Signals and systems

Homework #1

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Deadline : 13 Esfand, 1397 [23:55]

- Homeworks will not be accepted after the deadline.
- For theoretical problems, gather them in a single ***.pdf** file.
- For the matlab problems, provide both these materials:
  - **codes [*.m files]**
  - a simple **report** that includes all plots and screenshots.
- Notice that the homeworks will be **checked by plagiarism detectors**, avoid any similarities.
- Matlab problems and theoretical problems will be graded seperately (both will be graded out of 100), but their weights may be different and is determined by the course professor.

Question 1 (10 points)

find the even and odd components of the following signals.

[a] $x(t) = 1 + t\cos(t) + t^2\sin(t) + t^3\sin(t)\cos(t)$

[b] $x(t) = (1 + t^3)\cos^3(10t)$

[c] $x(t) = \Pi\left(t - \frac{1}{2}\right) = \begin{cases} 1\,, & 0 < t < 1 \\ 0\,, & \text{elsewhere} \end{cases}$

Question 2 (15 points)

For the following signals, determine whether they're periodic and find their fundamental period.

[a] $x(t) = \cos^2(2\pi t)$

[b] $x(t) = \sin^3(2t)$

[c] $x(t) = e^{-2t}\cos(2\pi t)$

[d] $x[n] = 5\cos[2n]$

[e] $x[n] = \sin\left[\frac{6\pi n}{35}\right]$

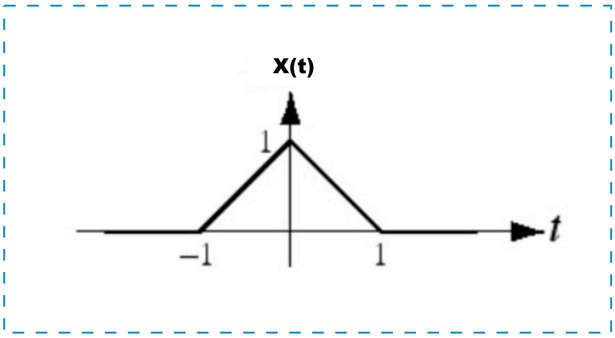
[f] $x[n] = e^{\frac{jn}{2}} + e^{\frac{jn}{3}}$

[g] $x[n] = e^{\frac{j\pi n}{2}} + e^{\frac{j\pi n}{3}}$

[h] $x[n] = \sin\left(\frac{3\pi}{5}n^2\right)$

Question 3 (10 points)

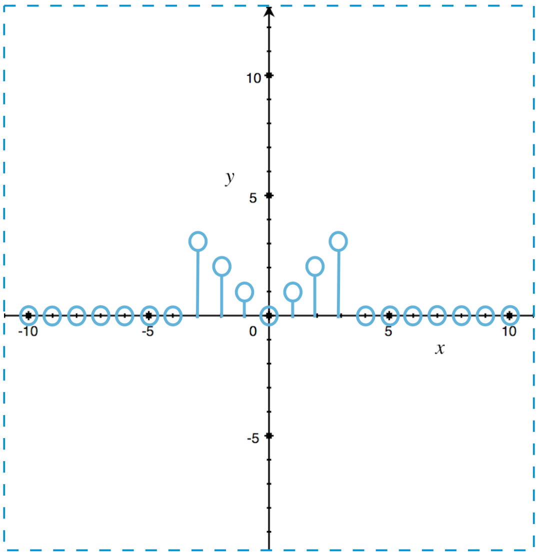
[a] a triangular pulse signal x(t) is drawn below :



draw the following signal :

$$x(2t) + x(2t-2)$$

[b] let x[n] be the following signal :

