سئوال ۴

مرحلهٔ ۱:

ID	Att	Class	Interval
1	1	A	[0, 2)
2	3	B	[2, 5)
3	7	A	[5, 7.5)
4	8	A	[7.5, 8.5)
5	9	A	[8.5, 10)
6	11	B	[10, 17)
7	23	B	[17, 30)
8	37	A	[30, 38)
9	39	B	[38, 42)
10	45	A	[42, 45.5]
11	46	A	[45.5, 52.5)
12	59	A	[52.5, 60)

	Class A (j = 1)	Class B (j = 2)	
Interval 1 (i = 1) :: [0, 2)	$A_{11} = 1$	$A_{12} = 0$	$R_1 = 1$
Interval 2 (i = 2) :: [2, 5)	$A_{21} = 0$	$A_{22} = 1$	$R_2 = 1$
	$C_1 = 1$	$C_2 = 1$	N = 2

$$E_{11} = (R_1 \times C_1)/N = (1\times1)/2 = 0.5$$
$$E_{12} = (R_1 \times C_2)/N = (1\times1)/2 = 0.5$$
$$E_{21} = (R_2 \times C_1)/N = (1\times1)/2 = 0.5$$
$$E_{22} = (R_2 \times C_2)/N = (1\times1)/2 = 0.5$$

$$\begin{aligned}\chi^2 &= (A_{11}-E_{11})^2/E_{11} + (A_{12}-E_{12})^2/E_{12} + (A_{21}-E_{21})^2/E_{21} + (A_{22}-E_{22})^2/E_{22} \\ &= (1-0.5)^2/0.5 + (0-0.5)^2/0.5 + (0-0.5)^2/0.5 + (1-0.5)^2/0.5 \\ &= 2 < 2.706\end{aligned}$$

ID	Att	Class	Interval
1	1	A	[0, 2)
2	3	B	[2, 5)
3	7	A	[5, 7.5)
4	8	A	[7.5, 8.5)
5	9	A	[8.5, 10)
6	11	B	[10, 17)
7	23	B	[17, 30)
8	37	A	[30, 38)
9	39	B	[38, 42)
10	45	A	[42, 45.5]
11	46	A	[45.5, 52.5)
12	59	A	[52.5, 60)

	Class A (j = 1)	Class B (j = 2)	
Interval 1 (i = 1) :: [2, 5)	$A_{11} = 0$	$A_{12} = 1$	$R_1 = 1$
Interval 2 (i = 2) :: [5, 7.5)	$A_{21} = 1$	$A_{22} = 0$	$R_2 = 1$
	$C_1 = 1$	$C_2 = 1$	N = 2

$$E_{11} = (R_1 \times C_1)/N = (1\times1)/2 = 0.5$$
$$E_{12} = (R_1 \times C_2)/N = (1\times1)/2 = 0.5$$
$$E_{21} = (R_2 \times C_1)/N = (1\times1)/2 = 0.5$$
$$E_{22} = (R_2 \times C_2)/N = (1\times1)/2 = 0.5$$

$$\begin{aligned}\chi^2 &= (A_{11} - E_{11})^2/E_{11} + (A_{12} - E_{12})^2/E_{12} + (A_{21} - E_{21})^2/E_{21} + (A_{22} - E_{22})^2/E_{22} \\ &= (0-0.5)^2/0.5 + (1-0.5)^2/0.5 + (1-0.5)^2/0.5 + (0-0.5)^2/0.5 \\ &= 2 < 2.706\end{aligned}$$

ID	Att	Class	Interval
1	1	A	[0, 2)
2	3	B	[2, 5)
3	7	A	[5, 7.5)
4	8	A	[7.5, 8.5)
5	9	A	[8.5, 10)
6	11	B	[10, 17)
7	23	B	[17, 30)
8	37	A	[30, 38)
9	39	B	[38, 42)
10	45	A	[42, 45.5]
11	46	A	[45.5, 52.5)
12	59	A	[52.5, 60)

	Class A (j = 1)	Class B (j = 2)	
Interval 1 (i = 1) :: [5, 7.5)	$A_{11} = 1$	$A_{12} = 0$	$R_1 = 1$
Interval 2 (i = 2) :: [7.5, 8.5)	$A_{21} = 1$	$A_{22} = 0$	$R_2 = 1$
	$C_1 = 2$	$C_2 = 0$	N = 2

$$E_{11} = (R_1 \times C_1)/N = (1 \times 2)/2 = 1$$
$$E_{12} = (R_1 \times C_2)/N = (1 \times 0)/2 \approx 0.1$$
$$E_{21} = (R_2 \times C_1)/N = (1 \times 2)/2 = 1$$
$$E_{22} = (R_2 \times C_2)/N = (1 \times 0)/2 \approx 0.1$$

$$\begin{aligned}\chi^2 &= (A_{11} - E_{11})^2/E_{11} + (A_{12} - E_{12})^2/E_{12} + (A_{21} - E_{21})^2/E_{21} + (A_{22} - E_{22})^2/E_{22} \\ &= (1-1)^2/1 + (0-0.1)^2/0.1 + (1-1)^2/1 + (0-0.1)^2/0.1 \\ &= 0.2 < 2.706\end{aligned}$$

