# *Electromagnesitm and Modern Physics*

PHY301M


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# Preface

This is a compilation of lecture notes with some books and my own thoughts. This document is not a holy text. So, if there is a mistake, solve it by your own judgement.

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# Part I

# Lecture/Class Note

# Sheet 1

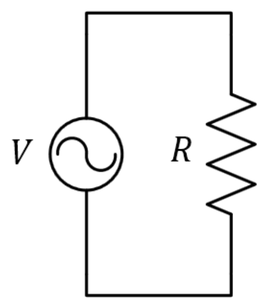
# RCL with AC

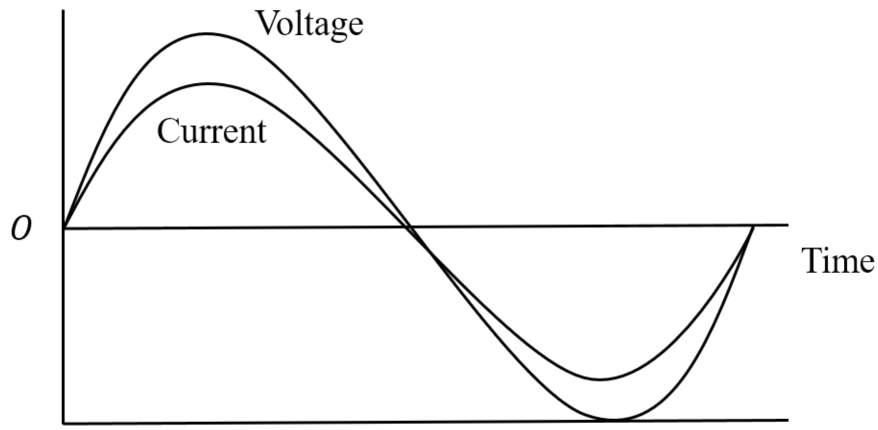
## 1.1 Alternating Current

Direct current (DC) circuits involve current flowing in one direction. In alternating current (AC) circuits, instead of a constant voltage supplied by a battery, the voltage oscillates in a sine wave pattern, varying with time as $V = V_0 \sin \omega t$.

In a household circuit, the frequency is 60 Hz. The angular frequency is related to the frequency, $f$, by $\omega = 2nfV_0$ represents the maximum voltage, which in a household circuit in North America is about 170 volts. We talk of a household voltage of 120 volts, though; this number is a kind of average value of the voltage. The particular averaging method used is something called root-mean-square (square the voltage to make everything positive, find the average, take the square root), or rms. Voltages and currents for AC circuits are generally expressed as r.m.s. values. For a sine wave, the relationship between the peak and the r.m.s. average is:

$$\text{r.m.s. value} = 0.707 \text{ peak value}$$



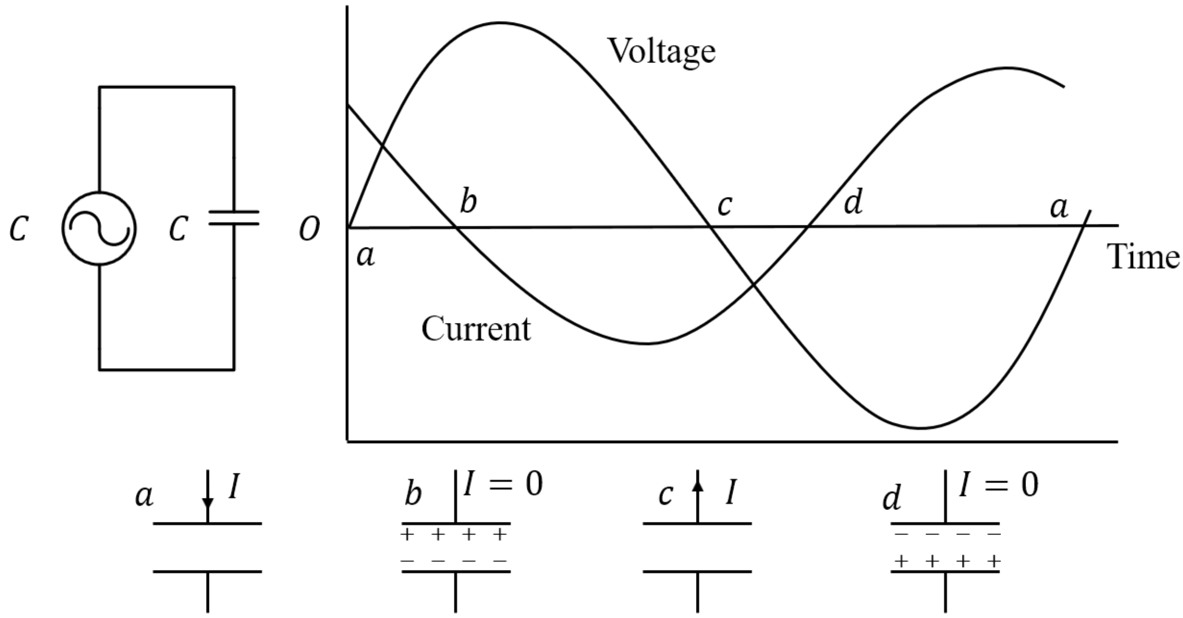

## 1.2 Resistance in an AC circuit

The relationship $V = IR$ applies for resistors in an AC circuit, so

$$I = \frac{V}{R} = \frac{V_0}{R}\sin(\omega t) = I_0 \sin(\omega t)$$

In AC circuits we'll talk a lot about the phase of the current relative to the voltage. In a circuit which only involves resistors, the current and voltage are in phase with each other, which means that the peak voltage is reached at the same instant as peak current. In circuits which have capacitors and inductors (coils) the phase relationships will be quite different.

## 1.3 Capacitance in an AC circuit




Consider now a circuit which has only a capacitor and an AC power source (such as a wall outlet). A capacitor is a device for storing charging. It turns out that there is a 90° phase difference between the current and voltage, with the current reaching its peak 90° (1/4 cycle) before the voltage reaches its peak. Put another way, the current leads the voltage by 90° in a purely capacitive circuit.

To understand why this is, we should review some of the relevant equations, including: Relationship between voltage and charge for a capacitor:

$$CV = Q$$

Relationship between current and the flow of change:

$$I = \frac{\Delta Q}{\Delta t}$$

The AC power supply produces an oscillating voltage. We should follow the circuit through one cycle of the voltage to figure out what happens to the current.

Step 1: At point $a$ (see diagram) the voltage is zero and the capacitor is uncharged, Initially, the voltage increases quickly. The voltage across the capacitor matches the power supply voltage, so the current is large to build up charge on the capacitor plates. The closer the voltage gets to its peak, the slower it changes, meaning less current has to flow. When the voltage reaches a peak at point $b$, the capacitor is fully charged and the current is momentarily zero.

Step 2: After reaching a peak, the voltage starts dropping. The capacitor must discharge now, so the current reverses direction. When the voltage passes through zero at point $c$, it's changing quite rapidly; to match this voltage the current must be large and negative.

Step 3: Between points $c$ and $d$, the voltage is negative. Charge builds up again on the capacitor plates, but the polarity is opposite to what it was in step 1. Again the current is negative, and as the voltage reaches its negative peak at point $d$ the current drops to zero.

Step 4: After point $d$, the voltage heads toward zero and the capacitor must discharge. When the voltage reaches zero it's gone through a full cycle so it's back to point $a$ again to repeat the cycle.

The larger the capacitance of the capacitor, the more charge has to flow to build up a particular voltage on the plates, and the higher the current will be. The higher the frequency of the voltage, the shorter the time available to change the voltage, so the larger the current has to be. The current, then, increases as the capacitance increases and as the frequency increases.

