Chapter 1

Predictor-Corrector Method

1.1 Predictor-Corrector Method

The combination of an explicit and implicit technique is called predictor-corrector method. The explicit method predicts an approximation and the implicit method corrects this prediction.

Consider the following 4th order method for solving an IVP: $y' = f(t, y); \quad a \leq t \leq b, \quad y(t_0) = y(a) = \alpha_0$

$$w_4^{(0)} = w_3 + \frac{h}{24}\left[55f(t_3, w_3) - 59f(t_2, w_2) + 37f(t_1, w_1) - 9f(t_0, w_0)\right]$$

The first step is to calculate the staring values w_0, w_1, w_2 and w_3 for the four-step Adams-Bashforth method. To do this, we use a fourth-order one-step method, the Runge-Kutta method of order four. The next step is to calculate an approximation, $w_4^{(0)}$ to $y(t_4)$ using the Adams-Bashforth method as predictor. This approximation is improved by inserting $w_4^{(0)}$ in the right-hand side of the three step Adams-Moulton method and using that method as a corrector.

$$w_4^{(1)} = w_3 + \frac{h}{24}\left[9f(t_4, w_4^{(0)}) + 19f(t_3, w_3) - 5f(t_2, w_2) + f(t_1, w_1)\right]$$

The only function evaluation is required in this procedure is $f(t_4, w_4^{(0)})$ in the corrector equation. All the other values of f have been calculated for earlier approximation.

The value $w_4^{(1)}$ is then used as the approximation to $y(t_4)$, and the technique of using the Adams-Bashforth method as a predictor and the Adams-Moulton method as a corrector is repeated to find $w_5^{(0)}$ and $w_5^{(1)}$, the initial and final approximation to $y(t_5)$ etc.

Improved approximation to $y(t_{i+1})$ can be obtained by iterating the Adams-Moulton formula

$$w_{i+1}^{(k+1)} = w_i + \frac{h}{24}\left[9f(t_{i+1}, w_{i+1}^{(k)}) + 19f(t_i, w_i) - 5f(t_{i-1}, w_{i-1}) + f(t_{i-2}, w_{i-2})\right]$$

However, $\left\{w_{i+1}^{(k+1)}\right\}$ converges to the approximation given by the implicit formula rather than to the solution $y(t_{i+1})$, and it is usually more effective to use reduction in the step size if improved accuracy is needed.

Example. For the IVP: $y' = y - t^2 + 1$, $0 \leq t \leq 2$, $y(0) = 0.5$ with $N = 10$.

The following table lists the results obtained by Adams fourth-order predictor corrector method:

t_i	$y_i = y(t_i)$	w_i	Error $\lvert y_i - w_i \rvert$
0.0	0.5000000	0.5000000	0.0000000
0.2	0.8292986	0.8292933	0.0000053
0.4	1.2140877	1.2140762	0.0000114
0.6	1.6489406	1.6489220	0.0000186
0.8	2.1272295	2.1272056	0.0000239
1.0	2.6408591	2.6408286	0.0000305
1.2	3.1799415	3.1799026	0.0000389
1.4	3.7324000	3.7323505	0.0000495
1.6	4.2834838	4.2834208	0.0000630
1.8	4.8151763	4.8150964	0.0000799
2.0	5.3054720	5.3053707	0.0001013