ID	Att	Class	Interval
1	1	A	[0, 2)
2	3	B	[2, 5)
3	7	A	[5, 7.5)
4	8	A	[7.5, 8.5)
5	9	A	[8.5, 10)
6	11	B	[10, 17)
7	23	B	[17, 30)
8	37	A	[30, 38)
9	39	B	[38, 42)
10	45	A	[42, 45.5]
11	46	A	[45.5, 52.5)
12	59	A	[52.5, 60)

	Class A (j = 1)	Class B (j = 2)	
Interval 1 (i = 1) :: [7.5, 8.5)	$A_{11} = 1$	$A_{12} = 0$	$R_1 = 1$
Interval 2 (i = 2) :: [8.5, 10)	$A_{21} = 1$	$A_{22} = 0$	$R_2 = 1$
	$C_1 = 2$	$C_2 = 0$	N = 2

$$E_{11} = (R_1 \times C_1)/N = (1 \times 2)/2 = 1$$
$$E_{12} = (R_1 \times C_2)/N = (1 \times 0)/2 \approx 0.1$$
$$E_{21} = (R_2 \times C_1)/N = (1 \times 2)/2 = 1$$
$$E_{22} = (R_2 \times C_2)/N = (1 \times 0)/2 \approx 0.1$$

$$\begin{aligned}\chi^2 &= (A_{11} - E_{11})^2/E_{11} + (A_{12} - E_{12})^2/E_{12} + (A_{21} - E_{21})^2/E_{21} + (A_{22} - E_{22})^2/E_{22} \\ &= (1-1)^2/1 + (0-0.1)^2/0.1 + (1-1)^2/1 + (0-0.1)^2/0.1 \\ &= 0.2 < 2.706\end{aligned}$$

ID	Att	Class	Interval
1	1	A	[0, 2)
2	3	B	[2, 5)
3	7	A	[5, 7.5)
4	8	A	[7.5, 8.5)
5	9	A	[8.5, 10)
6	11	B	[10, 17)
7	23	B	[17, 30)
8	37	A	[30, 38)
9	39	B	[38, 42)
10	45	A	[42, 45.5]
11	46	A	[45.5, 52.5)
12	59	A	[52.5, 60)

	Class A (j = 1)	Class B (j = 2)	
Interval 1 (i = 1) :: [8.5, 10)	$A_{11} = 1$	$A_{12} = 0$	$R_1 = 1$
Interval 2 (i = 2) :: [10, 17)	$A_{21} = 0$	$A_{22} = 1$	$R_2 = 1$
	$C_1 = 1$	$C_2 = 1$	N = 2

$$E_{11} = (R_1 \times C_1)/N = (1 \times 1)/2 = 0.5$$
$$E_{12} = (R_1 \times C_2)/N = (1 \times 1)/2 = 0.5$$
$$E_{21} = (R_2 \times C_1)/N = (1 \times 1)/2 = 0.5$$
$$E_{22} = (R_2 \times C_2)/N = (1 \times 1)/2 = 0.5$$

$$\begin{aligned}\chi^2 &= (A_{11} - E_{11})^2/E_{11} + (A_{12} - E_{12})^2/E_{12} + (A_{21} - E_{21})^2/E_{21} + (A_{22} - E_{22})^2/E_{22} \\ &= (1-0.5)^2/0.5 + (0-0.5)^2/0.5 + (0-0.5)^2/0.5 + (1-0.5)^2/0.5 \\ &= 2 < 2.706\end{aligned}$$

ID	Att	Class	Interval
1	1	A	[0, 2)
2	3	B	[2, 5)
3	7	A	[5, 7.5)
4	8	A	[7.5, 8.5)
5	9	A	[8.5, 10)
6	11	B	[10, 17)
7	23	B	[17, 30)
8	37	A	[30, 38)
9	39	B	[38, 42)
10	45	A	[42, 45.5]
11	46	A	[45.5, 52.5)
12	59	A	[52.5, 60)

	Class A (j = 1)	Class B (j = 2)	
Interval 1 (i = 1) :: [10, 17)	$A_{11} = 0$	$A_{12} = 1$	$R_1 = 1$
Interval 2 (i = 2) :: [17, 30)	$A_{21} = 0$	$A_{22} = 1$	$R_2 = 1$
	$C_1 = 0$	$C_2 = 2$	$N = 2$

$$E_{11} = (R_1 \times C_1)/N = (1 \times 0)/2 \approx 0.1$$
$$E_{12} = (R_1 \times C_2)/N = (1 \times 2)/2 = 1$$
$$E_{21} = (R_2 \times C_1)/N = (1 \times 0)/2 \approx 0.1$$
$$E_{22} = (R_2 \times C_2)/N = (1 \times 2)/2 = 1$$

$$\begin{aligned}\chi^2 &= (A_{11} - E_{11})^2/E_{11} + (A_{12} - E_{12})^2/E_{12} + (A_{21} - E_{21})^2/E_{21} + (A_{22} - E_{22})^2/E_{22} \\ &= (0 - 0.1)^2/0.1 + (1 - 1)^2/1 + (0 - 0.1)^2/0.1 + (1 - 1)^2/1 \\ &= 0.2 < 2.706\end{aligned}$$

