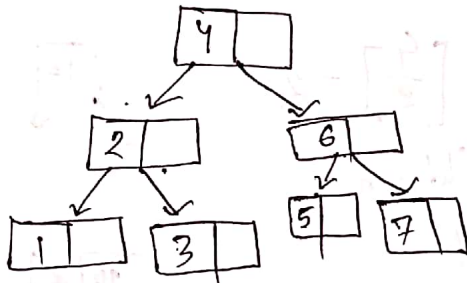


## 2-3 trees

min height and max  
and nodes

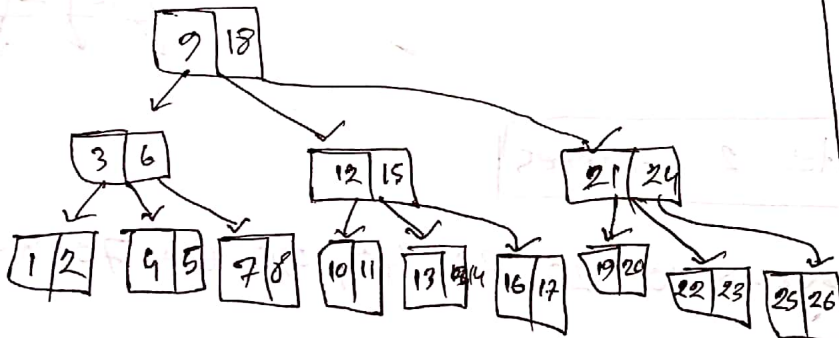


0  
1  
2

$$\min n = 1 + 2 + 2^2 + 2^3 + \dots$$

$$= 2^{h+1} - 1 \quad \left\lceil \frac{3}{2} \right\rceil = 2$$

$$\max h = \log_2(n+1) - 1$$



0  
1  
2

$$\max n = 1 + 3 + 3^2 + 3^3 + \dots$$

$$= \frac{3^{h+1} - 1}{3 - 1}$$

$$\min h = \log_3 \left[ \frac{n(3-1)+1}{3-1} \right]$$

Using B-trees

Databases

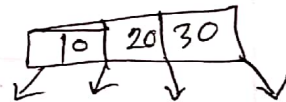
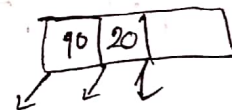
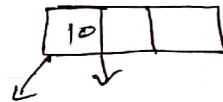
## 2-3-4 trees

### Lecture - 2

Height Balance Multway search tree.

- ① B-Tree of degree = 4
- ② Every node must have  $\lceil \frac{4}{2} \rceil = 2$  children
- ③ All leaf at same level

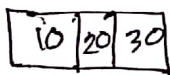
$d=4$



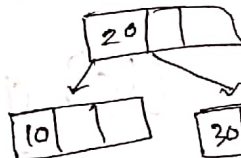
Let's create 2-3-4 trees

keys  $\rightarrow$  10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 110

Insert 10, 20, 30



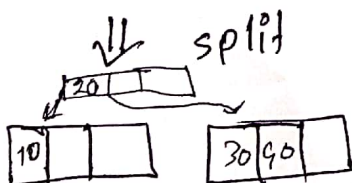
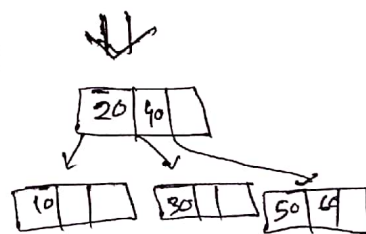
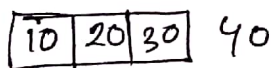
Insert 50, 60



