Towards a better reconstruction of deceleration parameter from SN data

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Abstract.
As a non-parametric approach, the method of smoothing has been proved to be a promising statistical tool for reconstruction of the history of the late universe from supenovae data. This method employs a Gaussian kernel associated with a width to smooth noisy data through an iterative procedure. The width of smoothing plays a key role in the final result and it particularly affects the precision of reconstruction. In this work, endeavored to achieve a better reconstruction, two different approaches are proposed to make the width of smoothing senstitive to the quality of data. This consideration ensures both approaches intend potentially to extract more information from data. In each approach, we build up a measure to quantify the quality of data in order to associate it with the smoothing width. Precisely, the first approach employes a variable smoothing width, and the second one impelements several constant smoothing widths for different segments of each data set. We challenge these modified methods by synthetic samples based on time varying equation of states like Kink, and CPL models along with $\omega = -0.9$. As a result of simulations, as long as data have enough quality, these modifications get conspicuously a better reconstruction in the deceleration parameter and decrease the variance in high redshifts comparing to the archetype smoothing method, which only employs constant smoothing widths. Finally, we employ this enhanced statistical to analyze the real data set, such as UNION 2.1 Compilation and PanSTARR. Additionally, to check all possible reconstruction, we start with a wide variety of cosmological models, and gather all results which possess a better χ^2 than flat ΛCDM.

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Contents

1 Introduction

In the early twentieth century, advancements in technologies, space sciences along with increasing computational capabilities opened a new window, *Precision Cosmology*, towards our perspective of the universe and ever since much progress have been made in cosmology to understand the expansion history of our universe. In spite of these impressive achievements, the nature of dark energy still remains as one of the most challenging questions about our universe that baffles cosmologists and particle physicists. In the last decade, many dark energy models were proposed to explain the accelerating expansion of our universe (see [1–7] for reviews). Moreover, many efforts have been made in numerical techniques to analyze data more accurately in order to reconstruct the desired cosmological quantities. These attempts can be generally categorized into parametric and non-parametric approaches (for instance, see [8] and references therein). Parametric methods tend to have small variance and they are very practical especially in cases which one has adequate knowledge about the underlying phenomenon. Nonetheless, it should be mentioned in case of dark energy they are severely prone to systematic errors, especially when analyst does not have enough information about the underlying model, and the actual behavior of the phenomenon may not be pronounced by the flexibility of the parametric model. For example, as mentioned in [9, 10], inappropriate prior assumptions of the dark energy equation of state (ω_{DE}) and of the matter density (Ω_M) are likely to end up with a fallacious estimation of the true underlying cosmology. On the other side, non-parametric methods can directly find the trend in our data and shed more light on our understanding, although they might suffer from larger variance in comparison to parametric methods. However, since one avoid the specific assumptions in the functionality of $\omega_{DE}(z)$, the results of non-parametric methods are less likely to be biased. Confidentially, a reliable non-parametric approach is complementary to other parametric methods, mainly because our current understanding of $\omega_{DE}(z)$ is still incomplete. Currently few methods have been able to reconstruct the expansion history of the universe directly from data. For instance in [11–13], non-parametric methods based on Gaussian processes are utilized to reconstruct the cosnological quantities, namely $\omega_{DE}(z)$ and the deceleration parameter $q(z)$, without assuming a cosmological model. As another non-parametric approach, in [14], method of smoothing is introduced to smooth noisy data using a Gaussian kernel associated with a smoothing width of (Δ). For the first time, in [15] this method is applied on real data and later on it is improved to be error-sensitive in [16]. The result of reconstruction depends on the width of smoothing and it should be noted that a small width[1] leads to a noisy reconstruction but small bias, while large width results in large bias. Δ plays an important role in this method, and the analyst should select an optimum width which not only avoids a noisy reconstruction but also ends up with a small bias. For most practical purposes, Δ is commonly chosen to be constant, not variable. Equivalently one can say that the current form of smoothing method is not quality sensitive, since data are neither uniformly distributed nor have equal errorbars. In this work, we aim at improving the method one step further to make it sensitive both to the pausity of data in high redshifts and to the error-bars. To this end, we propose two approaches that take into account these properties. The first approach employs a variable smoothing width ($\Delta_{(z)}$) so that this variability comes from data and errorbars. The second approach groups data points in bins and assigns a constant Δ for each bin connecting to the quality of that bin. The quality is connected to the number and error-bars of SN in each bin. Then smoothing procedures are applied in parallel for all values. Finally, for each bin, only result of reconstruction obtained from its specific width, is stored. In both approaches, the methods automatically use a small width for regions with high density and a large width for regions with high pausity.

In Sec. 2.1, the original error-sensitive method of smoothing is introduced. Following that, two proposal regarding the width of smoothing are presented. In subsection 2.3, our methodology for simulating SN data, data sets used for simulations and the true underlying models are discussed. Finally, in sec. 3 we sum up the comparison between results of our proposals and of the method with constant smoothing width.