ID	Att	Class	Interval
1	1	A	[0, 2)
2	3	B	[2, 5)
3	7	A	[5, 7.5)
4	8	A	[7.5, 8.5)
5	9	A	[8.5, 10)
6	11	B	[10, 17)
7	23	B	[17, 30)
8	37	A	[30, 38)
9	39	B	[38, 42)
10	45	A	[42, 45.5]
11	46	A	[45.5, 52.5)
12	59	A	[52.5, 60)

	Class A (j = 1)	Class B (j = 2)	
Interval 1 (i = 1) :: [17, 30)	$A_{11} = 0$	$A_{12} = 1$	$R_1 = 1$
Interval 2 (i = 2) :: [30, 38)	$A_{21} = 1$	$A_{22} = 0$	$R_2 = 1$
	$C_1 = 1$	$C_2 = 1$	$N = 2$

$$E_{11} = (R_1 \times C_1)/N = (1 \times 1)/2 = 0.5$$
$$E_{12} = (R_1 \times C_2)/N = (1 \times 1)/2 = 0.5$$
$$E_{21} = (R_2 \times C_1)/N = (1 \times 1)/2 = 0.5$$
$$E_{22} = (R_2 \times C_2)/N = (1 \times 1)/2 = 0.5$$

$$\begin{aligned}\chi^2 &= (A_{11} - E_{11})^2/E_{11} + (A_{12} - E_{12})^2/E_{12} + (A_{21} - E_{21})^2/E_{21} + (A_{22} - E_{22})^2/E_{22} \\ &= (0 - 0.5)^2/0.5 + (1 - 0.5)^2/0.5 + (1 - 0.5)^2/0.5 + (0 - 0.5)^2/0.5 \\ &= 2 < 2.706\end{aligned}$$

ID	Att	Class	Interval
1	1	A	[0, 2)
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3	7	A	[5, 7.5)
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6	11	B	[10, 17)
7	23	B	[17, 30)
8	37	A	[30, 38)
9	39	B	[38, 42)
10	45	A	[42, 45.5]
11	46	A	[45.5, 52.5)
12	59	A	[52.5, 60)

	Class A (j = 1)	Class B (j = 2)	
Interval 1 (i = 1) :: [30, 38)	$A_{11} = 1$	$A_{12} = 0$	$R_1 = 1$
Interval 2 (i = 2) :: [38, 42)	$A_{21} = 0$	$A_{22} = 1$	$R_2 = 1$
	$C_1 = 1$	$C_2 = 1$	$N = 2$

$$E_{11} = (R_1 \times C_1)/N = (1 \times 1)/2 = 0.5$$
$$E_{12} = (R_1 \times C_2)/N = (1 \times 1)/2 = 0.5$$
$$E_{21} = (R_2 \times C_1)/N = (1 \times 1)/2 = 0.5$$
$$E_{22} = (R_2 \times C_2)/N = (1 \times 1)/2 = 0.5$$

$$\begin{aligned}\chi^2 &= (A_{11} - E_{11})^2/E_{11} + (A_{12} - E_{12})^2/E_{12} + (A_{21} - E_{21})^2/E_{21} + (A_{22} - E_{22})^2/E_{22} \\ &= (1 - 0.5)^2/0.5 + (0 - 0.5)^2/0.5 + (0 - 0.5)^2/0.5 + (1 - 0.5)^2/0.5 \\ &= 2 < 2.706\end{aligned}$$

ID	Att	Class	Interval
1	1	A	[0, 2)
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3	7	A	[5, 7.5)
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5	9	A	[8.5, 10)
6	11	B	[10, 17)
7	23	B	[17, 30)
8	37	A	[30, 38)
9	39	B	[38, 42)
10	45	A	[42, 45.5]
11	46	A	[45.5, 52.5)
12	59	A	[52.5, 60)

	Class A (j = 1)	Class B (j = 2)	
Interval 1 (i = 1) :: [38, 42)	$A_{11} = 0$	$A_{12} = 1$	$R_1 = 1$
Interval 2 (i = 2) :: [42, 45.5)	$A_{21} = 1$	$A_{22} = 0$	$R_2 = 1$
	$C_1 = 1$	$C_2 = 1$	N = 2

$$E_{11} = (R_1 \times C_1)/N = (1 \times 1)/2 = 0.5$$
$$E_{12} = (R_1 \times C_2)/N = (1 \times 1)/2 = 0.5$$
$$E_{21} = (R_2 \times C_1)/N = (1 \times 1)/2 = 0.5$$
$$E_{22} = (R_2 \times C_2)/N = (1 \times 1)/2 = 0.5$$

$$\begin{aligned}\chi^2 &= (A_{11} - E_{11})^2/E_{11} + (A_{12} - E_{12})^2/E_{12} + (A_{21} - E_{21})^2/E_{21} + (A_{22} - E_{22})^2/E_{22} \\ &= (0 - 0.5)^2/0.5 + (1 - 0.5)^2/0.5 + (1 - 0.5)^2/0.5 + (0 - 0.5)^2/0.5 \\ &= 2 < 2.706\end{aligned}$$

