Chapter 1

Fuzzy Mapping

Definition 1 (Fuzzy Mapping)**.** Let X and Y be two non-empty set and let $f : X \to Y$ be an ordinary mapping. A fuzzy mapping $f^{\to} : \langle \mathcal{F}(X), \delta \rangle \to \langle \mathcal{F}(Y), \mu \rangle$ is defined by $f^{\to}(A)(y) = \bigvee\{A(x)|x \in X, f(x) = y\} \forall y \in Y$, and a fuzzy reverse mapping $f^{\leftarrow} : \langle \mathcal{F}(Y), \mu \rangle \to \langle \mathcal{F}(X), \delta \rangle$ is defined by $f^{\leftarrow}(B)(x) = B(f(x))\, \forall x \in X$.

Definition 2 (Continuous Fuzzy Mapping)**.** Let $\langle \mathcal{F}(X), \delta \rangle$ and $\langle \mathcal{F}(Y), \mu \rangle$ be two fuzzy topological space. A fuzzy mapping $f^{\to} : \langle \mathcal{F}(X), \delta \rangle \to \langle \mathcal{F}(Y), \mu \rangle$ is called continuous if for each $v \in \mu$, $f^{\to}(v) \in \delta$.

Definition 3 (Open Fuzzy Mapping)**.** Let $\langle \mathcal{F}(X), \delta \rangle$ and $\langle \mathcal{F}(Y), \mu \rangle$ be two fuzzy topological space. A fuzzy mapping $f^{\to}$ is called open if for each $u \in \delta$, $f^{\to}(u) \in \mu$.

Definition 4 (Closed Fuzzy Mapping)**.** Let $\langle \mathcal{F}(X), \delta \rangle$ and $\langle \mathcal{F}(Y), \mu \rangle$ be two fuzzy topological space. A fuzzy mapping $f^{\to}$ is called closed if for each closed set $F \in \delta$, $f^{\to}(F)$ is closed in μ.

Theorem 1.0.1. Let $\langle \mathcal{F}(X), \delta \rangle$ and $\langle \mathcal{F}(Y), \mu \rangle$ be two fuzzy topological spaces and $f : X \to Y$ be an ordinary mapping. Then for each $a \in [0, 1]$ and every $A \in \mathcal{F}(X)$, $f^{\to}(aA) = af^{\to}(A)$.

Proof. For all $a \in [0, 1]$, $\forall A \in \mathcal{F}(X)$ and $\forall y \in Y$ we have,

$$\begin{aligned} f^{\to}(aA)(y) &= \vee\{(aA)(x)|x \in X,\ f(x) = y\} \\ &= \vee\{a \wedge (A)(x)|x \in X,\ f(x) = y\} \\ &= a \wedge (\vee\{(A)(x)|x \in X,\ f(x) = y\}) \\ &= a \wedge f^{\to}(A)(y) \\ &= (af^{\to}(A))(y) \end{aligned}$$

Thus, $f^{\to}(aA) = af^{\to}(A)$. □