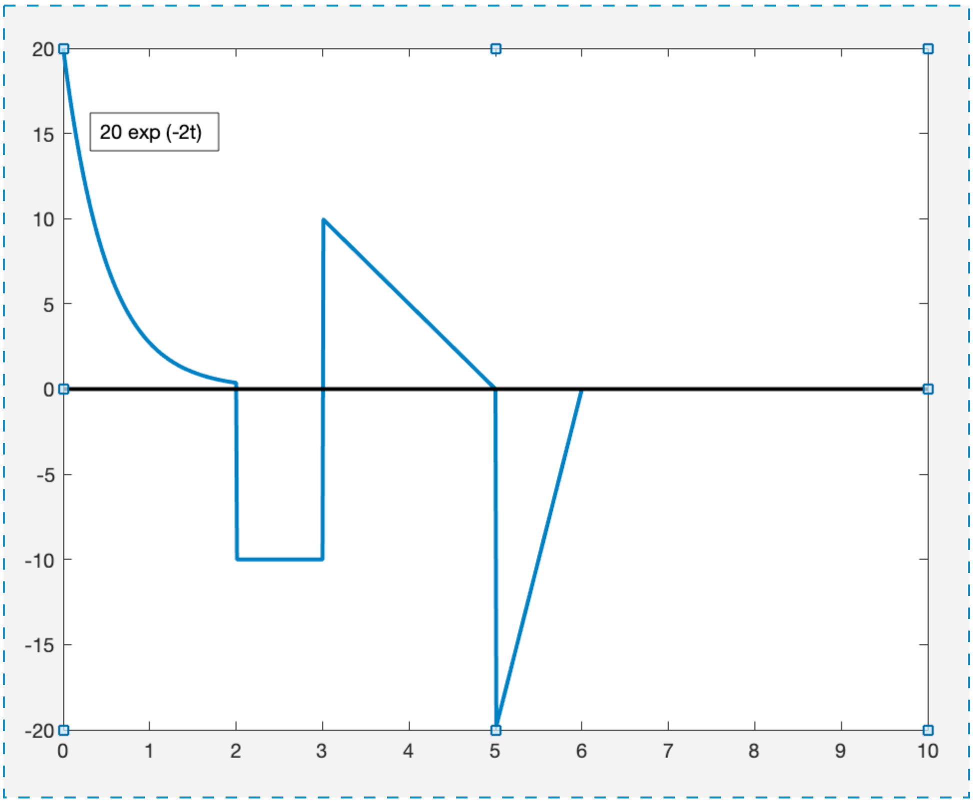


draw the following signal :

$$x[2n] - x[3-3n]$$

Question 4 (15 points)

express the following waveform as the sum of unit step functions and using the result, compute its derivative and draw it.



Question 5 (10 points)

consider the following signals, determine their average power :

[a] $x(t) = A\,cos(\omega t + \phi)$

[b]
$$x(t) = \begin{cases} 5\text{-}t & 4 \le t \le 5 \\ 1 & \text{-}4 \le t \le 4 \\ t+5 & \text{-}5 \le t \le \text{-}4 \\ 0 & otherwise \end{cases}$$

Question 6 (30 points)

determine and explain whether the following systems are

1. memoryless
2. stable
3. causal
4. linear
5. time-invariant

[a] $y(t) = x(5-t) + c$

[b] $y(t) = \sin(x(t))$

[c] $y[n] = -x[n]u[n]$

[d] $y(t) = x(\cos(t))$

[e] $y(t) = \frac{dx(t)}{dt}$

[f] $y[n] = \sum_{k=-\infty}^{n} x[k+2]$

[g] $y[n] = x[n] \sum_{k=-\infty}^{\infty} \delta[n-2k]$

[h] $y[n] = cos(2\pi x[n+1]) + x[n]$

[i] $y(t) = \int_{-\infty}^{\frac{t}{2}} x(\tau)\, d\tau$

[j] $y(t) = \begin{cases} x(t+2) & t > 0 \\ x(t-2) & t \le 0 \end{cases}$

[k] $y(t) = \frac{d}{dt}\{ e^{-t} x(t) \}$

[l] $y[n] = log_{10}(|x[n]|)$

Question 7 (10 points)

determine weather the following systems are invertable, and if they are, find out their inverted system.

[a] $y(t) = \frac{d(x(t))}{dt}$

[b] $y(t) = odd(x(t))$

[c] $y(t) = x\left(\frac{t}{3}\right)$

Matlab Question 1 (plotting continues time signals) (20 points)

