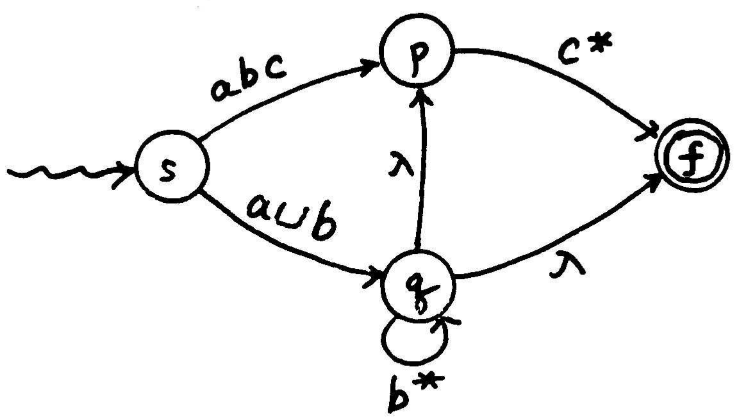
Finite Automata into Regular Expressions

To prove that every DFA language is regular we introduce an extension of finite automata.

Definition An extended finite automaton (EFA), M, is a quintuple $(Q,\ \Sigma,\ \delta,\ s,\ f)$ where
$Q,\ \Sigma,\ s$ **are as in λ-NFA,**
f **is the only final state, $f \neq s$,**
$\delta : Q \times Q \to R_\Sigma$ **is a total extended transition function.**

Example of an EFA:



$\delta(p, s) = \emptyset$
$\delta(s, f) = \emptyset$
......
One final state $f \neq s$

△ **A** <u>**configuration**</u> **is in** $Q\Sigma^*$

△ <u>**Move**</u>

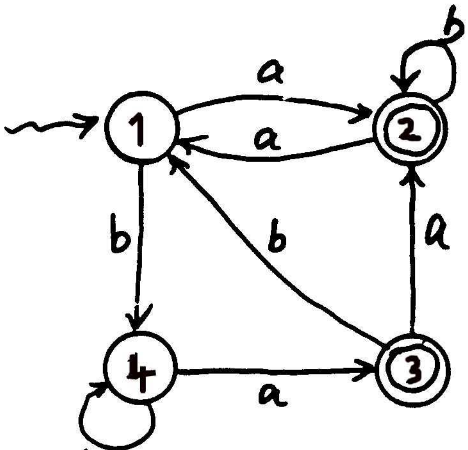
$$px \ \vdash \ qy \ \textbf{if}$$

(i) $x = wy, \ w \in \Sigma^*$,
(ii) $\delta(p, q) = E$, **and**
(iii) $w \in L(E)$.

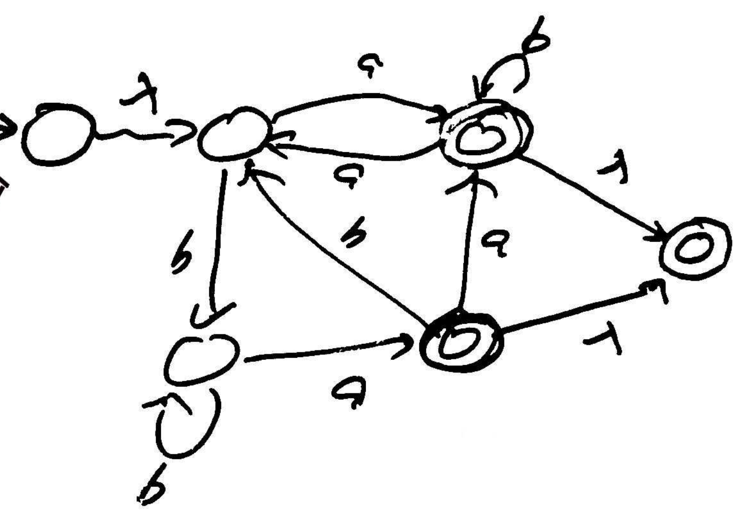
△ $\underline{\vdash^*}$, $\underline{\vdash^+}$ **are defined similarly as before.**

<u>**Lemma**</u> **If** M **is a DFA, Then there is an EFA** M' **with** $L(M') = L(M)$**.**

