n = iteration	x_n (upto 8D)
	$x_0 = 1.5$
0	$x_1 = 1.373\,333\,333$
1	$x_2 = 1.365\,262\,015$
2	$x_3 = 1.365\,230\,014$
3	$x_4 = 1.365\,230\,013$

n = iteration	x_n (upto 5D)
	$x_0 = 1.5$
0	$x_1 = 1.533\,333\,333$
1	$x_2 = 1.532\,090\,643$
2	$x_3 = 1.532\,088\,886$
3	$x_4 = 1.532\,088\,886$

x	e^{-x}
0	1
1	0.367 87
2	0.135 33
3	0.049 78
-1	2.718 20
-2	7.389 00

n = iteration	x_n (within 10^{-5})
	$x_0 = 0.79$
0	$x_1 = 0.739\,627\,55$
1	$x_2 = 0.739\,085\,198$
2	$x_3 = 0.739\,085\,133$

i	x_i	$f(x) = \frac{1}{x}$
0	$x_0 = 2$	$f(x_0) = 0.5$
1	$x_1 = 2.5$	$f(x_1) = 0.4$
2	$x_2 = 4$	$f(x_2) = 0.25$

x	$f(x)$
1.00	0.1924
1.05	0.2414
1.10	0.2933
1.15	0.3492

x	$f(x)$
1.0	1.000 00
1.1	1.233 68
1.2	1.552 71
1.3	1.493 72
1.4	2.611 70

x	$f(x)$
0.0	1.0
0.5	1.648 72
1.0	2.718 58
2.0	7.389 06

x_i	$f(x_i)$
$x_0 = 1.2$	$f(x_0) = 0.079\,181$
$x_1 = 1.4$	$f(x_1) = 0.146\,128$
$x_2 = 1.6$	$f(x_2) = 0.204\,120$
$x_3 = 1.8$	$f(x_3) = 0.255\,273$

	Zeroth divided difference	First divided difference	Second divided difference	Third divided difference
x_i	$f(x_i) = f[x_i]$	$f[x_i, x_{i+1}] = \frac{f[x_i] - f[x_{i+1}]}{x_i - x_{i+1}}$	$f[x_i, x_{i+1}, x_{i+2}] = \frac{f[x_i, x_{i+1}] - f[x_{i+1}, x_{i+2}]}{x_i - x_{i+2}}$	$f[x_i, x_{i+1}, x_{i+2}, x_{i+3}] = \frac{f[x_i, x_{i+1}, x_{i+2}] - f[x_{i+1}, x_{i+2}, x_{i+3}]}{x_i - x_{i+3}}$
$x_0 = 1.2$	$f[x_0] = 0.079181$	$f[x_0, x_1] = \frac{f[x_0] - f[x_1]}{x_0 - x_1} = 0.334735$		
			$f[x_0, x_1, x_2] = \frac{f[x_0, x_1] - f[x_1, x_2]}{x_0 - x_2} = -0.1119375$	
$x_1 = 1.4$	$f[x_1] = 0.146128$	$f[x_1, x_2] = \frac{f[x_1] - f[x_2]}{x_1 - x_2} = 0.28996$		$f[x_0, x_1, x_2, x_3] = \frac{f[x_0, x_1, x_2] - f[x_1, x_2, x_3]}{x_0 - x_3} = 0.044083333$
			$f[x_1, x_2, x_3] = \frac{f[x_1, x_2] - f[x_2, x_3]}{x_1 - x_3} = -0.0854875$	
$x_2 = 1.6$	$f[x_2] = 0.204120$			
$x_3 = 1.8$	$f[x_3] = 0.255273$	$f[x_2, x_3] = \frac{f[x_2] - f[x_3]}{x_2 - x_3} = 0.255765$		

