."
- This is incorrect, because :
    - "When taken at the mean distance of the Sun" is not the correct translation of *samarkasamah*. The correct translation is: "The years are solar years" as interpreted by Bhaskara 15 and Someśvara; or "The years of a *yuga* are equal to the number of revolutions of the Sun in a *yuga* as translated by Clark and as interpreted by Suryadeva, Parameśvara and Raghunatha-raja.
    - The diameters of the planets stated in the stanza under consideration correspond to the mean distance of the Moon and not to the mean distance of the Sun as Sengupta has supposed. Sengupta's disagreement on this point from the commentator Parameśvara is unwarranted. All commentators agree with Paramelvara.

Obliquity Of Ecliptic And Inclinations Of Orbits

Stanza 8:

"The greatest declination of the Sun is 24°. The greatest celestial latitude (lit. deviation from the ecliptic) of the Moon is 4 and 1/2°; of Saturn, Jupiter and Mars, 2°, 1° and 1 and 1/2° respectively; and of Mercury and Venus (each), 2°. 96 *angulas* or 4 cubits make a *ny.*"

- The greatest declination of the Sun is the obliquity of the ecliptic. According to Aryabhata I and other Indian astronomers, its value is 24° and according to modern astronomers its value is 23° 27′ 8.26-46".84 T, where T is measured in Julian centuries from 1900 A.D. The value in common use is $23\frac{1}{2}°$.
- The greatest celestial latitude of a planet is the inclination of the planet's orbit to the ecliptic. The values of the inclinations of the orbits of the Moon and the planets as given in the above stanza and those given by the Greek astronomer Ptolemy and the modern astronomers are being exhibited in the following table:

Table 5. Inclinations of the Orbits

Inclination of the orbit				
Planet	Aryabhata I	Ptolemy	*PauSi and RoSi* of Varahamihira	Modern
Moon	4 ° 30’	5°	4° 40’	5° 9’
Mars	1° 30’	1°		1° 51’ 01’
Mercury	2°	7°		7° 00’ 01”
Jupiter	1°	1° 30’		1° 18’ 28”
Venus	2°	3° 30’		3° 23’ 38”
Saturn	2°	2° 30’		2° 29’ 20”

- In the case of Mercury and Venus, Aryabhata I's values differ significantly from those of Ptolemy and modern astronomers because the values given by Aryabhata I are geocentric and those given by Ptolemy and modern astronomers are heliocentric.
- By combining the instruction in the last quarter of the above verse with that in the first quarter of verse 7, we have
  - 24 *angulas* = 1 cubit (*hasta*)
  - 4 cubits=1 nr
  - 8000 nr = 1 *yojana*
- Since earth's (equatorial) diameter=1050 *yojanas*, according to Aryabhata I, and = 12757 km or 7927 miles, according to modern astronomy, it follows that Aryabhata I's *yojana* is approximately equal to $12\frac{1}{9}$ km or $7\frac{1}{2}$ miles. Likewise his nr=152 cm. or 5 ft approx, and cubit = $1\frac{1}{4}$ ft. approx. The length of a cubit in common use is $1\frac{1}{2}$ ft.

Ascending Node And Apogees(Aphelia)

Stanza 9:

“The ascending nodes of Mercury, Venus, Mars, Jupiter and Saturn having moved to 20°, 60°, 40°, 80° and 100° respectively (from the beginning of the sign Aries) (occupy those positions);" and the apogees of the Sun and the same planets (viz., Mercury, Venus, Mars, Jupiter and Saturn) having moved to 78°, 210°, 90°, 118, 180° and 236° respectively (from the beginning of the sign Aries) (occupy those positions)."

- The following table gives the longitudes of the ascending nodes and the apogees of the planets for A. D. 499 as given by Aryabhata I and as calculated by modern methods. The corresponding longitudes for A.D. 150, as stated by Ptolemy are also given for comparison.

Table 6. Longitudes of the Ascending Nodes for A.D. 499

Planet	Longitudes in the ascending nodes		
	Aryabhata I	Ptolemy (for A.D. 150)	By modern calculation
Mars	40°	25° 30’	37° 49’
Mercury	20°	10° 00’	30° 35’
Jupiter	80°	51° 00’	85° 13’
Venus	60°	55° 00’	63° 16’
Saturn	100°	183° 00’	100° 32’

Table 7. Longitudes of the Apogees for A.D. 499

Planet	Longitudes of the apogees (aphelia)			
	Aryabhata I	Ptolemy (for A.D. 150)	RoSi Of Varahmihira	By modern calculation
Sun	78°	65° 31’	75°	77° 15’
Mars	118°	115° 30’		128° 28’
Mercury	210°	190° 00’		234° 11’
Jupiter	180°	161° 00’		170° 22’
Venus	90°	55° 00’		290° 4’
Saturn	236°	233° 00’		243° 40’

- The word *gatva* (meaning ‘having moved’ or ‘having moved to’) is used in the text to show that the ascending nodes and the apogees of the planet are not stationary but have a motion.

- The commentator Bhaskara I said that by teaching their motion, Aryabhata I has specified by implication their revolution-numbers in a *yuga*. The aged, who preserve the tradition, says he, remembers those revolution-numbers by the continuity of tradition.
- The period of 35750224800 years, according to the tradition, is the common period of motion (*yuga*) of the ascending nodes of all the planets, in which the ascending nodes of Mars, Mercury, Jupiter, Venus and Saturn make 2, 1, 4, 3 and 5 revolutions, respectively.
- In the case of the apogees of the planets, the periods and the corresponding revolutions, as handed down to Bhaskara I by tradition, are shown in the following table:

Table 8. Periods and Revolutions numbers of the Apogees

Apogee of	Period in years	Revolution - number
Sun	119167416000	13
Mars	357502248000	59
Mercury	23833483200	7
Jupiter	3972247200	1
Venus	7944494400	1
Saturn	178751124000	59

- The commentators Suryadeva and Raghunatha-raja have also cited the above-mentioned periods and revolution numbers to preserve the continuity of tradition line of the stanza:

  खाकाशाष्टकृतद्विद्विव्योमेष्वद्रीषुवह्नयः ।
  युगं बुधादिपातानां विद्वद्भिः परिपठ्यते ॥
  एकद्वित्रिचतुष्पञ्च भगणाः परिकीर्तिताः ।
  सौम्यारशुक्रजीवार्किपातानां क्रमशो युगे ॥,

  which has been quoted in full by Suryadeva, has been cited by Bhaskara I too. This means that the passage was derived from some earlier source, and the tradition mentioned in the stanza is definitely older than Bhaskara I.

- Whosoever might be the founder of the tradition, it is based on the misunderstanding that the ascending nodes and the apogees, after having started their motion from the first point of Aries at the beginning of the current Kalpa, moved exactly through the degrees mentioned by Aryabhata I up to 499 A.D., the epoch mentioned by Aryabhata I.
- The motions of the nodes and the apogees of the planets ascribed to tradition by Bhaskara I and the other commentators are much less than their actual motions. For example, the

node of Mercury, which is the slowest, actually requires about 166000 years to complete a revolution.[1] Similar is the case with the apogees.

Manda and *Sighra* Epicycles (Odd Quadrants)

Stanza 10:
"The *manda* epicycles of the moon, the Sun, Mercury, Venus, Mars, Jupiter and Saturn (in the first and third anomalistic quadrants) are, respectively, 7, 3, 7, 4, 14, 7 and 9 (degrees) each multiplied by 4 and 1/2 (i.e., 31.5, 13.5, 31.5, 18, 63, 31.5 and 40.5 degrees, respectively); the *sighra* epicycles of Saturn, Jupiter, Mars, Venus and Mercury (in the first and third anomalistic quadrants) are, respectively, 9, 16, 53, 59 and 31 (degrees) each multiplied by 4 (i.e., 40.5, 72, 238.5, 265.5 and 139.5 degrees, respectively)."

Manda and *Sighra* Epicycles (Even Quadrants)

Stanza 10:
"The *manda* epicycles of the retrograding planets (viz., Mercury, Venus, Mars, Jupiter and Saturn) in the second and fourth anomalistic quadrants are, respectively, 5, 2, 18, 8 and 13 (degrees) each multiplied by $4\frac{1}{2}$(ie., 22.5, 9, 81, 36 and 58.5) degrees, respectively); and the *Sighra* epicycles of Saturn, Jupiter, Mars, Venus, and Mercury (in the second and fourth anomalistic quadrants) are, respectively, 8, 15, 51, 57 and 29 (degrees) each multiplied by 4 $\frac{1}{2}$ (i.e., 36, 67.5, 229.5, 256.5 and 130.5 degrees, respectively). 3375 is the outermost circumference of the terrestrial wind."

- The dimensions of the *manda* and *sighra* epicycles are stated in terms of degrees, where a degree stands for the 360th part of the circumference of the deferent (*kaksyavṛtta*). Thus, when an epicycle is stated to be A°, it means that its periphery is A/360 of the circumference of the deferent.
- The following table gives the *manda* and s*ighra* epicycles as stated above by Aryabhata I and also those given by Ptolemy:

Table 9. ***Manda*** **and** ***Sighra*** **epicycles of the planets**

	Manda **epicycles**			***Sighra*** **epicycles**		
Planet	**Aryabhata I**			**Aryabhata I**		
	Odd Quadrant	**Even Quadrant**	**Ptolemy**	**Odd Quadrant**	**Even Quadrant**	**Ptolemy**
Sun	13°.50	13°.50	15°.00			
Moon	31°.50	31°.50	31°.40			
Mars	63°.00	81°.00	72°.00	238°.50	229°.50	237°
Mercury	32°.50	22°.50	18°.00	139°.50	130°.50	135°
Jupiter	31°.50	36°.00	33°.00	72°.00	67°.50	69°
Venus	18°.00	9°.00	15°.00	265°.50	256°.50	259°
Saturn	40°.50	58°.50	41°.00	40°.50	36°.00	39°

- It is noteworthy that in stating the dimensions of the manda epicycles the planets have been mentioned in the order of decreasing velocities (*manda-gati-krama*), whereas in stating the dimensions of the *sighra* epicycles they have been mentioned in the order of increasing velocities (*fighra-gati-krama*).
- The use of ablative in *yathoktebhyah* is meant to indicate that in finding the manda anomaly the longitude of the apogee is to be subtracted from the longitude of the planet.
- Similarly, the use of the inverted forms *uccaŝighrebhyah* and *uccacchighrat* in place of *Sighroccebhyah* and *fighroccat*, respectively, shows, as remarked by Bhaskara I and Someśvara, that, in finding the sighra anomaly, the longitude of the planet has to be subtracted from the longitude of the *sighrocca*.
- According to Bhaskara I and Lalla, the *manda and fighra* epicycles stated above correspond to the beginnings of the respective anomalistic quadrant.
- The Kerala astronomer *Sankaranarayana* (A. D. 869) refers to some astronomers (without naming them) who said that there was also the view that the epicycles given by Aryabhata I corresponded to the end-points of the anomalistic quadrants and he Kerala astronomer Govinda-*svami*, who also refers to this controversy, is of the opinion that s*ighra* epicycles stated above correspond to the beginnings of the respective anomalistic quadrants, but the *manda* epicycles stated above correspond to the last points of the respective anomalistic quadrants, and, consequently, he has replaced the rules referred to

above by another rule which has been quoted by Udayadivakara (1073 A.D.) in his commentary.

- This controversy is due to the fact that Aryabhata I himself does not specify whether the epicycles given by him correspond to the initial points or last points of the anomalistic quadrants.

- Since the epicycles stated in the text correspond to the beginnings of the odd and even anomalistic quadrants, their values at other positions of the planets are to be derived by the rule of three. Bhaskara I has prescribed the following rule.

- Let α and β be the epicycles (*manda or sighra*) of a planet for the beginnings of the odd and even anomalistic quadrants, respectively. Then

    - If the planet be in the first anomalistic quadrant, say at P and its anomaly be θ,

$$\begin{aligned} \text{epicycle at } P &= \alpha + \frac{(\beta-\alpha)R\sin\theta}{R}, \text{ when } \alpha < \beta \\ &= \alpha - \frac{(\alpha-\beta)R\sin\theta}{R}, \text{ when } \alpha > \beta \end{aligned}$$

    - If the planet be in the second anomalistic quadrant, say at Q and its anomaly be $90° + \phi$,

$$\begin{aligned} \text{epicycle at } Q &= \beta - \frac{(\beta-\alpha)R\,\text{vers}\phi}{R}, \text{ when } \alpha < \beta \\ &= \beta + \frac{(\alpha-\beta)R\,\text{vers}\phi}{R}, \text{ when } \alpha > \beta \end{aligned}$$

    - Similarly if the third and fourth quadrants. The epicycles thus derived are called true epicycles (*spasta or sphuta-paridhi).*

- But the tabulated manda epicycles or the true manda epicycles derived from them are not the actual epicycles on which the true planet in the case of the Sun and Moon or the true mean planet in the case of the other planets is supposed to move. It is believed that they are the mean epicycles corresponding to the mean distances of the planets.
- In order to obtain the actual epicycles, one should either apply the formula:

$$\text{Actual manda epicycle} = \frac{\text{tabulated or true manda epicycle x } H}{R}$$

where H is the planet's true distance in minutes obtained by the process of iteration (*asakṛtkalākarna or mandakarna*), or apply the process, of iteration.

