Problem 0.1. Solve the wave equation

$$\frac{\partial^2 u}{\partial t^2} = 4\frac{\partial^2 u}{\partial x^2}$$

subject to $u(0,t) = u(4,t) = 0$, $u_t(x,0) = 0$ and $u(x,0) = x(4-x)$ by taking step length in x is $h = 1$. $[0 \le x \le 4,\ 0 \le t \le 2]$

Solution. Standard form of wave equation is

$$\frac{\partial^2 u}{\partial t^2} = c^2\frac{\partial^2 u}{\partial x^2}$$

so, $c^2 = 4 \ \Rightarrow\ c = \pm 2$
Given that step length in x is $h = 1$.
$\therefore$ step length in t is $k = h/c = 1/2 = 0.5$.
By boundary conditions, $0 \le x \le 4$ which is divided as,

$x_0 = 0,\ \ x_1 = x_0 + h = 1,\ \ x_2 = x_0 + 2h = x_1 + h = 2,\ \ x_3 = x_0 + 3h = x_2 + h = 3,\ \ x_4 = x_0 + 4h = x_3 + h = 4$

Now,

$$t_0 = 0,\ \ t_1 = t_0 + k = 0.5,\ \ t_2 = t_0 + 2k = 1,\ \ t_3 = t_0 + 3k = 1.5,\ \ t_4 = t_0 + 4k = 2$$

$c^2 = 4$; $h = 1$ choose $\lambda = 1$, $\therefore\ \ \lambda = \frac{ka}{h}\ \Rightarrow\ \ k = 1/2$
As $\lambda = 1$; explicit formula is used,

	$x_0 = 0$	$x_1 = 1$	$x_2 = 2$	$x_3 = 3$	$x_4 = 4$
$t_0 = 0$	$u_{0,0} = 0$	$u_{1,0} = 3$	$u_{2,0} = 4$	$u_{3,0} = 3$	$u_{4,0} = 0$
$t_1 = 0.5$	$u_{0,1} = 0$	$u_{1,1} = 2$	$u_{2,1} = 3$	$u_{3,1} = 2$	$u_{4,1} = 0$
$t_2 = 1$	$u_{0,2} = 0$	$u_{1,2} = 0$	$u_{2,2} = 0$	$u_{3,2} = 0$	$u_{4,2} = 0$
$t_3 = 1.5$	$u_{0,3} = 0$	$u_{1,3} = -2$	$u_{2,3} = -3$	$u_{3,3} = -2$	$u_{4,3} = 0$
$t_4 = 2$	$u_{0,4} = 0$	$u_{1,4} = -3$	$u_{2,4} = -4$	$u_{3,4} = -3$	$u_{4,4} = 0$