Sheet 1

Assignment

1.1 Questions

1. An X-ray photon is found to have its wavelength doubled on being scattered through 90°. Find the wavelength of the incident photon. $[m_0 = 9.11 \times 10^{-31}\,kg,\ h = 6.63 \times 10^{-34}\,Js]$

2. Monochromatic X-rays of wavelength $\lambda = 0.124\,$Åare scattered from a carbon block. Determine the wavelength of the X-ray scattered through 45°.

3. In an experiment, Tungsten cathode, which has a threshold 2300 Å, is irradiated by ultraviolet light of wavelength 1800 Å. Calculate

   (i) maximum energy of emitted photoelectron and

   (ii) work function for tungsten.

4. Calculate the time required for 10% of a sample of thorium to disintegrate. Assume the half-life of thorium to be 1.4×10^{10} years.

5. Write down the postulates of Bohr atomic model. Establish a relation between de-Broglie hypothesis and Bohr theory of atom.

6. State uncertainty principle. How does uncertainty principle prohibit an electron staying inside the nucleus of atom?

Problem 1.1.1. An X-ray photon is found to have its wavelength doubled on being scattered through $90°$. Find the wavelength of the incident photon. $[m_0 = 9.11 \times 10^{-31}\, kg,\, h = 6.63 \times 10^{-34}\, Js]$

Solution. Let,

$$\begin{aligned} \lambda &= \lambda \qquad \text{and} \\ \lambda' &= 2\lambda \end{aligned}$$

Given,

$$\begin{aligned} h &= 6.63 \times 10^{-34}\, Js \\ m_0 &= 9.11 \times 10^{-31}\, kg \\ c &= 3 \times 10^{8}\, ms^{-1} \\ \phi &= 90° \end{aligned}$$

We know,

$$\begin{aligned} \lambda' - \lambda &= \frac{h}{m_0 c}(1 - \cos\phi) \\ \Rightarrow 2\lambda - \lambda &= \frac{h}{m_0 c}(1 - \cos\phi) \\ \Rightarrow \lambda &= \frac{6.63 \times 10^{-34}}{9.11 \times 10^{-31} \times 3 \times 10^{8}}\, m \\ \therefore \lambda &= 2.4259 \times 10^{-12}\, m \end{aligned}$$

Problem 1.1.2. Monochromatic X-rays of wavelength $\lambda = 0.124$ Åare scattered from a carbon block. Determine the wavelength of the X-ray scattered through $45°$.

Solution. Given,

$$
\begin{aligned}
\lambda &= 0.124\,\text{Å} \\
&= 0.124 \times 10^{-10}\,m \\
h &= 6.63 \times 10^{-34}\,Js \\
m_0 &= 9.11 \times 10^{-31}\,kg \\
c &= 3 \times 10^{8}\,ms^{-1} \\
\phi &= 45°
\end{aligned}
$$

We know,

$$
\begin{aligned}
\lambda' - \lambda &= \frac{h}{m_0 c}(1 - \cos\phi) \\
\Rightarrow \lambda' &= \lambda + \frac{h}{m_0 c}(1 - \cos\phi) \\
\Rightarrow \lambda' &= 0.124 \times 10^{10} + \frac{6.63 \times 10^{-34}(1 - \cos 45°)}{9.11 \times 10^{-31} \times 3 \times 10^{8}}\,m \\
\therefore \lambda' &= 1.3111 \times 10^{-11}\,m
\end{aligned}
$$

Problem 1.1.3. In an experiment, Tungsten cathode, which has a threshold 2300Å, is irradiated by ultraviolet light of wavelength 1800Å. Calculate

(i) maximum energy of emitted photoelectron and

(ii) work function for tungsten.

Solution. Given,

$$\begin{aligned}
\lambda &= 1800\,\text{Å} \\
&= 1800 \times 10^{-10}\,m \\
\lambda_0 &= 2300\,\text{Å} \\
&= 2300 \times 10^{-10}\,m \\
h &= 6.63 \times 10^{-34}\,Js \\
c &= 3 \times 10^{8}\,ms^{-1}
\end{aligned}$$

Now, Work function for tungsten, $\Phi = \dfrac{hc}{\lambda_0}$

$$\begin{aligned}
&= \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{2300 \times 10^{-10}}\,J \\
&= 8.6478 \times 10^{-19}\,J
\end{aligned}$$

Again, Maximum energy, $KE = \dfrac{hc}{\lambda} - \Phi$

$$\begin{aligned}
&= \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{1800 \times 10^{-10}} - 8.6478 \times 10^{-19}\,J \\
&= 2.4022 \times 10^{-19}\,J
\end{aligned}$$

Problem 1.1.4. Calculate the time required for 10% of a sample of thorium to disintegrate. Assume the half-life of thorium to be 1.4×10^{10} years.

