Bohr's Thoery

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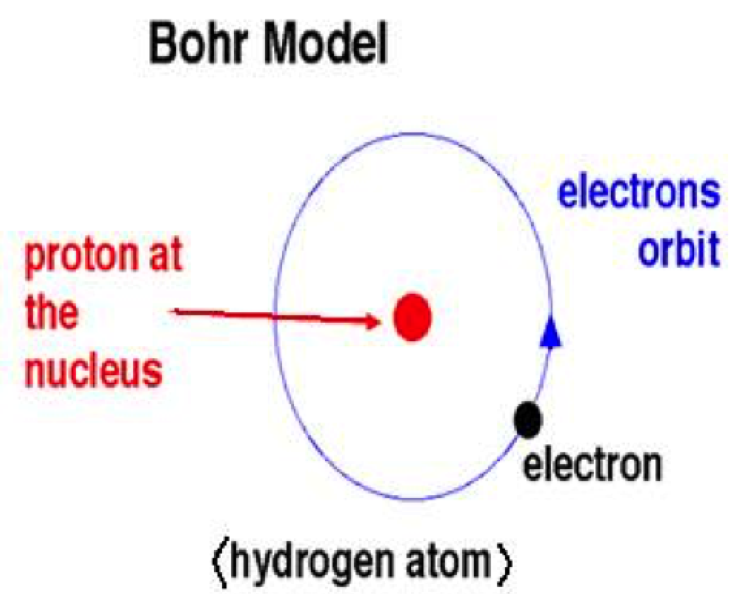
Bohr's Theory : a theory of atomic structure that explains the spectrum of hydrogen atoms. It assumes that the electron orbiting around the nucleus can exist only in certain energy states, a jump from one state to another being accompanied by the emission or absorption of a quantum of radiation.



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1 Derivation of Bohr's Theory

In an hydrogen atom, the centripetal force is being supplied by the coulomb force between it and the proton in the hydrogen nucleus.
F centripital = F electrostatic

$$F_{centripetal} = F_{electrostatic}$$
$$\frac{mv_n^2}{r_n} = \left| k\frac{-e(Ze)}{r_n^2} \right|$$
$$mv_n^2 = k\frac{Ze^2}{r_n}$$

[h!]

Remember that Z represents the atomic number (the number of protons), that electrons and protons have the same magnitude charge, e, and that a negative Felectrostatic merely means that the electrostatic force is attractive. Also note that the values of vn of rn are unknowns in this equation.

As a means of evaluating these two unknowns, Bohr first hypothesized that the electron's angular momentum was quantized.

$$L = n\left(\frac{h}{2\pi}\right)$$
$$I\omega = n\left(\frac{h}{2\pi}\right)$$
$$mr_n^2\left(\frac{v_n}{r_n}\right) = n\left(\frac{h}{2\pi}\right)$$
$$mv_n r_n = n\left(\frac{h}{2\pi}\right)$$

[h!]

Upon solving the angular momentum equation for vn, substituting it into the centripetal force equation yields the following expression for rn.

$$mv_n^2 = k\frac{Ze^2}{r_n}$$

$$m\left(\frac{nh}{2\pi mr_n}\right)^2 = k\frac{Ze^2}{r_n}$$

$$m\left(\frac{n^2h^2}{4\pi^2m^2r_n^2}\right) = k\frac{Ze^2}{r_n}$$

$$\frac{n^2h^2}{4\pi^2mr_n} = kZe^2$$

$$r_n = \frac{n^2h^2}{4\pi^2kmZe^2}$$

$$r_n = n^2\left(\frac{h^2}{4\pi^2kmZe^2}\right)$$

[h!]

2 F

or a ground state hydrogen electron,

$$n = 1 \ and \ Z = 1$$

$$r_1 = \frac{h^2}{4\pi^2kme^2}$$

$$r_1 = 0.53\times10^{-10} \ meters$$

[h!]

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r approximately half of an Angstrom.

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Bohr's second hypothesis in his model was that an electron only loses or releases energy (and therefore a photon) when it goes through de-excitation or drops from a higher energy state to a lower energy state. In order to determine the energy lost by the electron, an expression for an electron's total energy has to be developed.

Recall that the electric potential energy for an electron would equal

$$EPE = qV_{abs}$$
$$EPE = -e\left(\frac{k(Ze)}{r_n}\right)$$
$$EPE = -\frac{k(Ze^2)}{r_n}$$

[h!]

