[SADT2]

Proof. Suppose f has (at least) two fixed point a and b. Then by definition of a fixed point, $f(a) = a$ and $f(b) = b$.
By the mean value theorem, there is a c in (a, b) such that

$$\frac{f(b) - f(a)}{b - a} = f'(c)$$

Now,

$$\frac{b - a}{b - a} = f'(c)$$

So,

$$1 = f'(c)$$

That is, $f'(c) = 1$ However, by assumption, $f(x) \neq 1$ for all real number x. So, this is a contradiction.
Hence, f has at most one fixed point. □

[FAB4] Here, $f(x) = x^2 - 5x + 6$. $f(x)$ is a polynomial function, so it is continuous everywhere and has derivatives.
Now, $f'(x) = 2x - 5$. By setting $f'(x) = 0$ we get $x = 2.5$. Now we need to test for $(-\infty, 2.5)$ and $(2.5, \infty)$.
For $a = 2$, $f'(a) = -1 < 0$ and $b = 3$, $f(b) = 1 > 0$.
So, $f(x)$ has a relative minimum at $x = 2.5$.

[NAS1] We know, $L(x) = f(a) + f'(a)(x - a)$
Given,

$$f(x) = 5 - x^2$$

Now,

$$f'(x) = -2x$$

Again,

$$f(2) = 1 \qquad f'(2) = -4$$

So,

$$\begin{aligned} L(x) &= f(a) + f'(a)(x - a) \\ &= 1 - 4(x - 2) \\ &= 9 - 4x \end{aligned}$$