1 Assignment(updated)

Primal

$$\begin{aligned} \text{minimize} \quad & Z = -3x_1 + x_2 + x_3 \\ \text{subject to} \quad & x_1 - 2x_2 + x_3 \leq 11 \\ & -4x_1 + x_2 + 2x_3 \geq 3 \\ & 2x_1 - x_3 = -1 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

Standard form of primal

$$\begin{aligned} \text{minimize} \quad & Z = -3x_1 + x_2 + x_3 \\ \text{subject to} \quad & x_1 - 2x_2 + x_3 + x_4 = 11 \\ & -4x_1 + x_2 + 2x_3 - x_5 = 3 \\ & -2x_1 + x_3 = 1 \\ & x_1, x_2, x_3, x_4, x_5 \geq 0 \end{aligned}$$

Dual form

$$\begin{aligned} \text{maximize} \quad & W = 11y_1 + 3y_2 + y_3 \\ \text{subject to} \quad & y_1 - 4y_2 - 2y_3 \leq -3 \\ & \Rightarrow -y_1 + 4y_2 + 2y_3 \geq 3 \\ & -2y_1 + y_2 \leq 1 \\ & y_1 + 2y_2 + y_3 \leq 1 \\ & y_1 \leq 0 \\ & -y_2 \leq 0 \\ & \Rightarrow y_2 \geq 0 \\ & y_3 \text{ unrestricted} \end{aligned}$$

Standard form of dual form
Let $y_1 = -y_1^*$ and $y_3 = y_3^{'} - y_3^{''}$

$$\begin{aligned} \text{maximize} \quad & W = -11y_1^* + 3y_2 + y_3^{'} - y_3^{''} \\ \text{subject to} \quad & y_1^* + 4y_2 + 2y_3^{'} - 2y_3^{''} - y_4 = 3 \\ & 2y_1^* + y_2 + y_5 = 1 \\ & -y_1^* + 2y_2 + y_3^{'} - y_3^{''} + y_6 = 1 \\ & y_1^*,\, y_2,\, y_3^{'},\, y_3^{''},\, y_4,\, y_5,\, y_6 \geq 0 \end{aligned}$$

Taking $y_1^* = y_2 = y_3^{'} = y_3^{''} = 0$ we get $y_4 = -3$, $y_5 = 1$, $y_6 = 1$, which is basic non-feasible solution.
Adding artificial variables we get

$$\begin{aligned} \text{maximize} \quad & W = -11y_1^* + 3y_2 + y_3^{'} - y_3^{''} - My_7 \\ \text{subject to} \quad & y_1^* + 4y_2 + 2y_3^{'} - 2y_3^{''} - y_4 + y_7 = 3 \\ & 2y_1^* + y_2 + y_5 = 1 \\ & -y_1^* + 2y_2 + y_3^{'} - y_3^{''} + y_6 = 1 \\ & y_1^*,\, y_2,\, y_3^{'},\, y_3^{''},\, y_4,\, y_5,\, y_6,\, y_7 \geq 0 \end{aligned}$$

Taking $y_1^* = y_2 = y_3^{'} = y_3^{''} = y_4 = 0$ we get $y_7 = 3$, $y_5 = 1$, $y_6 = 1$, which is initial basic feasible solution.

Tab	C_B	basis \ $c_j \rightarrow$	-11	3	1	-1	0	0	0	-M	Constant/
			y_1^*	y_2	$y_3^{'}$	$y_3^{''}$	y_4	y_5	y_6	y_7	Solution
I	-M	y_7	1	4	2	-2	-1	0	0	1	3
	0	y_5	2	1	0	0	0	1	0	0	1
	0	y_6	-1	2	1	-1	0	0	1	0	1
		$\bar{c_j}$ row	-11+M	3+4M	1+2M	-1-2M	-M	0	0	0	W=-3M
II	-M	y_7	3	0	0	0	-1	0	-2	1	1
	0	y_5	5/2	0	-1/2	1/2	0	1	-1/2	0	1/2
	3	y_2	-1/2	1	1/2	-1/2	0	0	1/2	0	1/2
		$\bar{c_j}$ row	-19/2+3M	0	-1/2	1/2	-M	0	-3/2-2M	0	W=3/2-M
III	-M	y_7	0	0	3/5	-3/5	-1	-6/5	-7/5	1	2/5
	-11	y_1^*	1	0	-1/5	1/5	0	2/5	-1/5	0	1/5
	3	y_2	0	1	2/5	-2/5	0	1/5	2/5	0	3/5
		$\bar{c_j}$ row	0	0	3M/5-12/5	12/5-3M/5	-M	19/5-6M/5	-17/5-7M/5	0	W=-2/5-2M/5
IV	1	$y_3^{'}$	0	0	1	-1	-5/3	-2	-7/3	5/3	2/3
	-11	y_1^*	1	0	0	0	-1/3	0	-2/3	1/3	1/3
	3	y_2	0	1	0	0	2/3	1	4/3	-2/3	1/3
		$\bar{c_j}$ row	0	0	0	0	-4	-1	-9	4-M	W=-2

$\therefore \left\{y_1^*,\, y_2,\, y_3^{'},\, y_3^{''}\right\} = \left\{\frac{1}{3},\, \frac{1}{3},\, \frac{2}{3},\, 0\right\} \quad W_{max} = -2$

$\therefore \{y_1,\, y_2,\, y_3\} = \left\{-\frac{1}{3},\, \frac{1}{3},\, \frac{2}{3}\right\} \quad W_{max} = -2$