60  
Free space

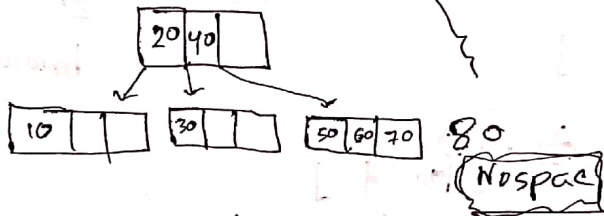


Insert 40

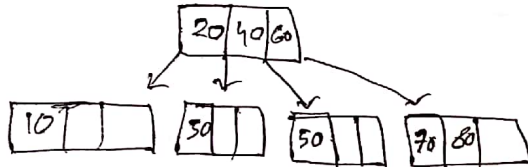


When Even number of key then consider Left bias / Right Bias

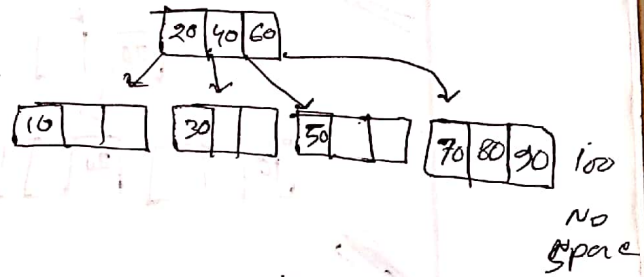
Insert 70, 80



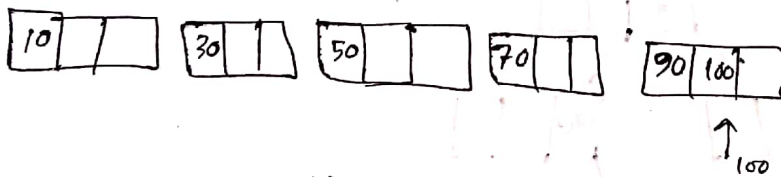
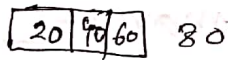
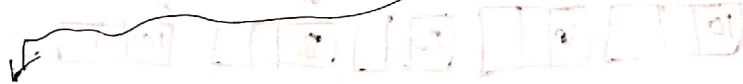
⇓ 80