By extending the centripetal force relationship, an expression can also be derived for the electron's kinetic energy

$$F_{centripetal} = F_{electrostatic}$$
$$\frac{mv_n^2}{r_n} = \left|k\frac{-e(Ze)}{r_n^2}\right|$$
$$mv_n^2 = k\frac{Ze^2}{r_n}$$
$$\frac{1}{2}mv_n^2 = k\frac{Ze^2}{2r_n}$$
$$KE = k\frac{Ze^2}{2r_n}$$

[h!]

Thus, the total energy, En, of an electron equals

$$E_n = EPE + KE$$
$$= -\frac{k(Ze^2)}{r_n} + k\frac{Ze^2}{2r_n}$$
$$= -k\frac{Ze^2}{2r_n}$$

[h!]

In this equation, notice that the total energy is negative. This is interpreting as meaning that the electron is trapped in an energy well about the nucleus; that is, it would take the addition of energy to ionize or free the electron. Substituting in the value for r1 into this total energy expression yields a ground state energy of 2.18 x 10-18 Joules or -13.6 eV for a hydrogen atom. Using the fact that rn = n2r1

$$r_n = n^2\left(\frac{h^2}{4\pi^2 kmZe^2}\right)$$

$$r_n = n^2 \; r_1$$

[h!]

we can now generated the first four energy levels for hydrogen.

E1 = -13.6 eV

E2 = E1 / 22 = -3.4 eV

E3 = E1 / 32 = -1.51 eV

E4 = E1 / 42 = -0.85 eV

Bohr's second hypothesis combined with Planck's formula for quantized energy (E = hf) will now allow us to derive Balmer's equation. Remember that the energy released by the electron during de-excitation equals the energy of the emitted photon.

Let's begin by assuming that an electron is falling from Ej, a high energy state, to Ei, a lower energy state.

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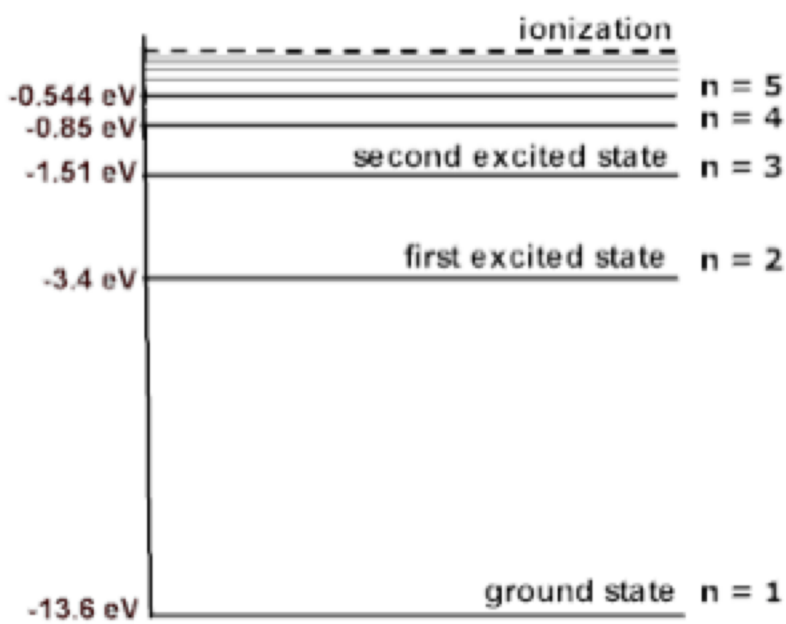

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$$\Delta E = E_j - E_i$$

$$\Delta E = -k\frac{Ze^2}{2r_j} - \left(-k\frac{Ze^2}{2r_i}\right)$$

$$\Delta E = -k\frac{Ze^2}{2}\left(\frac{1}{r_j} - \frac{1}{r_i}\right)$$

$$\textit{where}\ \ r_n = n^2\left(\frac{h^2}{4\pi^2 kmZe^2}\right)$$

$$\Delta E = -k\frac{Ze^2}{2}\left(\frac{1}{j^2\left(\frac{h^2}{4\pi^2 kmZe^2}\right)} - \frac{1}{i^2\left(\frac{h^2}{4\pi^2 kmZe^2}\right)}\right)$$

$$\Delta E = -k\frac{Ze^2}{2}\left(\frac{4\pi^2 kmZe^2}{h^2}\right)\left(\frac{1}{j^2} - \frac{1}{i^2}\right)$$

