[Skn2] given, $f(x,y) = 2x^3 + 6xy^2 - 3y^3 - 150x$

$$\begin{aligned}\frac{\partial f(x,y)}{\partial x} &= 6x^2 + 6y^2 - 150 \\ \frac{\partial f(x,y)}{\partial y} &= -9y^2 + 12xy\end{aligned}$$

Now,

$$\begin{aligned}6x^2 + 6y^2 - 150 = 0 \\ -9y^2 + 12xy = 0\end{aligned}$$

Solving we get $(x,y) = (3,4),\ (-3,-)$
This is critical point.

[FAB2] here $f(x,y) = 4x^3 + y^2$ and $2x^2 + y^2 = 1$
Now,

$$\begin{aligned}12x^2 &= 4\lambda x \\ 2y &= 2\lambda y \\ 2x^2 + y^2 &= 1\end{aligned}$$

From 2nd equation we must have $\lambda = 1$ or $y = 0$. If $y = 0$ then from 3rd equation $x = \pm\sqrt{\frac{1}{2}}$. If $\lambda = 1$ then from 1st equation $x = \frac{1}{3}$ and $y = \pm\frac{\sqrt{7}}{3}$. So the solutions are

$$(x,y) = \left(\sqrt{\frac{1}{2}}, 0\right), \left(-\sqrt{\frac{1}{2}}, 0\right), \left(\frac{1}{3}, \frac{\sqrt{7}}{3}\right), \left(\frac{1}{3}, -\frac{\sqrt{7}}{3}\right)$$

At these points,

$$\begin{aligned}f\left(\sqrt{\frac{1}{2}}, 0\right) &= \sqrt{2} \\ f\left(\sqrt{-\frac{1}{2}}, 0\right) &= -\sqrt{2} \\ f\left(\frac{1}{3}, \frac{\sqrt{7}}{3}\right) &= \frac{25}{27} \\ f\left(\frac{1}{3}, -\frac{\sqrt{7}}{3}\right) &= \frac{25}{27}\end{aligned}$$

[NAS] Here $f(x,y,z) = x^2y - yz^3 + z$
Now,

$$||a|| = \sqrt{4 + 1 - 4} = 1$$

Hence a is unit vector.
The directional derivative is

$$\begin{aligned}D_a f(x,y,z) &= 2(2xy) + 1(x^2 - z^3) - 2(1 - 3yz^2) \\ &= x^2 + 4xy + 6yz^2 - z^3 - 2 \\ D_a f(1,-2,0) &= -9\end{aligned}$$