ID	Att	Class	Interval
1	1	A	[0, 2)
2	3	B	[2, 5)
3	7	A	[5, 7.5)
4	8	A	[7.5, 8.5)
5	9	A	[8.5, 10)
6	11	B	[10, 17)
7	23	B	[17, 30)
8	37	A	[30, 38)
9	39	B	[38, 42)
10	45	A	[42, 45.5]
11	46	A	[45.5, 52.5)
12	59	A	[52.5, 60)

	Class A (j = 1)	Class B (j = 2)	
Interval 1 (i = 1) :: [42, 45.5)	$A_{11} = 1$	$A_{12} = 0$	$R_1 = 1$
Interval 2 (i = 2) :: [45.5, 52.5)	$A_{21} = 1$	$A_{22} = 0$	$R_2 = 1$
	$C_1 = 2$	$C_2 = 0$	N = 2

$$E_{11} = (R_1 \times C_1)/N = (1 \times 2)/2 = 1$$
$$E_{12} = (R_1 \times C_2)/N = (1 \times 0)/2 \approx 0.1$$
$$E_{21} = (R_2 \times C_1)/N = (1 \times 2)/2 = 1$$
$$E_{22} = (R_2 \times C_2)/N = (1 \times 0)/2 \approx 0.1$$

$$\begin{aligned}\chi^2 &= (A_{11} - E_{11})^2/E_{11} + (A_{12} - E_{12})^2/E_{12} + (A_{21} - E_{21})^2/E_{21} + (A_{22} - E_{22})^2/E_{22} \\ &= (1-1)^2/1 + (0-0.1)^2/0.1 + (1-1)^2/1 + (0-0.1)^2/0.1 \\ &= 0.2 < 2.706\end{aligned}$$

ID	Att	Class	Interval
1	1	A	[0, 2)
2	3	B	[2, 5)
3	7	A	[5, 7.5)
4	8	A	[7.5, 8.5)
5	9	A	[8.5, 10)
6	11	B	[10, 17)
7	23	B	[17, 30)
8	37	A	[30, 38)
9	39	B	[38, 42)
10	45	A	[42, 45.5]
11	46	A	[45.5, 52.5)
12	59	A	[52.5, 60)

	Class A (j = 1)	Class B (j = 2)	
Interval 1 (i = 1) :: [45.5, 52.5)	$A_{11} = 1$	$A_{12} = 0$	$R_1 = 1$
Interval 2 (i = 2) :: [52.5, 60)	$A_{21} = 1$	$A_{22} = 0$	$R_2 = 1$
	$C_1 = 2$	$C_2 = 0$	N = 2

$$E_{11} = (R_1 \times C_1)/N = (1 \times 2)/2 = 1$$
$$E_{12} = (R_1 \times C_2)/N = (1 \times 0)/2 \approx 0.1$$
$$E_{21} = (R_2 \times C_1)/N = (1 \times 2)/2 = 1$$
$$E_{22} = (R_2 \times C_2)/N = (1 \times 0)/2 \approx 0.1$$

$$\begin{aligned}\chi^2 &= (A_{11} - E_{11})^2/E_{11} + (A_{12} - E_{12})^2/E_{12} + (A_{21} - E_{21})^2/E_{21} + (A_{22} - E_{22})^2/E_{22} \\ &= (1-1)^2/1 + (0-0.1)^2/0.1 + (1-1)^2/1 + (0-0.1)^2/0.1 \\ &= 0.2 < 2.706\end{aligned}$$

مرحلة ٢:

ID	Att	Class	Interval
1	1	A	[0, 2)
2	3	B	[2, 5)
3	7	A	[5, 10)
4	8	A	
5	9	A	
6	11	B	[10, 30)
7	23	B	
8	37	A	[30, 38)
9	39	B	[38, 42)
10	45	A	[42, 60)
11	46	A	
12	59	A	

	Class A (j = 1)	Class B (j = 2)	
Interval 1 (i = 1) :: [0, 2)	$A_{11} = 1$	$A_{12} = 0$	$R_1 = 1$
Interval 2 (i = 2) :: [2, 5)	$A_{21} = 0$	$A_{22} = 1$	$R_2 = 1$
	$C_1 = 1$	$C_2 = 1$	N = 2

$$E_{11} = (R_1 \times C_1)/N = (1\times1)/2 = 0.5$$
$$E_{12} = (R_1 \times C_2)/N = (1\times1)/2 = 0.5$$
$$E_{21} = (R_2 \times C_1)/N = (1\times1)/2 = 0.5$$
$$E_{22} = (R_2 \times C_2)/N = (1\times1)/2 = 0.5$$

$$\begin{aligned}\chi^2 &= (A_{11}-E_{11})^2/E_{11} + (A_{12}-E_{12})^2/E_{12} + (A_{21}-E_{21})^2/E_{21} + (A_{22}-E_{22})^2/E_{22} \\ &= (1-0.5)^2/0.5 + (0-0.5)^2/0.5 + (0-0.5)^2/0.5 + (1-0.5)^2/0.5 \\ &= 2 < 2.706\end{aligned}$$

ID	Att	Class	Interval
1	1	A	[0, 2)
2	3	B	[2, 5)
3	7	A	[5, 10)
4	8	A	
5	9	A	
6	11	B	[10, 30)
7	23	B	
8	37	A	[30, 38)
9	39	B	[38, 42)
10	45	A	[42, 60)
11	46	A	
12	59	A	

