Exercise session on numerical solution of nonlinear equations, 2

October 21, 2021

Exercise 1

Consider the equation $f(x) = \cos(5x)x^{\frac{3}{7}} = 0$.

(a) Determine graphically an initial estimate of the nonzero solutions of the equation in $[0, 2]$.

(b) Compute the solutions with the Newton method, using as initial value the approximation determined in (a). Use the fact that

$$f'(x) = \frac{3}{7}\cos(5x)x^{-\frac{4}{7}} - 5x^{\frac{3}{7}}\sin(5x).$$

Use a tolerance 10^{-11} on the absolute error.

(c) Compute the solutions with those obtained by the bisection method with the same tolerance on the absolute error, comparing the number of iterations required by each method. Choose for each solution an appropriate initial interval for the bisection method.

Exercise 2

Consider the equation $f(x) = x^5 - 8x^4 + 21x^3 - 14x^2 - 20x + 24 = 0$.

(a) Determine graphically initial estimates of the solutions of the equation in $[1, 3.5]$.

(b) Compute the solutions with the secant method, using as initial values the approximation determined in (a). Use a tolerance 10^{-10} on the relative error.

(c) Repeat the computation by the Newton method.

(d) Compute the number of iterations required by the two methods for each solution. Explain the difference on the basis of the theory.

Exercise 3

Consider the nonlinear equation $x^3 - x^2 - 2x = 0$ on the interval $[-3, 3]$.

(a) Find graphically approximate values for the solutions of this equation. Use them as initial guesses for the Newton method, using a a tolerance on the residual of 10^{-7}.

(b) Say, on the basis of the theory, which of the solutions can be found employing the fixed point method with iteration function $\phi(x) = (x^3 - x^2)/2$. In the cases in which it is possible, compute the solutions also by the fixed point method with the same tolerance on the residual and compare the number of iterations required.

Exercise 4

Consider the nonlinear equation $e^x - 2x^2 = 0, \quad x \in [-2, 4]$. Determine graphically the number of solutions of the equation. Compute all the solutions of the equation using the following methods,

(a) bisection method,

(b) Newton method,

(c) chord method,

(d) secant method,

(e) fixed point method, after rewriting the problem as $x = \phi(x)$ using three different iteration functions, i.e. $\phi_1(x) = \log(2x^2)$, $\phi_2(x) = \sqrt{\frac{1}{2}e^x}$, $\phi_3(x) = -\sqrt{\frac{1}{2}e^x}$, checking graphically for which root each iteration function will converge.

For all methods, choose appropriate initial guesses and set the tolerance on the relative error l equal to 10^{-9} Compare the number of iterations needed by the methods to achieve an error under the selected tolerance.

Exercise 5

Consider the equation

$$f(x) = \frac{2x^2 - 3x - 2}{x - 1} = 0.$$

(a) Determine graphically approximate solutions of the equation in $[-3, 3]$,

(b) Consider the iteration functions

$$\phi_1 = \frac{3x^2 - 5x}{x - 1} \qquad \phi_2 = x - 2 + \frac{x}{x - 1}.$$

Determine which iteration function can be used to compute the positive solution x^* of the equation $f(x^*) = 0$ with the fixed point method.

(c) Approximate x^* with the fixed point method with a tolerance on the relative error equal to 10^{-8} using the initial guess $x^{(0)} = 1.2$.

(d) Compute x^* with the Newton method with a tolerance on the relative error equal to 10^{-8} using the initial guess $x^{(0)} = 1.2$

Exercise 6

Consider the equation $\sin x + \frac{1}{10} - 2x + x^3 = 0$, on the interval $[0, 2]$.

(a) Determine graphically two initial approximations for the solutions of this equation.

(b) Compute an approximation of these solutions using the previous initial estimates and the fixed point method, using the iteration function

$$\phi(x) = \frac{1}{2}(\sin x + x^3 + \frac{1}{10})$$

and a tolerance value of 10^{-9} on the relative error.

(c) Explain any differences between the results in the two cases on the basis of the theory of the fixed point method.

(d) Repeat the computation with the Newton method and with the `fsolve` function. Compare the number of iterations required to achieve the same error tolerance.