2 Methodology

2.1 Primitive Version of Smoothing Method

Supernovae Ia, as standard candles, are currently the best source of information for studying the expansion history of the universe. The measured quantity, distance modulus (μ) can be easily translated into Hubble parameter ($\mathrm{h}(z) \equiv H(z)/H_0$), and consequently to Deceleration parameter ($\mathrm{q}(z)$). That is to say, having distance moduli and redshift, one can easily solve the inverse problem and find out the trend of h(z) and q(z). This idea is known as a top-down recipe which is behind most non-parametric reconstruction approaches (2.1),

$$\mu_{(z)} \equiv 5 \ \log_{10}(\frac{d_L(z)}{Mpc}) + \ 25 \tag{2.1a}$$

$$\mathrm{h}(z)^{-1} = \frac{d}{dz}(\frac{d_L(z)}{1+z}) \tag{2.1b}$$

$$\mathrm{q}(z) = (1+z)\frac{h'(z)}{h(z)} - 1 \tag{2.1c}$$

Where d_L is the luminosity distance, and prime denotes derivative with respect to the redshift. Presumably, if one would find the best fit of SN data in an assumption free manner, then the

[1]smallness of width is defined by comparing to the density of data

best-fit curve could be used to reveal the expansion history of the universe in a model independent approach. However, since deceleration parameter embodies the second derivatives of distance modulus, any small weakness in the fitting procedure may be increased hugley and lead to a fallacious reconstruction. This is the nature of this work which makes it quite difficult in action. Additionally, parametric method are susceptible to systematic errors which stem from the fact that the behaviour of true model can not be spelled by the functionality of our model. That is to say, a non-parametric approach which is capable to find the trend of true model is a complementary tool to confirm or rule out the result of parametric approaches. In this regard, *Smoothing method* was initially introduced to reconstruct the cosmological parameters out of supernovae data. It is arguably considered as one of the best methods to derive distance modulus directly from SN data, in a model dependent manner. In practice, it initially assumes an smooth guess, to subtract from the data. Then it applies a Gaussian kernel, associated with a smoothing width of Δ, to smooth the noisy data in the residual space. Afterwards, the initial guess is added back in order to rebuild the smoothed distance modulus. This result then is considered as the initial guess for the next iteration, and this recipe is repeated up to a specific iteration. This idea ensures that the procedure avoids smoothing the luminosity distance, and more interestingly, this algorithm minimizes the noises in the final result. The error-sensitive version of smoothing method is as follows [16],

$$\ln d_L^s(z,\Delta) = \ln d_L^g(z) + N(z) \sum_i \frac{[\ln d_L(z_i) - \ln d_L^g(z_i)]}{\sigma^2_{d_L(z_i)}} \exp[-\frac{\ln^2(\frac{1+z_i}{1+z})}{2\Delta^2}] \tag{2.2a}$$

$$N(z)^{-1} \equiv \sum_i \exp[-\frac{\ln^2(\frac{1+z_i}{1+z})}{2\Delta^2}] \frac{1}{\sigma^2_{d_L(z_i)}} \tag{2.2b}$$

where $\ln d_L(z_i)$, $\sigma^2_{d_L(z_i)}$, $N(z)$, $\ln d_L^g(z_i)$ and Δ respectively denotes the data points, error-bars, the normalization factor, initial guess model, and width of smoothing.

2.2 Width of smoothing & our proposals

The smoothing width imposes how precise the smoothing procedure should work. Choosing the width of smoothing is the caveat of this method and one should select an optimum value of Δ which yields not only a small value of the bias, but also a reasonably small error. In fact, a small smoothing width results in a noisy reconstruction, although it is more accurate. On the other side, a large value of smoothing width decreases the error bars on the reconstructed quantities, however it ends up with a larger bias (a thorough discussion can be found in [14]). In practice, it sounds promising to use a small smoothing width when we have more data points or when the data points are close to each other. On the other hand, it is good to use a large smoothing width when the situation is adverse. In reality, SN data are not uniformly distributed over the redshifts, additionally the error bars are different. therefore, the probable information that one can extract from the data is somewhat redshift dependent. Indeed, more data points and less errorbars are equivalent to more information and more quality. In the following two approaches are introduced which both take into account the properties that quality of data are not uniform and our data are not uniformly distributed in redshift.

2.2.1 Approach A: $\Delta_{(z)}$ based on data quality

The first approach that we develop in this paper, is to build up a variable smoothing width which is particularly connected to the quality of data. To justify the idea behind this modification, it suffices to say that in reality, data are not uniformly spanned, so one should think of using a variable smoothing width in order to make the method more robust in those regions where more data points are available. Technically, applying a smaller smoothing, where more information are present in the data, results in a better reconstruction. Therefore, the primitive form of variable smoothing width is defined as follows,

$$\Delta_{(z)} \equiv \Delta_0[\text{ Quality of data }]^{\frac{-1}{p}} \tag{2.3}$$

Where the power "$\frac{-1}{p}$" enssures the fact that for those regions with more quality, Δ should be smaller and vice versa. In fact, for any Δ_0 there is an optimum iteration. The relationships between the number of data, the optimum constant smoothing width and the number of iterations are discussed in [14]. However, in our work, we employ simulations over a variety of cosmological models to check which combination of parameters gets the best result in overall. The proposed form of quality which includes the effects of errorbars and the sparsity of data is defined as,

$$\text{Quality}_{(z)} \equiv \sum_i \frac{1}{\sigma^2_{d_L(z_i)}} \exp[-\frac{(z-z_i)^2}{b^2}] \tag{2.4}$$

Where b, z_i & $\sigma^2_{d_L(z_i)}$ are respectively range of data, redshift and errorbar of the i'th SN. Each term in this quantity resembles a physical concept; the first term $1/\sigma^2_{d_L(z_i)}$ dictates that where our errors are lower, we have more quality, equivalently, more reliable information. The second term, $\exp[-\frac{(z-z_i)^2}{b^2}]$ describes how closely or sparsely the data points are distributed around the redshift z. For the sake of comparison, in Fig. 1b, "Quality" for two different samples used in our study, Joint Dark Energy Mission (JDEM) and Union 2.1 Compilation are depicted. This quantity is then employed in construction of the variable smoothing width.