- In the case of the *sighra* epicycles, however, the actual epicycles are the same as the tabulated epicycles.
- Brahmagupta criticises Aryabhata I for stating different epicycles for odd and even anomalistic quadrants.

- He wrote: "Since in (Aryabhata's) sunrise system of astronomy, the epicycle which is the multiplier of the Rsine of anomaly in the odd anomalistic quadrant is different from the epicycle I which is the multiplier of the Rsine of anomaly in the even anomalistic quadrant, the (*manda or sighra*) correction for the end of an odd anomalistic quadrant is not equal to that for the beginning of the (next) even anomalistic quadrant (as it ought to be). This discrepancy shows that the differing epicycles (stated by Aryabhata) are incorrect.

- "Since the epicycle which is the multiplier of the Rsine of anomaly in the odd anomalistic quadrant is different from the epicycle which is the multiplier of the Rversine of anomaly in the even anomalistic quadrant, the (manda or Sighra) correction for the anomaly amounting to half a circle, does not vanish (as it ought to). This discrepancy, too, shows that differing epicycles (stated by Aryabhata) are incorrect.

- "Since the epicycles (stated by Aryabhata) correspond to odd and even anomalistic quadrants (and not to their first or last points), the (so-called true) epicycle which is obtained by multiplying the Rsine of anomaly by the difference of the epicycles (for the odd and even quadrants) and dividing by the radius and then subtracting the resulting quotient from or adding that to the epicycle for the odd quadrant according as it is greater or less than the other, is not the correct epicycle.

- "If indeed there should be two different epicycles for the odd and even anomalistic quadrants, then, why have not two different epicycles been stated in the case of the Sun and the Moon. It simply shows that the process of planetary correction stated in (Aryabhata's) *Audayika-Tantra* (e., Aryabhattya) does, in either way, lead to a correct result."

- Had Aryabhata I specified that the epicycles stated by him corresponded to the first or last points of the respective anomalistic quadrants, there would not have been any occasion for such a criticism.

- The number 3375, denoting the length or the outer boundary of the terrestrial wind, has reminded Bhaskara I of the following formula which also involves that number :

$$Rsin\,\theta \;=\; \frac{4(180° - \theta)\,\theta.R}{12\,x\,3375 - (180° - \theta)\theta},$$

Where θ is in terms of degrees.

- Bhaskara I thought that the length of the outer boundary of the terrestrial winds has been stated simply to teach the method of finding Rsine without the use of the Rsine Table which is implied in the above formula.
- Brahmagupta (A. D. 628) misreads *giyiñasa* as *giyigasa* and unnecessarily criticised Aryabhata I for giving two different values of the Earth's diameter. He wrote
    - "The circumference being (stated as) 3393 *yojanas*, the Earth's diameter becomes equal to 1080 *yojanas*. By stating the same again as 1050 (*yojanas*) due to uncertainty of his mind, he (ie, Aryabhata I) has exposed his knowledge !"

Rsine - Differences

Stanza 12:
"225, 224, 222, 219, 215, 210, 205, 199, 191, 183, 174, 164,154, 143, 131, 119, 106, 93, 79, 65, 51, 37, 22, and 7- these are the Rsine.differences (at intervals of 225 minutes of arc) in terms of minutes of arc."

- The following table gives the Rsines and the Rsine-differences at intervals of 225' (or 3° 45') according to Aryabhata I and the corresponding modern values correct to three decimal places.

Table 10. Rsine and Rsine-differences at the intervals of 225' or 3°45'

Aruabhata I's values			Modern values	
Arc	**Rsine**	**Rsine-differences**	**Rsine**	**Rsine-differences**
225'	225'	225'	224'.856	224'.856
450'	449'	224'	448'.449	223'.893
674'	671'	222'	670'.720	221'.971
900'	890'	219'	889'.820	219'.100
1125'	1105'	215'	1105'.109	215'.289
1350'	1315'	210'	1315'.666	210'.557
1575'	1520'	205'	1520'.589	204'.923
1800'	1719'	199'	1719'.000	198'.411
2025'	1910'	191'	1910'.050	191'.050

2250’	2093’	183’	2092’.922	182’.872
2475’	2267’	174’	2266’.831	173’.909
2700’	2431’	164’	2431’.033	164’.202
2925’	2585’	154’	2584’.825	153’.792
3150’	2728’	143’	2727’.549	142’.724
3375’	2859’	131’	2858’.592	131’.043
3600’	2978’	199’	2977’.395	118’.803
3825’	3084’	106’	3083’.448	106’.053
4050’	3177’	93’	3176’.298	92’.850
4275’	3256’	79’	3255’.546	79’.248
4500’	3321’	65’	3320’.853	65’.307
4725’	3372’	51’	3371’.940	51’.087
4950’	3409’	37’	3408’.588	36’.648
5175’	3431’	22’	3430’.639	22’.051
5400’	3438’	7’	3438’.000	7’.361

- The twenty-four Rsines given in the Surya-siddhanta are exactly the same as those in column 2 above. P.C. Sengupta is of the opinion that the author of the Surya-siddhanta has based his Rsines on the Rsine-differences given by Aryabhata I.

- The 16th Rsine, viz., 2978, was modified by Aryabhata II (c. A.D. 950) who replaced it by the better value 2977. The table of Rsines given by Bhaskara II (A.D. 1150) is the same as that of Aryabhata II (c, A.D. 950).

Aim of the *Dasagitika-Sutra*

Stanza 13:
“Knowing the *Dasagitika-sutra*, (giving) the motion of the Earth and the planets, on the Celestial Sphere (Sphere of asterisms or Bhagola), one attains the Supreme Brahman after piercing through the orbits of the planets and stars.”

Ganita or Mathematics

The Ganita section consisting of 33 stanzas deals with mathematics.
The topics dealt with are:

- Geometrical figures and their properties
- Mensuration
- Problems on the shadow of the gnomon
- Series
- Interest
- Simple, simultaneous, quadratic and linear indeterminate equations.
- Arithmetical methods for extracting the square roots and cube roots
- Method of constructing the sine table

Invocation and Introduction

Stanza 1:

“(Having bowed with reverence to Brahma, Earth, Moon, Mercury, Venus, Sun, Mars, Jupiter, Saturn and the asterisms, Aryabhata sets forth before the knowledge honoured at Kusumapura.”

- Commenting on this stanza, Bhaskara I wrote:
    - Kusumapura is Pataliputra. (Aryabhata) sets forth the knowledge honoured there.
    - This is what one hears said:
        - Indeed this Svayambhuva-siddhanta was honoured by the learned people of Kusumapura, although the Paulisa-, Romaka-, Vasistha- and Saurya-Siddhantas were also (known) there. That is why (Aryabhata) says the knowledge is honoured at Kusumapura.

The First Ten Notational Places

Stanza 2:

“Eka (units place), dasa (tens place), fata (hundreds place), sahasra (thousands place), ayuta (ten thousands place), niyuta (hundred thousands place), prayuta (millions place), koti (ten millions place), arbuda (hundred millions place),and vynda (thousand millions place) are, respectively, from place to place, each ten times the preceding.”

- The notational places are denoted by writing zeros as follows:

0 0 0 0 0 0 0 0 0 0

- The zero on the extreme right denotes the unit's place, the next one (on its left) denotes the tens place, the next one denotes the hundreds place, and so on.

Square and Squaring

Stanza 3(a-b):

“(An equilateral quadrilateral with equal diagonals and also the area thereof are called 'square'. The product of two equal quantities is also 'square'.”

- The commentator Parameśvara explains the term *samacaturasra* as follows:
    - "That four-sided figure whose four sides are equal to one another and whose two diagonals are also equal to each other is called a *samacaturasra*."

- By defining a square as the product of two equal quantities the author has stated, by implication, the rule of squaring. That is, to find the square of a number, one should multiply that number by itself.

- The commentator Bhaskara I has given the terms *varga*, *karani, krti, vargana and yavakarana* as synonyms, meaning 'square or squaring'. The term *yavakarana* is derived from the fact that in Hindu algebra x is written as para (va standing for yavat-tavat, i.e., x, and va for *varga*, i.e., square).

- The terms for multiplication according to Bhaskara I are :
    - *Samvarga, ghata, gunana, hatiḥ and udvartana.*

- For the multiplication of equal quantities, Bhaskara I uses a special term, *gata*, meaning literally 'moved'>progressed>raised.

- "*Gunana* is the multiplication (*abhyasa*) of unequal quantities.
- *Gata*", "is the multiplication of equal quantities."
- The term *dvigata*, according to him, means square, trigata means 'cube'; and so on.
- According to this terminology, m^n will be expressed by saying 'nth gata of m', which corresponds to our present-day expression 'nth power of m'. Following the same terminology, the roots have been called gatamula.

Cube and Cubing

Stanza 3(c-d):

“(The continued product of three equals as also the (rectangular) solid having twelve (equal) edges is called a 'cube'.”

- The rule for cubing a number is implied as in the previous case.

- Here, Bhaskara I finds fault with the usual Hindu method of cubing a number for the reason that although it implies the use of the cubes of the digits 1 to 9, it neither states them nor tells how to find them out.

Square Root

Stanza 4:

“(Having subtracted the greatest possible square from the last odd place and then having written down the square root of the number subtracted in the line of the square root) always divide the even place (standing on the right) by twice the square root. Then, having subtracted the square (of the quotient) from the odd place (standing on the right), set down the quotient at the next place (i.e., on the right of the number already written in the line of the square root). This is the square root. (Repeat the process if there are still digits on the right).”

Example : Find the square root of 55,225

Solution : let the odd and even places be denoted as *o* and *e,* respectively.
Further steps -

235
line of square root

o e o e o
5 5 2 5 5

Subtract Square 4

Divide by twice the root

```
4) 1 5 (3
   1 2
   ---
     3 2
```

Subtract square of quotient 32 - 9

Divide by twice the root

```
46) 2 3 2 (5
    2 3 0
    -----
        2 5
        2 5
        ---
          0
```

The process ends. The square root is 235. The remainder being zero, the square root is exact.

- G.R. Kaye's statement that Aryabhata 1's method is algebraic in character and that it resembles the method given by Theon of Alexandria, are, as noted by W.E. Clark, B. Datta and A.N. Singh, incorrect.

Cube Root

Stanza 5:

"(Having subtracted the greatest possible cube from the last cube place and then having written down the cube root of the number subtracted in the line of the cube root), divide the second non-cube place (standing on the right of the last cube place) by thrice the square of the cube root (already obtained); (then) subtract from the first non-cube place (standing on the right of the second non-cube place) the square of the quotient multi- plied by thrice the previous (cube root); and (then subtract) the cube (of the quotient) from the cube place (standing on the right of the first non-cube place) (and write down the quotient on the right of the previous cube root in the line of the cube root, and treat this as the new cube root. Repeat the process if there are still digits on the right)."

- Beginning from the units place, the notational places are called cube place, first non-cube place, second non-cube place, cube place, first non-cube place, second non-cube place, cube place, and so on. Indicating the cube, first non-cube and second non-cube places by c, n and n', their positions may be shown as below:

c n' n c n' n c n' n c n' n c

0 0 0 0 0 0 0 0 0 0 0 0 0

Example : Find the cube root of 17,71,561

Solution :

121

line of cube root

c n' n c n' n c

1 7 7 1 5 6 1

Subtract 1^3	1
Divide by 3.1^2	3) 0 7 (2
	6
	1 7
Subtract $3.1.2^2$	1 2
	5 1
Subtract 2^3	8
Divide by 3.12^2	432) 4 3 5 (1
	4 3 2
	3 6
Subtract $3.12.1^2$	3 6
	0 1
Subtract 1^3	1
	0

The process ends. The required cube root is 121. The remainder being zero, the root is exact.

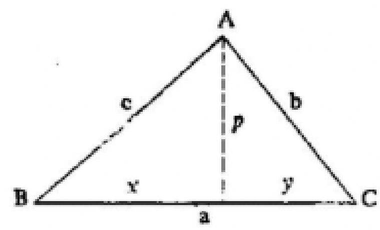
Area of a Triangle

Stanza 6 (a-b):

“The product of the perpendicular (dropped from the vertex on the base) and half the base gives the measure of the area of a triangle.”