<u>**Example**</u> **DFA into EFA.**



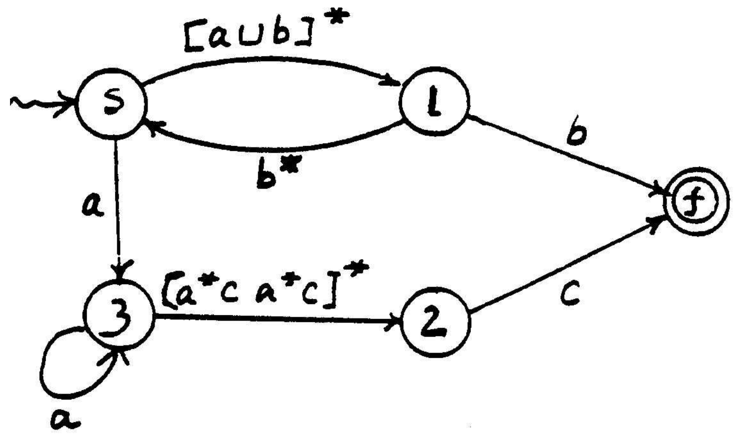
$\Rightarrow$



<u>Example</u>:

An extended finite Automaton (EFA).

M:



Check if the following words are in $L(M)$

(1) $bbabab$

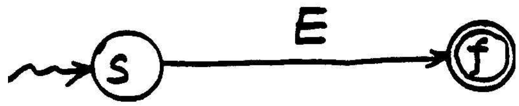
(2) $aabbc$

(3) $acccc$

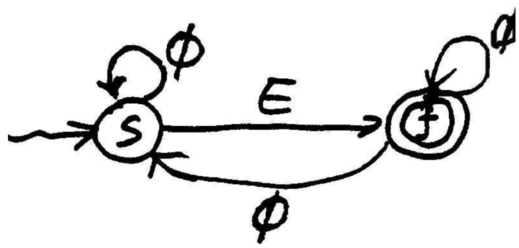
(4) $aaaaac$

<u>State Elimination Technique</u>

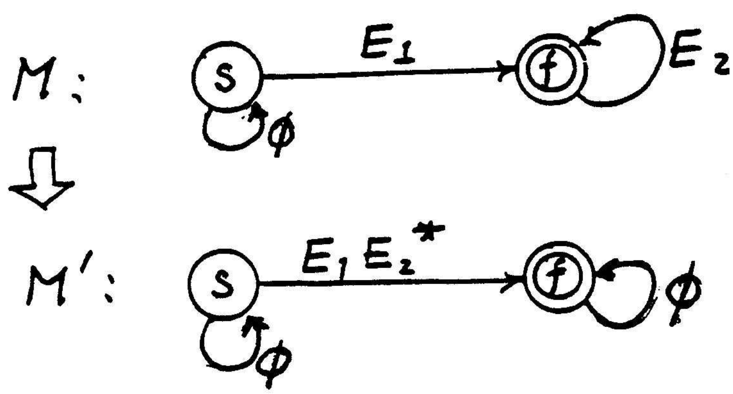
Goal of the technique:



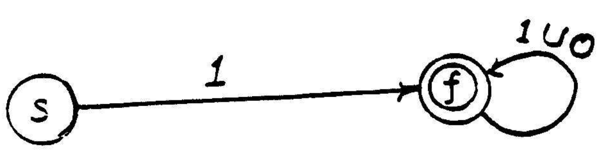
i.e.:



(1) <u>EFA has 2 states</u>

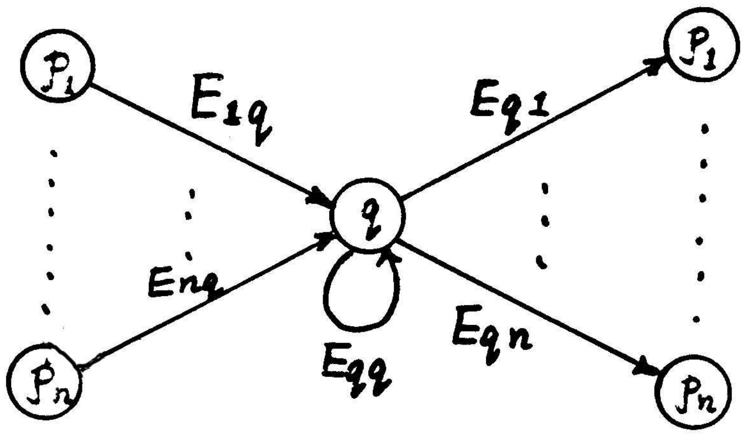


<u>Example</u>



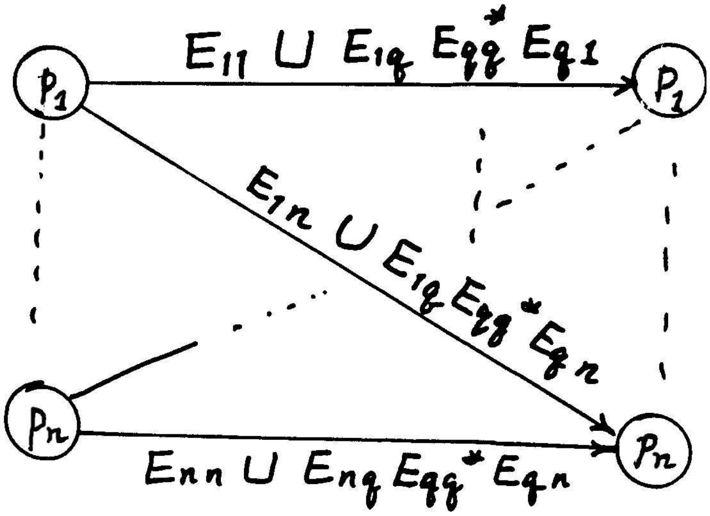
(2) EFA M has $k+1$ states, $k \geq 2$.
Then eliminate a state from M to form M':

M :



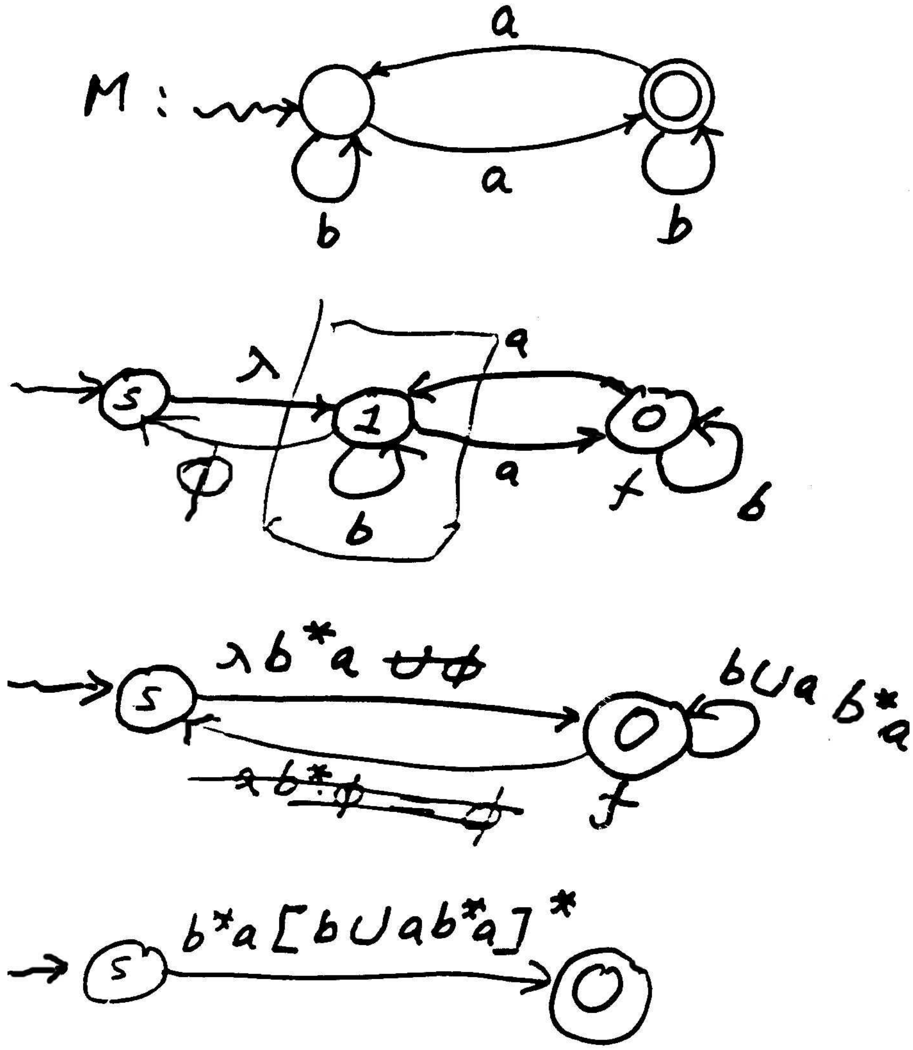
Note: $q \in Q - \{s,\ f\}$
Consider all transitions $(p_i,\ E_{iq},\ q)$
and $(q,\ E_{qi},\ p_i)$

M' :