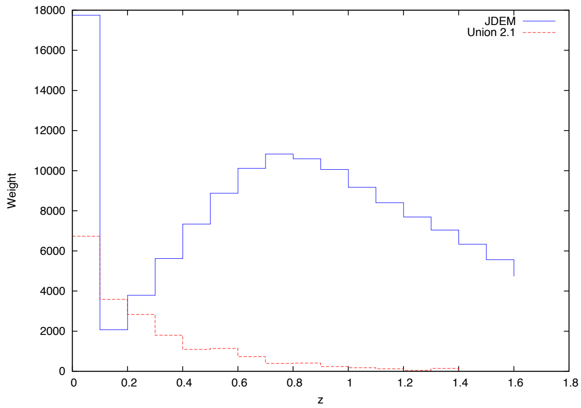
2.2.2 Approach B: A Mixture of Constant Δs

Even though the approach of using a mixture of constant Δ is apparantly different from the approach of variable $\Delta(z)$, however they both share the same idea effectively. They use a small smoothing width in regions with more data, and a large smoothing width in regions with the pausity of data. In this approach, at first we bin our data with a reasonable binsize[2] and compute the inverse of weighted error-bars in each bin (see Eq (2.5)). Then, we build up our $\Delta_{(bin)}$ from the inverse of weighted error-bars (see Eq (2.6)). Assuming the number of bins is m, we apply the smoothing method with constant smoothing width for m times, so that each time the constant Δ is allocated from $\Delta_{(bin)}$. We repeat the procedure up to 100 iterations and finally for each specific bin, let us say j'th bin, we keep only results from the smoothing with constant $\Delta = \Delta_{(j'th\ bin)}$. This approach resembles the fact that once we keep results of each bin corresponding to its weighted error, we are indeed extracting information from that bin as much as its potential quality .

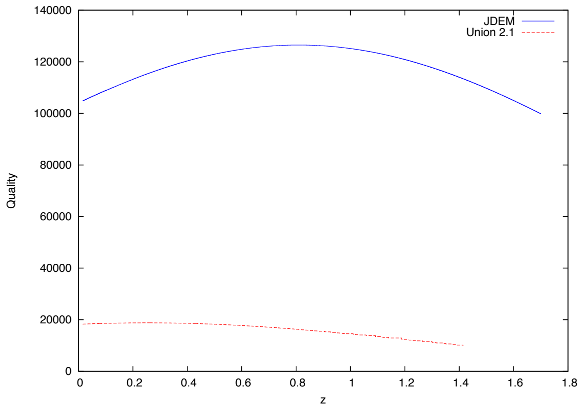
$$\text{Weight}_{(bin)} \equiv \sum_{bin} \frac{1}{\sigma_i^2} \tag{2.5}$$

$$\Delta_{(bin)} \sim [\text{Weight}_{(bin)}]^{(-1/p)} \tag{2.6}$$

[2]In general, applying a wider bins where the density is low decreases noise due to randomness of sampling. On the other side, using narrower bins where the density is high gives greater precision to the density estimation.



(a) Weight of bins



(b) Quality of data

Figure 1: (*left*) Weight of bins, (*right*) Quality of data in JDEM and Union 2.1. These quantity which somehow express the quality of data, are used to construct our smoothing width Δ. As illustrated in figures, the quality of data decreases vs. redshift

2.3 Simulations

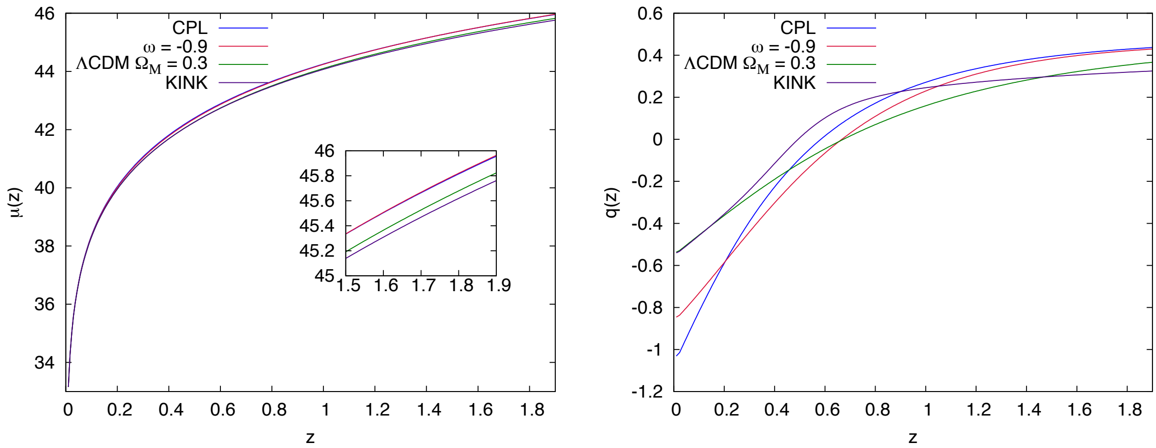
In this section, we introduce three different mock data sets, within different models, used to compare the robustness of proposed approaches to constant Δ for reconstructing the deceleration parameter. The main advantage of mock data sets is that one can easily produce sufficient samples based on an assumed true model. These samples can be employed to examine how well a method performs. The next advantage is that one can optimise the method with several mock samples mimicing the same quality as real data, before applying the method on real data.

