Exercise session on the numerical solution of linear systems, 1

November 8, 2021

Exercise 1

Solve all the linear systems defined in the previous exercise session by LU decomposition, checking whether pivoting has been performed. Compute the absolute and relative errors of the computed solutions with respect to $\mathbf{x}_{ex}$ in the l^2 and l^∞ norm.

Exercise 2

Build a matrix $\mathbf{A}$ of dimension 200×200 that has all elements equal to 5 on the main diagonal and all elements equal to -1 on the first and second subdiagonal and superdiagonal (use the command `diag`).

(a) Write a script to check that the matrix is diagonally dominant by rows.

(b) Compute the vector $\mathbf{b} = \mathbf{A}\mathbf{x}_{ex}$, where $\mathbf{x}_{ex} = [1, 1, \ldots, 1, 1]^T$, $\mathbf{x}_{ex} \in \mathbf{R}^{200}$.

(c) Solve the system $\mathbf{A}\mathbf{x} = \mathbf{b}$ using by the LU factorization method. Check that pivoting is not performed.

(d) Compute the absolute and relative error of the computed solution with respect to $\mathbf{x}_{ex}$ in the l^2 and l^∞ norm.

Exercise 3

Consider the $n \times n$ matrix $\alpha\mathbf{I} + \mathbf{H}_n$, where $\mathbf{H}_n$ the Hilbert matrix of dimension n, build using the command `hilb`. For the case $\alpha = 1$, $n = 20$, check if the matrix is diagonally dominant by rows and if the matrix is symmetric and positive definite; then

(a) compute the vector $\mathbf{b} = \mathbf{A}\mathbf{x}_{ex}$, where $\mathbf{x}_{ex} = [-1, 1, \ldots, 1, -1]^T$, $\mathbf{x}_{ex} \in \mathbf{R}^{20}$;

(b) solve the system $\mathbf{A}\mathbf{x} = \mathbf{b}$ using by the LU factorization method;

(c) compute the absolute and relative error of the computed solution with respect to $\mathbf{x}_{ex}$ in the l^2 and l^∞ norm. Repeat the exercise for the case $\alpha = 10^{-8}$.

Exercise 4

Using the command `vander`, build the 10×10 Vandermonde matrix associated to the vector $\mathbf{v} = [0.0001, 2, 3, 4, 5, 6, 7, 8, 9, 10]^T$. Compute the vector $\mathbf{b} = \mathbf{A}\mathbf{x}_{ex}$, where $\mathbf{x}_{ex} = [10, 9, \ldots, 2, 1]^T$, $\mathbf{x}_{ex} \in \mathbf{R}^{10}$.

(a) Solve the system $\mathbf{A}\mathbf{x} = \mathbf{b}$ using the LU factorization method; check if pivoting has been performed;

(b) compute the absolute and relative error of the computed solution with respect to $\mathbf{x}_{ex}$ in the l^2 and l^∞ norm.