- The term *samadalakoṭi* means 'the perpendicular dropped from the vertex on the base of a triangle', .e., 'the altitude of a triangle'. Bhaskara I criticises those who interpret it as meaning 'the upright which bisects the triangle into two equal parts', for, in that case, the above rule will be applicable only to equilateral and isosceles triangles.
- The word phalaśarīra means, according to Bhaskara I, *phala-pramana*, i.e., 'the measure or amount of the area'.
- The above rule is applicable when the base and the altitude of a triangle are known. When the three sides of a triangle are given but the altitude is not known, Bhaskara I gives the following formulae to get the segments of the base (called *abadha* or *abadhantara*) and the altitude:

Fig. 1



(1) $x=\frac{1}{2}\left(a+\frac{c^2-b^2}{a}\right)$

(2) $y=\frac{1}{2}\left(a-\frac{c^2-b^2}{a}\right)$

(3) $p=\sqrt{c^2-x^2}$ or $\sqrt{b^2-y^2}$

- It is remarkable that Bhaskara 1 doesn't mention the formula:

  Area of Triangle = √{s(s-a)(s-b)(s-c)} , 2s = a+b+c ,

  which his contemporary Brahmagupta states in his *Brahma-sphuta-siddhanta.*

Volume of Right Pyramids

Stanza 6 (c-d):

"Half the product of that area (of the triangular base) and the height is the volume of the six-edged solid."

- The rule is based on speculation on the analogy of the area of a triangle, and is inaccurate. The correct formula is found to occur in the *Brahma-sphuta-siddhanta* of Brahmagupta where it is stated as follows:
  - "The volume of a uniform excavation divided by three is the volume of the needle-shaped solid."
  - Simplifying to say, Volume of a cone or pyramid= (area of base) × (height).
- Bhaskara I seems to be unaware of this formula because he has no comment to make on the rule of Aryabhaṭa I. Even the commentators Someśvara and Suryadeva (b. A.D. 1191) have added nothing.

Area of a Circle

Stanza 7 (a-b):

“Half of the circumferences, multiplied by the semi-diameter certainly gives the area of a circle.”

- That is, area of a circle = ½ x circumference x radius.
- The same result in the form, area of a circle = ¼ x circumference x diameter, occurred earlier in the *Tattvärthadhigama-sutra-bhaṣya* of Umasvāti (1st century A.D.) and in the *Bṛhat-kşetra-samāsa* of Jinabhadra Gani (A.D. 609).

Volume of a Sphere

Stanza 7 (c-d):

“That area (of the diametral section) multiplied by its own square root gives the exact volume of a sphere.”

- That is, if r be the radius of a sphere, then, according to Aryabhata I:

  Volume of a sphere = $\pi r^2\sqrt{\pi r^2}$.

- This formula is based on speculation, and, as noted by Bhaskara I,is inaccurate, although called exact by Aryabhaṭa I.The probable rationale of Aryabhaṭa I's formula is as follows:The area of a circle of radius r

  = πr^2,

  = area of a square of side square root of area of circle

  On the analogy of this, Aryabhata I concludes that

  Volume of a sphere of radius r

  $$= \sqrt{\pi r^2} \times \sqrt{\pi r^2} \times \sqrt{\pi r^2}$$
  $$= \pi r^2 \times \sqrt{\pi r^2}.$$

- Although, Bhaskara I quotes the following formula from some earlier work, but he does not give it any credit and regards it as inferior to that given by Aryabhaṭa 1 :

  Volume of a sphere of radius r = $\frac{9}{2} r^3$

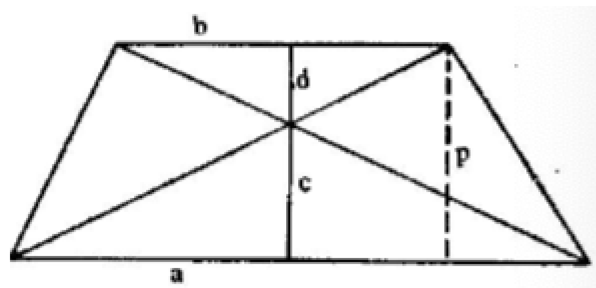
- Noting that Bhaskara 1's contemporary Brahmgupta, who has criticised Aryabhata 1 even for his minute errors, has not been able to make any improvement on Aryabhata's formula for the volume of the sphere.
- More noteworthy is that many mathematicians and astronomers in northern india regarded Aryabhata's formula accurate and went on using it even in the second half of the ninth century A.D.

Area of a Trapezium

Stanza 8:

"(Severally) multiply the base and the face (of the trapezium) by the height, and divide (each product) by the sum of the base and the face ; the results are the lengths of the perpendiculars on the base and the face (from the point of intersection of the diagonals). The results obtained by multiplying half the sum of the base and the face by the height is to be known as the area (of the trapezium)."

- Let a, b be the base and the face, p the height and c, d the lengths of the perpendiculars of the perpendiculars on the base and the face from the point where the diagonals intersect. Then,



$$c = \frac{ap}{a+b}$$

$$d = \frac{bp}{a+b}$$

$$\text{area} = \tfrac{1}{2}(a+b)\,p.$$

- The term '*ayama*' means breadth denotes the height of the trapezium.
- The term '*vistara*' means length denotes the face and base of the trapezium.
- The term '*vistarayogardha*' means half the sum of the base and the face.

- The term ‘*parsve’* means two sides of the trapezium lying on the two sides of the height, evidently they are the base and the face.

Area of Plane Figures

Stanza 9 (a-b):

“In the case of all the plane figures, one should determine the adjacent sides (of the rectangle into which that figure can be transformed) and find the area by taking their product.”

- According to Bhaskara 1, this rule is to calculate both the area and verifying the area of a plane figure.
- Bhaskara 1 puts up a doubt - “Now, the word all means 'everything without exception'; so, here, all (plane) figures are included. The area of all (plane) figures being thus determined by this rule, the statement of the previously stated rules becomes useless”.

  Answer being - This is not useless. Both the verification and the calculation of the areas are taught by this rule. The areas of the previously defined figures have to be verified. The mathematicians Maskari, Purana and Putana prescribe the verification of all the (plane) figures into a rectangular figure. Hence it was stated that -

  ‘Having determined the area in accordance with the prescribed rule, verification should always be made by (transforming the plane figure into) a rectangle, because it is only of the rectangle that the area is obvious.’

  ‘The determination of the area of the (plane) figures which have not been mentioned above is possible only by transforming them into rectangles.’

- There is no doubt that the above rule states clearly that all the plane figures can be transformed into rectangles.
- Bhaskara 1 has also found the area of the triangle, a quadrilateral, a drum-shaped figure, and a figure resembling the tusk of an elephant by transforming them into rectangles in his commentaries.

Chord of One-Sixth Circle

Stanza 9 (c-d):

“The chord of one-sixth of the circumference (of a circle) is equal to the radius.”

- Formula being -

$$\text{chord } 60° = R,$$

$$\text{or } R\sin 30° = R/2$$

Circumference-Diameter Ratio

Stanza 10:

“100 plus 4 , multiplied by 8, and added to 62,000 : this is the nearly approximate measure of the circumference of a circle whose diameter is 20,000.”

- This gives,

$$\pi = \text{circumference / diameter}$$

$$= 62832 / 20000$$

$$= 3.1416$$

- The value of pi does not occur in any earlier works of mathematics and hence is a very important contribution of Aryabhata 1.
- Noting that Aryabhata called the above value appropriate and hence the value still is used exactly described.

Computation of RSine Table geometrically

Stanza 11:

“Divide a quadrant of the circumference of a circle (into as many parts as desired). Then, from (right) triangles and quadrilaterals, one can find as many Rsines of equal areas as one likes, for any given radius.”

- As following Bhaskara 1, we explain the method using examples -

Example 1. Find second RSine at intervals of 15° in a circle of radius 3438'.

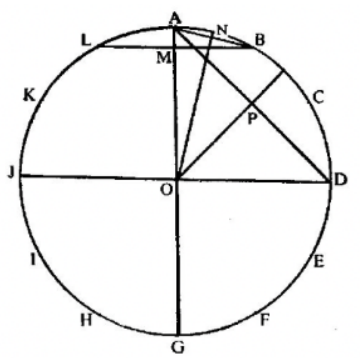
Solution: Let the following figure represent a circle of radius R = 3438'.

Divide its circumference into twelve equal parts by the points A, B, C, D, E, F,......., L.

Join BL.

This is equal to R and denotes chord 60°.

Half of this, i.e., MB, is Rsin 30°. Thus Rsin 30° = R/2 = 1719', This is second RSine.



By calculations, we get six Rsinees -

Rsin 15° = 890'

Rsin 30° = 1719'

Rsin 45° = 2431'

Rsin 60° = 2978'

Rsin 75° = 3321'

Rsin 90° = 3438'

After Calculation Analysis

Verse 9 (c-d) gives the second Rsine. This yields the first and the fourth Rsines. The first Rsine yields the fifth Rsine. The fourth and the fifth Rsines do not yield any other Rsines. This process ends here.

Again, the radius is the sixth Rsine. It yields the third Rsine. The third Rsine being odd, does not yield any further Rsine. So this process also ends.

Thus, from the second and the sixth Rsines, one gets all the six desired Rsines.

Derivation of RSine Difference

Stanza 12:

“The first RSine divided by itself and then diminished by the quotient gives the second Rsine - difference. The same first Rsine diminished by the quotients obtained by dividing each of the preceding RSines gives the remaining Rsine - differences.”

- Let $R_1, R_2, \ldots\ldots, R_{24}$ denote the twentyfour Rsines and δ_1 $(=R_1)$, $\delta_2, \delta_3, \ldots\ldots\ldots, \delta_{24}$ denote the twenty four Rsine-differences. Then, according to the above rule,

$$\delta_2 = R_1 - (R_1 / R_1) \qquad \ldots\ldots\ldots..(1)$$

$$\delta_{n+1} = R_1 - \{(R_1 + R_2 \ldots\ldots..+ R_n)/R_1\} \qquad \ldots\ldots\ldots(1)$$

- The above translation is based on Prabhakara's interpretation of the text. The same interpretation is given by the commentators Someśvara, Suryadeva (b. 1191 A. D.), Yallaya (1480 A. D.) and Raghunatha-rāja (1597 A.D.). It is interesting to note that this interpretation is also in agreement with the rule stated in the Surya-siddhānta (ii. 15-16), as interpreted by the commentator Ranganatha (1603 A.D.), viz..

$$R_{n+1} = R_n - R_1 - \{(R_1 + R_2 \ldots\ldots..+ R_n)/R_1\}$$

- Datta and Singh, following the commentator Parameśvara (1431 A.D.), have translated the text as follows:

  "The first Rsine divided by itself and then diminished by the quotient will give the second difference. For computing any other difference, (the sum of) all the preceding differences is divided by the first Rsine and the quotient is subtracted from the preceding difference. Thus, all the remaining differences (can be calculated)."

  That is,

$$\delta_2 = R_1 - (R_1 / R_1) \qquad \ldots\ldots\ldots(2)$$

$$\delta_{n+1} = \delta_n - \{(\delta_1 + \delta_2 \ldots\ldots..+ \delta_n)/R_1\} \qquad \ldots\ldots\ldots(2)$$

$$\text{or} \quad \delta_n - (R_n / R_1)$$

  This is also how the commentator Someśvara seems to have interpreted the text.

  One can easily see that (1) and (2) are equivalent.

- The commentator Nilakantha (c. 1500 A.D.) interprets the text as follows:

"The first Rsine divided by itself and then diminished by the quotient gives the second Rsine-difference. To obtain any other Rsine-difference, divide the preceding Rsine by the first Rsine and multiply the quotient by the difference between the first and second Rsine-differences and subtract the resulting product from the preceding Rsine-difference."

That is,

$$\delta_2 = R_1 - (R_1 / R_1) \qquad \text{..........(3)}$$

$$\delta_{n+1} = \delta_n - \{(R_n/R_1)(\delta_1-\delta_2)\} \qquad \text{..........(3)}$$

- This is the accurate form of the formula and reduces to the previous form because, according to Aryabhata 1,

$$\delta_1 - \delta_2 = 225 - 224 = 1$$

Construction of Circle, etc., and Testing of Level and Verticality

Stanza 13:

“A circle should be constructed by means of a pair of compasses; a triangle and a quadrilateral by means of the two hypotenuses (karna). The level of the ground should be tested by means of water; and verticality by means of a plumb.”