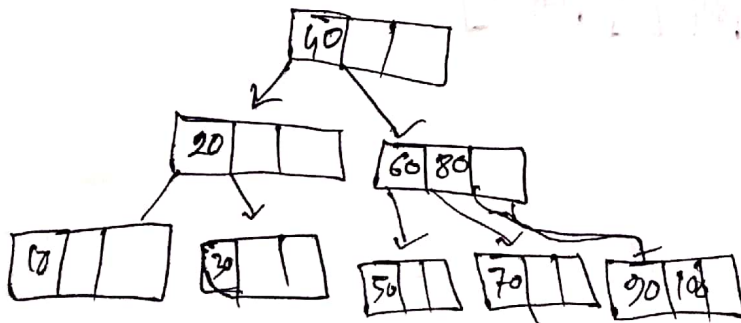
Insert 90, 100



⇓ 100



⇓

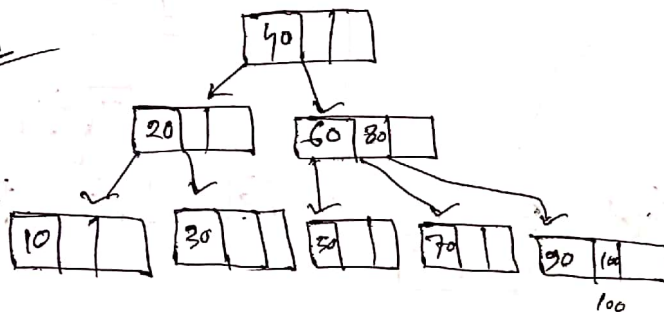


# How to delete from 2-3-4 trees

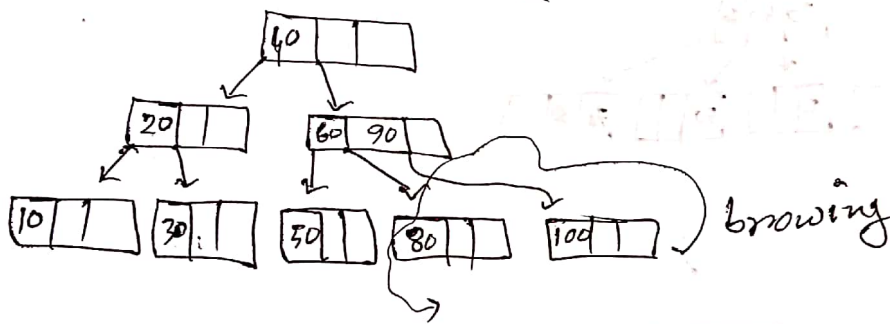
main

① case: 100

② case: 70  
browing key



↓ 70

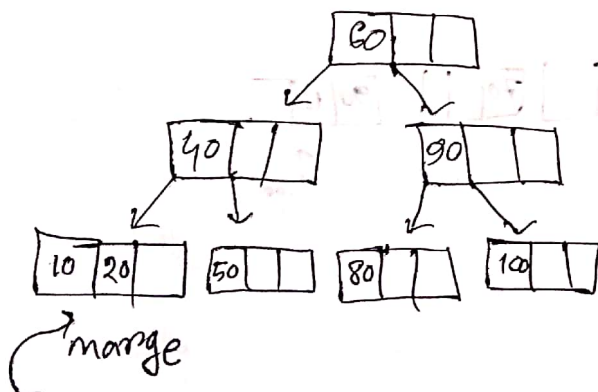


browing

↓

30

marge

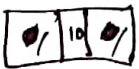



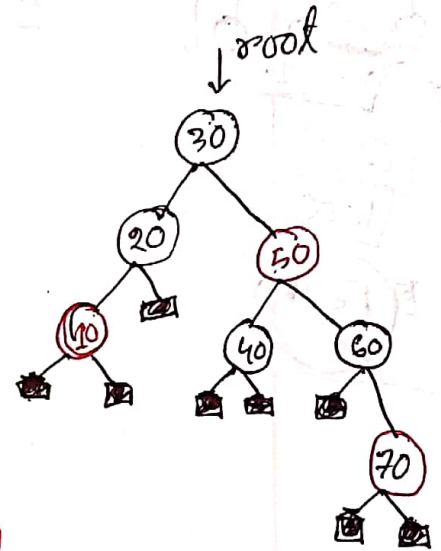
marge



# Red - Black - tree

## Lecture 3

- ① Its a Height balanced Binary search tree, similar to 2-3-4 tree
- ② Every Node is either Red or Black  $\Rightarrow \bigcirc \bigcirc$
- ~~\*\*~~ ③ Root of a tree is Black
- ④ Null is also Black 
- ⑤ Number of Black on path from root to leaf are same (3) like  $\rightarrow$
- ⑥ No 2 consecutive Red and parent and children of Red are black 
- ⑦ New Insert Node is Red  $\rightarrow \bigcirc$
- ⑧ Height in  $\log n \leq h \leq 2 \log n$



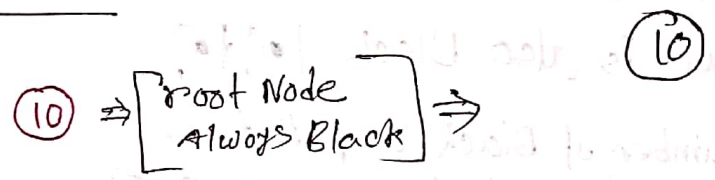
9+3=12  
40

Lecture 4

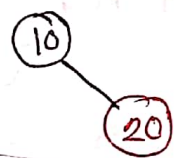
Red-Black tree  
\*  
create

keys : 10, 20, 30, 50, 40, 60, 70, 80, 4, 8

Insert 10

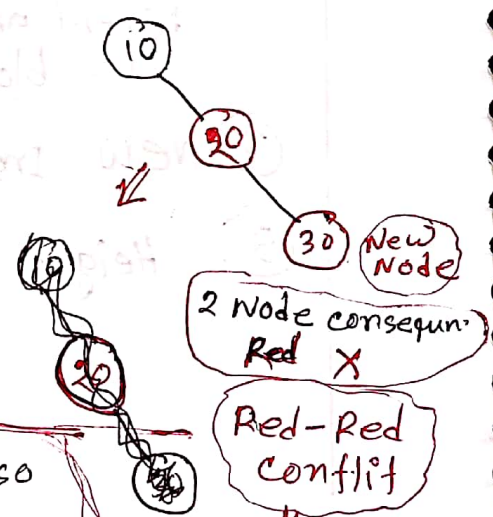


Insert 20



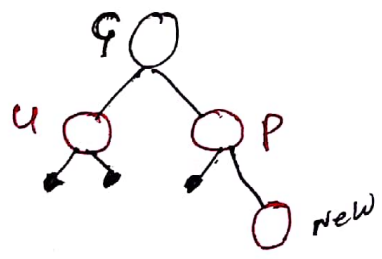
new Node always Black  
it is parents Black  
No problem