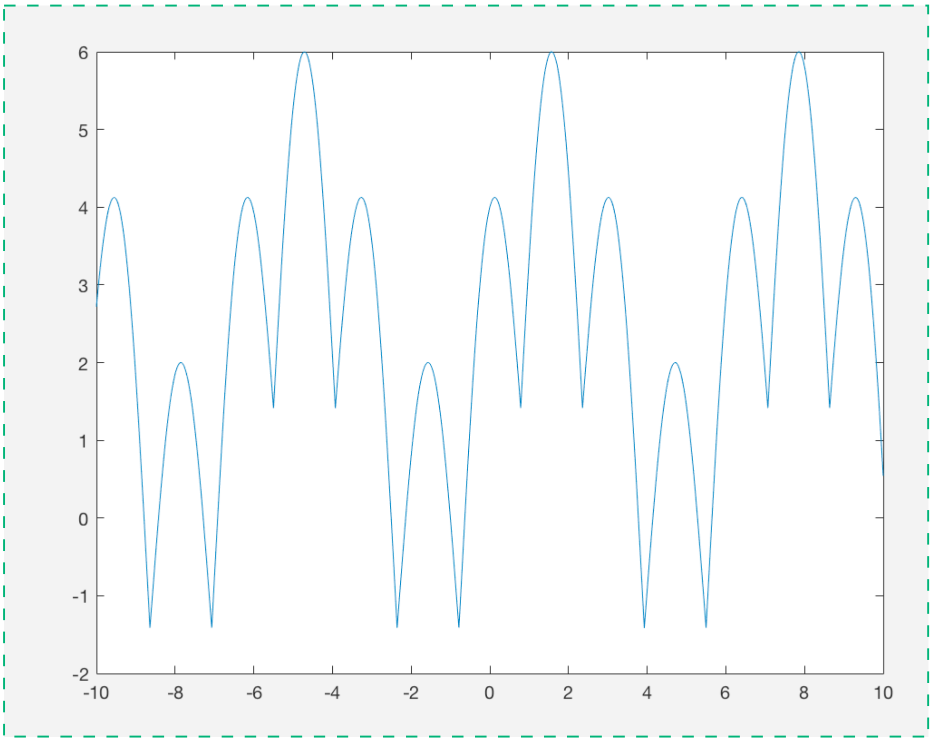
in this problem, we wanna use matlab **plot** function to plot continues-time signals.
consider the signal below :

x(t) = 2sin(t) + 4*|cos(2t)|

we're gonna plot this signal from t=-10 to t=10 with 1ms time-steps.
the matlab code for plotting this signal will be

```
%defining the independent variable
t = -10:0.001:10;
%defining the dependent variable
x = 2*sin(t) + 4*abs(cos(2*t));
%plotting the dependent variable with respect to independent variable
plot(t,x);
```

by the above code, we obtain the following figure :



Matlab Question 1

plot the following continues-time signals for t=-1 to t=10 with 1ms time-steps.

[a] $x(t) = 2e^{-3t}\, u(t)$

[b] $x(t) = e^{-2t} \sin(3t\text{-}1)\, u(t)$

[c] $x(t) = e^{-t}u(t) + 4\cos(2t\text{-}2)$

[d] $x\left(t\right) = \begin{cases} e^{-3t} - e^{-6t} & t \geq 0 \\ 0 & t < 0 \end{cases}$

hints :

- use matlab **heaviside( )** function for step functions
- for multi-conditional functions, you can use **FOR** to define dependent variable over different intervals of independent variable.

Matlab Question 2 (plotting discrete time signals) (20 points)