Usually this is thought of in terms of the effective resistance of the capacitor, which is known as the capacitive reactance, measured in ohms. There is an inverse relationship between current and resistance, so the capacitive reactance is inversely proportional to the capacitance and the frequency: A capacitor in an AC circuit exhibits a kind of resistance called capacitive reactance, measured in ohms. This depends on the frequency of the AC voltage, and is given by Capacitive reactance

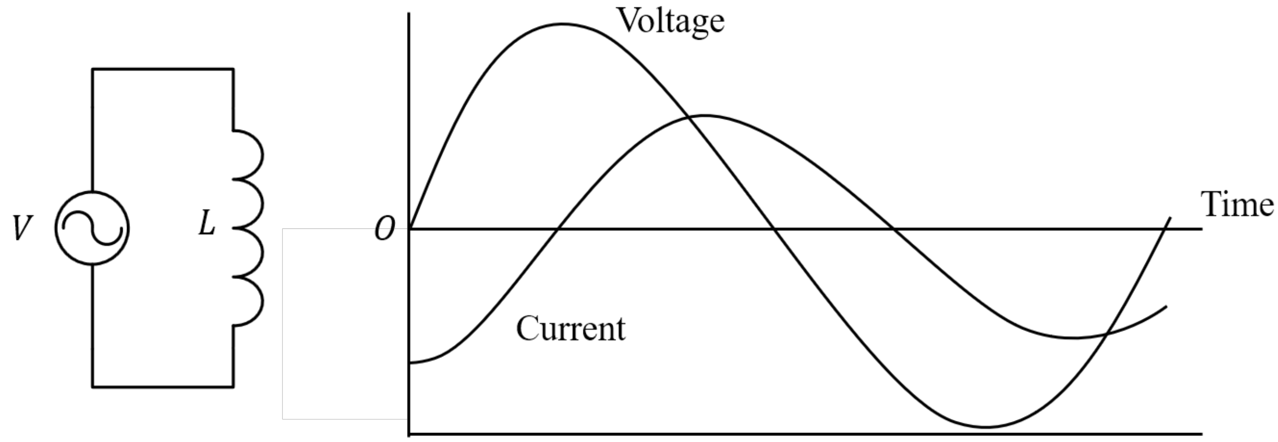
$$X_c = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

We can use this like a resistance (because, really, it is a resistance) in an equation of the form $V = IR$ to get the voltage across the capacitor:

$$V = IX_C$$

Note that $V$ and $I$ are generally the r.m.s. values of the voltage and current.

## 1.4 Inductance in an AC circuit




An inductor is simply a coil of wire (often wrapped around a piece of ferromagnet). If we now look at a circuit composed only of an inductor and an AC power source, we will again find that there is a 90° phase difference between the voltage and the current in the inductor. This time, however, the current lags the voltage by 90° so it reaches its peak 1/4 cycle after the voltage peaks.

The reason for this has to do with the law of induction:

$$e = -N\frac{\Delta\Phi}{\Delta t} \qquad \text{or} \qquad e = -L\frac{\Delta I}{\Delta t}$$

Applying Kirchhoff's loop rule to the circuit above gives:

$$V - L\frac{\Delta I}{\Delta t} = 0 \qquad \text{so} \qquad V = L\frac{\Delta I}{\Delta t}$$

As the voltage from the power source increases from zero, the voltage on the inductor matches it. With the capacitor, the voltage came from the charge stored on the capacitor plates (or, equivalently, from the electric field between the plates). With the inductor, the voltage comes from changing the flux through the coil, or, equivalently, changing the current through the coil, which changes the magnetic field in the coil.

To produce a Large positive voltage, a Large increase in current is required. When the voltage passes through zero, the current should stop changing just for an instant. When the voltage is Large and negative, the current should be decreasing quickly. These conditions can all be satisfied by having the current vary like a negative cosine wave, when the voltage follows a sine wave.

How does the current through the inductor depend on the frequency and the inductance? If the frequency is raised, there is less time to change the voltage. If the time interval is reduced, the change in current is also reduced, so the current is lower. The current is also reduced if the inductance is increased.

As with the capacitor, this is usually put in terms of the effective resistance of the inductor. This effective resistance is known as the inductive reactance. This is given by $X_L = \omega L = 2\pi f L$, where $L$ is the inductance of the coil (this depends on the geometry of the coil and whether it's got a ferromagnetic core). The unit of inductance is the henry.

As with capacitive reactance, the voltage across the inductor is given by:

$$V = IX_L$$

# Sheet 2

# RLC Circuit Analysis (Series and Parallel)

In *RLC circuit*, the most fundamental elements of a resistor, inductor and capacitor are connected across a voltage supply. All of these elements are linear and passive in nature. Passive components are ones that consume energy rather than producing it; linear elements are those which have a linear relationship between voltage and current.

There are a number of ways of connecting these elements across voltage supply, but the most common method is to connect these elements either in series or in parallel. The RLC circuit exhibits the property of resonance in same way as LC circuit exhibits, but in this circuit the oscillation dies out quickly as compared to LC circuit due to the presence of resistor in the circuit.

## 2.1 Series RLC Circuit

When a resistor, inductor and capacitor are connected in series with the voltage supply, the circuit so formed is called series RLC circuit.

Since all these components are connected in series, the current in each element remains the same,

$$I_R = I_L = I_C = I(t) \quad \text{where} \quad I(t) = I_M \sin(\omega t)$$

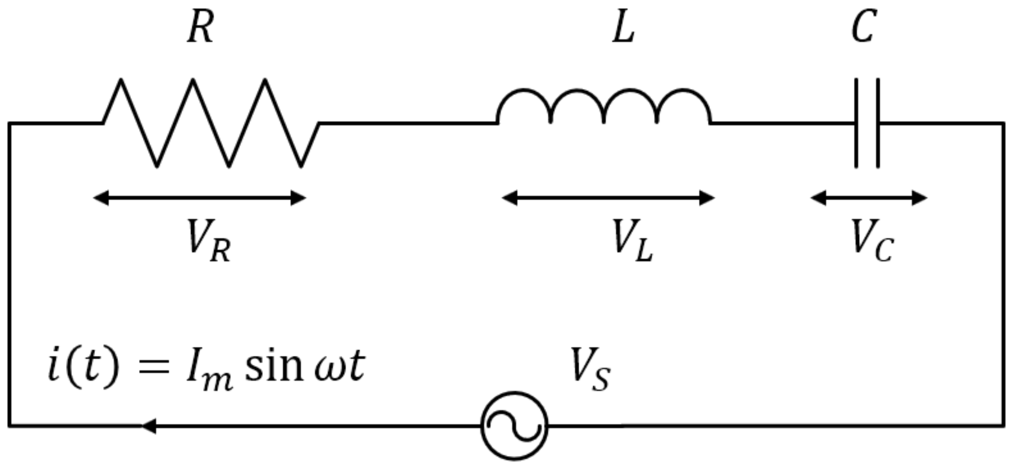
Let $V_R$ be the voltage across resistor, $R$.

$V_L$ be the voltage across inductor, $L$.

$V_C$ be the voltage across capacitor, $C$.

$X_L$ be the inductive reactance.

$X_C$ be the capacitive reactance. The total voltage in RLC circuit is not equal to algebraic sum



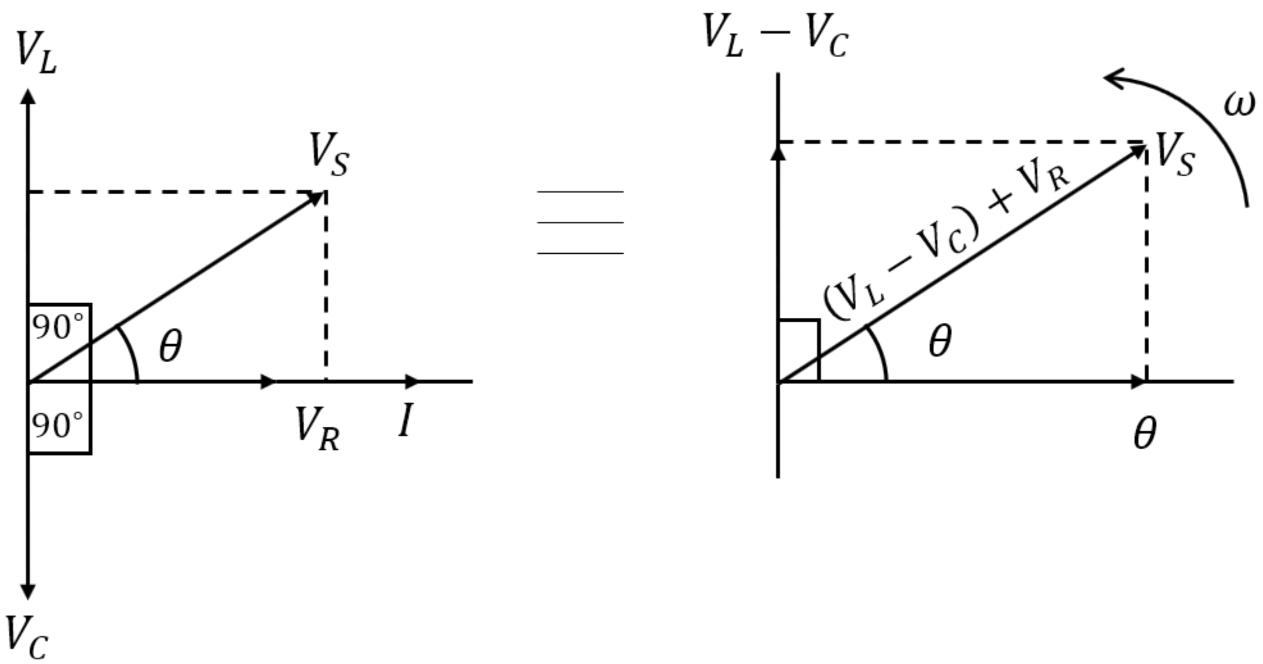

of voltages across the resistor, the inductor and the capacitor; but it is a vector sum because, in case of resistor the voltage is in phase with the current, for inductor the voltage leads the current by 90° and for capacitor, the voltage lags behind the current by 90°.

