Theory of Relativity and Cosmology

MAT518

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Preface

This is a compilation of lecture notes with some books and my own thoughts. If there are any mistake/typing error or, for any query mail me at mehedi12@student.sust.edu.

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CHAPTER 1

General Relativity

PROBLEM 1.1. Write down the principle of general relativity.

SOLUTION. Principle of general relativity:

(1) Principle of equivalence: The principle of equivalence which necessarily leads to the introduction of a curved space time.
In other words - The inertial mass and gravitational mass are same.

(2) Principle of covariance: The principle of covariance in the following form: The equations describing the laws of physics should have the same form in all coordinate system, or, the equations that describe the laws of physics should have tensorial form, since tensors are covariant quantities.

PROBLEM 1.2. Show that in General Relativity the space time metric has form $\mathrm{d}\,s^2 = g_{\mu\nu}\,\mathrm{d}\,x^\mu\,\mathrm{d}\,x^\nu$, where $g_{\mu\nu}$ is a gravitational potential.

SOLUTION. The special relativistic line element or proper time is given by

$$\begin{aligned}\mathrm{d}\,s^2 &= \eta_{\mu\nu}\,\mathrm{d}\,x^\mu\,\mathrm{d}\,x^\nu \\ &= c^2\,\mathrm{d}\,t^2 - \mathrm{d}\,x^2 - \mathrm{d}\,y^2 - \mathrm{d}\,z^2 \\ &= -c^2\,\mathrm{d}\,t^2 + \mathrm{d}\,x^2 + \mathrm{d}\,y^2 + \mathrm{d}\,z^2 \end{aligned} \tag{1}$$

where, $\eta_{\mu\nu}$ (metric tensor) is the flat space Minkowskian metric which is given by

$$\eta_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{2}$$

where we have used natural units: $c = 1$.

The form of the special relativistic proper time does not change if one goes from one inertial system of coordinates into another by means of Lorentz transformation.

Suppose that we from inertial system into a uniformly rotating (i.e. non- inertial) coordinate system. If the rotation b around the z-axis, then the transformation equations are

$$\left.\begin{aligned} x &= x'\cos\omega t - y'\sin\omega t \\ y &= x'\sin\omega t + y'\cos\omega t \\ z &= z' \end{aligned}\right\} \tag{3}$$