Normally, to generate a synthetic sample from a real data set, one first calculates the theoretical distance modulus at observed point z_i within an assumed cosmological model. Then a Gaussian random number scaled to the observed error $\sigma_{\mu_{z_i}}$, is added to the theory (see Eq. (2.7)). The validity of this recipe is based on the assumption that the errors are normally distributed and the data points are uncorrelated.

$$\mu^{\text{Synthetic}}(z_i) = \mu^{\text{theory}}(z_i) + \sigma_{\mu_{(z_i)}} N(0,1) \tag{2.7}$$

In our analysis, the mock samples are based on two data sets including JDEM, and Union 2.1 compilation. First data includes the measurement of 2298 supernovae, with an intrinsic σ of 0.13, expected from future surveys and spanned over a redshift of $0.015 < \text{z} < 1.7$. The next sample, includes 580 supenovae distributed over redshifts of $0 < \text{z} < 1.4$. Given redshift and errorbars of these data sets, we generate several samples based on three dark energy models all within flat universe framework with $\Omega_M = 0.3$. (i) First model has a constant equation of state, $\omega = -0.9$. (ii)The second model (CPL) has a time varying equation of state, in which ω starts from -1.5 and approaches to -1 [17, 18],

$$\omega_{(z)} = -1.5 + 0.5(\frac{z}{1+z}) \tag{2.8}$$



(a) Distance modulus

(b) Deceleration Parameter

Figure 2: Two couples of cosmological models, which deceptively resemble same distance modulus ($\mu(z)$). These models are considered to generate synthetic data sets. Flat ΛCDM model is a fiducial model for the sake of comparison. The main advantage of mock data sets is that the analyst is able to check the capability of the statistics in catching the true underlying model.

(iii) the third model also has a time varying equation of state, see (2.9), which leads to a special feature, Kink, in the deceleration parameter [19].

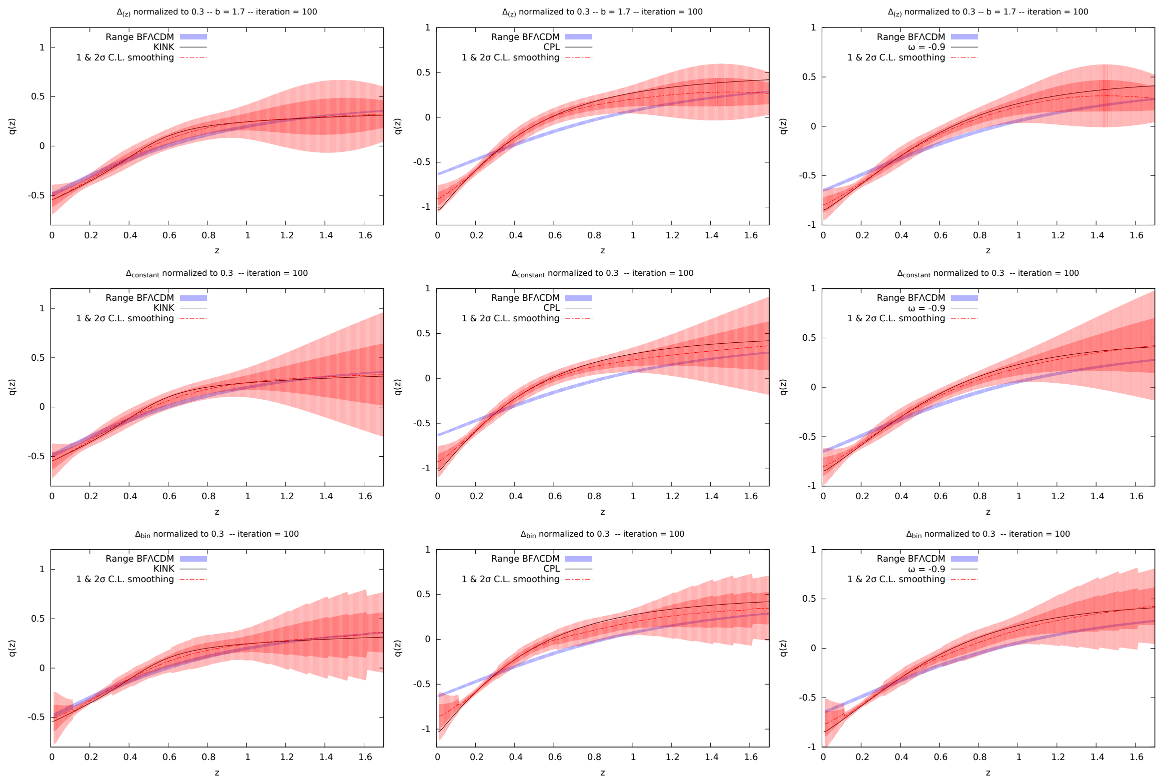
$$\omega_{(z)} = \omega_0 + (\omega_m - \omega_0)\frac{1+\exp(\Delta_t^{-1}(1+z_t)^{-1})}{1-\exp(\Delta_t^{-1})} \\ \times[1 - \frac{\exp(\Delta_t^{-1}) + \exp(\Delta_t^{-1}(1+z_t)^{-1})}{\exp(\Delta_t^{-1}(1+z)^{-1}) + \exp(\Delta_t^{-1}(1+z_t)^{-1})}] \tag{2.9}$$

With constants values of $\omega_0 = -1.0$, $\omega_m = -0.5$, $z_t = 0.5$ and $\Delta_t = 0.05$. Distance modulus and deceleration parameter of the three considered true models along with ΛCDM $\Omega_M = 0.3$ as fiducial model, are depicted in Fig. 2. It is shown that descriminating these models from distance modulus is quite a difficult job.