- The two hypotenuses (*karnas*)
    - In the case of triangle are the two lateral sides above the base
    - In the case of rectangle are the two diagonals
    - In the case of trapezium are the two lateral sides.
- The reference is to the usual methods constructing a triangle when
    - The three sides (i.e., the base and the two lateral sides) are given
    - A parallelogram, when one side and two diagonals are given
    - A trapezium, when the base, height and the two lateral sides (called hypotenuse) are given.
- Bhaskara 1 observes that “In no wind, when a jar full of water is placed upon tripod of the ground which has been made plane by means of eye or thread, and a bore hole, so that water may have continuous flow, when the water falling on the ground spreads in a circle, the ground is in perfect level; and the water accumulates after departing from the circle of the water, there it is low, and where water doesn't reach”

Radius of the Shadow Sphere

Stanza 14:

“Add the square of the height of the gnomon to the square of its shadow. The square root of that sum is the semi-diameter of the circle of the shadow.”

- "The semi-diameter of the circle of shadow is taken here", the commentator Bhaskara I says that in order to accomplish the rule of three, viz. if these are the values of the gnomon and the shadow corresponding to the radius of the circle of shadow, the question is, what will correspond to the radius of the celestial sphere. Thus the Rsines of the Sun's altitude and zenith distance are obtained. At an equinox, these are called the Rsines of colatitude and latitude (respectively).
- As regards the shape of a gnomon, Bhaskara I informs us that the Hindu astronomers differed from one another. Some took a gnomon with one third at the bottom of the shape of a right prism on a square base, one third in the middle of the shape of a cylinder, and one third at the top of the shape of a cone. Others took a gnomon of the shape of a right prism on a square base.
- The followers of Aryabhata I, writes Bhaskara I, preferred a cylindrical gnomon, made of excellent timber, free from holes, knots and scars, with large diameter and height. In order to get a prominent tip of the shadow, a cylindrical needle (of height greater than the radius of the gnomon) made of timber or iron was fixed vertically at the top of the gnomon in the middle. Such a gnomon being large and massive was unaffected by the wind; being cylindrical, it was easy to manufacture; being surmounted by a needle of small diameter, the tip of the shadow was easily perceived.
- A gnomon was generally divided into 12 equal parts called angulas, but, according to Bhaskara I, there was no such hard and fast rule. A gnomon could be of any length with any number of divisions.

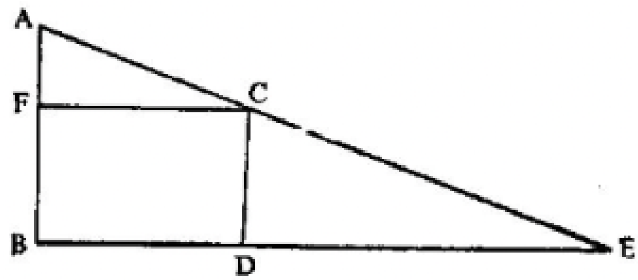
Gnomonic Shadow due to a Lamp - Post

Stanza 15:

“Multiply the distance between the gnomon and the lamp-post (the latter being regarded as base) by the height of the gnomon and divide (the product) by the difference between (the heights of) the lamp-post (base) and the gnomon. The quotient (thus obtained) should be known as the length of the shadow measured from the foot of the gnomon.”

- In the following figure,
    - let AB be the lamp-post, CD the gnomon
    - E the point where AC and BD produced meet.

- Then DE is the shadow cast by the gnomon due to light from the lamp A
- Let FC be parallel to BD
- Then, comparing the similar triangles CDE and AFC, we have



$$DE = (FC \times CD)/AF$$

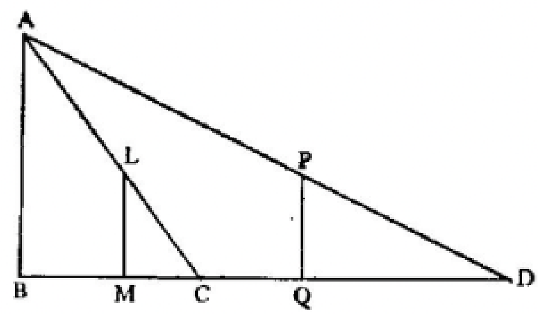
$$= (BD \times CD)/(AB-CD)$$

Tip of the Gnomonic Shadow from the lamp-post and Height of the latter

Stanza 16:

"(When there are two gnomons of equal height in the same direction from the lamp-post), multiply the distance between the tips of the shadows (of the two gnomons) by the (larger or shorter) shadow and divide by the larger shadow diminished by the shorter one: the result is the upright (i.e., the distance of the tip of the larger or shorter shadow from the foot of the lamp- post). The upright multiplied by the height of the gnomon and divided by the (larger or shorter) shadow gives the base (i.e., the height of the lamp-post)."

- In the following figure,
  - let AB be the lamp-post (BASE),
  - BC or BD is the upright
  - LM and PQ are the gnomons of equal height



We have,

$$AB/PQ = BD/QD \qquad (i)$$

$$AB/LM = BC/MC \qquad (ii)$$

Since PQ = LM, therefore,

$$BD/QD = BC/MC = CD/(QD-MC) \qquad (iii)$$

Hence from (iii), (i) and (ii), we have,

$$BD = (CDXQD)/(QD-MC) \qquad (1)$$

$$BC = (CDXMC)/(QD-MC) \qquad (2)$$

and $$AB = (BDXPQ)/QD = (BCXLM)/MC \qquad (3)$$

Theorems on Square of Hypotenuse and on Square of Half-Chord

Stanza 17:

“(In a right-angled triangle) the square of the base plus the square of the upright is the square of the hypotenuse.”

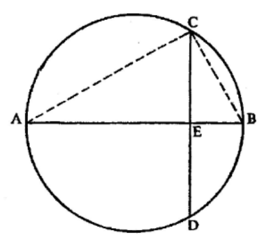
“In a circle (when a chord divides it into two arcs), the product of the arrows of the two arcs is certainly equal to the square of half the chord.”

- Of the two theorems stated above,
    - the first one is "the theorem of the Square of the Hypotenuse", as Hankel has called it. This theorem has been known in India since very early times. Baudhayana (c. 800 B.C.), the author of the Baudhayana-śulba-sutra, has enunciated it thus :

      "The diagonal of a rectangle produces both (areas) which its length and breadth produce separately."

      This theorem is now universally associated with the name of the Greek Pythagoras (c. 540 B.C.), though "no really trustworthy evidence exists that it was actually discovered by him." It was certainly the Hindus who enunciated the property of the right-angled triangle in its most general form. No other ancient nation is known to have made any attempt in this direction.

    - The second theorem states that if, in a circle, a chord CD and a diameter AB intersect each other at right-angles at E, then



$AE \times EB = CE^2$

This result easily follows from the comparison of the similar triangles CAE and CEB.

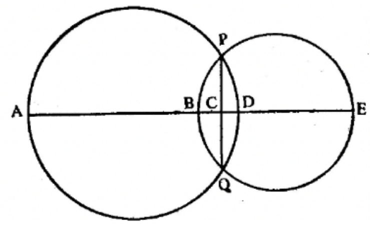
This second theorem occurs earlier in the works of Umasvati (1st century A.D.). It has been mentioned by Jinabhadra Gani (A.D. 609) and brahmagupta (A.D. 628) also.

Arrows of Intercepted Arcs of Intersecting Circles

Stanza 18:

"(When one circle intersects another circle) multiply the diameters of the two circles each diminished by the erosion, by the erosion and divide (each result) by the sum of the diameters of the two circles after each has been diminished by the erosion then are obtained the arrows of the arcs (of the two circles) intercepted in each other."

- Let two circles intersect at P and Q and let ABCDE be the line passing through the centres of the two circles. Then BD is the erosion (*grāsa*), and BC, CD are the arrows of the intercepted arcs.



  - The rule states that,

    BC = [{(AD - BD).BD}/(AD-BD)+(BE-BD)] ………(1)

    and CD = [{(BE - BD).BD}/(AD-BD)+(BE-BD)] ………(2)

  - These formulae may be derived as follows:

    Since AC x CD = BC x CE, therefore

    (AC - BD + BC)(BD - BC) = BC(BE-BC) ………(3)

    (AC - CD)CD = (BD - CD)(BE - BD + CD) ………(4)

    Solving (3) for BC, we get (1), and solving (4) for CD, we get (2).

Sum (or partial sum) of a series in A.P.

Stanza 19:

"Diminish the given number of terms by one, then divide by two, then increase by the number of the preceding terms (if any), then multiply by the common difference, and then increase by the result is the the first term of the (whole) series: the result arithmetic mean (of the given number of terms). This multi- plied by the given number of terms is the sum of the given terms. Alternatively, multiply the sum of the first and last terms (of the series or partial series which is to be summed up) by half the number of terms."

- Let an arithmetic series be

  a + (a+d) + (a+2d) + ……..

- The rule says that
    - The arithmetic mean of the n terms

$$(a+pd) + \{a+(p+1)d\} + \ldots\ldots.. + \{a+(p+n-1)d\} = a + [\{(n-1)/2\} + p]d$$

    - The sum of the n terms

$$(a+pd) + \{a+(p+1)d\} + \ldots\ldots.. + \{a+(p+n-1)d\} = n\,[a + (\{(n-1)/2\} + p)d]$$

In particular (when p = 0)

- The arithmetic mean of the series

$$a + (a+d) + \ldots.. +\{a+(n-1)d\} = a + \{(n-1)/2\}d$$

- The sum of the series

$$a + (a+d) + \ldots.. +\{a+(n-1)d\} = n\,[a + \{(n-1)/2\}d]$$

Number of terms of a series in A.P.

Stanza 20:

"The number of terms (is obtained as follows): Multiply (the sum of the series) by eight and by the common difference, increase that by the square of the difference between twice the first term and the common difference, and then take the square root; then subtract twice the first term, then divide by the. common difference, then add one (to the quotient), and then divide by two."

- Let S be the sum of the series

$$a + (a+d) + (a+2d) + (a+3d) + \ldots\ldots\ldots\ldots\ldots\ldots \text{ to n terms.}$$

Then

$$x = \frac{1}{2}\left[\frac{\sqrt{8dS + (2a-d)^2} - 2a}{d} + 1\right]$$

$$n = \frac{1}{2}\left[\frac{\sqrt{8dS + (2a-d)^2} - 2a}{d} + 1\right].$$

Sum of the series 1+(1+2)+(1+2+3)+........ To N Terms

Stanza 21:

"Of the series (*upaciti*) which has one for the first term and one for the common difference, take three terms in continuation, of which the first is equal to the given number of terms, and find their continued product. That (product), or the number of terms plus one subtracted from the cube of that, divided by 6, gives the citighana."

- The term upaciti or citi is used in the sense of a series in general. The series 1+2+3+ n, which has one for the first term and one for the common difference, is called *ekottarādi-upaciti.* The sum of this series is generally called *sankalita*. Bhaskara I call it *sankalanā.*
- The term citighana is used in the sense of the sum of the series

$$1+(1+2)+(1+2+3)+........ \qquad (1)$$

- to any number of terms. This sum is generally called *sankalita-sankalita*. Bhaskara I have called it s*ankalana-sankalanā*.
- The above rule gives the sum to n terms of the series (1) in two forms:
    - $\{n(n + 1)(n+2)\} / 6$
    - $\{(n+1)^3—(n+1)\} / 6$
- The term *citighana* literally means 'the solid contents of a pile (of balls) in the shape of a pyramid on a triangular base'. The pyramid is so constructed that there is 1 ball in the topmost layer, 1+2 balls in the next lower layer, 1+2+3 balls in the further next lower layer, and so on. In the nth layer, which forms the base, there are

1+2+3+...+n balls.

- The number of balls in the solid pyramid,

i.e., *citighana* $S_1 + S_2 + ... + S_r + ... + S_n$

where

$$S_r=1+2+3+...+r.$$

- The base of the pyramid is called *upaciti*, so

$$\textit{upaciti}\ 1+2+3+...+n.$$

Sum of the series ΣN^2 and ΣN^3

Stanza 22:

"The continued product of the three quantities, viz., the number of terms plus one, the same increased by the number of terms, and the number of terms, when divided by 6 gives the sum of the series of squares of natural numbers (vargacitighana). The square of the sum of the series of natural numbers (citi) gives the sum of the series of cubes of natural numbers *(ghanacitighana*)."

- The term *vargachtighana* is used in the sense of the sum of the series

$$1^2+2^2+3^2+.........+ n^2,$$

i.e., the sum of the series of squares of natural numbers; and the term *ghanacitighana* is used in the sense of the sum of the series

$$1^3+2^3+3^3+.........+n^3,$$

i.e., the sum of the series of cubes of natural numbers. Bhaskara I has called these sums by the terms *vargasankalanā* and *ghanasankalana*, respectively. Other mathematicians have called them *vargasankalita* and *ghanasankalita*, respectively. According to the above rule

$$1^2+2^2+3^2+.........+ n^2 = \{n(n+1)(2n+1)\} / 6$$

$$1^3+2^3+3^3+.........+ n^3 = (1+2+3+.....+n)^2 = [\{n(n+1)\}/2]^2$$

- The term *vargacitighana* literally means 'the solid contents of a pile (of balls) in the shape of a pyramid on a square base'. It is so constructed that there is 1 ball in the topmost layer, 2^2 balls in the next lower layer, 3^2 balls in the further next lower layer, and so on, nth layer from the top, which forms the base of the pile, n^2 balls.
- The term *ghanacitighana* similarly means 'the solid contents of a pile (of cuboidal bricks) in the shape of a pyramid having cuboidal layers'. It is so constructed that there is 1 brick in the topmost layer, 2^3 bricks in the next lower layer, 3^8 bricks in the further next lower layer, and so on. In the nth layer (from the top), which forms the base of the pile, there are n^3 bricks, n bricks in each edge of the cuboidal base.

