Homework No.1

Machine Learning 2021

Information Theory

1. Expressing mutual information in terms of entropies Show that

$$I(X, Y) = H(X) - H(X|Y) = H(Y) - H(Y|X)$$

2. A measure of correlation (normalized mutual information)
Let X and Y be discrete random variables which are identically distributed (so H(X) = H(Y)) but not necessarily independent. Define

$$r = 1 - \frac{H(Y|X)}{H(X)}$$

   (a) Show $r = \frac{I(X,Y)}{H(X)}$
   (b) Show $0 \leq r \leq 1$
   (c) When is $r = 0$?
   (d) When is $r = 1$?

3. Consider two binary variables x and y having the joint distribution given in the following table.

		y	
		0	1
x	0	1/3	1/3
	1	0	1/3

Evaluate the following quantities

   (a) $H[x]$
   (b) $H[y]$
   (c) $H[y \mid x]$
   (d) $H[x \mid y]$
   (e) $H[x, y]$
   (f) $I[x, y]$

Draw a diagram to show the relationship between these various quantities.

4. Suppose that the conditional entropy H$[y \mid x]$ between two discrete random variables x and y is zero. Show that, for all values of x such that $p(x) > 0$, the variable y must be a function of x, in other words for each x there is only one value of y such that $p(y \mid x) \neq 0$.