3 Results and Discussion

3.1 Reconstruction of q(z) from sythetic data sets

In this section, we present results of our comparison between our modifications and the original type of smoothing method, based on the analysis of the synthetic data sets. To analysis each sample, we start finding the best fit of ΛCDM from a simple grid based method. Then we assume the best fit as our initial guess for smoothing procedure ($\ln d_L^g(z) = \ln d_L^{BF-\Lambda CDM}(z)$). The number of iteration is fixed on 100, and for all approaches, mean values of Δ are 0.3 and 0.6, respectively for JDEM and Union samples. We compare results of modified approaches to constant Δ approach. Additionally, we include the reconstruced range obtained by best fit of ΛCDM to all data sets. It worths to point out that



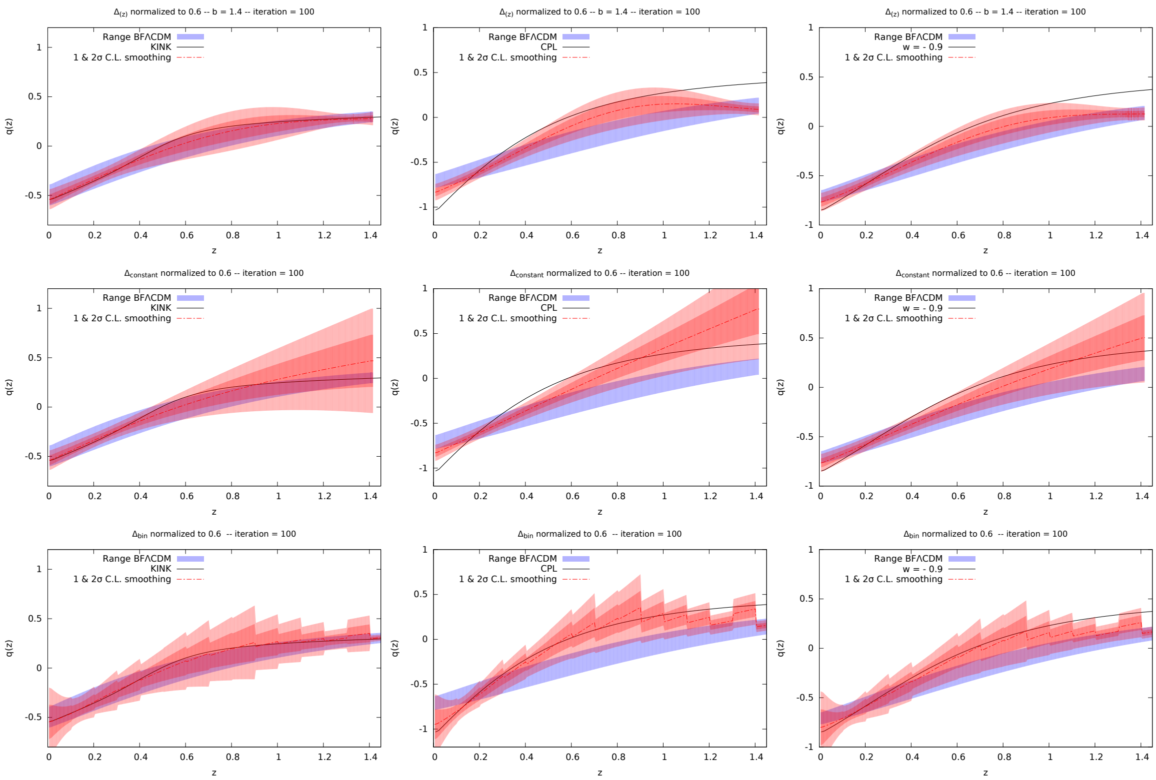
(a) Kink (b) CPL (c) ω = -0.9

Figure 3: Reconstructed $q(z)$ for synthetic samples from JDEM. From left to right, different models are presented. From top to bottom, approach A, constant Δ and approach B, consecutively are depicted. As shown in plots, both modifications tend to have a smaller variance in high redshift (z $>$ 1.0). This is because both methods utilize a larger width for that region comparing to the case of constant smoothing width.

- Both approaches succeed to have a smaller variance in high redshifts comparing to the archetype version of smoothing method, based on constant smoothing width (see Fig. 3). This result is simply because in both approaches, the method employ a larger smoothing width in regions with low quality. In [14] it is discussed that large smoothing widths do have smaller variance.

- Interestingly, approach B shows a promising tool which can catch the true model, even if 1) true model is strangely different from ΛCDM 2) the quality of data are not that much good (for instance, see low redshifts in Fig. 4)

- In regions that we have pausity in data, it might happen that neither of approaches can completely catch the true model through 100 iterations. This is because the method of smoothing, like all non-parametric methods, is highly based on data. For instance, in Union 2.1 Compilation, we only have four data points in redshifts between 1.2 and 1.3 (see Fig. 1a). This deficiency leads to the result that the approach B could not catch the true model in high redshifts.



(a) Kink (b) CPL (c) ω = -0.9

Figure 4: Reconstructed $q(z)$ for synthetic samples from Union 2.1. The order of models and approaches are as the same as Fig. 3. Since the number of data in Union 2.1 are different from JDEM sample, we need to go for higher iteration to catch the true models completely. However, even with 100 iterations, the appraoch B works very well (neglecting the part in high z)

3.2 Reconstruction of q(z) from Union 2.1 Compilation (real data)

After showing the results of simulations, we apply all three methods on real data to reconstruct the deceleration parameter directly from data. To find the potential trend around our estimation, we follow two approaches; first, as shown in Fig. 5, we utilize Jackknife method to derive the trend around our estimation. To do this, we resample the data set by drawing one random data point. However, since the total number of data is comparably larger than 1, the band is very narrow. As second alternative, we simulate data sets around the best fit curves obtained by all three types of smoothing methods. The logic behind this is that we believe the distribution of real data around the true underlying model is nearly the same as the distribution of mock data, around our best fit curve. Then we re-apply the method to derive the confidence regions. As shown in figures (5 and 6) the results are more or less consistent with the best fit of ΛCDM.