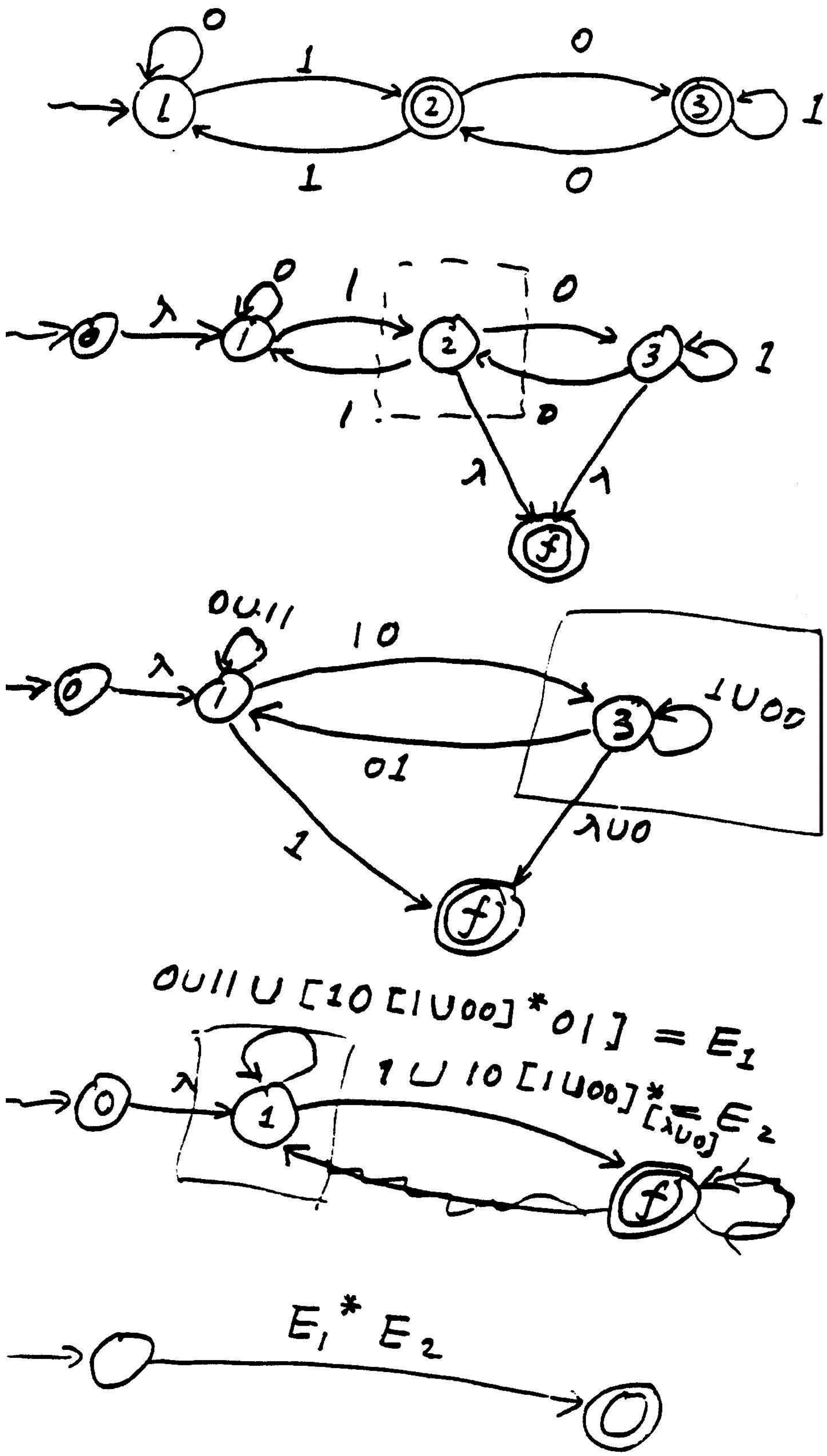
$$\delta'(p_i, p_j) = \delta(p_i, p_j) \cup \delta(p_i, q)(\delta(q, q))^* \delta(q, p_j)$$

Example



$b^*a[b \cup ab^*a]^*$

Example



Summary of the State Elimination Technique

(0) **Change FA into EFA**

(1) **Add a new start state if the original one has incoming transitions.**

(2) **Add a new final state if there are more than one final states originally. Old final states become non-final states.**

(3) **Eliminate the states in $Q - \{s,\ f\}$ one by one.**

(4) **Eliminate the transition $\delta(f, f)$.**