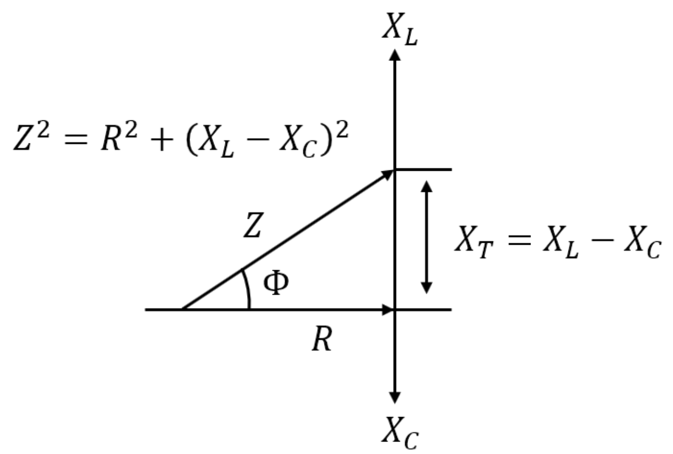
So, voltages in each component are not in phase with each other; so they cannot be added arithmetically. The figure below shows the phasor diagram of series RLC circuit. For drawing the phasor diagram for RLC series circuit, the current is taken as reference because, in series circuit the current in each element remains the same and the corresponding voltage vectors for each component are drawn in reference to common current vector.

$$V_S{}^2 = V_R{}^2 + (V_L - V_C)^2 \qquad (\text{if } V_L > V_C)$$

$$V_S{}^2 = V_R{}^2 + (V_L - V_C)^2 \qquad (\text{if } V_L > V_C)$$

Where, $V_R = IR$, $V_L = IX_L$, $V_C = IX_C$ The Impedance for a Series RLC Circuit The impedance




$Z$ of a series RLC circuit is defined as opposition to the flow of current due circuit resistance $R$, inductive reactance, $X_L$ and capacitive reactance, $X_C$. If the inductive reactance is greater than the capacitive reactance i.e. $X_L > X_C$, then the RLC circuit has lagging phase angle and if the capacitive reactance is greater than the inductive reactance i.e. $X_C > X_L$ then, the RLC circuit have leading phase angle and if both inductive and capacitive are same i.e. $X_L = X_C$ then circuit will behave as purely resistive circuit.

We Know that

$$V_S{}^2 = V_R{}^2 + (V_L - V_C)^2$$

Where,

$$V_R = IR,\ V_L = IX_L,\ V_C = IX_C$$

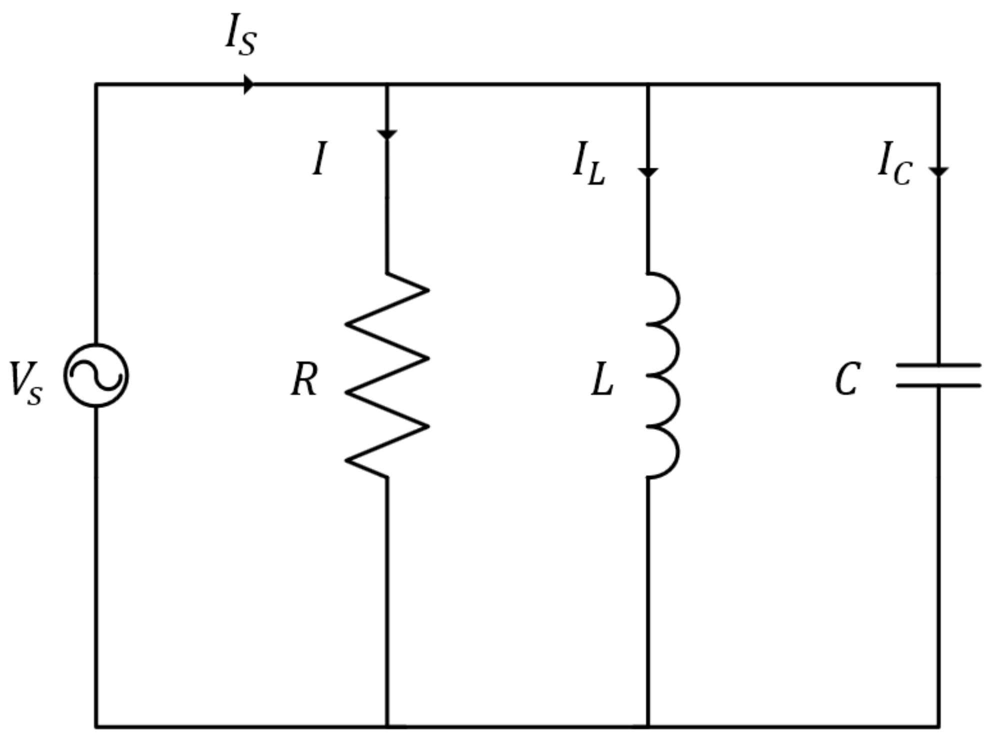
Substituting the values

$$V_S{}^2 = (IR)^2 + (IX_L - IX_C)^2$$
$$\Rightarrow V_S = I\sqrt{R^2 + (X_L - X_C)^2}$$
$$\text{or, impedance } Z = \sqrt{R^2 + (X_L - X_C)^2}$$

## 2.2 Parallel RLC Circuit

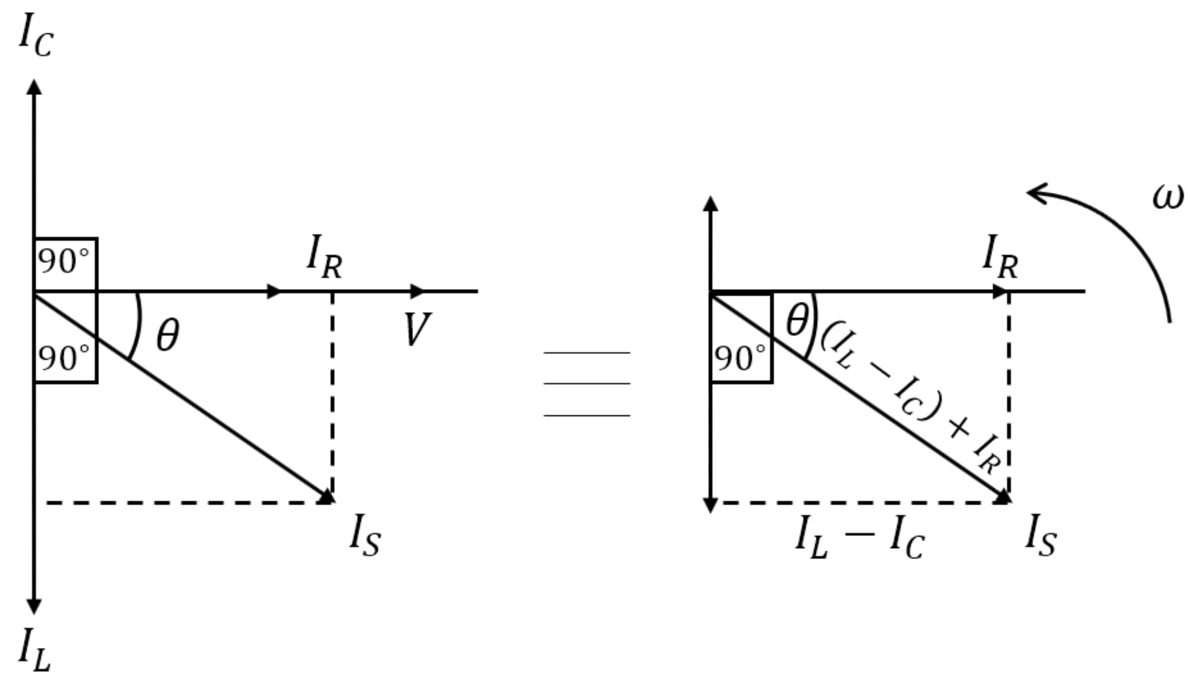
In parallel RLC Circuit the resistor, inductor and capacitor are connected in parallel across a voltage supply. The parallel RLC circuit is exactly opposite to the series RLC circuit. The applied voltage remains the same across all components and the supply current gets divided.