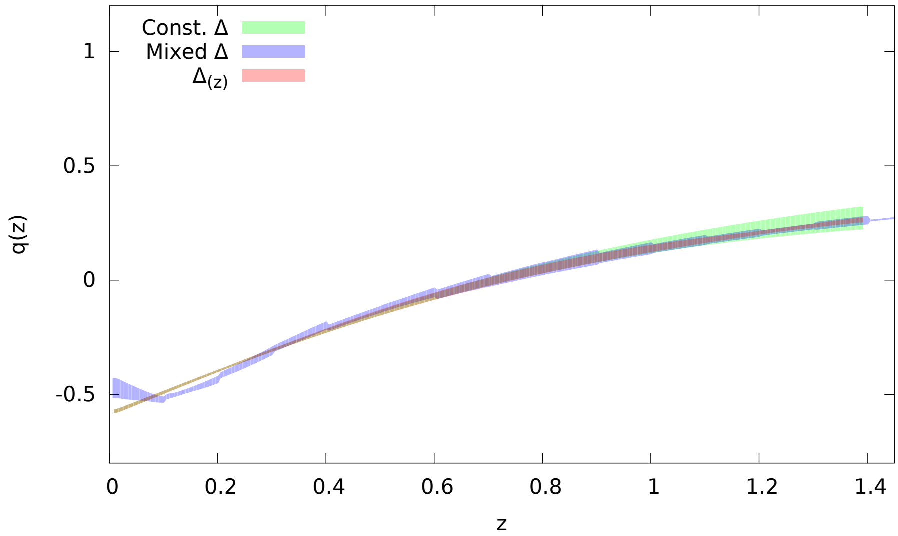


Figure 5: Reconstructed $q(z)$ for Union 2.1 Compilation, C.L obtained from Jackknife method. We resample the real data sets by picking up a data point from the whole sample. This recipe is employed for all data points and the result of reconstruction for samples are presented.

3.3 Reconstruction of q(z) from different initial guesses

To finalize our analysis, we follow the same approach in [15]. Given two real data sets, PanSTARR and Union 2.1 Compilation, we find the best fit of flat ΛCDM[3] to the data sets. Afterwards, starting with various initial guess models ranging from flat, curved universes to CPL universe, we discovered all possible reconstructions which all possess smaller χ^2 than simple flat ΛCDM universe. The difference in χ^2 is colour coded in the plots (see Fig. 7 & 8). As these figures show, the variance of reconstruction is larger for PanSTARR sample and it stems from the fact that it only includes 313 supernovae. In the following (Fig. 9) the histogram of data density for both data sets are presented to demonstrate the difference between these two samples. As shown in Fig. 9, more data we have, more reliable our estimation is; due to data paucity in high-z, the reconstructed results have some signatures of initial guesses.

[3]This model only includes one parameter, Ω_M. H_0 is a nuisance parameter to which does not affect q(z)

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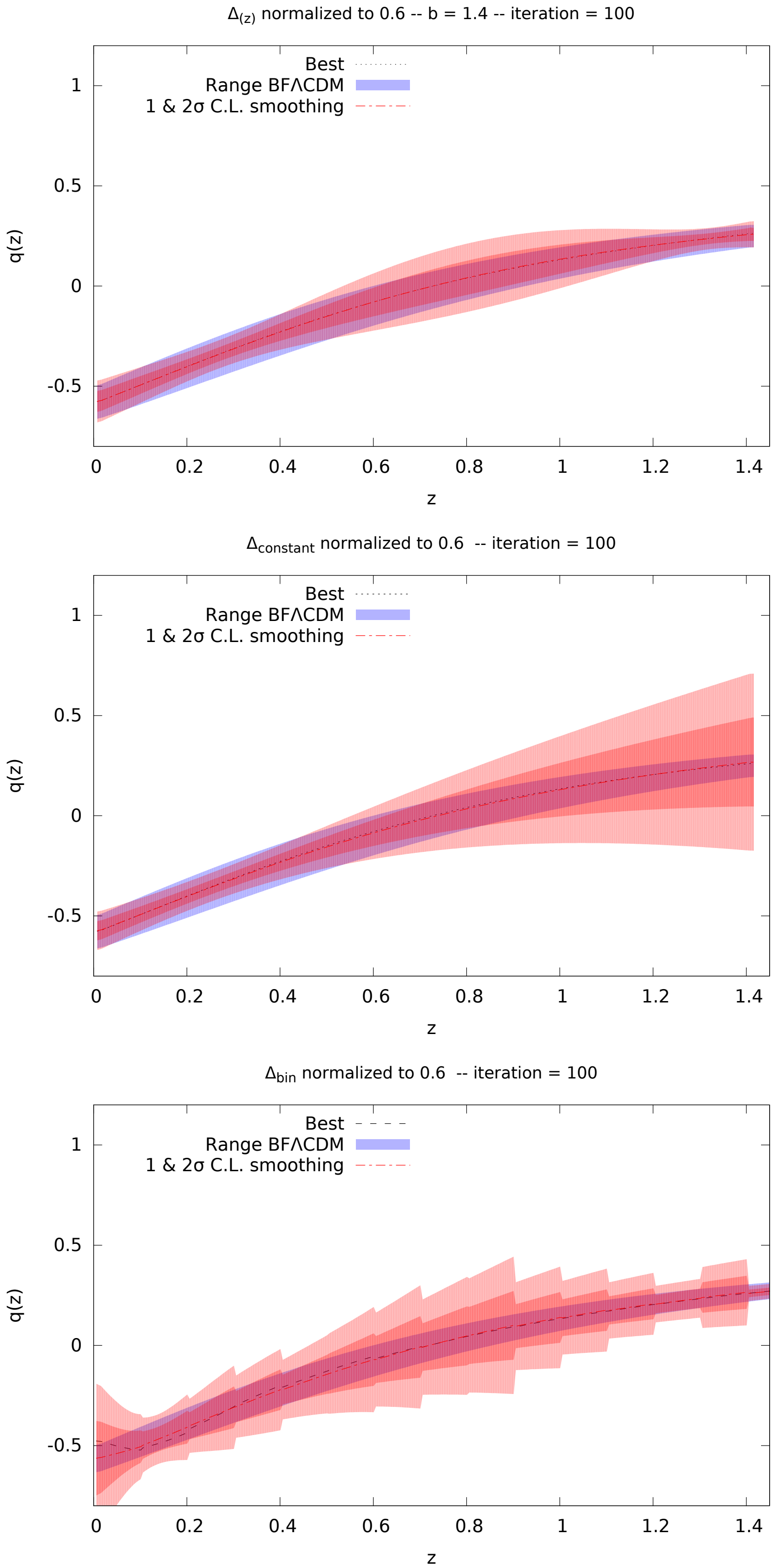