	Class A (j = 1)	Class B (j = 2)	
Interval 1 (i = 1) :: [2, 5)	$A_{11} = 0$	$A_{12} = 1$	$R_1 = 1$
Interval 2 (i = 2) :: [5, 10)	$A_{21} = 3$	$A_{22} = 0$	$R_2 = 3$
	$C_1 = 3$	$C_2 = 1$	N = 4

$$E_{11} = (R_1 \times C_1)/N = (1\times 3)/4 = 0.75$$
$$E_{12} = (R_1 \times C_2)/N = (1\times 1)/4 = 0.25$$
$$E_{21} = (R_2 \times C_1)/N = (3\times 3)/4 = 2.25$$
$$E_{22} = (R_2 \times C_2)/N = (3\times 1)/4 = 0.75$$

$$\begin{aligned}\chi^2 &= (A_{11}-E_{11})^2/E_{11} + (A_{12}-E_{12})^2/E_{12} + (A_{21}-E_{21})^2/E_{21} + (A_{22}-E_{22})^2/E_{22} \\ &= (0-0.75)^2/0.75 + (1-0.25)^2/0.25 + (3-2.25)^2/2.25 + (0-0.75)^2/0.75 \\ &= 4 > 2.706\end{aligned}$$

ID	Att	Class	Interval
1	1	A	[0, 2)
2	3	B	[2, 5)
3	7	A	[5, 10)
4	8	A	
5	9	A	
6	11	B	[10, 30)
7	23	B	
8	37	A	[30, 38)
9	39	B	[38, 42)
10	45	A	[42, 60)
11	46	A	
12	59	A	

	Class A (j = 1)	Class B (j = 2)	
Interval 1 (i = 1) :: [5, 10)	$A_{11} = 3$	$A_{12} = 0$	$R_1 = 3$
Interval 2 (i = 2) :: [10, 30)	$A_{21} = 0$	$A_{22} = 2$	$R_2 = 2$
	$C_1 = 3$	$C_2 = 2$	N = 5

$$E_{11} = (R_1 \times C_1)/N = (3\times 3)/5 = 1.8$$
$$E_{12} = (R_1 \times C_2)/N = (3\times 2)/5 = 1.2$$
$$E_{21} = (R_2 \times C_1)/N = (2\times 3)/5 = 1.2$$
$$E_{22} = (R_2 \times C_2)/N = (2\times 2)/5 = 0.8$$

$$\begin{aligned}\chi^2 &= (A_{11}-E_{11})^2/E_{11} + (A_{12}-E_{12})^2/E_{12} + (A_{21}-E_{21})^2/E_{21} + (A_{22}-E_{22})^2/E_{22} \\ &= (3-1.8)^2/1.8 + (0-1.2)^2/1.2 + (0-1.2)^2/1.2 + (2-0.8)^2/0.8 \\ &= 5 > 2.706\end{aligned}$$

ID	Att	Class	Interval
1	1	A	[0, 2)
2	3	B	[2, 5)
3	7	A	[5, 10)
4	8	A	
5	9	A	
6	11	B	[10, 30)
7	23	B	
8	37	A	[30, 38)
9	39	B	[38, 42)
10	45	A	[42, 60)
11	46	A	
12	59	A	

	Class A (j = 1)	Class B (j = 2)	
Interval 1 (i = 1) :: [10, 30)	$A_{11} = 0$	$A_{12} = 2$	$R_1 = 2$
Interval 2 (i = 2) :: [30, 38)	$A_{21} = 1$	$A_{22} = 0$	$R_2 = 1$
	$C_1 = 1$	$C_2 = 2$	N = 3

$$E_{11} = (R_1 \times C_1)/N = (2 \times 1)/3 = 0.66$$
$$E_{12} = (R_1 \times C_2)/N = (2 \times 2)/3 = 1.33$$
$$E_{21} = (R_2 \times C_1)/N = (1 \times 1)/3 = 0.33$$
$$E_{22} = (R_2 \times C_2)/N = (1 \times 2)/3 = 0.66$$

$$\begin{aligned}\chi^2 &= (A_{11} - E_{11})^2/E_{11} + (A_{12} - E_{12})^2/E_{12} + (A_{21} - E_{21})^2/E_{21} + (A_{22} - E_{22})^2/E_{22} \\ &= (0 - 0.66)^2/0.66 + (2 - 1.33)^2/1.33 + (1 - 0.33)^2/0.33 + (0 - 0.66)^2/0.66 \\ &= 3.0178 > 2.706\end{aligned}$$

ID	Att	Class	Interval
1	1	A	[0, 2)
2	3	B	[2, 5)
3	7	A	[5, 10)
4	8	A	
5	9	A	
6	11	B	[10, 30)
7	23	B	
8	37	A	[30, 38)
9	39	B	[38, 42)
10	45	A	[42, 60)
11	46	A	
12	59	A	