x_i	Zeroth Divided Difference $f(x_i)=f[x_i]$	First Divided Difference $f[x_i,x_{i+1}] = \dfrac{f[x_i]-f[x_{i+1}]}{x_i-x_{i+1}}$	Second divided difference $f[x_i,x_{i+1},x_{i+2}] = \dfrac{f[x_i,x_{i+1}]-f[x_{i+1},x_{i+2}]}{x_i-x_{i+2}}$	Third divided difference $f[x_i,x_{i+1},x_{i+2},x_{i+3}] = \dfrac{f[x_i,x_{i+1},x_{i+2}]-f[x_{i+1},x_{i+2},x_{i+3}]}{x_i-x_{i+3}}$
$x_0=1.2$	$f[x_0]=0.079181$			
		$f[x_0,x_1] = \dfrac{f[x_0]-f[x_1]}{x_0-x_1}$ =0.334735		
$x_1=1.4$	$f[x_1]=0.146128$		$f[x_0,x_1,x_2] = \dfrac{f[x_0,x_1]-f[x_1,x_2]}{x_0-x_2}$ $=-0.1119375$	
		$f[x_1,x_2] = \dfrac{f[x_1]-f[x_2]}{x_1-x_2}$ =0.28996		$f[x_0,x_1,x_2,x_3] = \dfrac{f[x_0,x_1,x_2]-f[x_1,x_2,x_3]}{x_0-x_3}$ =0.044083333
$x_2=1.6$	$f[x_2]=0.204120$		$f[x_1,x_2,x_3] = \dfrac{f[x_1,x_2]-f[x_2,x_3]}{x_1-x_3}$ $=-0.0854875$	
		$f[x_2,x_3] = \dfrac{f[x_2]-f[x_3]}{x_2-x_3}$ =0.255765		
$x_3=1.8$	$f[x_3]=0.255273$			

x_i	$f(x_i)$
$x_0=1.05$	$f(x_0)=1.7433$
$x_1=1.20$	$f(x_1)=2.5722$
$x_2=1.30$	$f(x_2)=3.6021$
$x_3=1.43$	$f(x_3)=8.2381$

x_i	Zeroth divided difference $f(x_i)=f[x_i]$	First divided difference $f[x_i,x_{i+1}] = \dfrac{f[x_i]-f[x_{i+1}]}{x_i-x_{i+1}}$	Second divided difference $f[x_i,x_{i+1},x_{i+2}] = \dfrac{f[x_i,x_{i+1}]-f[x_{i+1},x_{i+2}]}{x_i-x_{i+2}}$	Third divided difference $f[x_i,x_{i+1},x_{i+2},x_{i+3}] = \dfrac{f[x_i,x_{i+1},x_{i+2}]-f[x_{i+1},x_{i+2},x_{i+3}]}{x_i-x_{i+3}}$
$x_0=1.2$	$f[x_0]=0.079181$			
		$f[x_0,x_1] = \dfrac{f[x_0]-f[x_1]}{x_0-x_1}$ =0.334735		
$x_1=1.4$	$f[x_1]=0.146128$		$f[x_0,x_1,x_2] = \dfrac{f[x_0,x_1]-f[x_1,x_2]}{x_0-x_2}$ $=-0.1119375$	
		$f[x_1,x_2] = \dfrac{f[x_1]-f[x_2]}{x_1-x_2}$ =0.28996		$f[x_0,x_1,x_2,x_3] = \dfrac{f[x_0,x_1,x_2]-f[x_1,x_2,x_3]}{x_0-x_3}$ =0.044083333
$x_2=1.6$	$f[x_2]=0.204120$		$f[x_1,x_2,x_3] = \dfrac{f[x_1,x_2]-f[x_2,x_3]}{x_1-x_3}$ $=-0.0854875$	
		$f[x_2,x_3] = \dfrac{f[x_2]-f[x_3]}{x_2-x_3}$ =0.255765		
$x_3=1.8$	$f[x_3]=0.255273$			