Insert 30

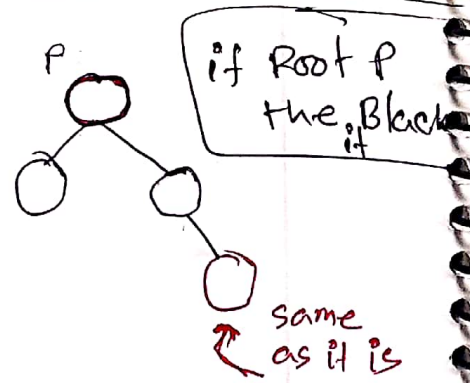


color  
① New Node is Red it's parent also Red then check  
Uncle is Red

- ① Recolor
- ② Rotation

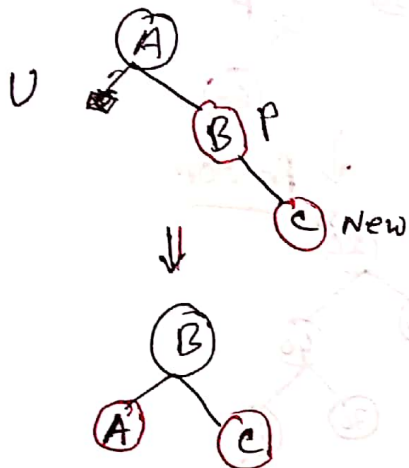


color  
⇒

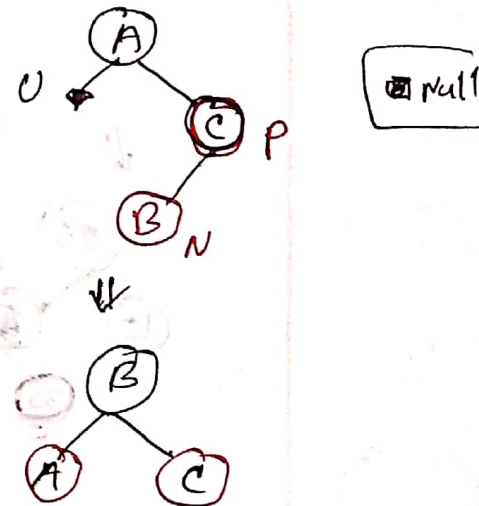


# Uncle is Black Rotation

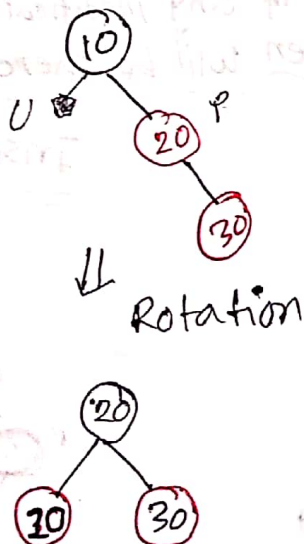
Zig-Zig LL/RR



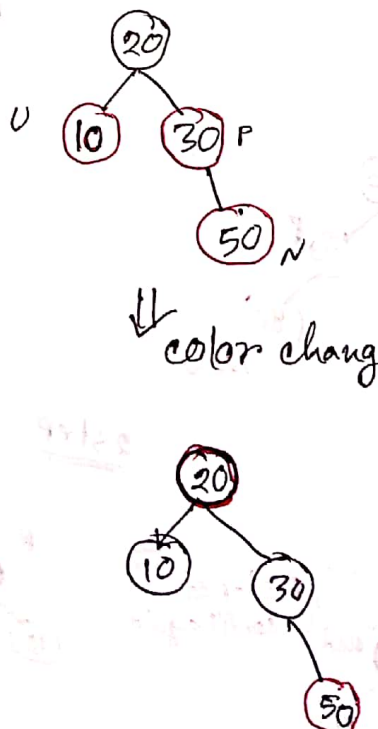
Zig-Zag RL/LR



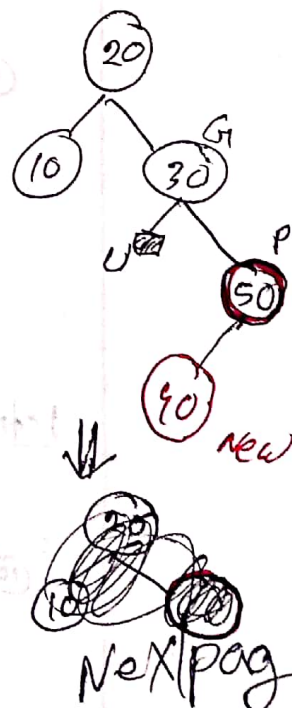
Insert 30



Insert 50



