Chapter 1

Questions

1.1 General Relativity

1. Marks: 7+7

   (a) What are Bianchi identities, Einstein tensor and Ricci tensor? Show that the covariant derivatives of Einstein tensor is zero.

   (b) Define a flat space-time. Show that the vanishing of curvature tensor is a necessary and sufficient condition for a space-time to be flat.

2. Marks: 6+8

   (a) Give an account of Einstein's principle of equivalence. What are observable consequences of General theory of Relativity?/ Discuss that it acts as a bridge to pass from special to general theory of Relativity.

   (b) Write a short note on the energy momentum tensor $T^{\mu\nu}$ and discuss the reasons which led Einstein to choose the field equations in the form

   $$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = kT_{\mu\nu}$$

   or,

   $$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -8\pi T_{\mu\nu}$$

   show further that these equations reduce in linear approximation to Newtonian equations $\nabla^2\psi = -8\pi$

3. Marks: 7+7

   (a) What is meant by geodesics? Derive the equations of geodesics with the help of a variational principle.

   (b) Show that the curvature tensor may be contracted in two ways. One of these leads to a zero tensor and the other method leads to Ricci tensor.

4. Marks:

(a) State and comment on the basic hypothesis and postulates of the general theory of relativity and discuss how the principle of equivalence and covariance follow from the guiding principle in the development if general relativity.

(b) Show that geodesics equations of motion are reducible to Newtonian equations of motion in case of a weak static field.

5. Marks: 7+7

(a) Derive Schwarzchild interior solution for a spherically symmetric distribution of matter with constant density.

(b) Deduce Einstein's field equations for interior material world in the form

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -8\pi T_{\mu\nu}$$

explaining the significance of the symbols used. Hence obtain Poissions equations on approximation for a very weak static field.

6. Marks: 14
Obtain Schwarzchild's exterior solution for the gravitational field of a single mass at rest and explain on the basis of this solution the advanced of perihelion of the planet Mercury.