Product of Factors from their Sum and Squares

Stanza 23:

"From the square of the sum of the two factors subtract the sum of their squares. One- half of that (difference) should be known as the product of the two factors."

- That is,

$$A \, x \, B \;=\; \frac{(A+B)^2 - (A^2+B^2)}{2}$$

$$A \times B = \frac{(A+B)^2 - (A^2+B^2)}{2}.$$

Quantities from their Difference and Product

Stanza 24:

"Multiply the product by four, then add the square of the difference of the two (quantities), and then take the square root. (Set down this square root in two places). (In one place) increase it by the difference (of the two quantities), and (in the other place) decrease it by the same. The results thus obtained, when divided by two, give the two factors (of the given product)."

- That is, if

$$x - y = a$$

$$xy = b$$

Then,

$$x \;=\; \frac{\sqrt{4b + a^2} + a}{2}$$

$$y \;=\; \frac{\sqrt{4b - a^2} + a}{2}$$

Interest on Principal

Stanza 25:

"Multiply the interest on the principal plus the interest on that interest by the time and by the principal; (then) add the square of half the principal; (then) take the square root; (then) subtract half the principal; and (then) divide by the time: the result is the interest on the principal."

- Taking a problem and understanding the rule -

  The problem envisaged is : A principal P is lent out at a certain rate of interest per month. At the expiry of one month, the interest 'I' which accrues on P in one month is given on loan at the same rate of interest for T months. After T months 'I' amounts to A. The problem is to find 'I' when A is given.

  Solution:

  $$I = \frac{\sqrt{PTA + (P/2)^2} - (P/2)}{T}.$$ as stated in the rule.

Rule of Three

Stanza 26:

"In the rule of three, multiply the 'fruit' (*phala*) by the 'requisition' (*iccha*) and divide the resulting product by the 'argument' (*pramana*). Then is obtained the 'fruit corresponding to the requisition' (*icchaphala*)."

- Understanding the rule by an example -

  Example 1. If A books cost P rupees, what will R books cost ?

  Solution : Here A is the 'argument', P the 'fruit' and R the 'requisition'.

  So the answer will be

  P x R / A rupees

  Example 2. If the interest on Rs. 100 for 2 months is Rs. 5, find the interest on Rs. 25 invested for 8 months.

Solution : Here we have two arguments, viz, Rs. 100 and 2 months ; and two requisitions viz., Rs. 25 and 8 months. The fruit is Rs. 5. So the required will be

$$25 \times 8 \times 5 / 100 \times 2 \quad \text{or} \quad 5 \text{ rupees}$$

Simplification of the Quotients of Fractions

Stanza 27(a-b):

“The numerators and denominators of the multipliers and divisors should be multiplied by one another.”

- Understanding with an example -
  - $(a/b) / (c/d) = ad/bc$
  - $(ac/bd) / (eg/fh) = (ac)(fh)/(bd)(eg)$

Reduction of Two Fractions to a Common Denominator

Stanza 27(c-d):

“Multiply the numerator as also the denominator of each fraction by the denominator of the other fraction; then the (given) fractions are reduced to a common denominator.”

- That is,

$$a/b + c/d = ad/bd + bc/bd = (ad + bc)/bd,$$

$$a/b - c/d + ad/bd - bc/bd = (ad-bc)/bd$$

Example. Add $\frac{1}{2}$, $\frac{1}{6}$ and $\frac{1}{3}$.

Solution : $\frac{1}{2} + \frac{1}{6} = 6/12 + 2/12 = 6+2 / 12 = \frac{2}{3}$

Now adding $\frac{2}{3}$ and $\frac{1}{3}$, we get

$$\frac{2}{3} + \frac{1}{3} = 2+1 / 3 = 1$$

Method of Inversion

Stanza 28:

"In the method of Inversion multipliers become divisors and divisors become multipliers, additive becomes subtractive and subtractive becomes additive."

- Understanding the rule by example -

  Example. A number is multiplied by 2 ; then increased by 1 ; the divided by 5 ; then multiplied by 3 ; then diminished by 2 ; and then divided by 7 ; the result (thus obtained) is 1. Say what is the initial number.

  Solution : Starting from the last number 1, in the reverse order, inverting the operations, the result is

$$1 \text{ x } 7, + 2, \% \ 3, \text{ x } 5, - 1, \% \ 2, \text{ i.e. } 7.$$

Unknown Quantities from Sums of All but One

Stanza 29:

"The sums of all (combinations of) the (unknown) quantities except one (which are given) separately should be added together; and the sum should be written down separately and divided by the number of (unknown) quantities less one : the quotient thus obtained is certainly the total of all the (unknown) quantities. (This total severally diminished by the given sums gives the various unknown quantities)."

- That is, if

$$(x_1 + x_2 + \ldots\ldots\ldots\ldots\ldots + x_n) - x_1 = a_1$$

$$(x_1 + x_2 + \ldots\ldots\ldots\ldots\ldots + x_n) - x_2 = a_2$$

$$\ldots\ldots\ \ldots\ldots\ldots\ \ldots\ldots\ldots\ \ldots\ldots\ldots\ \ldots\ldots$$

$$(x_1 + x_2 + \ldots\ldots\ldots\ldots\ldots + x_n) - x_n = a_n$$

  then

$$x_1 + x_2 + \ldots\ldots\ldots\ldots\ldots + x_n = (a_1 + a_2 + \ldots\ldots\ldots + a_n) / (n-1),$$

  so that

$$x_1 = \{(a_1 + a_2 + \dots\dots\dots + a_n) / (n-1)\} - a_1$$

$$x_2 = \{(a_1 + a_2 + \dots\dots\dots + a_n) / (n-1)\} - a_2$$

……. ……… ……… ……… ……. ……

$$x_n = \{(a_1 + a_2 + \dots\dots\dots + a_n) / (n-1)\} - a_n$$

where

$x_1 + x_2 + \dots\dots\dots + x_n$ are the unknown quantities

$a_1 + a_2 + \dots\dots\dots + a_n$ are the given sums

Unknown Quantities from Equal Sums

Stanza 30:

"Divide the difference between the *rupakas* with the two persons by the difference between their *gulikas.* The quotient is the value of one *gulika*, if the possessions of the two persons are of equal value."

- The rule teaches us to find the value of one *gulika* in terms of *rūpakas*.

  Example. Two people are equally rich. Of them, one possesses *a gulikas* and *b rupakas* (coins), and the other possesses *c gulikas* and *d rupakas*. Find the value of one *gulika* in terms of *rūpakas*.

  Solution : Algebraically, if

  $$ax+b=cx+d,$$

  then

  $$x = (d-b) / (a-c)$$

- Bhaskara 1 (629 A.D.) described the term *gulika* stands for 'a thing of unknown value.
- *Gulika* and *yavattavat* (commonly used in Hindu algebra for an unknown value) are used as synonyms.
- Hence Bhaskara I wrote that "These very *gulikas* of unknown value are called *yavattavat.*"
- Bhaskara 1 describes the term *rūpaka* a coin.

Meeting of Two Moving Bodies

Stanza 31:

"Divide the distance between the two bodies moving in the opposite directions by the sum of their speeds, and the distance between the two bodies moving in the same direction by the difference of their speeds; the two quotients will give the time elapsed since the two bodies met or to elapse before they will meet."

- The following cases may arise :
  - Case 1. When the two bodies are moving in the opposite directions.

  If the bodies are facing each other, i.e., if they have not already met, the distance between them when divided by the sum of their velocities will give the time to elapse before they meet.

  If the bodies have already met and moving away from each other, the distance between them when divided by the sum of their velocities will give the time elapsed since they met each other.

  - Case 2. When the two bodies are moving in the same direction.

  If the fast-moving body is behind, i,e., if they have not already met, the distance between them when divided by the difference of their velocities will give the time to elapse before they meet.

  If the slow-moving body is behind, i.e., if they have already met, the distance between them when divided by the difference of their velocities will give the time elapsed since they met each other.

Pulveriser

- **Residual Pulveriser**

  Stanza 32-33:

"Divide the divisor corresponding to the greater remainder by the divisor corresponding to the smaller remainder. (Discard the quotient). Divide the remainder obtained (and the divisor) by one another (until the number of quotients of the mutual division is even and the final remainder is small enough). Multiply the final remainder by an optional number and to the product obtained add the difference of the remainders (corresponding to the greater and smaller divisors; then divide this sum by the last divisor of the mutual division.

The optional number is to be chosen so that this division is exact. Now place the quotients of the mutual division one below the other in a column; below them write the optional number and underneath it the quotient just obtained. Then reduce the chain of numbers which have been written down one below the other, as follows): Multiply by the last but one number (in the bottom) the number just above it and then add the number just below it (and then discard the lower number). (Repeat this process until there are only two numbers in the chain). Divide (the upper number) by the divisor corresponding to the smaller remainder, then multiply the remainder obtained by the divisor corresponding to the greater remainder, and then add the greater remainder: the result is the dvicched agra (i.e., the number answering to the two divisors). (This is also the remainder corresponding to the divisor equal to the product of the two divisors)."

- The rule stated is meant for solving a residual pulveriser (*sāgra-kuṭṭakara*), i.e., a problem of the following type:

  > A number leaves 1 as the remainder when divided by 5, and 2 (as the remainder) when divided by 7. Calculate what that number is.

- It may be pointed out that when the quotients of the mutual division are odd in number, the difference of the greater and smaller remainders is subtracted from the product of the last remainder of the mutual division and the optional number.
- To illustrate the above rule, we solve the following example.

  Example. Find the number which yields 5 as the remainder when divided by 8, 4 as the remainder when divided by 9, and 1 as the remainder when divided by 7.

  Solution :

	(i)	(ii)	(iii)
Remainder	5	4	1
Divisor	8	9	7

  To begin with, we apply the process of the pulveriser on the first two pairs of remainder and divisor, viz., (i) and (ii).

  - ➢ Dividing 8 (the divisor corresponding to the greater remainder) by 9 (the divisor corresponding to the smaller remainder)
  - ➢ we get 8 as the remainder and 0 as the quotient
  - ➢ We discard the quotient 0 and divide the remainder 8 and the divisor 9 mutually until there are even number of quotients and the final remainder is small :

8) 9 (1

8

1) 8 (8

<u>8</u>

0

- We choose 1 as the optional number and multiply the remainder 0 by it and add 1 (the difference of the greater and smaller remainders) to it.
- The result is 1. Dividing this 1 by 1 (the final divisor of the mutual division), the quotient obtained is 1.
- Now, we write the quotients of the mutual division, viz., 1 and 8 one below the other and below them the optional number 1 and then the quotient 1 just obtained.

Thus we get

1

8

1

1

- Reducing this chain, we successively get

1	1	10
8	9	9
1	1	
1		

- Now dividing the upper number 10 by 9 (the divisor corresponding to the smaller remainder), we get 1 as the remainder. Multiplying this 1 by 8 (the divisor corresponding to the greater remainder), we get 8. Adding the greater remainder 5 to this 8, we get 13.
- This 13 is obviously the number which is divided by 8 leaves 5 as remainder and divided by 9 leaves 4 as remainder. This number is called *dvicchedägra* because it answers to two divisors.
- This 13 is also the remainder corresponding to the divisor 8x9 (i.e., 72).
- We now apply the process of the pulveriser on the following pairs of remainder and divisor

	(i)	(ii)
Remainder	13	1
Divisor	72	7

- ➢ Proceeding as above, we get 85. This is called *tricchedagra* because this answers to three divisors (viz., those in the example). One can easily see that 85 leaves 5 as remainder when divided by 8, 4 as remainder when divided by 9, and 1 as remainder when divided by 7.
- ➢ This is the least integral solution of the problem. The general solution is 8 x 9 x 7 m + 85, i.e., 504 m + 85, m = 0, 1, 2, 3, ……

- **Non-Residual Pulveriser**

Stanza 32-33:

"Divide the greater number (denoting the divisor) bt the smaller number (denoting the dividend) (and by the remainder obtained the smaller number and so on. Dividing the greater and the smaller numbers by the last non-zero remainder of the mutual division, reduce them to their lowest terms.) Divide the resulting numbers mutually (until the number of quotients of the mutual division is even and the final remainder is small enough). Multiply the final remainder by an optional number and to the product obtained add the (given) additive (or subtract the subtractive).2 (Divide this sum or difference by the last divisor of the mutual division. The optional number is so chosen that this division is exact. Now place the quotients of the mutual division one below the other in a column; below them write the optional number and underneath it the quotient just obtained. Then reduce this chain of numbers as follows). Multiply by the last but one number (in the bottom) the number just above it and then add the number just below it (and then discard the lower number). (Repeat this process until there are only two numbers in the chain). Divide (the upper number by the abraded greater number and the lower number) by the abraded smaller number. (The remainders thus obtained are the required values of the unknown multiplier and quotient)."