The total current drawn from the supply is not equal to mathematical sum of the current flowing in the individual component, but it is equal to its vector sum of all the currents, as the current flowing in resistor, inductor and capacitor are not in the same phase with each other; so they cannot be added arithmetically. Phasor diagram of parallel RLC circuit,




$I_R$ is the current flowing in the resistor, $R$ in amps.
$I_C$ is the current flowing in the capacitor, $C$ in amps.
$I_L$ is the current flowing in the inductor, $L$ in amps.
$I_S$ is the supply current in amps.
In the parallel RLC circuit, all the components are connected in parallel; so the voltage across each element is same. Therefore, for drawing phasor diagram, take voltage as reference vector and all the other currents i.e $I_R$, $I_C$, $I_L$ are drawn relative to this voltage vector. The current through each element can be found using Kirchhoff's Current Law, which states that the sum of currents entering a junction or node is equal to the sum of current leaving that node.

$$I_S{}^2 = I_R{}^2 + (I_L - I_C)^2$$




Now,

$$I_R = \frac{V}{R}, \quad I_C = \frac{V}{X_C}, \quad I_L = \frac{V}{X_L}$$

$$I_S = \sqrt{\frac{V^2}{R^2} + \left(\frac{V}{X_L} - \frac{V}{X_C}\right)^2}$$

So,

$$\text{admitance, } \frac{1}{Z} = \frac{I_S}{V} = Y = \sqrt{\frac{1}{R^2} + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2}$$

As shown above in the equation of impedance, $Z$ of a parallel RLC circuit; each element has reciprocal of impedance $(1/Z)$ i.e. admittance, $Y$. So in parallel RLC circuit, it is convenient to use admittance instead of impedance.

# Sheet 3

# De Broglie Waves (Matter Waves)

In 1924, Lewis de-Broglie proposed that matter has dual nature just like photon. His concept about the dual nature of matter was based on the following observations:

- The whole universe is composed of matter and electromagnetic radiations. Since both are forms of energy so can be transformed into each other.
- The nature loves symmetry. As the radiation has dual nature, matter should also possess dual character.

According to the de Broglie concept of matter waves the matter has dual nature. It means when the matter is moving it shows the wave properties are associated with it and when it is at rest then it shows particle properties. Thus, the matter has dual nature. The waves associated with moving particles are called matter waves or de-Broglie waves.

## 3.1 De Broglie Wave Length

The energy of a photon can be written as $E = hv = \frac{hc}{\lambda}$, if a photon possess mass (rest mass is zero), then according to the theory of relativity its energy is given by,

$$E = mc^2 = mc \cdot c = pc$$

Hence the momentum of a photon

$$p = \frac{E}{c} = \frac{hc}{\lambda c} = \frac{h}{\lambda}$$

wavelength of a photon,

$$\lambda = \frac{h}{p}$$

Instead of photon, we consider a particle of mass $m$ moving with velocity $v$, then the momentum of the particle $p = mv$ then wavelength of that particle will be

$$\boxed{\lambda = \frac{h}{mv}} \longrightarrow \text{de-Broglie wavelength}$$

where, $m = \dfrac{m_0}{\sqrt{1 - v^2/c^2}}$, the relativistic mass.

## 3.2 Phase Velocity and Group Velocity

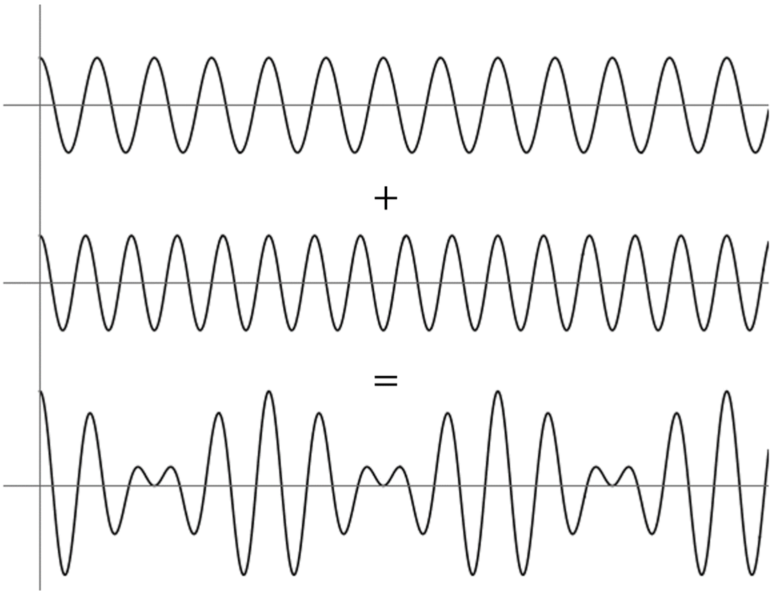
According to de-Broglie's concept of matter waves, each particle of matter (like electron, proton etc) while in potion, may be regarded as consisting of a group of waves or a wave packet as it is called the wave packet, formed by superposition of a number of waves and traveling with the velocity of the particles, behaves very much like a corpuscle. Each component wave propagates with a definite velocity called the wave velocity or phase velocity. But when a disturbance consists of a number of component waves, each travelling with slightly different velocity, the resultant velocity will be that of a periodicity. The velocity with which the periodicity advances is called the Group velocity.

### 3.2.1 Derivation of the formula of phase and group velocity

Let us consider that the wave group arises from the combination of two waves that have the same amplitude. A bit differs by an amount $\Delta\,\omega$ in angular frequency and an amount $\Delta\,k$ in wave number. We may suspect the original waves by the formulas,

$$\begin{aligned} y_1 &= A\cos(\omega\,t - kx) \\ y_2 &= A\cos\left[(\omega + \Delta\,\omega)t - (k + \Delta\,k)x\right] \end{aligned}$$

The resultant displacement $y$ at time $t$ and any position $x$ is the sum of $y_1$ and $y_2$.



$$\begin{aligned} y &= y_1 + y_2 \\ &= A\cos(\omega\,t - kx) + A\cos\left[(\omega + \Delta\,\omega)t - (k + \Delta\,k)x\right] \\ &= 2A\cos\frac{\omega\,t - kx + \omega\,t + \Delta\,\omega\,t - kx - \Delta\,kx}{2} \cdot \cos\frac{\omega\,t - kx - \omega\,t - \Delta\,\omega t + kx + \Delta\,kx}{2} \\ &= 2A\cos\frac{1}{2}\left[(2x + \Delta\,\omega)t - (2k + \Delta\,k)x\right] \times \cos\frac{1}{2}\left(\Delta\,\omega\,t - \Delta\,kx\right) \end{aligned}$$

Since $\Delta\,\omega$ and $\Delta\,k$ are small compared with $\omega$ and $k$.

$$2\omega + \Delta\,\omega \approx 2\omega \qquad \text{and} \qquad 2k + \Delta\,k = 2k$$

So,

$$y = 2A\cos\left(\omega\,t - kx\right) \cdot \cos\left(\frac{\Delta\,\omega}{2}t - \frac{\Delta\,k}{2}x\right)$$

This equation represent a wave of angular frequency $\omega$ and wave number $k$ that has superimposed upon it a modulation of angular frequency $\frac{1}{2}\Delta\,\omega$ and of wave number $\frac{1}{2}\Delta\,k$.

The effect of the modulation is thus to produce successive wave group. The phase velocity, $v_p$ is,

$$\boxed{v_p = \frac{\omega}{k}} \longrightarrow \text{Phase Velocity}$$

and the group velocity, $v_g$ is,

$$\boxed{v_g = \frac{\Delta\,\omega}{\Delta\,k}} \longrightarrow \text{Group Velocity}$$

when $\omega$ and $k$ have continuous speeds then,

$$\boxed{v_g = \frac{\mathrm{d}\,\omega}{\mathrm{d}\,k}}$$

Depending on how phase velocity varies with wave number in a particular situation, the group velocity may greater or less than the phase velocities of its member waves.