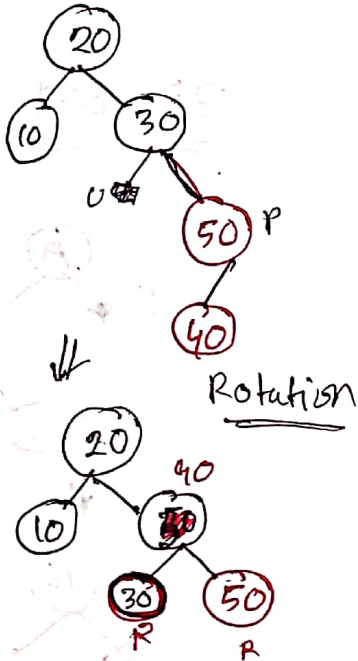
Insert 40



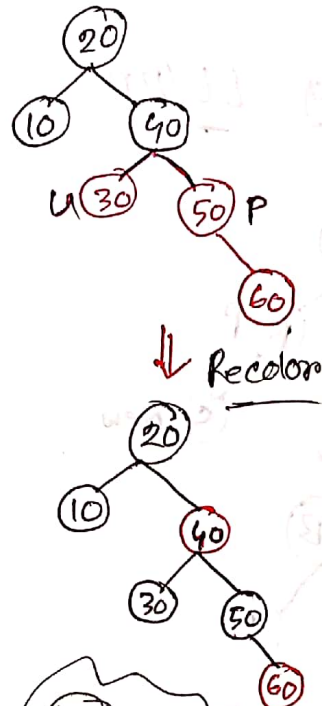
if G parent is  
 20 Root then make it black



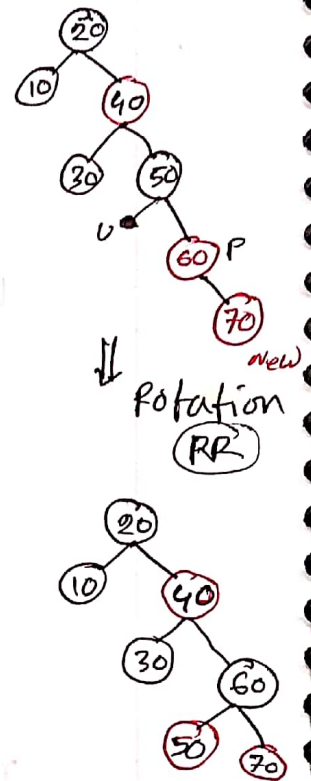
## Insert 40



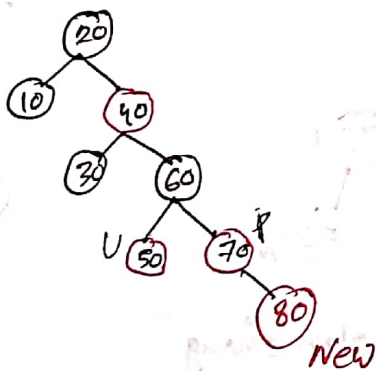
## Insert 60



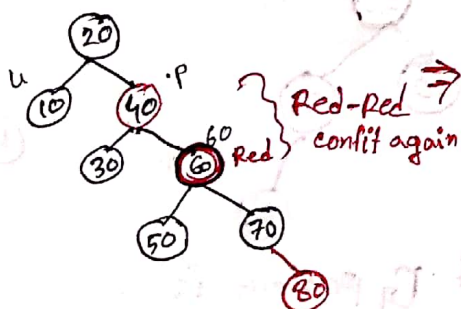
## Insert 70



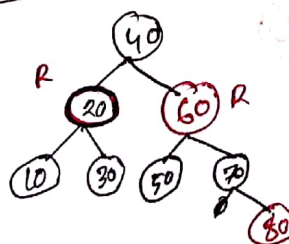
## Insert 80



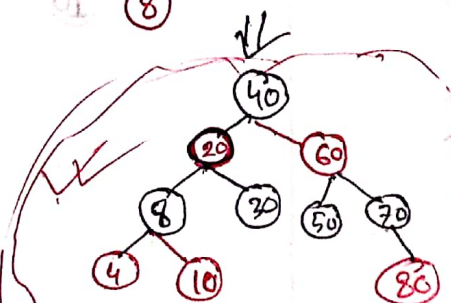
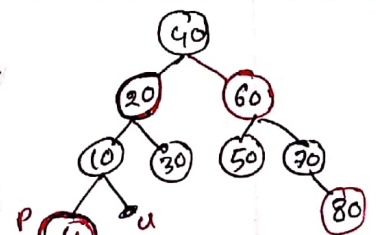
1 step