$$\Delta E = -k^2\,\frac{2\pi^2 mZ^2e^4}{h^2}\left(\frac{1}{j^2} - \frac{1}{i^2}\right)$$

$$\Delta E = k^2\,\frac{2\pi^2 mZ^2e^4}{h^2}\left(\frac{1}{i^2} - \frac{1}{j^2}\right)$$

$$\Delta E_{\textit{lost by the electron}} = E_{photon} = hf = h\frac{c}{\lambda}$$

$$h\frac{c}{\lambda} = k^2\,\frac{2\pi^2 mZ^2e^4}{h^2}\left(\frac{1}{i^2} - \frac{1}{j^2}\right)$$

$$\frac{1}{\lambda} = k^2\,\frac{2\pi^2 mZ^2e^4}{ch^3}\left(\frac{1}{i^2} - \frac{1}{j^2}\right)$$

$\textit{For } Z = 1,$

$$\frac{1}{\lambda} = k^2\,\frac{2\pi^2 me^4}{ch^3}\left(\frac{1}{i^2} - \frac{1}{j^2}\right)$$

$$\frac{1}{\lambda} = 1.097\times 10^7\left(\frac{1}{i^2} - \frac{1}{j^2}\right)$$

$$\frac{1}{\lambda} = R\left(\frac{1}{i^2} - \frac{1}{j^2}\right)$$

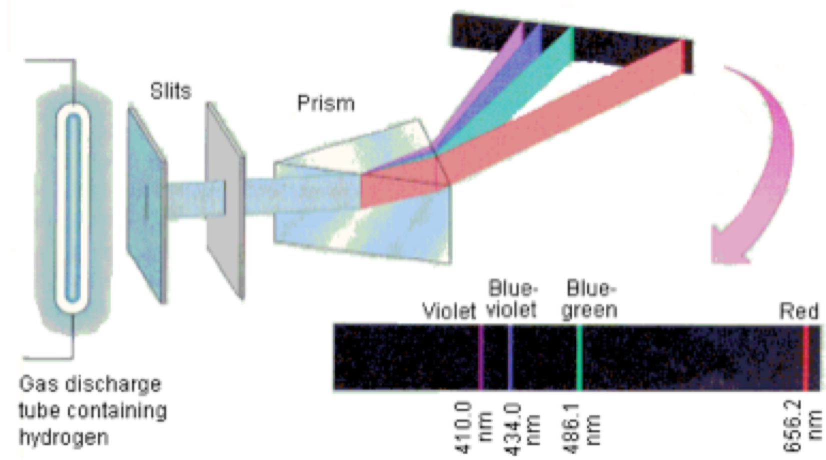
If we let i = 2, and j 3, 4, 5, 6 then we have derived Balmer's empirical formula!

Source :

http://dev.physicslab.org/document.aspx?doctype=3filename=atomicnuclear$_bohrmodelderivation.xml$

4 Hydrogen Spectra

When an electric current is passed through a glass tube that contains hydrogen gas at low pressure the tube gives off blue light. When this light is passed through a prism (as shown in the figure below), four narrow bands of bright light are observed against a black background.



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These narrow bands have the characteristic wavelengths and colors shown in the table below.

Wavelength	Color
656.2	red
486.1	blue-green
434.0	blue-violet
410.1	violet

Four more series of lines were discovered in the emission spectrum of hydrogen by searching the infrared spectrum at longer wave-lengths and the ultraviolet spectrum at shorter wavelengths. Each of these lines fits the same general equation, where n1 and n2 are integers and RH is 1.09678 x 10-2 nm-1.

$$\frac{1}{\lambda} = R_H\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

[h!]

According to the Bohr model, the wavelength of the light emitted by a hydrogen atom when the electron falls from a high energy (n = 4) orbit into a lower energy (n = 2) orbit. Substituting the appropriate values of RH, n1, and n2 into the equation shown above gives the following result.

$$\frac{1}{\lambda} = (\ 1.09678x10^{-2}\ nm^{-1})\left[\frac{1}{2_1^2} - \frac{1}{4_2^2}\right]$$

[h!]