The angular frequency and wave number of the de-Broglie waves associated with a body at rest mass $m_0$ moving with velocity $v$ are,

$$\omega = 2\pi v = \frac{2\pi mc^2}{h}$$

high frequency of de-Broglie waves,

$$\omega = \frac{2\pi m_0 c^2}{h\sqrt{1 - v^2/c^2}}$$

Both $\omega$ and $k$ are functions of the body's velocity $v$.
The group velocity $v_g$ of the de-Broglie waves associated with body is,

$$v_g = \frac{\mathrm{d}\,\omega}{\mathrm{d}\,k} = \frac{\frac{\mathrm{d}\,\omega}{\mathrm{d}\,v}}{\frac{\mathrm{d}\,k}{\mathrm{d}\,v}} \left| \begin{array}{l} \dfrac{\mathrm{d}\,\omega}{\mathrm{d}\,v} = \dfrac{2\pi m_0 v}{h\left(1 - \frac{v^2}{c^2}\right)^{\frac{3}{2}}} \\ \dfrac{\mathrm{d}\,k}{\mathrm{d}\,v} = \dfrac{2\pi m_0}{h\left(1 - \frac{v^2}{c^2}\right)^{\frac{3}{2}}} \end{array} \right.$$

De Broglie group velocity

$$\boxed{v_g = v}$$

The de Broglie wave group associated with moving body travels with the same velocity as the body,

$$\boxed{v_p = \frac{\omega}{k} = \frac{c^2}{v}} \longrightarrow \text{De Broglie Phase Velocity}$$

**Problem 3.2.1.** An electron has a de Broglie wavelength of $2pm$. Find its kinetic energy and the phase and group velocities of its de-Broglie waves. [The rest energy of electron$= 511KeV$]

**Solution.**

$$\begin{aligned} pc = \frac{hc}{A} &= \frac{(4.13 \times 10^{-15})\,(3 \times 10^8)}{2 \times 10^{-12}} \\ &= 6.20 \times 10^5 eV \\ &= 620KeV \end{aligned}$$

Now,

$$\begin{aligned} KE &= E - E_0 \\ &= \sqrt{E_0^2 + (pc)^2} - E_0 \\ &= \sqrt{(511)^2 + (620)^2} - 511 \\ &= 292KeV \end{aligned}$$

The electron velocity can be found from

$$\begin{aligned} E &= \frac{E_0}{\sqrt{1 - v^2/c^2}} \\ \Rightarrow\ v &= c\sqrt{1 - E_0^2/E^2} \\ \Rightarrow\ v &= c\sqrt{1 - \frac{511^2}{803^2}} \\ &= 0.771c \end{aligned}$$

Hence,

$$\begin{aligned} \text{the group velocity, } v_g &= v = 0.771c \\ \text{the phase velocity, } v_p &= \frac{c^2}{v} = 1.30c \end{aligned}$$

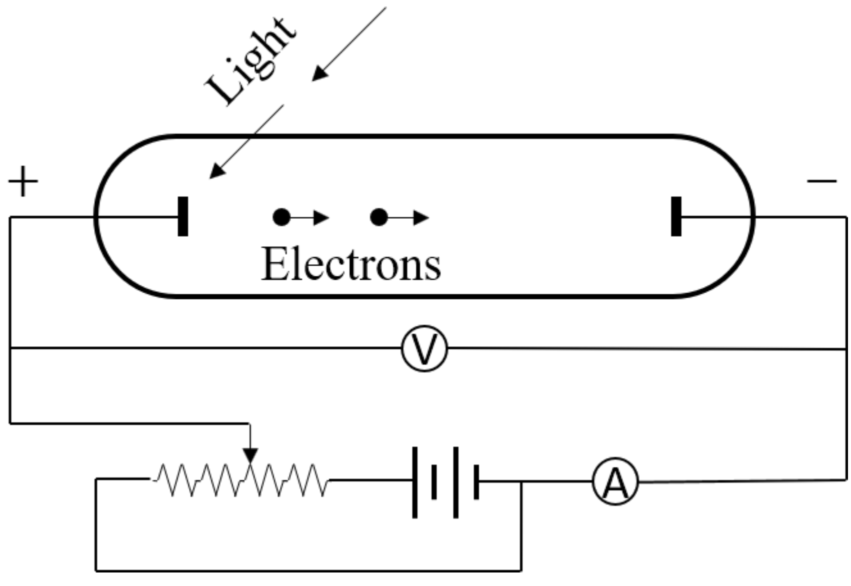
# Sheet 4

# Photoelectric Effect

When a beam of light strikes on surface of metals, electrons are emitted, this phenomenon is known as the photoelectric effect of light. The emitted electrons are called photoelectrons.

## 4.1 Experiment

An evacuated tube contains two electrodes connected to a source of variable voltage with the metal plate whose surface is irradiated as the anode, some of the photoelectrons that emerge from this surface have enough energy to reach the cathode despite its negative polarity and they constitute the measured current. The slower photoelectrons are repelled before they get to cathode. When the




voltage is increased to a certain value $v_0$, of the order of several volts, no more photoelectrons arrive as indicated by the current dropping to zero.

Light waves carry energy and some of the energy absorbed by the metal may somehow concentrate on individual electrons and reappear as their kinetic energy.

| |
|---|
| In the absence of light no electrons flow in the circuit and ammeter reads zero, because it is an open circuit. |

## 4.2 Wave Theory of Light (when light behaves like a wave, not a particle)

If monochromatic light hits the plate behaves as a wave, wave theory of light predicts

1. If intensity of light is increased, then the number of electron emitted and their maximum kinetic energy will increase.

2. Frequency of light does not affect kinetic energy of electrons, only the intensity does.

## 4.3 Quantum Theory of Light

Quantum theory of light proposed by Albert Einstein predicts that,

- A beam of light with frequency $v$ consists of photons that each carry the same energy $(hv)$. By increasing the intensity of beam it is possible to increase the number of photons, not the energy of each photon.

- According to quantum theory, a single photon collides with the electron in the metal plate. In order to emit an electron, the attractive electric forces between the electron and photon in the atom of metal plate must be overcome, that is the photon must do same amount of minimum work $(w_0)$ to release the electron. This is known as work function.

If the energy carried by the photon is less than the work function electron will not be emitted.

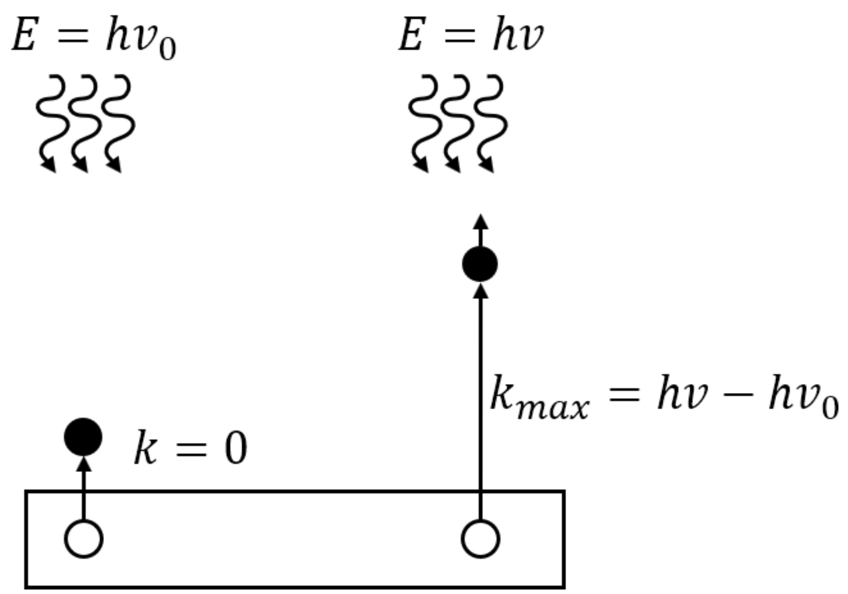
- If $hv \leq w_0$, electrons are not emitted from metal

- If $hv > w_0$, electrons are emitted and the remaining energy goes into the kinetic energy.

- $w_0 = hv_0$ here $v_0$ is critical frequency below which no photoelectrons are emitted.

$$\boxed{hv = k_{max} + w_0 \qquad \text{or,} \qquad hv - w_0 = k_{max}}$$

**Conclusion:**




1. If the frequency of light is less than frequency required to emit electron from metal, increasing the intensity will have no effect on emitting electrons.

2. If the frequency is large enough to emit electrons increasing the intensity increase the number of electrons emitted but $K_{max}$ of electron will remain unchanged.