	Class A (j = 1)	Class B (j = 2)	
Interval 1 (i = 1) :: [30, 38)	$A_{11} = 1$	$A_{12} = 0$	$R_1 = 1$
Interval 2 (i = 2) :: [38, 42)	$A_{21} = 0$	$A_{22} = 1$	$R_2 = 1$
	$C_1 = 1$	$C_2 = 1$	$N = 2$

$$E_{11} = (R_1 \times C_1)/N = (1 \times 1)/2 = 0.5$$
$$E_{12} = (R_1 \times C_2)/N = (1 \times 1)/2 = 0.5$$
$$E_{21} = (R_2 \times C_1)/N = (1 \times 1)/2 = 0.5$$
$$E_{22} = (R_2 \times C_2)/N = (1 \times 1)/2 = 0.5$$

$$\begin{aligned}\chi^2 &= (A_{11} - E_{11})^2/E_{11} + (A_{12} - E_{12})^2/E_{12} + (A_{21} - E_{21})^2/E_{21} + (A_{22} - E_{22})^2/E_{22} \\ &= (1 - 0.5)^2/0.5 + (0 - 0.5)^2/0.5 + (0 - 0.5)^2/0.5 + (1 - 0.5)^2/0.5 \\ &= 2 < 2.706\end{aligned}$$

ID	Att	Class	Interval
1	1	A	[0, 2)
2	3	B	[2, 5)
3	7	A	[5, 10)
4	8	A	
5	9	A	
6	11	B	[10, 30)
7	23	B	
8	37	A	[30, 38)
9	39	B	[38, 42)
10	45	A	[42, 60)
11	46	A	
12	59	A	

	Class A (j = 1)	Class B (j = 2)	
Interval 1 (i = 1) :: [38, 42)	$A_{11} = 0$	$A_{12} = 1$	$R_1 = 1$
Interval 2 (i = 2) :: [42, 60)	$A_{21} = 3$	$A_{22} = 0$	$R_2 = 3$
	$C_1 = 3$	$C_2 = 1$	N = 4

$$E_{11} = (R_1 \times C_1)/N = (1 \times 3)/4 = 0.75$$
$$E_{12} = (R_1 \times C_2)/N = (1 \times 1)/4 = 0.25$$
$$E_{21} = (R_2 \times C_1)/N = (3 \times 3)/4 = 2.25$$
$$E_{22} = (R_2 \times C_2)/N = (3 \times 1)/4 = 0.75$$

$$\begin{aligned}\chi^2 &= (A_{11} - E_{11})^2/E_{11} + (A_{12} - E_{12})^2/E_{12} + (A_{21} - E_{21})^2/E_{21} + (A_{22} - E_{22})^2/E_{22} \\ &= (0 - 0.75)^2/0.75 + (1 - 0.25)^2/0.25 + (3 - 2.25)^2/2.25 + (0 - 0.75)^2/0.75 \\ &= 4 > 2.706\end{aligned}$$

مرحلة ٣:

ID	Att	Class	Interval
1	1	A	[0, 5)
2	3	B	
3	7	A	[5, 10)
4	8	A	
5	9	A	
6	11	B	[10, 30)
7	23	B	
8	37	A	[30, 42)
9	39	B	
10	45	A	[42, 60)
11	46	A	
12	59	A	

	Class A (j = 1)	Class B (j = 2)	
Interval 1 (i = 1) :: [0, 5)	$A_{11} = 1$	$A_{12} = 1$	$R_1 = 2$
Interval 2 (i = 2) :: [5, 10)	$A_{21} = 3$	$A_{22} = 0$	$R_2 = 3$
	$C_1 = 4$	$C_2 = 1$	N = 5

$$E_{11} = (R_1 \times C_1)/N = (2\times 4)/5 = 1.6$$
$$E_{12} = (R_1 \times C_2)/N = (2\times 1)/5 = 0.4$$
$$E_{21} = (R_2 \times C_1)/N = (3\times 4)/5 = 2.4$$
$$E_{22} = (R_2 \times C_2)/N = (3\times 1)/5 = 0.6$$

$$\begin{aligned}\chi^2 &= (A_{11}-E_{11})^2/E_{11} + (A_{12}-E_{12})^2/E_{12} + (A_{21}-E_{21})^2/E_{21} + (A_{22}-E_{22})^2/E_{22} \\ &= (1-1.6)^2/1.6 + (1-0.4)^2/0.4 + (3-2.4)^2/2.4 + (0-0.6)^2/0.6 \\ &= 1.8750 < 2.706\end{aligned}$$

ID	Att	Class	Interval
1	1	A	[0, 5)
2	3	B	
3	7	A	[5, 10)
4	8	A	
5	9	A	
6	11	B	[10, 30)
7	23	B	
8	37	A	[30, 42)
9	39	B	
10	45	A	[42, 60)
11	46	A	
12	59	A	

	Class A (j = 1)	Class B (j = 2)	
Interval 1 (i = 1) :: [5, 10)	$A_{11} = 3$	$A_{12} = 0$	$R_1 = 3$
Interval 2 (i = 2) :: [10, 30)	$A_{21} = 0$	$A_{22} = 2$	$R_2 = 2$
	$C_1 = 3$	$C_2 = 2$	$N = 5$

$$E_{11} = (R_1 \times C_1)/N = (3\times 3)/5 = 1.8$$
$$E_{12} = (R_1 \times C_2)/N = (3\times 2)/5 = 1.2$$
$$E_{21} = (R_2 \times C_1)/N = (2\times 3)/5 = 1.2$$
$$E_{22} = (R_2 \times C_2)/N = (2\times 2)/5 = 0.8$$

$$\begin{aligned}\chi^2 &= (A_{11}-E_{11})^2/E_{11} + (A_{12}-E_{12})^2/E_{12} + (A_{21}-E_{21})^2/E_{21} + (A_{22}-E_{22})^2/E_{22} \\ &= (3-1.8)^2/1.8 + (0-1.2)^2/1.2 + (0-1.2)^2/1.2 + (2-0.8)^2/0.8 \\ &= 5 > 2.706\end{aligned}$$