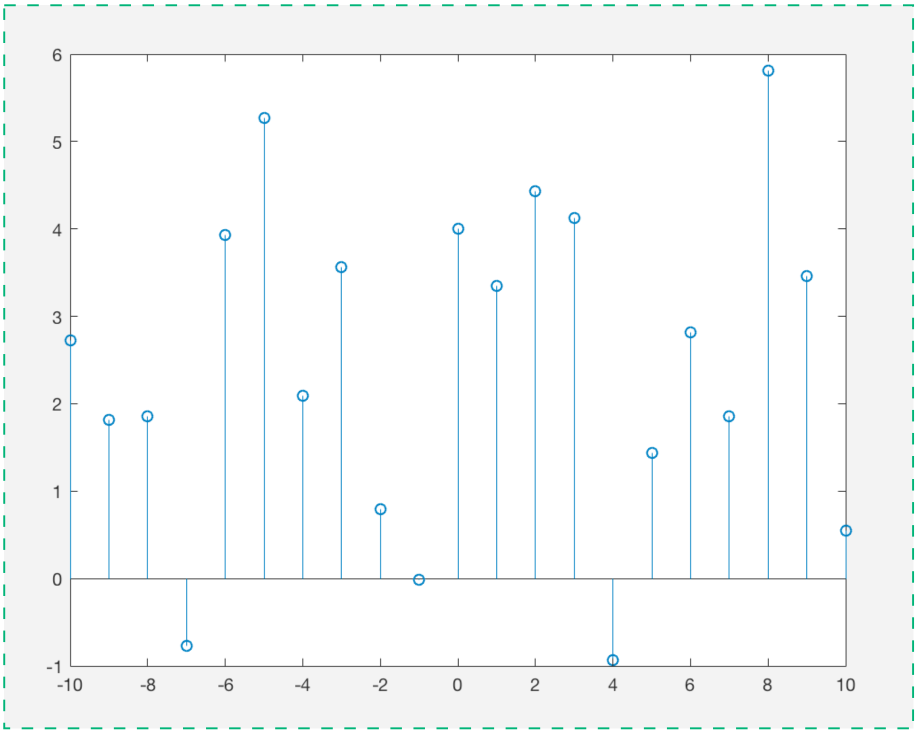
in this problem, we wanna use matlab **stem** function to plot discrete-time signals.
consider the signal below :

x[n] = 2sin[n] + 4*|cos[2n]|

we're gonna plot this signal from t=-10 to t=10 with 1ms time-steps.
the matlab code for plotting this signal will be

```
%defining the independent variable
t = -10:1:10;
%defining the dependent variable
x = 2*sin(t) + 4*abs(cos(2*t));
%plotting the dependent variable with respect to independent variable
stem(t,x);
```

by the above code, we obtain the following figure :



Matlab Question 2

now plot the following discrete-time signals using for n=-10 to t=50

[a] $$x[n] = (0.5)^n \left(\sin\left(\frac{\pi n}{4}\right) + \cos\left(\frac{n\pi}{4}\right)\right)$$

[b] $$x[n] = 3\,u[n-2] + \left(1 - e^{0.2n}\right) u[-n+1]$$

Matlab Question 3 (basic interfacing with matlab) (20 points)

Consider a complex exponential signal $x[n] = Ae^{-an}$ where

$$A = 5\cos\left(\frac{\pi}{12}n\right)$$

and a is a complex number in form of M + (M/2) j

we want to findout the minimum amount for M so that the energy of our signal over n = 0 to 100 interval not exceed 300.

write down a script to find the answer, then plot the real and imaginary parts of the resultant signal.

hints :

- initialize M with 1 and reduce it by 0.001 at each iteration of a while loop and in each iteration, check if the energy of the signal exceeds 300.

Matlab Question 4 (writing matlab functions) (20 points)

sometimes, we need to extract statistical features of a specific signal. The most important statistical features that a signal contains are including :

- mean
- mode
- variance
- median
- min
- max

Matlab provides functions to calculate these values:

```
x = [ 1 2 3 4 5 1 2 3];
disp(mean(x));
disp(median(x));
disp(mode(x));
disp(var(x));
disp(min(x));
disp(max(x));
```

the results will be :

```
2.6250

2.5000

 1

1.9821

 1

 5
```

to get familiar with matlab programming, you need to write down a **single function** that calculates all these six statistical values for a given input vector without using matlab functions.

Matlab Question 5 (handling files and variables) (20 points)

In this problem, we wanna get more familiar with matlab signal processing features.
for now, we're gonna just load and play with a simple sound file.

load the appended sound file using matlab **uiopen( )** function.
2 different variables will be added to your workspace, **fs** is sampling frequency of the sound and **data** is the sound itself. **data** has 2 columns as the sound file is stereo-typed.
use matlab **sound( )** function to play the sound. the **sound( )** function gets 2 arguments, first is the sound data and second is sampling frequency.

[a] set fs=1000, fs=20000, fs=40000 and fs=100000 and play it. listen to the results and explain whats happening.

[b] now plot the data for right and left stereos, seperately. include the result in your answers.

for the following parts, consider fs=44100Hz (which is the default sampling frequency of this sound file)

[c] now, we wanna add a **fading effect** to the sound, so that the sound starts normally, and instead of finishing suddenly, its starts to fade some seconds before the sound ends. write a simple code to do so, then save the result as **.mat** file and include it in your answers (codes also must be included).