## 4.4 Compton Effect

The Compton effect was an experiment conducted by Arthur H. Compton in 1923 which is the further confirmation of the photon model (quantum theory of light).

An X-ray photon strikes an electron (assumed to be initially at rest in laboratory co-ordinate system) and is scattered away from its original direction of motion while the electron receives an impulse and begins to move. If the initial photon has the frequency $v$ associated with it, the scattered photon has lower frequency $v'$, where,

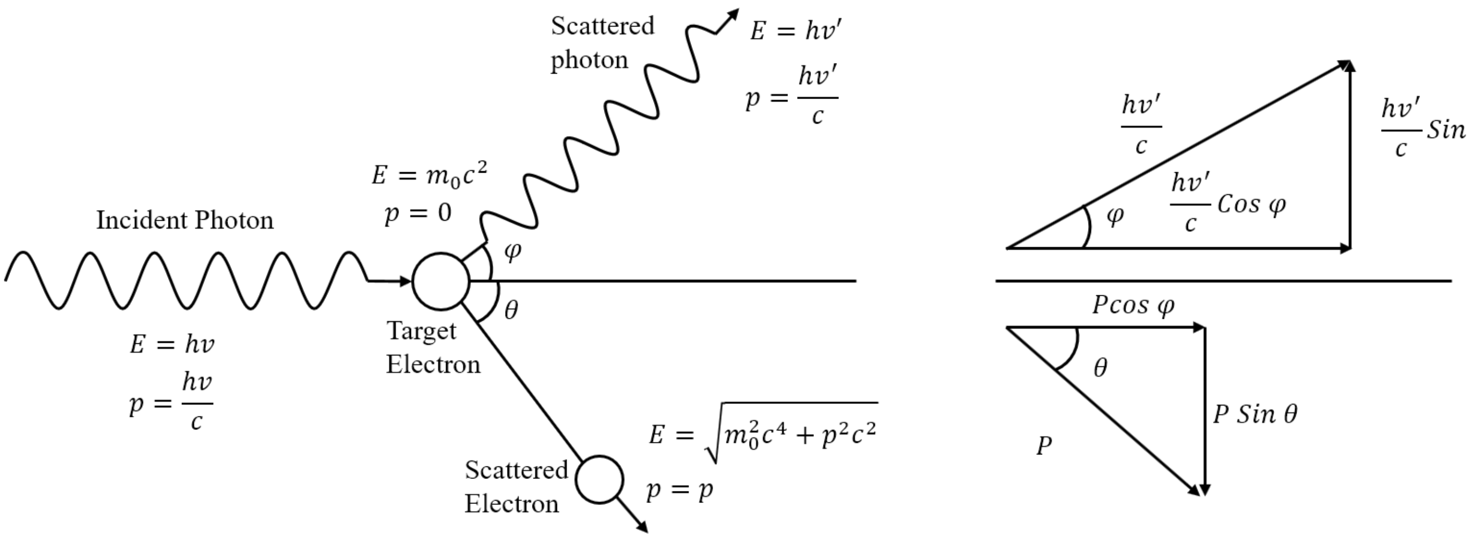
Loss in photon energy = Gain in electron energy

$$hv - hv' = K_E \tag{4.1}$$

We know, the momentum of a massless particle is related to its energy by the formula $E = pc$. Since the energy of photon is $hv$, its momentum is,

$$p = \frac{E}{c} = \frac{hv}{c} \tag{4.2}$$

The angle $\varphi$ is that between the directions of the initial and scattered photons and $\theta$ is that between




the directions of the initial photon and the recoil electron.

Here, momentum is vector quantity. In the collision momentum must be conserved in each of two mutually perpendicular directions. The initial photon momentum is $\frac{hv}{c}$, the scattered photon momentum is $\frac{hv'}{c}$ and the initial and the final electron momentum are respectively 0 and $p$.

In the original photon direction:

$$\text{Initial momentum} = \text{Final momentum}$$
$$\frac{hv}{c} + 0 = \frac{hv'}{c}\cos\varphi + p\cos\theta \tag{4.3}$$

and perpendicular to this direction:

$$\text{Initial momentum} = \text{Final momentum}$$
$$0 = \frac{hv'}{c}\sin\varphi - p\sin\theta \tag{4.4}$$

Multiplying equation (4.3) and equation (4.4) by $c$ we get

$$Pc\cos\theta = hv - hv'\cos\varphi$$
$$Pc\sin\theta = hv'\sin\varphi$$

Now, squaring each equation and adding then we get

$$p^2c^2 = (hv)^2 - 2(hv)(hv')\cos\varphi + (hv')^2 \tag{4.5}$$

The total energy of a particle can be written as

$$E = K_E + m_0c^2 \tag{4.6}$$

$$E = \sqrt{m_0^2c^4 + p^2c^2} \tag{4.7}$$

Now, equation (4.7) can be expressed as,

$$\begin{aligned} E^2 &= {m_0}^2c^4 + p^2c^2 \\ \Rightarrow \left(K_E + m_0c^2\right)^2 &= {m_0}^2c^4 + p^2c^2 \\ \Rightarrow p^2c^2 &= {K_E}^2 + 2m_0c^2K_E \\ \Rightarrow p^2c^2 &= (hv - hv')^2 + 2m_0c^2\left(hv - hv'\right) \end{aligned} \tag{4.8}$$

Substituting equation (4.5) in equation (4.8) we get,

$$\begin{aligned} (hv)^2 - 2(hv)(hv')\cos\varphi + (hv')^2 &= (hv)^2 - 2(hv)(hv') + (hv')^2 + 2m_0c^2(hv - hv') \\ \Rightarrow 2m_0c^2(hv - hv') &= 2(hv)(hv')(1 - \cos\varphi) \end{aligned} \tag{4.9}$$

Dividing equation (4.9) by $2h^2c^2$

$$\frac{m_0c}{h}\left(\frac{v}{c} - \frac{v'}{c}\right) = \frac{v}{c}\frac{v'}{c}(1 - \cos\varphi)$$

$$\frac{m_0c}{h}\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right) = \frac{1}{\lambda}\frac{1}{\lambda'}(1 - \cos\varphi)$$

$$\text{or, } \boxed{\lambda - \lambda' = \frac{h}{m_0c}(1 - \cos\varphi)} \tag{4.10}$$

Equation (4.10) is known as Compton effect, which shows the change in wavelength expected for a photon that is scattered through the angle $\varphi$ by a particle of rest mass $m_0$. This change is independent of wavelength $\lambda$ of the incident photon.
The quantity $\frac{h}{m_0c} = \lambda_c$ is known as Compton wavelength

$$\lambda' - \lambda = \lambda_c(1 - \cos\varphi)$$

For $\varphi = 180°$, the wavelength change will be twice the Compton wavelength $\lambda_c$.

## 4.5 Mathematical Problem

**Problem 4.5.1.** UV light of wavelength 350nm and intensity $1.00\,W/m^2$ is directed at a potassium surface.

(a) Find the maximum $K_E$ of the photoelectrons.

(b) If 0.50 percent of the incident photons produce photoelectrons, how many are emitted per second if the potassium surface has an area if $1.00\,cm^2$? [work function of potassium $= 2.2\,eV$]

**Solution.** (a) We know, energy of photon is

$$\begin{aligned} E &= hv \\ \Rightarrow E &= h\frac{c}{\lambda} \\ \Rightarrow E &= \frac{6.626\times10^{-34}Js}{1.602\times10^{-19}J/eV} \times \frac{2.998\times10^{8}m/s}{350\times10^{-9}m} \\ \Rightarrow E &= 3.5\,eV \end{aligned}$$

Now,

$$K_{max} = hv - \varphi = 3.5\,eV - 2.2\,eV = 1.3eV$$

(b) The photon energy $= 3.5\,eV = 5.68 \times 10^{-19}\,J$
The number of photons that reach the surface per second is

$$h_p = \frac{E/t}{E_p} = \frac{(p/A)A}{E_p} = \frac{(1.00\,W/m^2)(1.00 \times 10^{-4}m^2)}{5.68 \times 10^{-19}} = 1.76 \times 10^{14} \text{ photons/s}$$

The rate at which photo electrons are emitted is therefore,

$$n_e = (0.0050)h_p = 8.8 \times 10^{11} \text{ photoelectrons/s}$$

**Problem 4.5.2.** X-rays of wavelength $10.0\,pm$ are scattered from a target.

(a) Find the wavelength of x-rays scattered through $45°$

(b) Find the maximum wavelength present in the scattered x-rays.