Marks	No. of students
30 – 40	31
40 – 50	42
50 – 60	51
60 – 70	35
70 – 80	31

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$	$\Delta^5 f(x)$
40	31					
50	73	42	9	−25		
60	124	51	−16	12	37	
70	159	35	−4			
80	190	31				

Year	Population (in thousands)
1911	12
1921	15
1931	20
1941	27
1951	39
1961	52

Age (x)	Premium $f(x)$
20	0.014 27
24	0.1581
28	0.017 72
32	0.019 96

Age x	Premium $f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
20	0.014 27			
		0.001 55		
24	0.015 81		0.0037	
		0.001 91		−0.000 04
28	0.017 72		0.000 33	
		0.002 24		
32	0.019 96			

k→	0	1	2	3	4	5	6
$x_1^{(k)}$	0	0.6000	1.0302	1.0066	1.0004	1.0001	1.0000
$x_2^{(k)}$	0	2.3273	2.0370	2.0036	2.0003	2.0000	2.0000
$x_3^{(k)}$	0	−0.9873	−1.0145	−1.0025	−1.0003	−1.0000	−1.0000
$x_4^{(k)}$	0	0.8789	0.4843	0.9984	0.9998	1.0000	1.0000

Operation	Coefficient of			R.H.S.	Eq. #	Check Sum
	x	y	z			
	16	5	7	29	$Eq1$	57
	5	12	9	5	$Eq2$	31
	8	11	20	35	$Eq3$	74
Eq 1/16	1	0.3125	0.4375	1.8125	$Eq4*$	3.5625
Eq 2/5	1	2.4000	1.8000	1.000	$Eq5$	6.2000
Eq 3/8	1	1.3750	2.5000	4.3750	$Eq6$	9.2500
Eq 5 - Eq 4		2.0875	1.3625	-0.8125	$Eq7$	2.6375
Eq 6 - Eq 4		1.0625	2.0625	2.5625	$Eq8$	5.6875
Eq 7/2.0875		1	0.6527	-0.3892	$Eq9*$	1.2635
Eq 8/1.0625		1	1.9412	2.4118	$Eq10$	5.3530
Eq 10 - Eq 9			1.2885	2.8010	$Eq11*$	4.0895

Operation	Coefficient of			R.H.S	Eq. #	Check sum
	x	y	z			
	16.0000	5.0000	7.0000	29.0000	Eq1	57.0000
	5.0000	12.0000	9.0000	5.0000	Eq2	31.0000
	8.0000	11.0000	20.0000	35.0000	Eq3	74.0000
Eq 1/16	1.0000	0.3125	0.4375	1.8125	Eq4*	3.5625
Eq 2/5	1.0000	2.4000	1.8000	1.0000	Eq5	6.2000
Eq 3/8	1.0000	1.3750	2.5000	4.3750	Eq6	9.2500
Eq 5 - Eq 4		2.0875	1.3625	-0.8125	Eq7	2.6375
Eq 6 - Eq 4		1.0625	2.0625	2.5625	Eq8	5.6875
Eq 7/2.0875		1.0000	0.6527	-0.3892	Eq9*	1.2635
Eq 8/1.0625		1.0000	1.9412	2.4118	Eq10	5.3530
Eq 10 - Eq 9			1.2885	2.8010	Eq11*	4.0895

Operation	Coefficient of			R.H.S	Eq. #	Check sum
	x	y	z			
	5.0000	12.0000	9.0000	5.0000	Eq1	31.0000
	8.0000	11.0000	20.0000	35.0000	Eq2	74.0000
	16.0000	5.0000	7.0000	29.0000	Eq3	57.0000
Eq 2/20	0.4000	0.5500	1.0000	1.7500	Eq4*	3.7000
Eq 3/7	0.2857	0.7143	1.0000	4.1429	Eq5	8.1429
Eq 1/9	0.5556	1.3333	1.0000	0.5556	Eq6	3.4445
Eq 5 - Eq 4	1.8857	0.1643	0.0000	2.3929	Eq7	4.4429
Eq 6 - Eq 4	0.1556	0.7833	0.0000	-1.1944	Eq8	-0.2555
Eq 7/1.8857	1.0000	0.0871	0.0000	1.2690	Eq9*	2.3561
Eq 8/0.1556	1.0000	5.0341	0.0000	-7.6761	Eq10	-1.6420
Eq 10 - Eq 9	0.0000	4.9470	0.0000	-8.9451	Eq11*	-3.9981