- The commentators Bhaskara I, Suryadeva and others have also interpreted vss. 32-33 as a rule for solving a non-residual pulveriser (*niragra-kuṭṭākāra*), i.e., a problem of the following type:

  > 11 is multiplied by a certain number, the product is diminished by 3, and the difference thus obtained being divided by 23 is found to be exactly divisible. Find the multiplier and the quotient.

- In the above translation, the word *agra* has been taken to mean 'number' and *agrântara* to mean 'the given additive or subtractive'.
- The operations mentioned in *adhikāgracchedaguṇaṁ dvicchedāgram adhikāgrayutam* are, as remarked by the commentator Süryadeva, not needed in the case of a non-residual pulveriser. He wrote: "*Adhi- kāgracchedaguṇam ityādi niragrakuṭṭakāreṣu nopayujyate*."
- This rule might be illustrated by an example:

  Example. Solve

$$(16 x-138) / 487 = y$$

Solution :

- ➢ Here, the divisor = 487, dividend = 16 and subtractive = 138. Since 487 and 16 are already prime to each other, we proceed with their mutual division. The mutual division runs as follows:

16) 487 (30

<u>480</u>

7) 16 (2

<u>14</u>

2 x 76 - 138 = 14 (2

<u>14</u>

0

- ➢ The chain of the quotients of the mutual division, the optimal number and the final quotient is reduced as follows:

30	30	4696
2	154	154
76	76	
2		

- ➢ Dividing 4696 by the divisor 487, the remainder is 313: this is the value of x.
- ➢ Dividing 154 by the dividend 16, the remainder is 10: this is the value of y.
- ➢ Hence,

$$x = 313$$

$$y = 10$$

This is the least integral solution of the problem.

➢ The general solution will be

$$x = 487\,\lambda + 313$$

$$y = 16\,\lambda + 10$$

where, $\lambda = 0, 1, 2, 3\ldots\ldots$

- The commentator Somesvara, does not interpret the text in a different way, rather he interprets a non-residual pulveriser itself as a residual pulveriser. Thus he interprets the non-residual pulveriser

$$(ax - c) / b = y$$

as the residual pulveriser

$$N = by + c = ax + 0$$

in which

c is the *adhikāgra* (i.e., the greater remainder),

b is the *adhikāgrabhāgahāra* (i.e., divisor corresponding to the greater remainder),

0 is the *unōgra* (i.e., the smaller remainder), and

a is the (*ūnāgrabhāgahāra*, i.e., the divisor corresponding to the smaller remainder).

The Kalakriya or The Reckoning of Time

The aim of this section is to teach theoretical astronomy as far as the determination of true positions of the planets is concerned.

The section focuses on the explaining the following statements -

- Time and Circular Divisions
- Conjunctions of Two Planets in a *Yuga*
- *Vyatipatas* in a *Yuga*
- Anomalistic and Synodic Revolutions
- Jovian Years in a *Yuga*
- Solar, Lunar, Civil and Sidereal Years
- Intercalary Months and Omitted Lunar Days
- Days of Men, Manes Gods and of *Brahma*
- *Utsarpini, Apsarpini, Susama* and *Dussama*
- Date of Aryabhata 1
- Beginning of the *Yuga*, Year, Month, and Day
- Equality of the Linear Motion of the Planets
- Consequence of Equal Linear Motion of the Planets
- Non-Equality of the Linear Measures of the Circular Divisions
- Relative Positions of Asterisms and Planets
- Lords of the Hours and Days
- Motion of the Planets Explained through Eccentric Circles
- Motion of the Planets explained through Epicycles
- Motion of Epicycles
- Addition and Subtraction of *Mandaphala* and *Sighraphala*
- A special pre-correction of the Superior Planets
- Procedure of *Mandaphala* and *Sighraphala* corrections for Superior Planets
- *Mandaphala* and *Sighraphala* corrections for Inferior Planets
- Distance and Velocity of a Planet.

The section intotal contains 25 stanzas which describe, calculate and explain the above statements in detail.

Gola or The Celestial Sphere

In order to demonstrate the motion of the heavenly bodies, the Hindu astronomers make use of spheres constructed by means of circles made of flexible wooden sticks or bamboo strips. These are called ***Gola*** and correspond to the Celestial Sphere of modern astronomy. The section deals with motion of the Sun, Moon and the planets on the celestial sphere. It describes the various circles of the celestial sphere and indicates the method of automatically rotating the sphere once in 24 hours, the motion of the celestial sphere as seen by those on the equator and those on the north and south poles and also deals with the calculation and graphical representation of the ellipses and the visibility of the planets.

The *Gola* which is supposed to be centred at the Earth's centre is called *Bhagola* ('Sphere of the asterisms'). It is used to demonstrate the motion of the Sun, the Moon and the planets in their orbits. The principal circles of this sphere are:

1. the celestial equator
2. the ecliptic
3. the orbits of the Moon and the planets
4. the day-circles, etc.

The *Gola* which is supposed to be centred at the observer is called *Khagola* ('Sphere of the sky'). It is fixed in position and is used to demonstrate the diurnal motion of the heavenly bodies; the principal circles of this sphere are:

1. the horizon,
2. the meridian,
3. the prime vertical
4. the six o'clock circle, etc.

In the present Section, Aryabhata aims at teaching spherical astronomy. He begins by giving a brief description of the *Bhagola* and the *Khagola* and then, with their help, demonstrates the motion of the heavenly bodies in a total of 50 stanzas.

WORK

After meeting the first aim of the project, the literature review of The Aryabhatiya, we started working on the website demonstration of the same. We have created a website that demonstrates and depicts The Aryabhatiya in a simple, interactive and visually interesting manner.

We have added the snapshots of the website made below in the Appendix section.

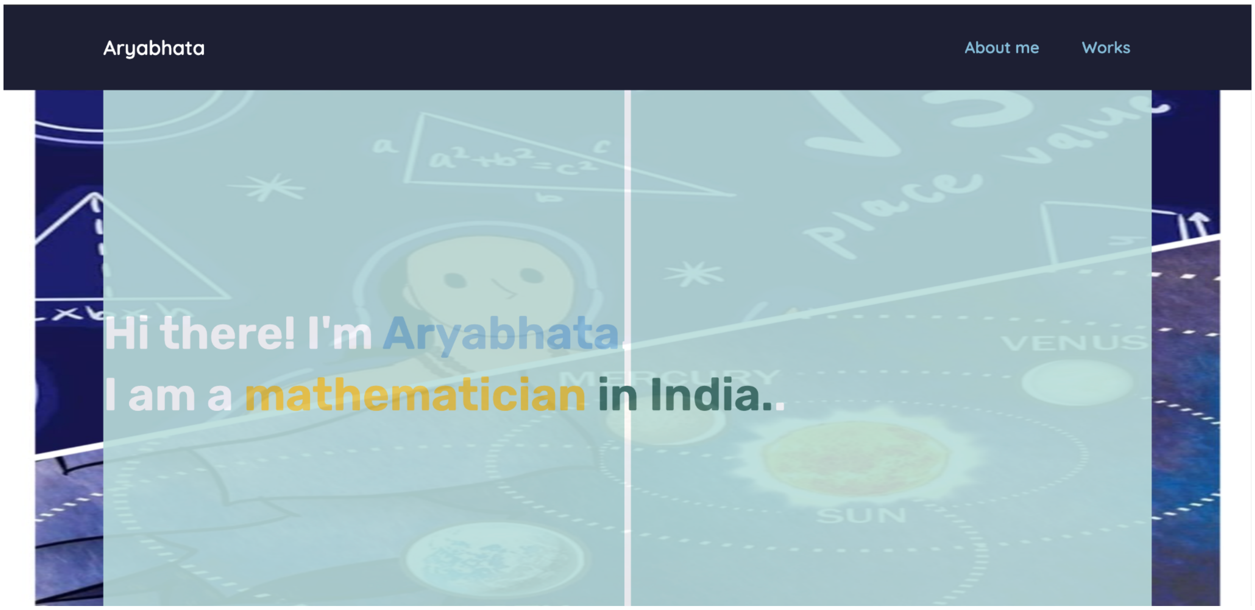
FUTURE SCOPE

In future,

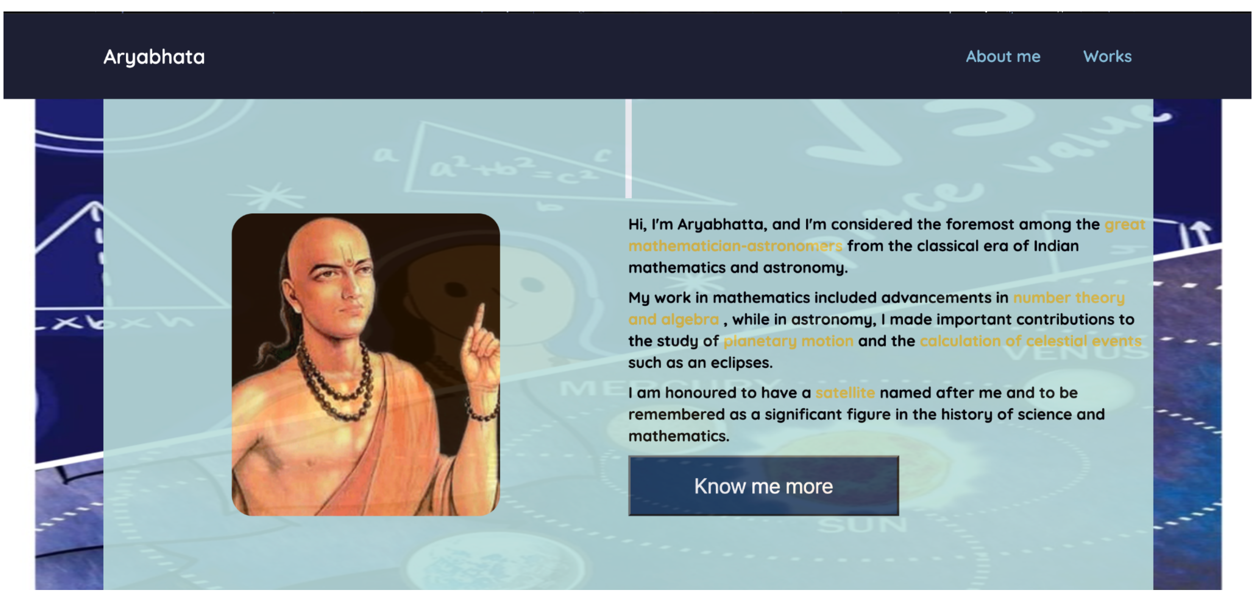
- We aim to work on the literature review of the last two sections of The Aryabhatiya, i.e., The Kalakriya or The Reckoning Time and Gola or The Celestial Sphere.
- Also, we aim to add the content for the last two sections as mentioned above and to increase, improve and visualise the content of the existing website.

Appendix

1. Website



1. **Landing Page**



2. **About Aryabhata**

All About Me

476 CE

My Time

- When - I was born on March 21, AD 476.
- Where - Pataliputra (now Patna), Bihar, India.
- Death - I passed away in India in 550 AD.

