Transforming λ-NFA to NFA

Two steps:
Step I: $\lambda-$ completion

Step II: $\lambda-$ transition removal

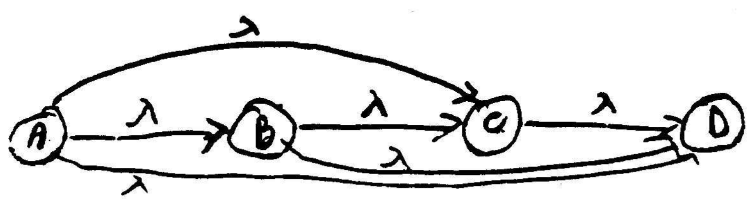
(I). λ-Completion

Given a λ-NFA $M = (Q, \Sigma, \delta, s, F)$
perform the following process:

For all $p, q, r \in Q$**:**
  whenever $(p, \lambda, q), (q, \lambda, r)$ **are in** δ
    add (p, λ, r) **to** δ
until **no new transitions are added to** δ
and let this be $\underline{\delta'}$**.**

Let the new $\lambda-$NFA be
$M' = (Q, \Sigma, \delta', s, F')$
where $F' = F \cup \{p \mid (p, \lambda, f) \in \delta$ **and** $f \in F\}$
and $\delta' = \delta \cup \{(p, \lambda, q) \mid p \vdash^{+} q\}$

Example:



Claim 1: **For any** $p,\ q \in Q$**,**

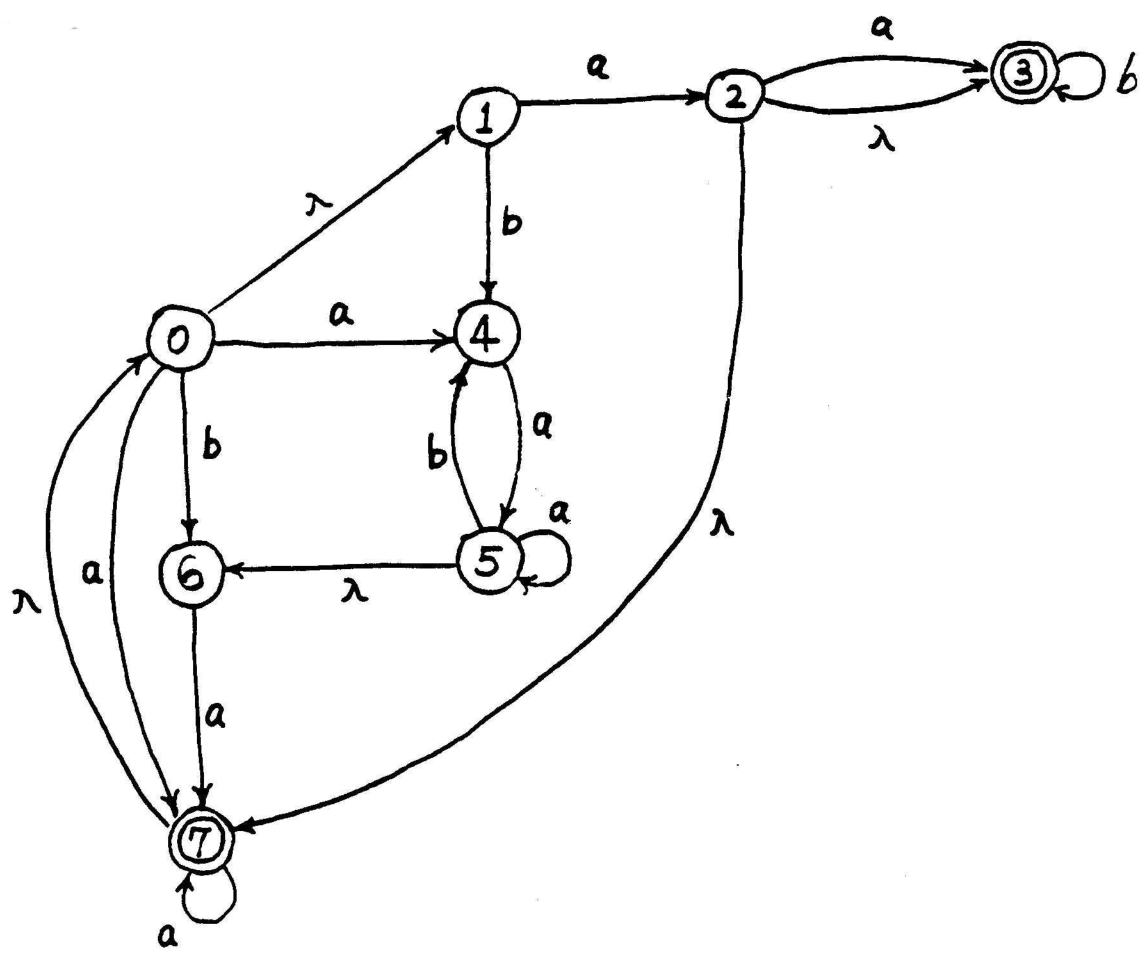
$$p \ \vdash_M^+ \ q \text{ if and only if } p \ \vdash_{M'} \ q$$

Claim 2: **For any** $p,\ q \in Q,\ x \in \Sigma^*$**,**

$$px \ \vdash_M^* \ q \text{ if and only if } px \ \vdash_{M'}^* \ q$$

Theorem: $L(M') = L(M)$

Example:



(II) λ–Transition Removal

Given a λ-completed λ-NFA

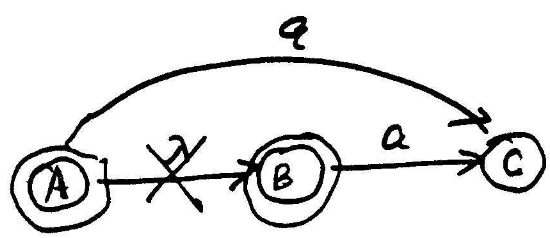
$$M = (Q,\ \Sigma,\ \delta,\ s,\ F),$$

perform the following process:

$$\begin{array}{ll}
(0) & \delta' = \delta; \\
(i) & \textbf{For all } p, q, r \in Q, \\
 & \quad \textbf{if } \underline{(p, \lambda, q)} \textbf{ and } \underline{(q, a, r)} \textbf{ in } \delta \\
 & \quad \textbf{then } \underline{\textbf{add}}\ \underline{(p, a, r)} \textbf{ to } \delta'; \\
(ii) & \underline{\textbf{Delete}} \textbf{ all } \underline{\lambda\textbf{-transitions}} \textbf{ from } \delta'.
\end{array}$$

Now we got $M' = (Q,\ \Sigma,\ \delta',\ s,\ F)$
where $\delta' = (\delta \cup \{(p, a, r) \mid (p, \lambda, q), (q, a, r) \in \delta\})$
$-\{(p, \lambda, q) \mid p, q \in Q\}$

Example



Claim **Whenever**

$$sx \ \vdash_M^* \ f$$

for some $f \in F$**, we have**

$$sx \ \vdash_{M'}^* \ f$$

and vice versa.

Claim $L(M') = L(M)$

Theorem

$$\mathcal{L}_{\lambda-NFA} = \mathcal{L}_{NFA}$$