Solving for the wavelength of this light gives a value of 486.3 nm, which agrees with the experimental value of 486.1 nm for the blue line in the visible spectrum of the hydrogen atom

5 Hydrogen Spectra Series

Lyman Series (n‘=1) The series is named after its discoverer, Theodore Lyman, who discovered the spectral lines from 1906–1914. All the wavelengths in the Lyman series are in the ultraviolet band.
Balmer Series (n‘=2) Named after Johann Balmer, who discovered the Balmer formula, an empirical equation to predict the Balmer series, in 1885. Balmer lines are historically referred to as "H-alpha", "H-beta", "H-gamma" and so on, where H is the element hydrogen.[8] Four of the Balmer lines are in the technically "visible" part of the spectrum, with wavelengths longer than 400 nm and shorter than 700 nm. Parts of the Balmer series can be seen in the solar spectrum. H-alpha is an important line used in astronomy to detect the presence of hydrogen.
Paschen series (n‘=3) Named after the German physicist Friedrich Paschen who first observed them in 1908. The Paschen lines all lie in the infrared band.[9] This series overlaps with the next (Brackett) series, i.e. the shortest line in the Brackett series has a wavelength that falls among the Paschen series. All subsequent series overlap.

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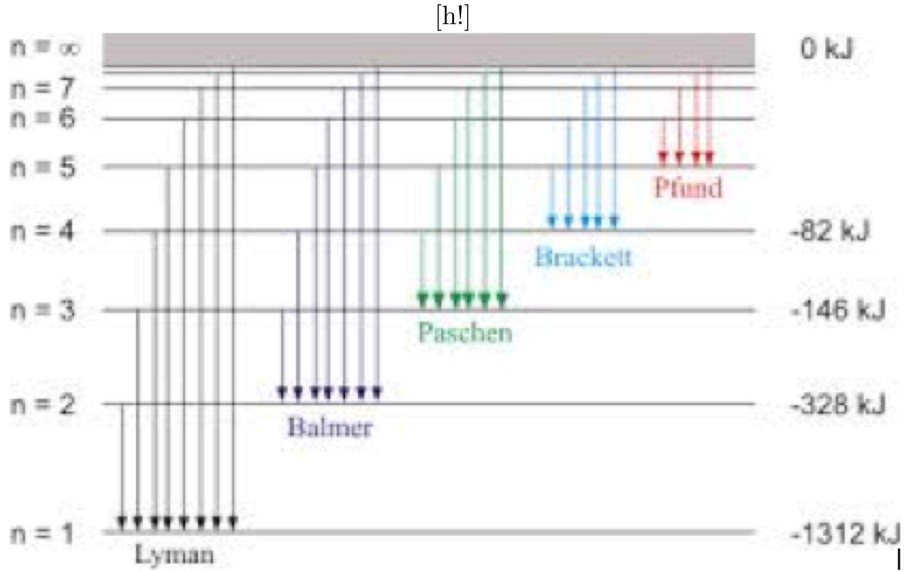

6 Relevant Formulas

In the Bohr model, the wavelength associated with the electron is given by the DeBroglie relationship

$$\lambda = \frac{h}{mv}$$

[h!]

and the standing wave condition that circumference = whole number of wavelengths. In the hydrogenic case, the number n is the principal quantum number.

[h!] $$2\pi r = n\lambda_n$$

These can be combined to get an expression for the angular momentum of the electron in orbit. (Note that this assumes a circular orbit, a generally unwarranted assumption.)

$$L = mvr = \frac{hr}{\lambda} = \frac{hr}{\left[\frac{2\pi r}{n}\right]} = \frac{nh}{2\pi}$$

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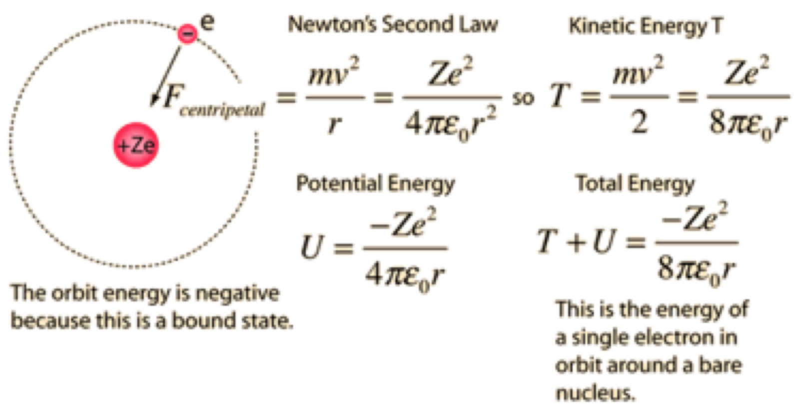
Classical Electron Orbit



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