ID	Att	Class	Interval
1	1	A	[0, 5)
2	3	B	
3	7	A	[5, 10)
4	8	A	
5	9	A	
6	11	B	[10, 30)
7	23	B	
8	37	A	[30, 42)
9	39	B	
10	45	A	[42, 60)
11	46	A	
12	59	A	

	Class A (j = 1)	Class B (j = 2)	
Interval 1 (i = 1) :: [10, 30)	$A_{11} = 0$	$A_{12} = 2$	$R_1 = 2$
Interval 2 (i = 2) :: [30, 42)	$A_{21} = 1$	$A_{22} = 1$	$R_2 = 2$
	$C_1 = 1$	$C_2 = 3$	N = 4

$$E_{11} = (R_1 \times C_1)/N = (2 \times 1)/4 = 0.5$$
$$E_{12} = (R_1 \times C_2)/N = (2 \times 3)/4 = 1.5$$
$$E_{21} = (R_2 \times C_1)/N = (2 \times 1)/4 = 0.5$$
$$E_{22} = (R_2 \times C_2)/N = (2 \times 3)/4 = 1.5$$

$$\begin{aligned}\chi^2 &= (A_{11} - E_{11})^2/E_{11} + (A_{12} - E_{12})^2/E_{12} + (A_{21} - E_{21})^2/E_{21} + (A_{22} - E_{22})^2/E_{22} \\ &= (0 - 0.5)^2/0.5 + (2 - 1.5)^2/1.5 + (1 - 0.5)^2/0.5 + (1 - 1.5)^2/1.5 \\ &= 1.3333 < 2.706\end{aligned}$$

ID	Att	Class	Interval
1	1	A	[0, 5)
2	3	B	
3	7	A	[5, 10)
4	8	A	
5	9	A	
6	11	B	[10, 30)
7	23	B	
8	37	A	[30, 42)
9	39	B	
10	45	A	[42, 60)
11	46	A	
12	59	A	

	Class A (j = 1)	Class B (j = 2)	
Interval 1 (i = 1) :: [30, 42)	$A_{11} = 1$	$A_{12} = 1$	$R_1 = 2$
Interval 2 (i = 2) :: [42, 60)	$A_{21} = 3$	$A_{22} = 0$	$R_2 = 3$
	$C_1 = 4$	$C_2 = 1$	N = 5

$$E_{11} = (R_1 \times C_1)/N = (2 \times 4)/5 = 1.6$$
$$E_{12} = (R_1 \times C_2)/N = (2 \times 1)/5 = 0.4$$
$$E_{21} = (R_2 \times C_1)/N = (3 \times 4)/5 = 2.4$$
$$E_{22} = (R_2 \times C_2)/N = (3 \times 1)/5 = 0.6$$

$$\begin{aligned}\chi^2 &= (A_{11} - E_{11})^2/E_{11} + (A_{12} - E_{12})^2/E_{12} + (A_{21} - E_{21})^2/E_{21} + (A_{22} - E_{22})^2/E_{22} \\ &= (1-1.6)^2/1.6 + (1-0.4)^2/0.4 + (3-2.4)^2/2.4 + (0-0.6)^2/0.6 \\ &= 1.8750 < 2.706\end{aligned}$$

مرحلة ۴:

ID	Att	Class	Interval
1	1	A	[0, 5)
2	3	B	
3	7	A	[5, 10)
4	8	A	
5	9	A	
6	11	B	[10, 42)
7	23	B	
8	37	A	
9	39	B	
10	45	A	[42, 60)
11	46	A	
12	59	A	

	Class A (j = 1)	Class B (j = 2)	
Interval 1 (i = 1) :: [0, 5)	$A_{11} = 1$	$A_{12} = 1$	$R_1 = 2$
Interval 2 (i = 2) :: [5, 10)	$A_{21} = 3$	$A_{22} = 0$	$R_2 = 3$
	$C_1 = 4$	$C_2 = 1$	N = 5

$$E_{11} = (R_1 \times C_1)/N = (2\times 4)/5 = 1.6$$
$$E_{12} = (R_1 \times C_2)/N = (2\times 1)/5 = 0.4$$
$$E_{21} = (R_2 \times C_1)/N = (3\times 4)/5 = 2.4$$
$$E_{22} = (R_2 \times C_2)/N = (3\times 1)/5 = 0.6$$

$$\begin{aligned}\chi^2 &= (A_{11}-E_{11})^2/E_{11} + (A_{12}-E_{12})^2/E_{12} + (A_{21}-E_{21})^2/E_{21} + (A_{22}-E_{22})^2/E_{22} \\ &= (1-1.6)^2/1.6 + (1-0.4)^2/0.4 + (3-2.4)^2/2.4 + (0-0.6)^2/0.6 \\ &= 1.8750 < 2.706\end{aligned}$$

