$\boxed{\text{Sep 9}}$ n men & n women : n^2 pairs $(m,w) \in M \times W$

set of men → M; set of women → W

\# pairs of men $\binom{n}{2} = \frac{n(n-1)}{2}$

"$|\{(m,m') \mid m \neq m';\ m,m' \in M\}|$

Stable Matching / Marriage Problem (NOT feminist)

→ n men $M = \{m_1, \dots, m_n\}$

→ n women $W = \{w_1, \dots, ~~w_m~~ w_n\}$

$\boxed{n=2}$

$M = \{BP, BBT\}$

$W = \{JA, AJ\}$

Matching

Def (Matching) A set $S \subseteq M \times W = \{(m,w) \mid m \in M, w \in W\}$ is a matching if

(i) $\forall m \in M$, m is assigned ≤ 1 woman ($\exists \leq 1$ woman w s.t $(m,w) \in S$)

AND

(ii) $\forall w \in W$, $\exists \leq 1$ man m s.t $(m,w) \in S$.

Def Perfect matching : Replace "≤ 1" by "exactly 1"

✓ ~~Not~~ ✓ Perfect Matching

× Perfect Matching ✓

× Matching

P× M✓

P✓ M✓

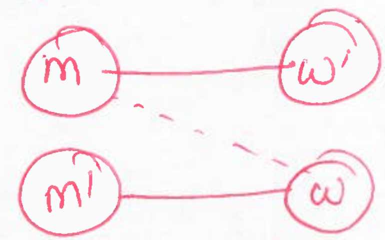
Preference Lists: $\forall\, m \in M$, $L_m \to$ total ranking of all women
$\forall\, w \in W$, $L_w \to$ ———————— men

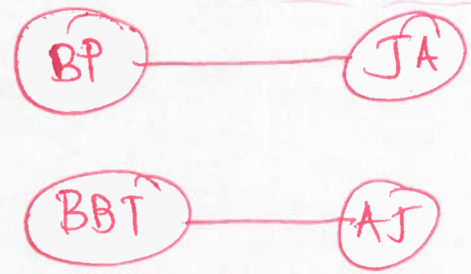
L_{BP}: $AJ > JA$ | L_{JA}: $BP > BBT$ | L'_{JA}: $BBT > BP$

L_{BBT}: $AJ > JA$ | L_{AJ}: $BP > BBT$ | L'_{BBT}: $JA > AJ$

Def: A stable matching is (i) a perfect matching
(ii) has no instability

Def: (Given pref lists & a perfect matching S)
$(m, w) \notin S$ is an instability:
(1) In L_m: $w > w'$ $\boxed{\text{AND}}$
(2) In L_w: $m > m'$





(BP, AJ) is an instability

Problem:
Input: M, W $\quad \forall\, m \in M$, L_m
$\forall\, w \in W$, L_w

Output: A stable matching (if one $\exists$)