499 CE

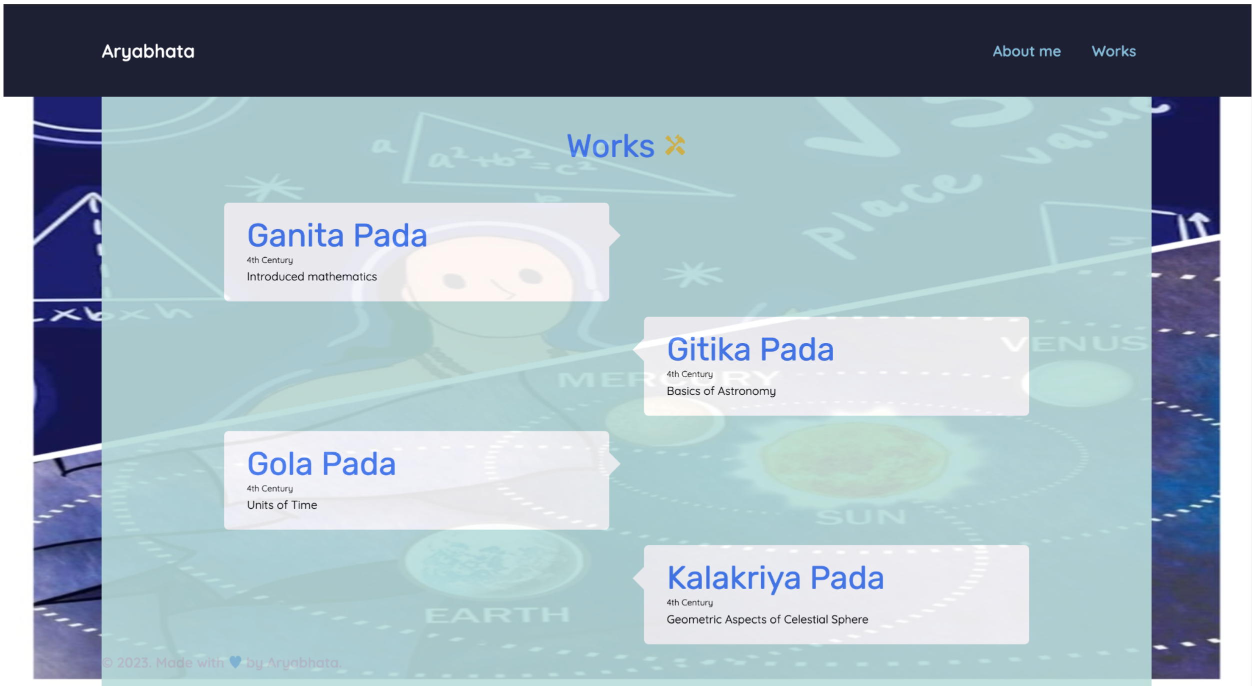
My Place

- I was referred to as Asmaka and I wrote Aryabhatiya in Patliputra, Magadha.
- The University of Nalanda, Patna had a special provision for the study of astronomy. I was designated as the Kulapa (meaning Kulapati or Head of a University).

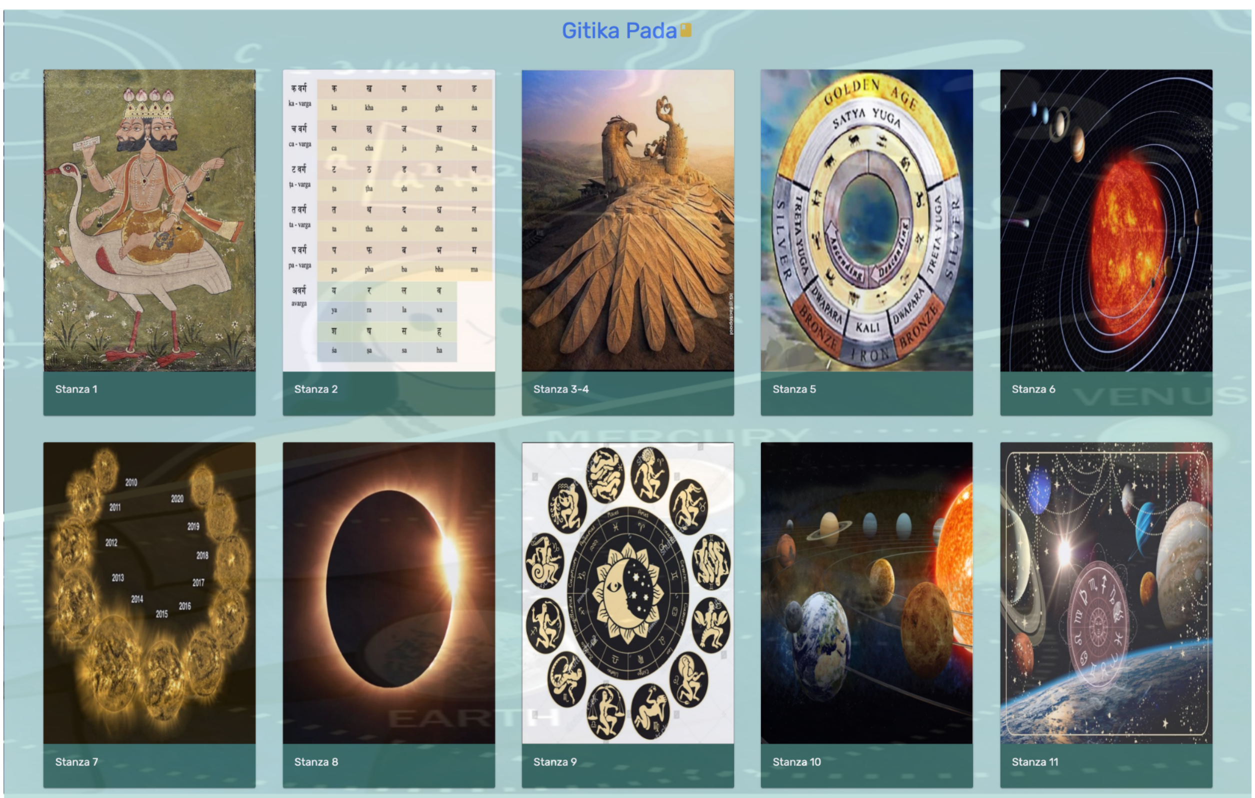
499 CE

My Work

- I wrote two works on astronomy: 1. Aryabhatiya 2. Aryabhata-Siddhanta
- My profession was teaching.
- A couple of my pupils were Panduranga-svami, Latadeva and Nisanku.
- Latadeva went on to become a great scholar and teacher of astronomy.



3. **Works of Aryabhata**



4. **Gitka Pada**

Verse 1

BHASKARA I

BIBHUIBHUSHAN DATTA

- Obeisance to Brahma points to the school to which Aryabhata belongs.
- When viewed as the unchangeable and unsustained God, He is one, but when taken to reside in the bodies of so many living beings, He is many.
- Or, in the beginning He was only one, but later He became twofold - man and woman - and created all living beings and became many.
- Or, viewed as the omnipresent God (visvarnpa), He is unquestionably one and many.
- He is called the real God, because the other gods having been created by Him are not real gods. He is called the 'Supreme Brahman' (param brahma), because He is the root cause of the world.

5. Verse 1- Gitika Pada

Table 3. Linear diameters of the Earth etc.

	Linear diameter in *yojanas*	Linear diameter in *yojanas* (at the moon's mean distance)
Earth	1050	
Sun	4410	
Moon	315	
Mars		12.60
Mercury		21.00
Jupiter		31.50
Venus		63.00
Saturn		15.75

Table 4. Mean angular diameters of the planets

Planet	Mean angular diameter according to		
	Aryabhata I	Greek astronomers	Modern
Moon	31' 20"	35' 20" (Ptolemy) Tycho Brahe (15-46-1631)	31' 8"
Mars	1' 15".6	1' 40"	
Mercury	2' 6"	2' 10"	
Jupiter	3' 9"	2' 45"	
Venus	6' 18"	3' 15"	
Saturn	1' 34".5	1' 50"	

Verse 7

LINEAR DIAMETERS

- 'Ny' is a unit of length whose measure is equal to the height of a man. Ny is also known as nara, puruşa, dhanu and danda, "Puruşa, dhanu, danda and nara are synonyms"
- P. C. Sengupta translates the second half of the stanza as follows:

"The diameters of Venus, Jupiter, Mercury, Saturn and Mars are, respectively, 1/5, 1/10, 1/15, 1/20 and 1/25 of the diameter of the Moon, when taken at the mean distance of the Sun."

- This is incorrect, because the diameters of the planets stated in the stanza under consideration correspond to the mean distance of the Moon and not to the mean distance of the Sun as Sengupta has supposed.

6. Verse 7- Gitika Pada

Table 5. Inclinations of the Orbits

Planet	Inclination of the orbit			
	Aryabhata I	Ptolemy	*PauSi and RoSi* of Varahamihira	Modern
Moon	4 ° 30’	5°	4° 40’	5° 9’
Mars	1° 30’	1°		1° 51’ 01’
Mercury	2°	7°		7° 00’ 01”
Jupiter	1°	1° 30’		1° 18’ 28”
Venus	2°	3° 30’		3° 23’ 38”
Saturn	2°	2° 30’		2° 29’ 20”

Verse 8

OBLIQUITY OF ECLIPTIC AND INCLINATIONS OF ORBITS

- The greatest declination of the Sun is the obliquity of the ecliptic. According to Aryabhata I and other Indian astronomers, its value is 24° and according to modern astronomers its value is 23° 27' 8.26-46”.84 T, where T is measured in Julian centuries from 1900 A.D.

- In the case of Mercury and Venus, Aryabhata I's values differ significantly from those of Ptolemy and modern astronomers because the values given by Aryabhata I are geocentric and those given by Ptolemy and modern astronomers are heliocentric.

- By combining the instruction in the last quarter of the above verse with that in the first quarter of verse 7, we have

24 angulas = 1 cubit (hasta)

4 cubits=1 ny

8000 nr = 1 yojana

7. Verse 8 - Gitika Pada

Table 6. Longitudes of the Ascending Nodes for A.D. 499

Planet	Longitudes in the ascending nodes		
	Aryabhata I	Ptolemy (for A.D. 150)	By modern calculation
Mars	40°	25° 30’	37° 49’
Mercury	20°	10° 00’	30° 35’
Jupiter	80°	51° 00’	85° 13’
Venus	60°	55° 00’	63° 16’
Saturn	100°	183° 00’	100° 32’

Table 7. Longitudes of the Apogees for A.D. 499

Planet	Longitudes of the apogees (aphelia)			
	Aryabhata I	Ptolemy (for A.D. 150)	RoSi Of Varahmihira	By modern calculation
Sun	78°	65° 31’	75°	77° 15’
Mars	118°	115° 30’		128° 28’
Mercury	210°	190° 00’		234° 11’
Jupiter	180°	161° 00’		170° 22’
Venus	90°	55° 00’		290° 4’
Saturn	236°	233° 00’		243° 40’

Verse 9

ASCENDING NODE AND APOGEES(APHELIA)

- The table gives the longitudes of the ascending nodes and the apogees of the planets as given by Aryabhata I and as calculated by modern methods. The corresponding longitudes for A.D. 150, as stated by Ptolemy are also given for comparison.

- The word gatva (meaning ‘having moved’) is used in the text to show that the ascending nodes and the apogees of the planet are not stationary but have motion.

- Commentator Bhaskara I said that by teaching their motion, Aryabhata I has specified by implication their revolution-numbers in a yuga.

- The period of 35750224800 years, according to the tradition, is the common period of motion (yuga) of the ascending nodes of all the planets, in which the ascending nodes of Mars, Mercury, Jupiter, Venus and Saturn make 2, 1, 4, 3 and 5 revolutions, respectively.

8. Verse 9 - Gitika Pada

Table 9. ***Manda*** **and** ***Sighra*** **epicycles of the planets**

	Manda epicycles			*Sighra* epicycles		
Planet	Aryabhata I			Aryabhata I		
	Odd Quadrant	Even Quadrant	Ptolemy	Odd Quadrant	Even Quadrant	Ptolemy
Sun	13°.50	13°.50	15°.00			
Moon	31°.50	31°.50	31°.40			
Mars	63°.00	81°.00	72°.00	238°.50	229°.50	237°
Mercury	32°.50	22°.50	18°.00	139°.50	130°.50	135°
Jupiter	31°.50	36°.00	33°.00	72°.00	67°.50	69°
Venus	18°.00	9°.00	15°.00	265°.50	256°.50	259°
Saturn	40°.50	58°.50	41°.00	40°.50	36°.00	39°

Verse 10

ODD QUADRANTS BHASKARA'S RULE

- The dimensions of the manda and sighra epicycles are stated in terms of degrees, where a degree stands for the 360th part of the circumference of the deferent (kaksyavṛtta).

- In stating the dimensions of the manda epicycles the planets have been mentioned in the order of decreasing velocities (manda-gati-krama)

- Whereas in stating the dimensions of the sighra epicycles they have been mentioned in the order of increasing velocities (fighra-gati-krama).

