Chapter 1

Calculations

1.1 Calculations of Mid-point Method

$$w_{i+1} = w_i + hf\left(t_i + \frac{h}{2},\, w_i + \frac{h}{2}f(t_i, w_i)\right)$$

Here $t_i = 0.2i,\ h = 0.2,\ N = 10,\ f(t_i,\ w_i) = w_i - t_i^2 + 1$

$$\begin{aligned} w_{i+1} &= w_i + \frac{h}{2}f(t_i, w_i) \\ &= w_i + \frac{0.2}{2}\left[w_i - 0.04i^2 + 1\right] \\ &= w_i + 0.1w_i - 0.004i^2 + 0.1 \\ &= 1.1w_i - 0.004i^2 + 0.1 \end{aligned}$$

Again,

$$\left(t_i + \frac{h}{2}\right) = 0.2i + 0.1$$

$$\begin{aligned} \therefore\ w_{i+1} &= w_i + 0.2f\left(t_i + \frac{h}{2}, w_i + \frac{h}{2}f(t_i, w_i)\right) \\ &= w_i + 0.2f\left(0.2i + 0.1,\ 1.1w_i - 0.004i^2 + 0.1\right) \\ &= w_i + 0.2\left[1.1w_i - 0.004i^2 + 0.1 - (0.2i + 0.1)^2 + 1\right] \\ &= w_i + 0.22w_i - 0.0008i^2 + 0.02 - 0.2(0.04i^2 + 0.04i + 0.01) + 0.2 \\ &= 1.22w_i - 0.0008i^2 + 1.02 - 0.008i^2 - 0.008i + 0.002 \\ &= 1.22w_i - 0.0088i^2 - 0.008i + 0.218 \end{aligned}$$

1.2 Modified Euler's Method

$$y_{i+1}^{(n+1)} = y_i + \frac{1}{2}h\left[f(x_i, y_i) + f(x_{i+1}, y_{i+1}^{(n)})\right] \tag{1.1}$$

where

$$y_{i+1}^{(n)} = y_i + hf(x_i, y_i) \quad i = 0, 1, 2, \ldots, N-2; \quad n = 0, 1, 2, \ldots \tag{1.2}$$

For $i = 0$

$$y_1^{(n+1)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(n)})\right] \tag{1.3}$$

and

$$y_1^{(n)} = y_0 + hf(x_0, y_0); \qquad n = 0, 1, 2, \ldots \tag{1.4}$$

Let $n = 0$, then from (1.3) and (1.4) we have

$$y_1^{(1)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(0)})\right] \tag{1.5}$$

and

$$y_1^{(0)} = y_0 + hf(x_0, y_0) \tag{1.6}$$

Using (1.6) in (1.5), $y_1^{(1)}$ can be evaluated.
Let $n = 1$, then from (1.3), we have

$$y_1^{(2)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(1)})\right] \tag{1.7}$$

Let $n = 2$, then from (1.3), we have

$$y_1^{(3)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(2)})\right] \tag{1.8}$$

Similarly, $y_1^{(4)}$, $y_1^{(5)}, \ldots$ etc. can be evaluated until we reach the desired accuracy.

For $i = 1$, we have from (**??**),

$$y_2^{(n+1)} = y_1 + \frac{h}{2}\left[f(x_1, y_1) + f(x_2, y_2^{(n)})\right] \tag{1.9}$$

and

$$y_2^{(n)} = y_1 + hf(x_1, y_1); \qquad n = 0, 1, 2, \ldots \tag{1.10}$$

For $n = 0$, we have from (1.10),

$$y_2^{(1)} = y_1 + \frac{h}{2}\left[f(x_1, y_1) + f(x_2, y_2^{(0)})\right] \tag{1.11}$$

where

$$y_2^{(0)} = y_1 + hf(x_1, y_1) \tag{1.12}$$

By (1.12), $y_2^{(1)}$ is known from (1.11).
For $n = 1$, from (1.10),

$$y_2^{(2)} = y_1 + \frac{h}{2}\left[f(x_1, y_1) + f(x_2, y_2^{(1)})\right] \tag{1.13}$$

$y_2^{(1)}$ is known from (1.11). Similarly, $y_2^{(3)}$, $y_2^{(4)}, \ldots$ etc. can be calculated.