	Using Maxima	with Partial Pivoting	with Total Pivoting
x	1.4265	1.4264	1.4265
y	-1.8081	-1.8080	-1.8082
z	2.1739	2.1738	2.1739

n	0	1	2	3	4	...	9	10	11
x_n	0	1.67	1.81	2.48	2.33	...	2.29	2.29	2.29
y_n	0	2.71	2.10	1.78	1.52	...	1.62	1.61	1.61
z_n	0	0.73	-1.11	-0.89	-0.98	...	-0.84	-0.84	0.84

Table 1: Successive iterates of solution (Jacobi Method)

x	$f(x)$
0.8	1.5505
0.9	1.5289
1.0	1.4687
1.1	1.3627
1.2	1.2031

t	$D(t)$
8.0	17.453
9.0	21.460
10.0	25.752
11.0	30.301
12.0	35.084

t	$D(t)$
8.0	17.453
9.0	21.460
10.0	25.752
11.0	30.301
12.0	35.084

(a)

x	$f(x)$
0.7	1.297
0.8	1.597
1.0	2.287
1.2	3.094
1.3	3.536

(b)

x	$f(x)$
0.7	1.297
0.9	1.927
1.0	2.287
1.1	2.677
1.3	3.536

1.

x	$f(x)$
0.7	1.297
0.8	1.597
1.0	2.287
1.2	3.094
1.3	3.536

2.

x	$f(x)$
0.7	1.297
0.9	1.927
1.0	2.287
1.1	2.677
1.3	3.536

x	$y = f(x) = \ln x$
$x_0 = 4.0$	$y_0 = 1.386\,294\,4$
$x_0 + h = 4.2$	$y_1 = 1.435\,084\,5$
$x_0 + 2h = 4.4$	$y_2 = 1.481\,604\,5$
$x_0 + 3h = 4.6$	$y_3 = 1.526\,056\,3$
$x_0 + 4h = 4.8$	$y_4 = 1.568\,615\,9$
$x_0 + 5h = 5.0$	$y_5 = 1.609\,437\,9$
$x_0 + 6h = 5.2$	$y_6 = 1.648\,658\,6$

Description of the strategy	Equations involving M_0 and M_n
Natural cubic spline "a relaxed curve": $S'(x_0)$ and $S''(x_n)$.	$M_0 = 0$, $M_n = 0$
Clamped cubic spline: specify $S'(x_0) = A$ and $S'(x_n) = B$.	$2M_0 + M_1 = \frac{6}{h_0}\left[\frac{\Delta y_0}{h_0} - A\right]$ $M_n + 2M_{n-1} = \frac{6}{h_{n-1}}6\left[B - \frac{\Delta y_{n-1}}{h_{n-1}}\right]$
Extrapolated cubic spline: M_0 as linear extrapolation from M_1 and M_2 : $\frac{M_1 - M_0}{h_0} = \frac{M_2 - M_1}{h_1}$ M_n as linear extrapolation from M_{n-1} and M_{n-2}: $\frac{M_n - M_{n-1}}{h_{n-1}} = \frac{M_{n-1} - M_{n-2}}{h_{n-2}}$	$M_0 = M_1 - \frac{h_0(M_2 - M_1)}{h_1}$ $M_n = M_{n-1} - \frac{h_{n-1}(M_{n-1} - M_{n-2})}{h_{n-2}}$
Parabolically terminated spline ($S''(x)$ is constant near the end points)	$M_0 = M_1, \quad M_n = M_{n-1}$