

Circular Motion and Gravity

Orbital Simulation

PHYS 442

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Partners: Whole class
Instructor: Me

1 Objective

Explored the motion of a particle under the influence of a gravitational force. Specifically we look at attractive inverse square distance forces, Hookean forces, escape velocity, circular orbits, kinetic energy, potential energy and elliptical orbits. These are defined in 1.1:

1.1 Definitions

Law of Universal Gravitation The law of universal gravitation states the force of gravity between two point masses is directly proportional to each mass and inversely proportional to the distance between them. This is also true for masses outside of spherically symmetric mass distributions. (Homer, 2014)

$$F_g = \frac{mMG}{r^2}$$

Hookean Forces Inside a uniformly dense sphere of mass the force is Hookean, with an attractive force proportional to the displacement from equilibrium. The effective spring constant is $K = \frac{mMG}{R^3}$.

$$F_g = \frac{mMG}{R^3}r$$

Gravitational Constant The universal gravitation constant G determines the strength of the gravity force from a given mass. It may also be considered

as the force that 1 kg exerts on another 1 kg mass separated by 1 meter.

$$G = 6.67 \times 10^{-11} \frac{Nm^2}{kg^2}$$

Escape Velocity Escape velocity is the initial velocity required to escape gravitational attraction. An object launched at the escape velocity will never come back (escape).

$$v_{escape} = \sqrt{\frac{2MG}{r}}$$

Kinetic Energy Kinetic energy is the energy associated with motion.

$$KE = \frac{mv^2}{2}$$

Potential Energy The potential associated with the universal gravitation force is written as follows.

$$PE = -\frac{mMG}{r}$$

Circular Orbit A circular orbit is an orbit with a constant radius r .

Elliptic Orbit An elliptic orbit is a closed orbit with changing radius r .

2 Simulation

The simulation applies a central acceleration to the orbiting particle. Outside the boundary of the central mass we have the following acceleration.

$$a = \frac{K}{r^2}$$

Inside the boundary of the central mass ($r < R$) we have the following acceleration.

$$a = \frac{K}{R^3}r$$

Here R is the radius of the central mass and K is a constant determined by the user of the simulation. K is MG . For this simulation the radius was set to $R = 6$ and the constant was set to $K = -0.1$ making the acceleration attractive.

The initial position \vec{r}_0 and initial velocity \vec{v}_0 are set by the user.

3 Sample Calculation

3.1 Circular Orbit

Given $\vec{r}_0 = (10, 0)$ and $K = -0.1$ we find the \vec{v}_0 for circular orbit.

$$\begin{aligned}F_{net} &= ma \\ \frac{mMG}{r^2} &= m \frac{v^2}{r} \\ \frac{K}{r^2} &= \frac{v^2}{r} \\ v &= \sqrt{\frac{K}{r}} \\ v &= \sqrt{\frac{0.1}{10}} = 0.1\end{aligned}$$

The velocity must be tangential and therefore \vec{v}_0 must be perpendicular to \vec{r}_0 .

$$\vec{v}_0 = (0, 0.1)$$

3.2 Escape Velocity

Given $\vec{r}_0 = (10, 0)$ and $K = -0.1$ we find the \vec{v}_0 for escape from the central mass' gravitational attraction. Escape is associated with a total mechanical energy of zero.

$$\begin{aligned}PE + KE &= 0 \\ -\frac{mMG}{r} + \frac{mv^2}{2} &= 0 \\ -\frac{K}{r} + \frac{v^2}{2} &= 0 \\ v_{escape} &= \sqrt{\frac{2K}{r}} \\ v_{escape} &= \sqrt{\frac{2(0.1)}{10}} = 0.14\end{aligned}$$

4 Results and Conclusions

4.1 Circular Orbit

For the conditions $\vec{r}_0 = (10, 0)$ and $K = -0.1$ we calculate an initial velocity $\vec{v}_0 = (0, 0.1)$ will give a circular orbit. Running the simulation yields the following orbit. We can see it is circular.

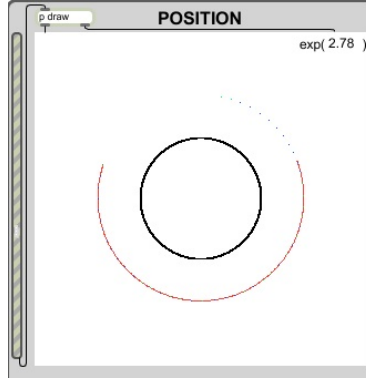


Figure 1: Circular Orbit

4.2 Escape Velocity

For the conditions $\vec{r}_0 = (10, 0)$ and $K = -0.1$ we calculate an initial velocity $\vec{v}_0 = (0, 0.14)$ will give an escape from the gravitational attraction. We can observe in the simulation the object indeed escapes and the total energy is zero. See the following figure.

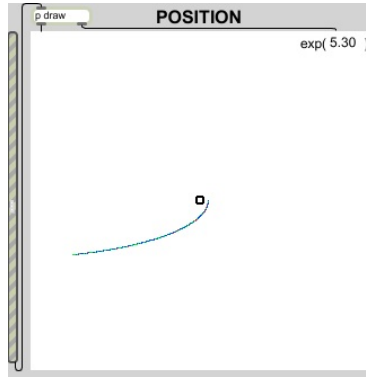


Figure 2: Escape Velocity

4.3 Elliptical Orbit

For the conditions $\vec{r}_0 = (10, 0)$ and $K = -0.1$ we use an initial velocity $\vec{v}_0 = (0, 0.12)$ that is between the one for circular orbit and escape. As the figure shows the resulting orbit is elliptical.

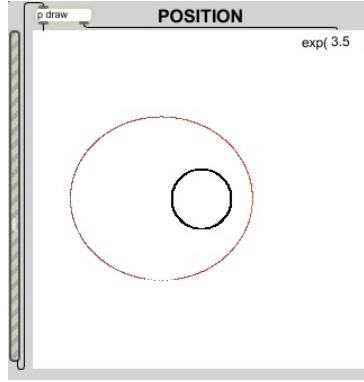


Figure 3: Elliptic Orbit

5 Discussion

Overall this simulation satisfied its objective: observing the motion of a particle under the influence of a gravitational force. By initially defining the law of Universal Gravitation, Hookean Force, Gravitational Constant, Escape/Circular/Elliptical orbit, and Kinetic/Potential Energy, this simulation establishes a connection between different formulas that lead to the mathematical reasoning of other formulas (This can be observed through the method the circular and escape velocities are obtained)(Ex : $mMG/r^2 = mv^2/2 > V_{\text{circular}}$, $PE + KE = 0 \hat{=}$ V_{escape}), hence asserting once again that Physics as a science is interrelated. The figures depicting the different velocities serve as visual aids, helping the reader to conceptualize this phenomenon. It became clear that Circular and Escape velocities share to some extent a minimum/maximum relationship where their range represents an elliptical orbit. ($CircularVelocity < EllipticalVelocity < EscapeVelocity$)

The importance of this expertise has achieved an unprecedented value since billion dollar industries have emerged heavily relying on the basis of this simulation. (Commercial rockets, planned trip to mars, satellite launches etc.).

References

Homer, J. (2014). *Physics*. Oxford, 3rd edition.