Figure 6: Reconstructed $q(z)$ for Union 2.1 Compilation, CL. obtained from MC simulation. To get the confidence region around our estimation, we make adequate simulations around our best fit, and then re-apply the method on the simulations.

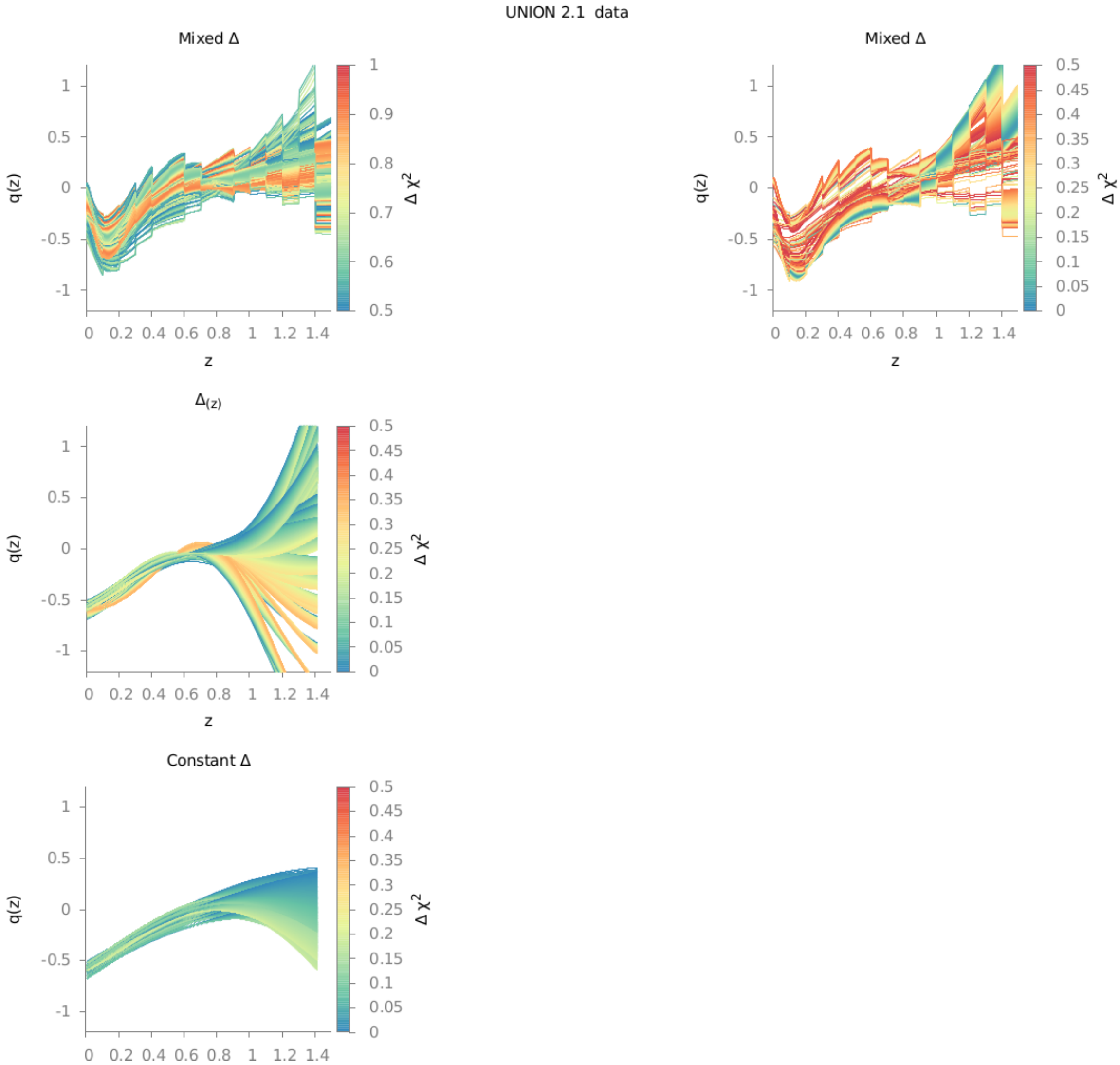


Figure 7: Reconstructed $q(z)$ for Union 2.1 sample. All reconstructed curves have a smaller χ^2 than best fit of ΛCDM. As shown in figures, the difference of $\chi^2_{\Lambda CDM} - \chi^2_{smoothed}$ is colour coded.

