Chapter 1

Separation Axioms

1.1 T_0 Space

Definition 1.1. A topological space (X, τ) is said to be a T_0 space iff given any two distinct points x, y in X, there exists either an open set U containing x but not y or an open set v containing y but not x.

Example 1.1. Every discrete space is a T_0 space. Because for any two points $x \neq y$ in X, there always exists an open $\{x\}$ containing x but not y. But indiscrete space is not T_0 space because the only nonempty open set in this space is X itself.

Example 1.2. Any co-finite topological space xt is a T_0 space for

Case i. X is finite. In this case $\tau = \mathcal{P}(X)$, a discrete space, and so (X, τ) is T_0 space.

Case ii. X is infinite. In this case, let $x, y \in X$ with $x \neq y$. Then $X - \{x\}$ is an open set containing y but not x.

Example 1.3. Every metric space is T_0 space. For let (X, d) be a metric space and τ be a topology induced by the metric d.
The open set