Fuzzy Topology

MAT514

PROF. DR SHAMSUN NAHER BEGUM

Shahjalal University of Science and Technology

EDITED BY
MEHEDI HASAN

Preface

This is a compilation of lecture notes with some books and my own thoughts. If there are any mistake/typing error or, for any query mail me at mehedi12@student.sust.edu.

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Part I

Sheet

Chapter 1

Fuzzy Sets

Definition 1 (Characteristic function)**.** Let X be a universal set and $A \subseteq X$. Then the function[1]

$$\chi_A(x) = \begin{cases} 1; & x \in A \\ 0; & x \notin A \end{cases}$$

is characteristic function of A in X.

Definition 2 (Fuzzy Set)**.** A fuzzy set[2] $A \subseteq X$ is a mapping $A : X \to [0,1]$, where, $A(x) = y \in [0,1]$ is called the membership function or, grade of membership of x in A. The collection of all fuzzy sets of X is denoted by $\mathcal{F}(x)$.

Definition 3 (Fuzzy subset)**.** A fuzzy set A is called a fuzzy subset of another fuzzy set B if $A(x) \leq B(x)$ $\forall x \in X$. We denote it by $A \leq B$.

Definition 4 (Empty fuzzy set)**.** A fuzzy set A is called empty fuzzy set if $\forall x \in X$ $A(x) = 0$. The empty fuzzy set is denoted by $\underline{0}$. Thus, $\underline{0}(x) = 0$ $\forall x \in X$.

Definition 5 (Total fuzzy set)**.** The total fuzzy set $\underline{1}$ is defined by $\underline{1}(x) = 1$ $\forall x \in X$.

Definition 6 (Equality of two fuzzy sets)**.** Two fuzzy sets A and B of X is said to be equal iff $A \leq B$ and $B \leq A$.

Example (Empty and Total fuzzy set)**.** Suppose, $A : X \to [0,1]$ where $X = [20, 80]$. Then,

$$\underline{0}(x) = \begin{cases} 0 & \text{if } 15 < x < 90 \\ 1 & \text{otherwise} \end{cases} \qquad \text{and} \qquad \underline{1}(x) = \begin{cases} 1 & \text{if } 20 \leq x < 90 \\ 0 & \text{otherwise} \end{cases}$$

Example (Fuzzy subset)**.** Suppose, $A : X \to [0,1]$ where, $X = [0, 100]$ defined by

$$A(x) = \begin{cases} 0; & \text{if } 0 \leq x < 40 \\ \frac{x}{75}; & \text{if } 40 \leq x < 75 \\ 1; & \text{if } 75 \leq x \leq 100 \end{cases}$$

and $B : X = [0, 100] \to [0,1]$ defined by

$$B(x) = \begin{cases} 0; & \text{if } 0 \leq x < 40 \\ \frac{x}{95}; & \text{if } 40 \leq x < 95 \\ 1; & \text{if } 95 \leq x \leq 100 \end{cases}$$

Then, $B(x)$ is a subset of $A(x)$. Since, $B(x) \leq A(x)$ $\forall x \in X$.

[1] Some authors use μ as characteristic function.

[2] Sometimes fuzzy set is denoted by $\underset{\sim}{A}$.

1.1 Fuzzy Set Operations

Definition 7 (Union of Fuzzy Sets)**.** Let $A,\ B \in \mathcal{F}(x)$. Then the union of A and B is denoted and defined by, $(A \vee B)(x) = \max\{A(x),\ B(x)\}\,,\ \forall x \in X$.

Definition 8 (Intersection of Fuzzy Sets)**.** Let $A,\ B \in \mathcal{F}(x)$. Then the intersection of A and B is denoted and defined by, $(A \wedge B)(x) = \min\{A(x),\ B(x)\}\,,\ \forall x \in X$.

Definition 9 (Complement of Fuzzy Set)**.** Let A be a fuzzy set of X. Then, the complement of A is denoted by A^c and defined by $A^c(x) = 1 - A(x),\ \forall x \in X$.

Example. Given,

$$A_1 = \begin{cases} 1; & \text{if } 40 \leq x < 50 \\ 1 - \dfrac{x-50}{10}; & \text{if } 50 \leq x < 60 \\ 0; & \text{if } 60 \leq x \leq 100 \end{cases} \qquad \text{and} \qquad A_2 = \begin{cases} 0; & \text{if } 40 \leq x < 50 \\ \dfrac{x-50}{10}; & \text{if } 50 \leq x < 60 \\ 1 - \dfrac{x-60}{10}; & \text{if } 60 \leq x < 70 \\ 0; & \text{if } 70 \leq x \leq 100 \end{cases}$$

1. Find the complement of A_1 and A_2.
2. Find $(A_1 \wedge A_2)(x)$ and $(A_1 \vee A_2)(x)$

Solution:

1. Complement of

$$A_1, A_1^c = \begin{cases} 0; & \text{if } 40 \leq x < 50 \\ \dfrac{x-50}{10}; & \text{if } 50 \leq x < 60 \\ 1; & \text{if } 60 \leq x \leq 100 \end{cases}$$

Complement of

$$A_2, A_2^c = \begin{cases} 1; & \text{if } 40 \leq x < 50 \\ \dfrac{60-x}{10}; & \text{if } 50 \leq x < 60 \\ \dfrac{x-60}{10}; & \text{if } 60 \leq x < 70 \\ 1; & \text{if } 70 \leq x \leq 100 \end{cases}$$

2.

$$(A_1 \wedge A_2)(x) = \begin{cases} 0; & \text{if } 40 \leq x < 50 \\ \dfrac{x-50}{10}; & \text{if } 50 \leq x \leq 55 \\ 1 - \dfrac{x-50}{10}; & \text{if } 55 \leq x \leq 60 \\ 0; & \text{if } 60 \leq x \leq 100 \end{cases}$$

$$(A_1 \vee A_2)(x) = \begin{cases} 1; & \text{if } 40 \leq x \leq 50 \\ 1 - \dfrac{x-50}{10}; & \text{if } 50 \leq x \leq 55 \\ \dfrac{x-50}{10}; & \text{if } 55 \leq x < 60 \\ 1 - \dfrac{x-60}{10}; & \text{if } 60 \leq x < 70 \\ 0; & \text{if } 70 \leq x < 100 \end{cases}$$

Chapter 2

Separation Axioms

Definition 10 (Quasi T_0-space)**.** Let, $\langle \mathcal{F}(x), \delta \rangle$ be a fuzzy topological space. Then, $\langle \mathcal{F}(x), \delta \rangle$ is called a quasi T_0-space, if for every two distinct fuzzy points x_a and x_b with same support point x, there exists $U \in Q_\delta(x_a)$ such that $x_b U$ or, there exists $V \in Q_\delta(x_b)$ such that $x_a \not\propto V$.