2 step



## Insert 4, then 8



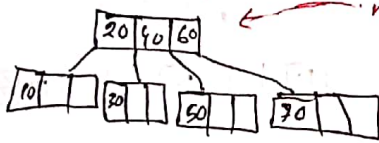
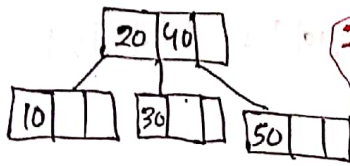
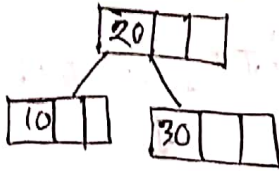


# Red-Black tree

## Lecture 5

## 2-3-4 tree and Red-Black tree

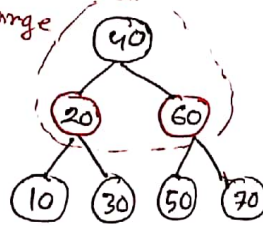
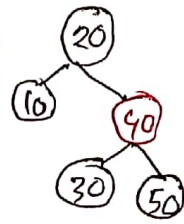
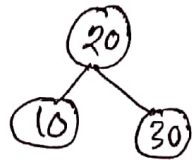
### 2-3-4 tree



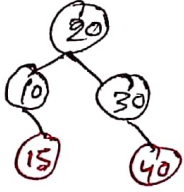
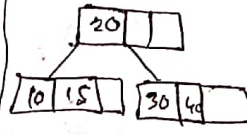
20, 40 in same node

merge

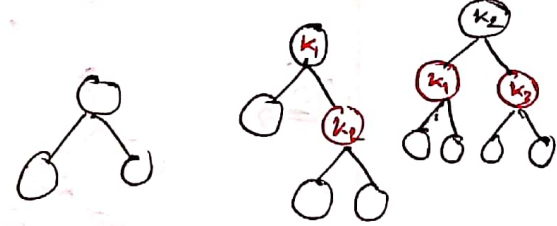
### Red-Black tree



### Example 2-3-4 tree      Red-Black



### General formula:



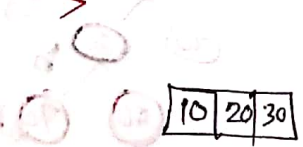
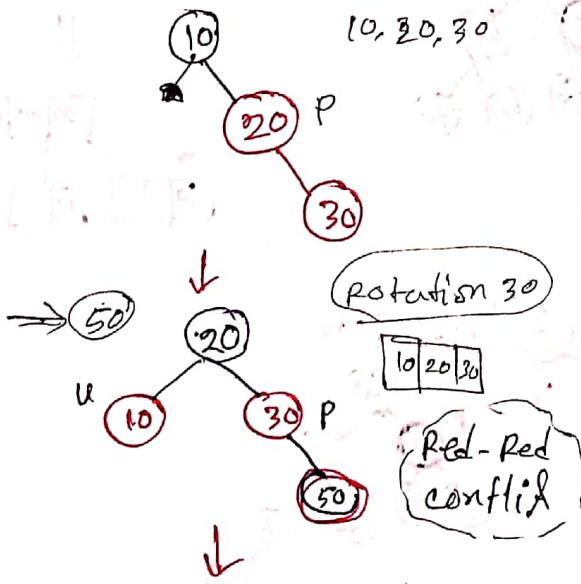
two children  
black color