ID	Att	Class	Interval
1	1	A	[0, 5)
2	3	B	
3	7	A	[5, 10)
4	8	A	
5	9	A	
6	11	B	[10, 42)
7	23	B	
8	37	A	
9	39	B	
10	45	A	[42, 60)
11	46	A	
12	59	A	

	Class A (j = 1)	Class B (j = 2)	
Interval 1 (i = 1) :: [5, 10)	$A_{11} = 3$	$A_{12} = 0$	$R_1 = 3$
Interval 2 (i = 2) :: [10, 42)	$A_{21} = 1$	$A_{22} = 3$	$R_2 = 4$
	$C_1 = 4$	$C_2 = 3$	$N = 7$

$$E_{11} = (R_1 \times C_1)/N = (3 \times 4)/7 = 1.71$$
$$E_{12} = (R_1 \times C_2)/N = (3 \times 3)/7 = 1.29$$
$$E_{21} = (R_2 \times C_1)/N = (4 \times 4)/7 = 2.29$$
$$E_{22} = (R_2 \times C_2)/N = (4 \times 3)/7 = 1.71$$

$$\begin{aligned}\chi^2 &= (A_{11} - E_{11})^2/E_{11} + (A_{12} - E_{12})^2/E_{12} + (A_{21} - E_{21})^2/E_{21} + (A_{22} - E_{22})^2/E_{22} \\ &= (3 - 1.71)^2/1.71 + (0 - 1.29)^2/1.29 + (1 - 2.29)^2/2.29 + (3 - 1.71)^2/1.71 \\ &= 3.9630 > 2.706\end{aligned}$$

ID	Att	Class	Interval
1	1	A	[0, 5)
2	3	B	
3	7	A	[5, 10)
4	8	A	
5	9	A	
6	11	B	[10, 42)
7	23	B	
8	37	A	
9	39	B	
10	45	A	[42, 60)
11	46	A	
12	59	A	

	Class A (j = 1)	Class B (j = 2)	
Interval 1 (i = 1) :: [10, 42)	$A_{11} = 1$	$A_{12} = 3$	$R_1 = 4$
Interval 2 (i = 2) :: [42, 60)	$A_{21} = 3$	$A_{22} = 0$	$R_2 = 3$
	$C_1 = 4$	$C_2 = 3$	N = 7

$$E_{11} = (R_1 \times C_1)/N = (4 \times 4)/7 = 2.29$$
$$E_{12} = (R_1 \times C_2)/N = (4 \times 3)/7 = 1.71$$
$$E_{21} = (R_2 \times C_1)/N = (3 \times 4)/7 = 1.71$$
$$E_{22} = (R_2 \times C_2)/N = (3 \times 3)/7 = 1.29$$

$$\begin{aligned}\chi^2 &= (A_{11} - E_{11})^2/E_{11} + (A_{12} - E_{12})^2/E_{12} + (A_{21} - E_{21})^2/E_{21} + (A_{22} - E_{22})^2/E_{22} \\ &= (1 - 2.29)^2/2.29 + (3 - 1.71)^2/1.71 + (3 - 1.71)^2/1.71 + (0 - 1.29)^2/1.29 \\ &= 3.9630 > 2.706\end{aligned}$$

مرحلهٔ۵:

ID	Att	Class	Interval
1	1	A	[0, 10)
2	3	B	
3	7	A	
4	8	A	
5	9	A	
6	11	B	[10, 42)
7	23	B	
8	37	A	
9	39	B	
10	45	A	[42, 60)
11	46	A	
12	59	A	

	Class A (j = 1)	Class B (j = 2)	
Interval 1 (i = 1) :: [0, 10)	$A_{11} = 4$	$A_{12} = 1$	$R_1 = 5$
Interval 2 (i = 2) :: [10, 42)	$A_{21} = 1$	$A_{22} = 3$	$R_2 = 4$
	$C_1 = 5$	$C_2 = 4$	$N = 9$

$$E_{11} = (R_1 \times C_1)/N = (5\times5)/9 = 2.78$$
$$E_{12} = (R_1 \times C_2)/N = (5\times4)/9 = 2.22$$
$$E_{21} = (R_2 \times C_1)/N = (4\times5)/9 = 2.22$$
$$E_{22} = (R_2 \times C_2)/N = (4\times4)/9 = 1.78$$

$$\begin{aligned}\chi^2 &= (A_{11}-E_{11})^2/E_{11} + (A_{12}-E_{12})^2/E_{12} + (A_{21}-E_{21})^2/E_{21} + (A_{22}-E_{22})^2/E_{22} \\ &= (4-2.78)^2/2.78 + (1-2.22)^2/2.22 + (1-2.22)^2/2.22 + (3-1.78)^2/1.78 \\ &= 2.7125 > 2.706\end{aligned}$$

نتیجه نهایی:

ID	Att	Class	Interval
1	1	A	[0, 10)
2	3	B	
3	7	A	
4	8	A	
5	9	A	
6	11	B	[10, 42)
7	23	B	
8	37	A	
9	39	B	
10	45	A	[42, 60)
11	46	A	
12	59	A	