(c) Find the maximum kinetic energy of the recoil electrons.$[\lambda_c = 2.426\,pm]$

**Solution.** (a) From the equation of Compton effect we get

$$\begin{aligned} \lambda' - \lambda &= \lambda_c(1 - \cos\varphi) \\ \lambda' &= \lambda + \lambda_c(1 - \cos 45°) \\ &= 10.0\,pm + 0.293\lambda_c \\ &= 10.7\,pm \end{aligned}$$

(b) $\lambda' - \lambda$ is a maximum when $(1 - \cos\varphi) = 2\tan\varphi = 180°$

$$\lambda' = \lambda + 2\lambda_c = 10.0\,pm + 4.9\,pm = 14.9\,pm$$

(c) The maximum recoil kinetic energy is equal to the different between the energies of the incident and scattered photons. So,

$$\begin{aligned} KE_{max} &= hv - hv' \\ &= hc\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right) \\ &= \frac{(6.623 \times 10^{-34}\,Js)\,(3 \times 10^8 m/s)}{\times 10^{-12}\,m/pm}\left(\frac{1}{10\,pm} - \frac{1}{14.9\,pm}\right) \\ &= 6.54 \times 10^{-15}\,pm \\ &= 40.8\,KeV \end{aligned}$$

# Sheet 5

# Radioactivity

## 5.1 Application of Radioactivity

1. Nuclear radiation like $\gamma$-rays have been utilized for the preservation of food. Food stuff mainly meat, poultry, fish, fruits etc are exposed to $\gamma$ rays from cobalt-60 or caesium-137. A dose of about 2 to 5 million rads is sufficient to destroy almost all bacteria in food.

2. Radiation is used for producing new and improved varieties of plants.

3. Gamma radiation from cobalt-60 is used in hospitals to sterilize materials like hypodermic syringe, surgical instruments, dressings etc.

4. Radiation can also be used as pesticide.

5. Gamma radiation from cobalt-60, iridium-192 are used in industrial radiography i.e, for investigating the interiors metallic castings for detecting any flaws or defects.

6. A carefully prepared mixture of radio thorium ($\alpha$-emitter) with zinc sulphide exhibits a more or less permanent luminescence and is used for coating the pointers and figures of clocks and watches, for rendering visible signs in theaters and so on.

7. Radioisotopes are used to diagnose the nature of blood circulatory disorder, defects of bone metabolism, to locate tumors, etc. Radio-sodium is used to study the circulatory disorder in blood vessels while radioactive iodine is used to study any disorder in thyroid gland. $Tc^{99m}$ is used to study the functioning of different organs like liver, kidney and spleen under normal and diseased conditions.

$$\boxed{[Tc^{99m} \text{ metastable nuclear isomer of technisium-99}]}$$

## 5.2 Activity

The activity of a sample of any radioactive nuclide is the rate at which the nuclei of its constituent atom decay. If $N$ is the number of Nuclei present in the sample at a certain time, its activity $R$ is given by

$$R = \frac{-\,\mathrm{d}\,N}{\mathrm{d}\,t}$$
$$\text{or, } R = \lambda N$$

SI unit of activity is Becquerel.

$$1\text{Becquerel} = 1\text{Bq} = 1\text{decay/s}$$

## 5.3 Radioactive Dating: Age of The Earth

The age of earth is estimated from the relative abundance of the two isotopes of uranium $U-238$ and $U-235$. The present abundance ratio of $U-235$ and $U-238$ is 1.140 (0.7% to 99.3%). The half-lives of U-235 and U-238, according to the best estimate are $7.07 \times 10^8$ years and $4.5 \times 10^9$ years respectively. Assuming that at the beginning the proportions of the two isotopes were equal, the present relative abundance of U-238 and U-235 may be expressed as

$$\frac{N_1}{N_2} = \frac{99.3}{0.7} = \frac{N_0 e^{\lambda_1 t}}{N_0 e^{\lambda_2 t}} = e^{\lambda_1 - \lambda_2 t}$$

Where

$$\lambda_1 = \frac{0.693}{4.5 \times 10^9} y^{-1} \quad \text{and} \quad \lambda_2 = \frac{0.693}{7.07 \times 10^8} y^{-1}$$

Now,

$$\begin{aligned} \ln \left[\frac{99.3}{0.7}\right] &= (\lambda_2 - \lambda_1)\, t \\ t &= \frac{1}{\lambda_2 - \lambda_1} \ln \left[\frac{99.3}{0.7}\right] \\ &\approx 5.93 \times 10^9 \text{ years} \\ &\approx 5000 \text{ million years} \end{aligned}$$

This values agrees nearly with that given by astronomical evidence for the age of the universe.

## 5.4 Solve the following Problems

1. The half-life of a radioactive substance is 30 days. Calculate
   (a) the radioactive decay constant,
   (b) the mean life,
   (c) the time taken for 3/4 of the original number of atoms to disintegrate and
   (d) the time for 1/8 of the original number of atoms to remain unchanged.
2. The half-life of radium is 1620 years. In how many years will one gram of pure element
   (a) lose one centigram and
   (b) be reduced to one centigram?
3. 1 gram of radium is reduced by $2.1\,mg$ in 5 years by $\alpha$-decay. Calculate the half-life of radium.
4. The alpha decay of $Rn-222$ to $Po-218$ has half-life of 3.8 days.
   (a) How long does it take for 60% of sample of radon to decay (initial mass of $1.0\,mg$)
   (b) Find the activity of $1.0\,mg$ of Radon-222, whose atomic mass is $222u$.

# Part II

# Suggestions

# Sheet 6

# Suggestion

## 6.1 Electromagnetism and Modern Physics

Electromagnetism:

- Coulombs Law.
- Define point Charge, Conservation of Charge, Quantization of Charge [Proton and neutron are made of fractional electric charges, how charges remain quantized?]
- Electric field and a dipole in an electric field

$$E = \frac{1}{4\pi\epsilon_0}\frac{p}{x^3}; \quad \vec{\tau} = -\vec{p} \times \vec{E}; \quad U = -\vec{p} \cdot \vec{E}$$

- Gauss's Law and its applications; Gauss's Law $\leftrightarrow$ Coulomb's Law
- Define Electric potential and Electric Potential energy, Potential due to point charge and dipole.
- Capacitance of different types of Capacitor. Energy $U = 1/2CV^2$ or $U = 1/2\epsilon_0 E^2$
- Effect of dielectric in capacitor
- Difference between electric field and magnetic field, Magnetic Force $F = ILB$, $F = qvB$
- Torque on a current loop, HALL effect and voltage
- Biot-Savart Law and applications
- Define Ampere and Ampere's Law, magnetic Flux
- Magnetic field for solenoid and toroid
- Brief idea on Faraday's Law, Lenz Law, Self and mutual inductance
- r.m.s value of $I$ and $E$, RLC circuit in AC circuit.

Modern Physics:

- Photoelectric effect, Compton effect, de-Broglie wave, Find the expression of Phase velocity and group velocity
- Brief idea about Uncertainty principle, different types of atomic model, nuclear size, binding energy.
- Properties of $\alpha$, $\beta$, $\gamma$; Radioactive decay law, decay constant, mean life, half-life, Application of Radioactivity, Radioactive dating - the age of earth calculation.

$\boxed{\text{Content related all mathematical problems}}$

# Part III

# Assignment

# Sheet 7

# Assignment

## 7.1 Questions

1. An X-ray photon is found to have its wavelength doubled on being scattered through 90°. Find the wavelength of the incident photon. $[m_0 = 9.11 \times 10^{-31}\, kg,\ h = 6.63 \times 10^{-34}\, Js]$

2. Monochromatic X-rays of wavelength $\lambda = 0.124$ Åare scattered from a carbon block. Determine the wavelength of the X-ray scattered through 45°.

3. In an experiment, Tungsten cathode, which has a threshold 2300 Å, is irradiated by ultraviolet light of wavelength 1800 Å. Calculate

   (i) maximum energy of emitted photoelectron and

   (ii) work function for tungsten.

4. Calculate the time required for 10% of a sample of thorium to disintegrate. Assume the half-life of thorium to be $1.4 \times 10^{10}$ years.

5. Write down the postulates of Bohr atomic model. Establish a relation between de-Broglie hypothesis and Bohr theory of atom.