Solution. Let,

Initial mass $= N_0$

If in time t, 10% of thorium is disintegrated, then the amount of thorium that disintegrate $= N_0 \times \frac{10}{100} = 0.1\,N_0$.

Thorium left, $N = N_0 - 0.1N_0$

$= 0.9N_0$

Now,

$$
\begin{aligned}
N &= N_0 e^{-\lambda t} \\
\Rightarrow e^{-\lambda t} &= \frac{N_0}{N} \\
\Rightarrow \lambda t &= \ln\left(\frac{0.9N_0}{N_0}\right) \\
\Rightarrow \frac{0.693}{T} t &= \ln(0.9) \qquad \text{Here, } T = \text{half life} \\
\Rightarrow t &= \frac{T}{0.693} \times 0.1053 \\
\Rightarrow t &= \frac{1.4 \times 10^{10} \times 0.1053}{0.693} \\
\therefore t &= 2.1272 \times 10^{6} \text{ years}
\end{aligned}
$$

Problem 1.1.5. Write down the postulates of Bohr atomic model. Establish a relation between de-Broglie hypothesis and Bohr theory of atom.

Solution. Postulates of Bohr atomic model:

1. First postulate (relating Angular momentum):
   While orbiting in a permanent orbit total angular momentum of an electron will be an integer of $\frac{h}{2\pi}$ i.e., $L = \frac{nh}{2\pi}$, here h is the plank's constant.

2. Second postulate (relating energy state):
   Electrons in an atom revolve round the nucleus in all probable orbits rather they rotate in certain fixed prescribed circular orbits. These orbits are called permanent and non-radiating orbits.

3. Third postulate (relating frequency):
   Whenever an electron jumps from a convenient orbit to another convenient orbit, then radiation of energy takes place.

   The amount of this radiated or absorbed energy is equal to the difference of the energies of these two orbits between which transition takes place and its value is one quantum. i.e., hv.

$$\therefore E = E_2 - E_1 = hv$$

Relation between de-Broglie hypothesis and Bohr's theory of atom:
de-Broglie came up with an explanation for why the angular momentum might be quantized in the manner Bohr assumed it was. de-Broglie realized that if you use the wavelength associated with the electron and assume that an integer number of wavelengths must fit in the circumference of an orbit, you get the same quantized angular momentum that Bohr did.

The derivation works like this, starting from the idea that the circumference of the circular orbit must be an integer number of wavelengths.

$$2\pi r = n\lambda$$

Taking the wavelength to be de-Broglie wavelength $\lambda = \frac{h}{p}$, this becomes,

$$2\pi r = \frac{nh}{p}$$

the momentum p is simply mv as long as we are talking about non-relativistic speeds, so this becomes,

$$2\pi r = \frac{nh}{mv}$$

rearranging this a little gives the Bohr relationship.

$$L_n = mvr = \frac{nh}{2\pi}$$

Problem 1.1.6. State uncertainty principle. How does uncertainty principle prohibit an electron staying inside the nucleus of atom?

Solution. Uncertainty principle:
The Heisenberg's uncertainty principle states that:

"Position and momentum of a particle cannot be simultaneously measured accurately."

Mathematically the principle of uncertainty can be expressed as,

$$\Delta x \Delta p \leq \frac{\hbar}{2}$$

Here, $\hbar = \frac{h}{2\pi}$ = plank's reduced constant.

Electron cannot be found inside nucleus:

Radius of a nucleus is approximately $\times 10^{-14}\, m$. So for an electron to stay inside the nucleus, uncertainty in the position cannot be more than $2 \times 10^{-14}\, m$.

Now if Δx and Δp are uncertainties of the position and momentum respectively. Then,

$$\begin{aligned}\Delta x \Delta p &= \frac{\hbar}{2} \\ \Rightarrow\ \Delta p &= \frac{h}{2 \times 2\pi \times \Delta x} \\ &= \frac{6.63 \times 10^{-34}}{4 \times 3.14 \times 2 \times 10^{-14}} \\ &= 2.64 \times 10^{-22}\, kgms^{-1}\end{aligned}$$

Now, if the uncertainty in the momentum is of this magnitude then momentum of the electron must be of this magnitude. i.e., $p = 2.6 \times 10^{-22}\, kgms^{-1}$.
The kinetic energy of electron is

$$\begin{aligned}E &= \frac{p^2}{2m} \\ &= \frac{(2.64 \times 10^{-22})^2}{2 \times 9.1 \times 10^{-31}} \\ &= 3.83 \times 10^{-12}\, J \\ &= 23.93\, MeV\end{aligned}$$

This means for the electron to stay inside the nucleus, it has to have $23.93\, MeV$ energy. But from experiment result it is found that kinetic energy of electron is not more than $4\, MeV$. So electron cannot stay inside the nucleus.