## Lecture 6

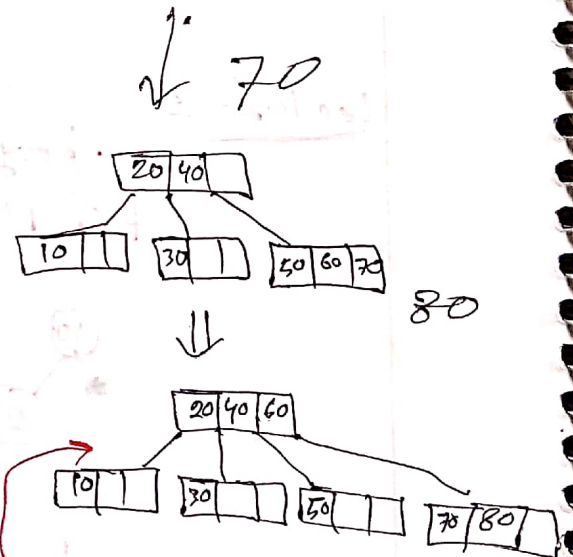
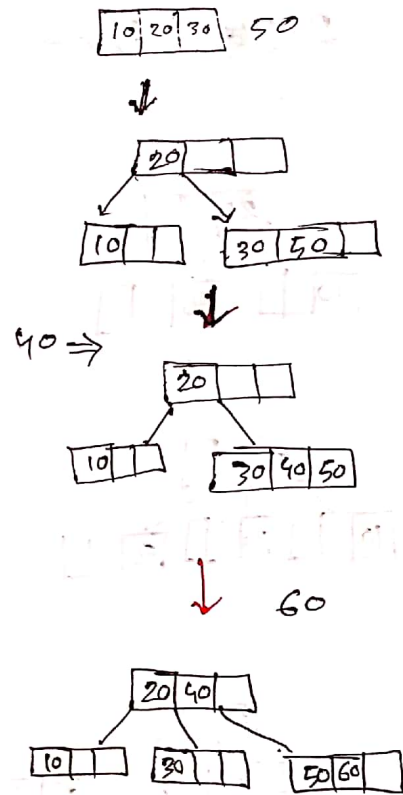
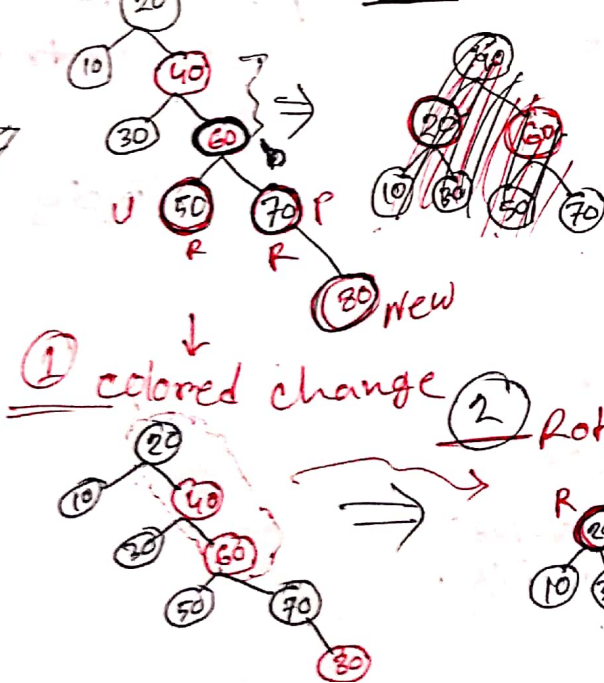
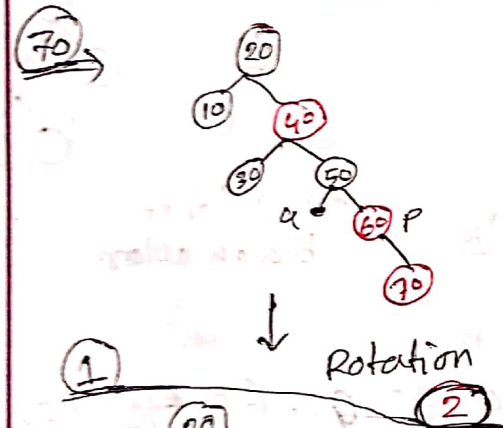
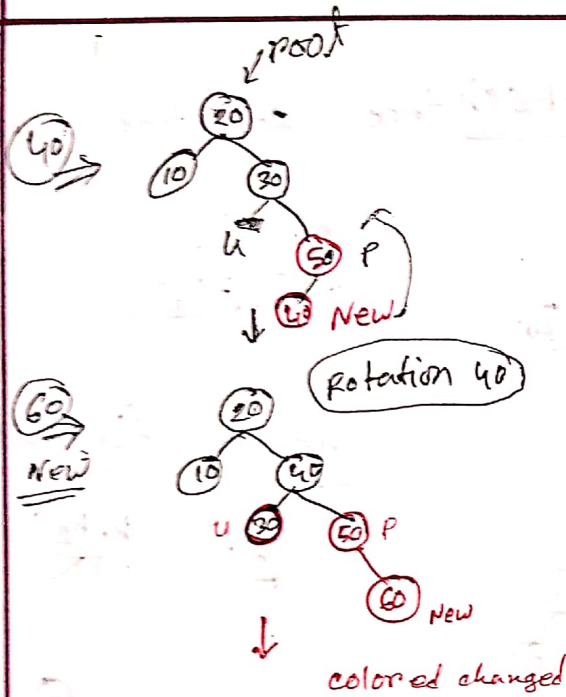
keys: 10, 20, 30, 50, 60, 70, 80, 4, 8