9. Verse 10 - Gitika Pada

Table 10. Rsine and Rsine-differences at the intervals of 225' or 3°45'

Aruabhata I's values			Modern values	
Arc	Rsine	Rsine-differences	Rsine	Rsine-differences
225'	225'	225'	224'.856	224'.856
450'	449'	224'	448'.449	223'.893
674'	671'	222'	670'.720	221'.971
900'	890'	219'	889'.820	219'.100
1125'	1105'	215'	1105'.109	215'.289
1350'	1315'	210'	1315'.666	210'.557
1575'	1520'	205'	1520'.589	204'.923
1800'	1719'	199'	1719'.000	198'.411
2025'	1910'	191'	1910'.050	191'.050
2250'	2093'	183'	2092'.922	182'.872
2475'	2267'	174'	2266'.831	173'.909
2700'	2431'	164'	2431'.033	164'.202
2925'	2585'	154'	2584'.825	153'.792
3150'	2728'	143'	2727'.549	142'.724
3375'	2859'	131'	2858'.592	131'.043
3600'	2978'	199'	2977'.395	118'.803
3825'	3084'	106'	3083'.448	106'.053
4050'	3177'	93'	3176'.298	92'.850
4275'	3256'	79'	3255'.546	79'.248
4500'	3321'	65'	3320'.853	65'.307
4725'	3372'	51'	3371'.940	51'.087
4950'	3409'	37'	3408'.588	36'.648
5175'	3431'	22'	3430'.639	22'.051
5400'	3438'	7'	3438'.000	7'.361

Verse 12

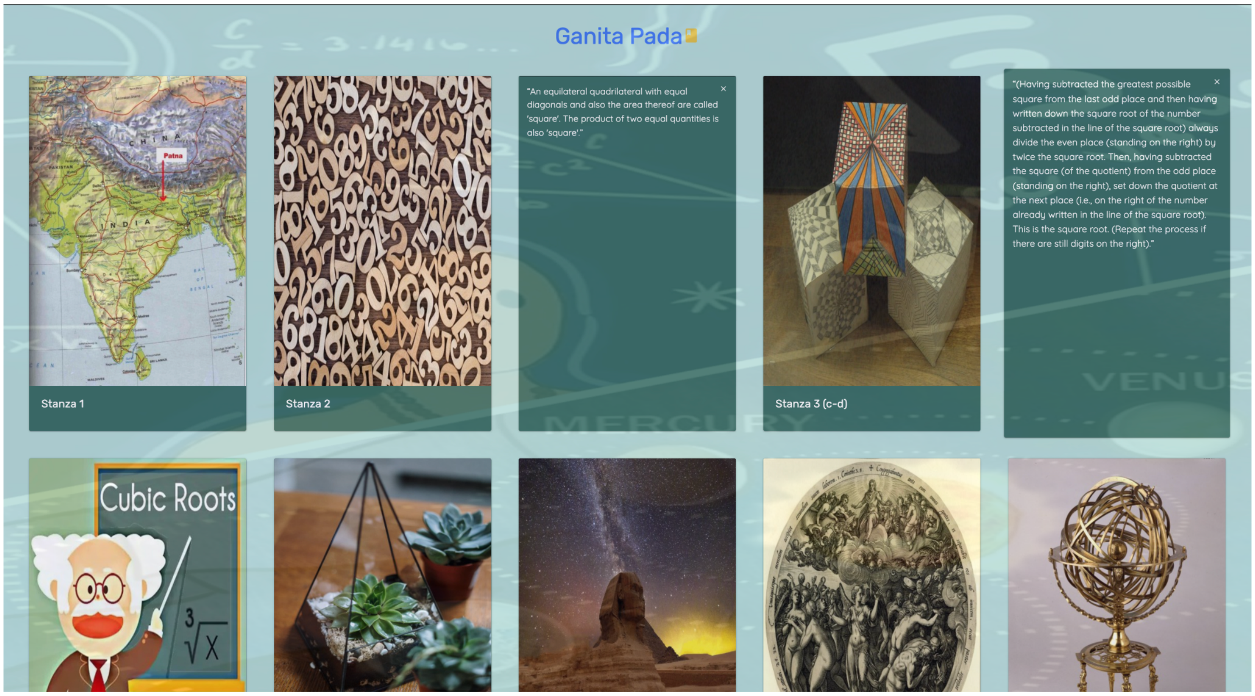
RSINE-DIFFERENCES

- The following table gives the Rsines and the Rsine-differences at intervals of 225' (or 3° 45') according to Aryabhata I and the corresponding modern values correct to three decimal places.

- The twenty-four Rsines given in the Surya-siddhanta are exactly the same as those in column 2 in the table.

- P.C. Sengupta is of the opinion that the author of the Surya-siddhanta has based his Rsines on the Rsine-differences given by Aryabhata I.

10. Verse 12 - Gitika Pada



11. Ganita Pada

Varga	Letters and their numerical values				
ka-varga	*k* = 1	*kh* = 2	*g* = 3	*gh* = 4	*ṅ* = 5
ca-varga	*c* = 6	*ch* = 7	*j* = 8	*jh* = 9	*ñ* = 10
ṭa-varga	*ṭ* = 11	*ṭh* = 12	*ḍ* = 13	*ḍh* = 14	*ṇ* = 15
ta-varga	*t* = 16	*th* = 17	*d* = 18	*dh* = 19	*n* = 20
pa-varga	*p* = 21	*ph* = 22	*b* = 23	*bh* = 24	*m* = 25

Verse 2

CLASSIFICATION NOTATIONAL PLACES EXAMPLE

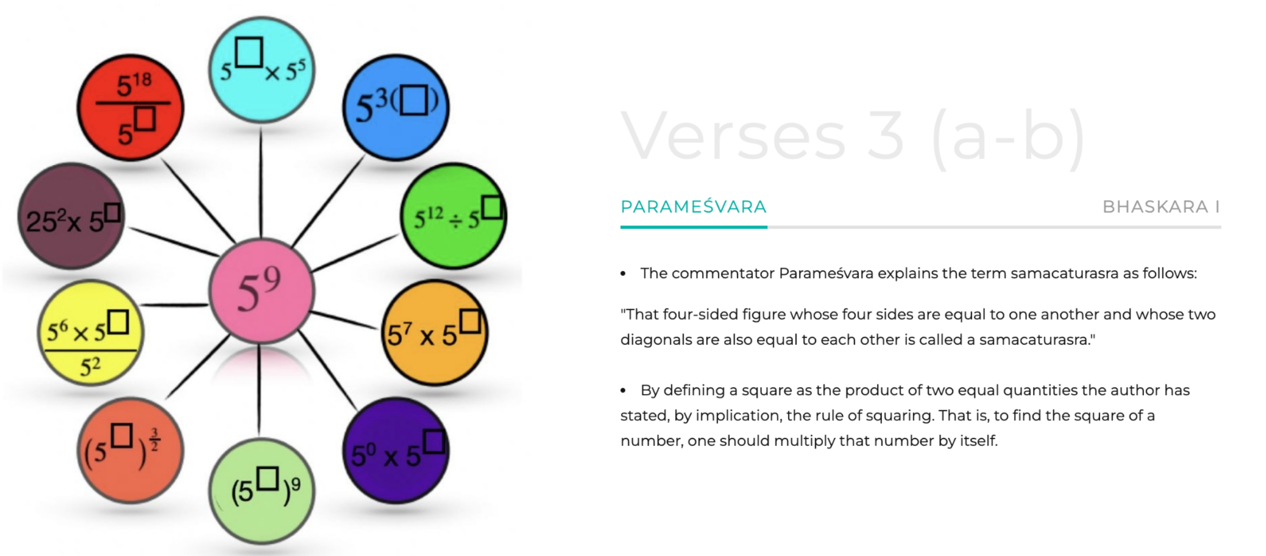
ṅiśibuṇḷṣkhṛ (=ṅ+i, ś+i, b+u, ṇ+ḷ, ṣ+kh+ṛ)

- It denotes the number which is obtained by writing the ṅ in the varga place and ś in the avarga place occupied by the vowel i; b in the varga place occupied by the vowel u; ṇ in the avarga place occupied by the vowel ḷ; and ṣ in the avarga place and kh in the varga place occupied by the vowel ṛ as follows:

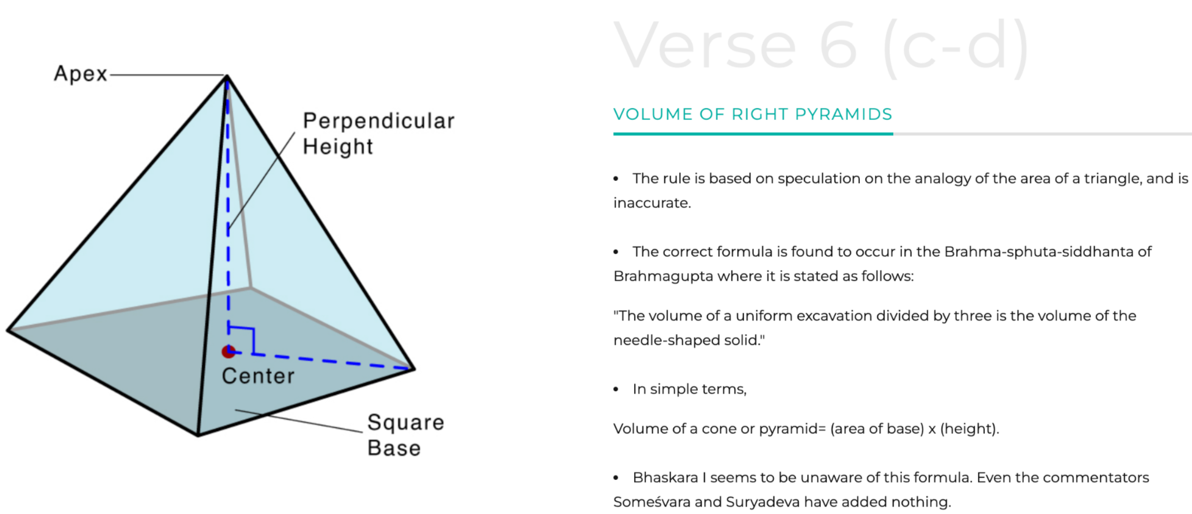
ḷ	ḷ	ṛ	ṛ	u	u	i	i	a	a
	ṇ	*ṣ*	*kh*	0	*b*	*ś*	*ṅ*	0	0
=1	5	8	2	2	3	7	5	0	0

- Thus ṅiśibuṇḷṣkhṛ = 1582237500

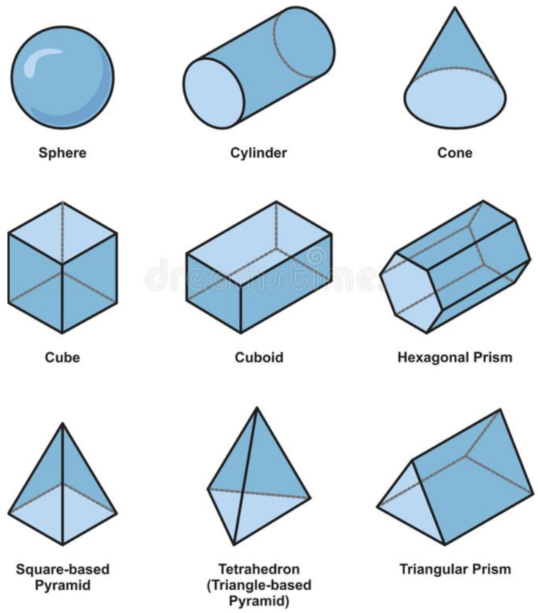
12. Verse 3- Ganita Pada



13. Verse 3- Ganita Pada



14. Verse 6- Ganita Pada



Verse 9 (a-b)

AREA OF A PLANE FIGURES | BHASKARA'S DOUBT

- According to Bhaskara I, this rule is to calculate both the area and verifying the area of a plane figure.
- It states clearly that all the plane figures can be transformed into rectangles.
- Bhaskara 1 has also found the area of the triangle, a quadrilateral, a drum-shaped figure, and a figure resembling the tusk of an elephant by transforming them into rectangles in his commentaries.

15. Verse 9- Ganita Pada

Table 9. ***Manda*** **and** ***Sighra*** **epicycles of the planets**

	Manda **epicycles**			***Sighra*** **epicycles**		
Planet	**Aryabhata I**			**Aryabhata I**		
	Odd Quadrant	**Even Quadrant**	**Ptolemy**	**Odd Quadrant**	**Even Quadrant**	**Ptolemy**
Sun	13°.50	13°.50	15°.00			
Moon	31°.50	31°.50	31°.40			
Mars	63°.00	81°.00	72°.00	238°.50	229°.50	237°
Mercury	32°.50	22°.50	18°.00	139°.50	130°.50	135°
Jupiter	31°.50	36°.00	33°.00	72°.00	67°.50	69°
Venus	18°.00	9°.00	15°.00	265°.50	256°.50	259°
Saturn	40°.50	58°.50	41°.00	40°.50	36°.00	39°

Verse 11

EVEN QUADRANTS | BHASKARA'S RULE

- The dimensions of the manda and sighra epicycles are stated in terms of degrees, where a degree stands for the 360th part of the circumference of the deferent (kaksyavṛtta).
- In stating the dimensions of the manda epicycles the planets have been mentioned in the order of decreasing velocities (manda-gati-krama)
- Whereas in stating the dimensions of the sighra epicycles they have been mentioned in the order of increasing velocities (fighra-gati-krama).

16. Verse 11- Ganita Pada

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