6. State uncertainty principle. How does uncertainty principle prohibit an electron staying inside the nucleus of atom?

**Problem 7.1.1.** An X-ray photon is found to have its wavelength doubled on being scattered through $90°$. Find the wavelength of the incident photon. $[m_0 = 9.11 \times 10^{-31}\,kg,\ h = 6.63 \times 10^{-34}\,Js]$

**Solution.** Let,

$$\begin{aligned} \lambda &= \lambda \qquad \text{and} \\ \lambda' &= 2\lambda \end{aligned}$$

Given,

$$\begin{aligned} h &= 6.63 \times 10^{-34}\,Js \\ m_0 &= 9.11 \times 10^{-31}\,kg \\ c &= 3 \times 10^{8}\,ms^{-1} \\ \phi &= 90° \end{aligned}$$

We know,

$$\begin{aligned} \lambda' - \lambda &= \frac{h}{m_0 c}(1 - \cos\phi) \\ \Rightarrow 2\lambda - \lambda &= \frac{h}{m_0 c}(1 - \cos\phi) \\ \Rightarrow \lambda &= \frac{6.63 \times 10^{-34}}{9.11 \times 10^{-31} \times 3 \times 10^{8}}\,m \\ \therefore \lambda &= 2.4259 \times 10^{-12}\,m \end{aligned}$$

**Problem 7.1.2.** Monochromatic X-rays of wavelength $\lambda = 0.124$ Åare scattered from a carbon block. Determine the wavelength of the X-ray scattered through 45°.

**Solution.** Given,

$$\begin{aligned}
\lambda &= 0.124\,\text{Å} \\
&= 0.124 \times 10^{-10}\,m \\
h &= 6.63 \times 10^{-34}\,Js \\
m_0 &= 9.11 \times 10^{-31}\,kg \\
c &= 3 \times 10^{8}\,ms^{-1} \\
\phi &= 45^\circ
\end{aligned}$$

We know,

$$\begin{aligned}
\lambda' - \lambda &= \frac{h}{m_0 c}(1 - \cos\phi) \\
\Rightarrow \lambda' &= \lambda + \frac{h}{m_0 c}(1 - \cos\phi) \\
\Rightarrow \lambda' &= 0.124 \times 10^{10} + \frac{6.63 \times 10^{-34}(1 - \cos 45^\circ)}{9.11 \times 10^{-31} \times 3 \times 10^{8}}\,m \\
\therefore \lambda' &= 1.3111 \times 10^{-11}\,m
\end{aligned}$$

**Problem 7.1.3.** In an experiment, Tungsten cathode, which has a threshold 2300Å, is irradiated by ultraviolet light of wavelength 1800Å. Calculate

(i) maximum energy of emitted photoelectron and

(ii) work function for tungsten.

**Solution.** Given,

$$\begin{aligned}
\lambda &= 1800\,\text{Å} \\
&= 1800 \times 10^{-10}\, m \\
\lambda_0 &= 2300\,\text{Å} \\
&= 2300 \times 10^{-10}\, m \\
h &= 6.63 \times 10^{-34}\, Js \\
c &= 3 \times 10^{8}\, ms^{-1}
\end{aligned}$$

Now, Work function for tungsten,
$$\begin{aligned}
\Phi &= \frac{hc}{\lambda_0} \\
&= \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{2300 \times 10^{-10}}\, J \\
&= 8.6478 \times 10^{-19}\, J
\end{aligned}$$

Again, Maximum energy,
$$\begin{aligned}
KE &= \frac{hc}{\lambda} - \Phi \\
&= \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{1800 \times 10^{-10}} - 8.6478 \times 10^{-19}\, J \\
&= 2.4022 \times 10^{-19}\, J
\end{aligned}$$

**Problem 7.1.4.** Calculate the time required for 10% of a sample of thorium to disintegrate. Assume the half-life of thorium to be $1.4 \times 10^{10}$ years.

**Solution.** Let,

Initial mass $= N_0$

If in time $t$, 10% of thorium is disintegrated, then the amount of thorium that disintegrate $= N_0 \times \frac{10}{100} = 0.1\, N_0$.

Thorium left, $N = N_0 - 0.1N_0$

$= 0.9N_0$

Now,

$$
\begin{aligned}
N &= N_0 e^{-\lambda t} \\
\Rightarrow e^{-\lambda t} &= \frac{N_0}{N} \\
\Rightarrow \lambda t &= \ln\left(\frac{0.9N_0}{N_0}\right) \\
\Rightarrow \frac{0.693}{T} t &= \ln(0.9) \qquad \text{Here, } T = \text{half life} \\
\Rightarrow t &= \frac{T}{0.693} \times 0.1053 \\
\Rightarrow t &= \frac{1.4 \times 10^{10} \times 0.1053}{0.693} \\
\therefore t &= 2.1272 \times 10^6 \text{ years}
\end{aligned}
$$

**Problem 7.1.5.** Write down the postulates of Bohr atomic model. Establish a relation between de-Broglie hypothesis and Bohr theory of atom.

**Solution.** Postulates of Bohr atomic model:

1. First postulate (relating Angular momentum):
   While orbiting in a permanent orbit total angular momentum of an electron will be an integer of $\frac{h}{2\pi}$ i.e., $L = \frac{nh}{2\pi}$, here $h$ is the plank's constant.

2. Second postulate (relating energy state):
   Electrons in an atom revolve round the nucleus in all probable orbits rather they rotate in certain fixed prescribed circular orbits. These orbits are called permanent and non-radiating orbits.

3. Third postulate (relating frequency):
   Whenever an electron jumps from a convenient orbit to another convenient orbit, then radiation of energy takes place.

   The amount of this radiated or absorbed energy is equal to the difference of the energies of these two orbits between which transition takes place and its value is one quantum. i.e., $hv$.

$$\therefore E = E_2 - E_1 = hv$$

Relation between de-Broglie hypothesis and Bohr's theory of atom:
de-Broglie came up with an explanation for why the angular momentum might be quantized in the manner Bohr assumed it was. de-Broglie realized that if you use the wavelength associated with the electron and assume that an integer number of wavelengths must fit in the circumference of an orbit, you get the same quantized angular momentum that Bohr did.

The derivation works like this, starting from the idea that the circumference of the circular orbit must be an integer number of wavelengths.

$$2\pi r = n\lambda$$

Taking the wavelength to be de-Broglie wavelength $\lambda = \frac{h}{p}$, this becomes,

$$2\pi r = \frac{nh}{p}$$

the momentum $p$ is simply $mv$ as long as we are talking about non-relativistic speeds, so this becomes,

$$2\pi r = \frac{nh}{mv}$$

rearranging this a little gives the Bohr relationship.

$$L_n = mvr = \frac{nh}{2\pi}$$

**Problem 7.1.6.** State uncertainty principle. How does uncertainty principle prohibit an electron staying inside the nucleus of atom?

**Solution.** Uncertainty principle:
The Heisenberg's uncertainty principle states that:

"Position and momentum of a particle cannot be simultaneously measured accurately."

Mathematically the principle of uncertainty can be expressed as,

$$\Delta x \Delta p \leq \frac{\hbar}{2}$$

Here, $\hbar = \frac{h}{2\pi}$ = plank's reduced constant.

Electron cannot be found inside nucleus:

Radius of a nucleus is approximately $\times 10^{-14}\,m$. So for an electron to stay inside the nucleus, uncertainty in the position cannot be more than $2 \times 10^{-14}\,m$.

Now if $\Delta x$ and $\Delta p$ are uncertainties of the position and momentum respectively. Then,

$$\begin{aligned}
\Delta x \Delta p &= \frac{\hbar}{2} \\
\Rightarrow \ \Delta p &= \frac{h}{2 \times 2\pi \times \Delta x} \\
&= \frac{6.63 \times 10^{-34}}{4 \times 3.14 \times 2 \times 10^{-14}} \\
&= 2.64 \times 10^{-22}\,kgms^{-1}
\end{aligned}$$

Now, if the uncertainty in the momentum is of this magnitude then momentum of the electron must be of this magnitude. i.e., $p = 2.6 \times 10^{-22}\,kgms^{-1}$.
The kinetic energy of electron is

$$\begin{aligned}
E &= \frac{p^2}{2m} \\
&= \frac{(2.64 \times 10^{-22})^2}{2 \times 9.1 \times 10^{-31}} \\
&= 3.83 \times 10^{-12}\,J \\
&= 23.93\,MeV
\end{aligned}$$

This means for the electron to stay inside the nucleus, it has to have $23.93\,MeV$ energy. But from experiment result it is found that kinetic energy of electron is not more than $4\,MeV$. So electron cannot stay inside the nucleus.