Chapter 1

Butcher's Table

1.1 Using Butcher's Table

Generalized Runge-Kutta methods are given as,

$$y_{i+1} = y_i + \sum_{i=1}^{s} b_i k_i$$

where,

$$\begin{aligned}
k_1 &= f(t_i,\, y_i),\\
k_2 &= f(t_i + c_2 h,\, y_i + h(a_{21}k_1)),\\
k_3 &= f(t_i + c_3 h,\, y_i + h(a_{31}k_1 + a_{32}k_2)),\\
&\ \vdots\\
k_s &= f(t_i + c_s h,\, y_i + h(a_{s1}k_1 + a_{s2}k_2 + \cdots + a_{s,s-1}k_{s-1})).
\end{aligned}$$

The constants are given as a table known as *Butcher tableau*.

$$\begin{array}{c|ccccc}
0 & & & & & \\
c_2 & a_{21} & & & & \\
c_3 & a_{31} & a_{32} & & & \\
\vdots & \vdots & & \ldots & & \\
c_s & a_{s1} & a_{s2} & \ldots & a_{s,s-1} & \\
\hline
 & b_1 & b_2 & \ldots & b_{s-1} & b_s
\end{array}$$

1.2 Butcher's Table for some methods

1.2.1 Midpoint Method

$$\begin{array}{c|cc}
0 & & \\
\frac{1}{2} & \frac{1}{2} & \\
\hline
 & 0 & 1
\end{array}$$

1.2.2 Modified Euler Method

$$\begin{array}{c|cc}
0 & & \\
1 & 1 & \\
\hline
 & \frac{1}{2} & \frac{1}{2}
\end{array}$$

1.2.3 Heun' Method/Ralston's Method

$$
\begin{array}{c|cc}
0 & & \\
\frac{2}{3} & \frac{2}{3} & \\
\hline
 & \frac{1}{4} & \frac{3}{4}
\end{array}
$$

1.2.4 Runge-Kutta Order 4

$$
\begin{array}{c|cccc}
0 & & & & \\
\frac{1}{2} & \frac{1}{2} & & & \\
\frac{1}{2} & 0 & \frac{1}{2} & & \\
1 & 0 & 0 & 1 & \\
\hline
 & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} & \frac{1}{6}
\end{array}
$$