### Red-Black tree

### 2-3-4 trees



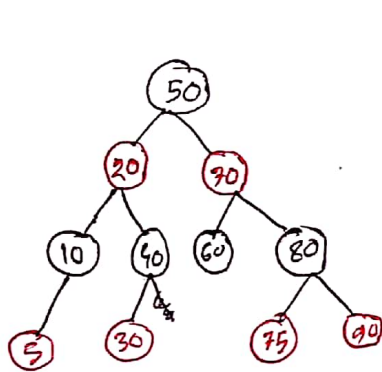
Keys : 10, 20, 30, 50, 40, 60, 70, 80, 4, 8



# Deletion from Red-Black tree \*

## Lecture 7

Red-Black-tree ~~and~~ deletion similar Binary search tree



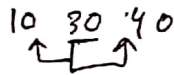
90  $\Rightarrow$  simply delete core leaf node

20  $\Rightarrow$  We don't delete the node

\* we delete the value

\* who will take its place

inorder predecessor or inorder successor



Inorder: 5, 10, 20, 30, 40, 50, 60, 70, 75, 80, 90

take place (20)

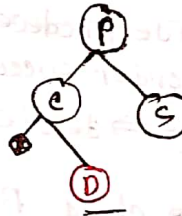
\* when ever deleted A node from BST then leaf node will be deleted or who ever only on child.

case 1

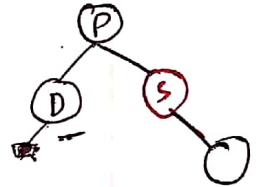


when Red Node and don't have any children delete it

Case 2



case 2

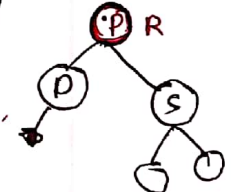


when deleted Node is Black the check sibling color it is Red then perform Rotation

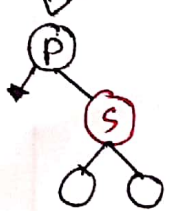


case 3

①

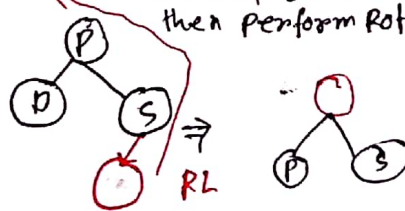


when delete Black Node and its siblings also Black (S) then check the siblings children both the color is black then change the color.



②

siblings color both Not Black then perform Rotation

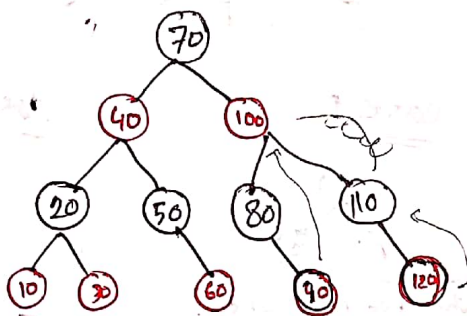




## Lecture 8

## Deletion

7x15  
105



90  $\Rightarrow$  simply deleted

100  $\Rightarrow$  Find inorder predecessor  
or inorder successor

90  $\Rightarrow$  delete 90 node

110  $\Rightarrow$  have one child Bring 120  
the color Black then  
delete 120 Red node

Lecture: 9

Red-Black and Deletion  
\*  
2-3-4-trees