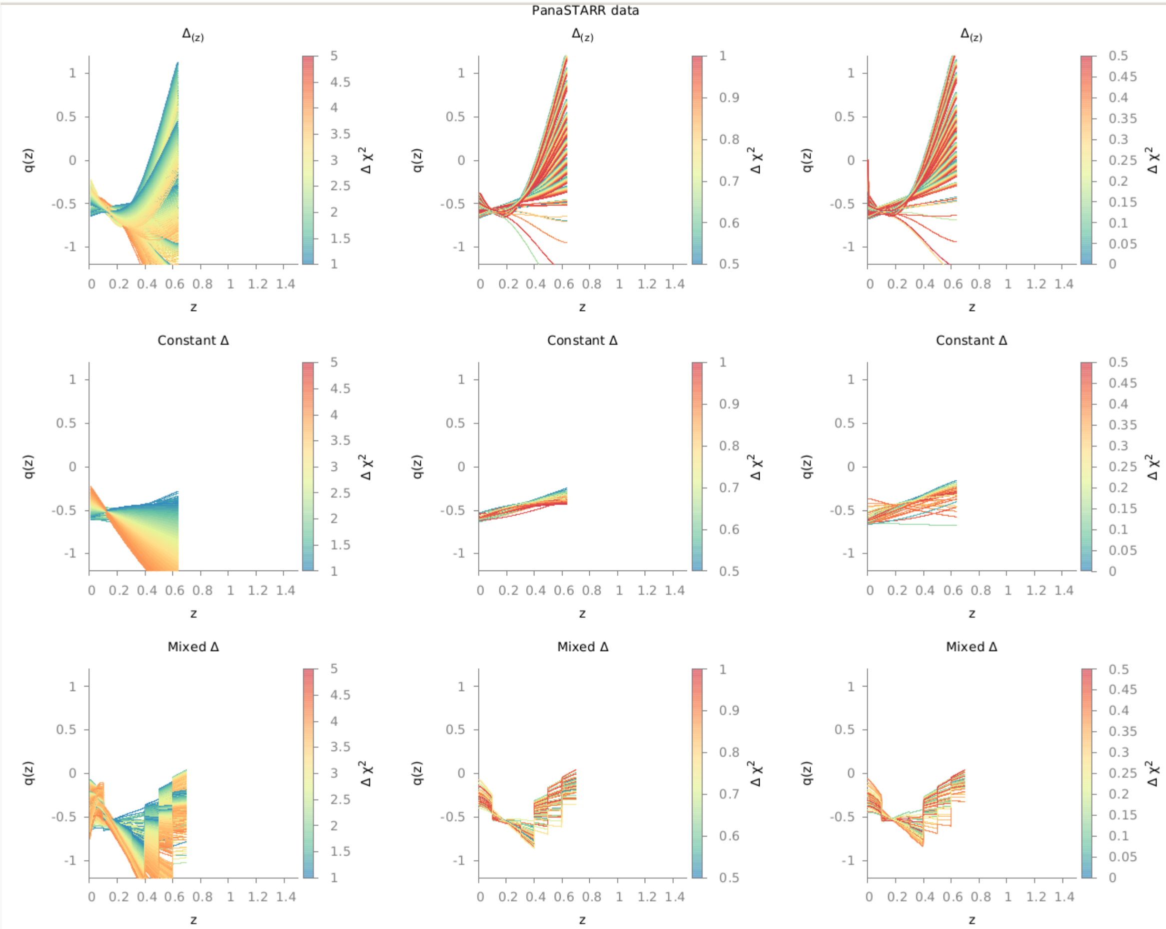
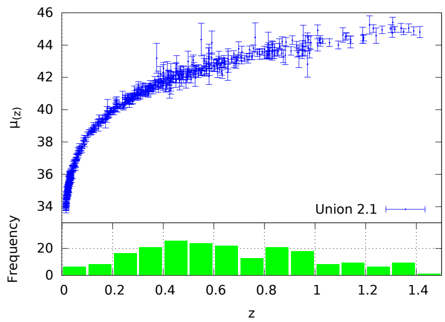
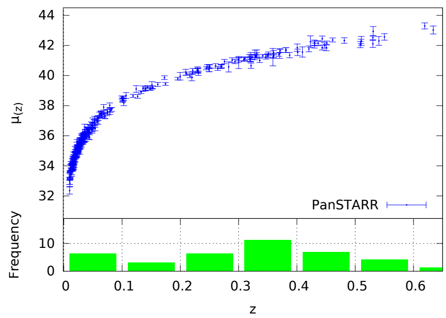


Figure 8: Reconstructed $q(z)$ for PanSTARR sample. All reconstructed curves have a smaller χ^2 than best fit of ΛCDM. As shown in figures, the difference of $\chi^2_{\Lambda CDM} - \chi^2_{smoothed}$ is colour coded.



(a) Union 2.1



(b) PanSTARR

Figure 9: Real data sets: distance modulus vs. redshift alongside histogram of the number of data points in each bin. Binsize is fixed at 0.0.1