Chapter 1

Finite Element Solution of Two-dimensional BVPs

1.1 The Galerkin formulation in 2D (Matrix formulation)

Here we shall formulate the matrix for Laplace and Poisson equation because almost all types of 2D problems arise in physical problems are Laplace or Poisson equation.

In 2D, domain is a region (area) closed by a curve. Suppose $(x, y) \in R$ [Domain] with condition, essential at C, then $u(c) = u(x, y)$ (x, y lies on C). Suppressible at C^*, then $\frac{\partial u}{\partial n} = K^*$ (K^* depends on the point of C^*) and whole domain boundary $B(C + C^*)$.

1.1.1 Green's theorem in the plane

This theorem states that, if A is a region in the xy-plane bounded by a closed curve Γ and if $F(x, y)$ and $G(x, y)$ are suitably "smooth functions" then,

$$\iint\limits_{A} \left\{ \frac{\partial G}{\partial x} - \frac{\partial F}{\partial y} \right\} \mathrm{d}\,x \,\mathrm{d}\,y = \oint\limits_{\Gamma} (F \,\mathrm{d}\,x + G \,\mathrm{d}\,y)$$

Problem 1.1. Solve the two-dimensional BVP: where, u denotes the temperature at any point of the plate. Hence, determine an approx. using the trial function, $\bar{u}(x, y) = x(y^2 - 4)(a_1 + a_2 x)$