0.1 Predictor-Corrector Method

Remainder:
Adams-Bashforth four-step method:

$$
\begin{aligned}
&w_0 = \alpha, \quad w_1 = \alpha_1, \quad w_2 = \alpha_2, \quad w_3 = \alpha_3 \\
&w_{i+1} = w_i + \frac{h}{24}\left[55f(t_i, w_i) - 59f(t_{i-1}, w_{i-1}) + 37f(t_{i-2}, w_{i-2}) - 9f(t_{i-3}, w_{i-3})\right] \quad \text{for } i = 3, 4, 5, \ldots, N-1
\end{aligned}
$$

Adams-Moulton three-step method:

$$
\begin{aligned}
&w_0 = \alpha, \quad w_1 = \alpha_1, \quad w_2 = \alpha_2 \\
&w_{i+1} = w_i + \frac{h}{24}\left[9f(t_{i+1}, w_{i+1}) + 19f(t_i, w_i) - 5f(t_{i-1}, w_{i-1}) + f(t_{i-2}, w_{i-2})\right] \quad \text{for } i = 2, 3, \ldots, N-1
\end{aligned}
$$

IVP: $\frac{\mathrm{d}y}{\mathrm{d}t} = y' = f(t, y); \quad a \le t \le b, y(a) = y(t_0) = \alpha$
For $i = 3$, from Adams-Bashforth four-step method, we have,

$$w_4 = w_3 + \frac{h}{24}\left[55f(t_3, w_3) - 59f(t_2, w_2) + 37f(t_1, w_1) - 9f(t_0, w_0)\right] \tag{1}$$

For Predictor-Corrector method, we denote $w_4^{(0)}$ for w_4 so

$$w_4^{(0)} = w_3 + \frac{h}{24}\left[55f(t_3, w_3) - 59f(t_2, w_2) + 37f(t_1, w_1) - 9f(t_0, w_0)\right] \tag{2}$$

Here we know only $w_0 = \alpha$, so, first we need to calculate w_1, w_2, w_3 using RK-4. Then using the values of w_0, w_1, w_2, w_3 we will get $w_4^{(0)}$ from (2).
Now, we use Adams-Moulton three-step method as a corrector:

$$w_4^{(1)} = w_3 + \frac{h}{24}\left[9f(t_4, w_4^{(0)}) + 19f(t_3, w_3) - 5f(t_2, w_2) + f(t_1, w_1)\right] \tag{3}$$

Here $f(t_4, w_4^{(0)})$ will evaluate using the value of $w_4^{(0)}$ that already calculated using (2).
After calculation of (3), we will get $w_4^{(1)}$. If we will not achieve required accuracy, e.g., $\left|w_4^{(1)} - w_4^{(0)}\right| < 10^{-4}$, then we have to proceed in the following way:

$$w_4^{(2)} = w_3 + \frac{h}{24}\left[9f(t_4, w_4^{(1)}) + 19f(t_3, w_3) - 5f(t_2, w_2) + f(t_1, w_1)\right] \tag{4}$$

Note that only $f(t_4, w_4^{(1)})$ have to evaluate but $f(t_3, w_3)$, $f(t_2, w_2)$, $f(t_1, w_1)$ already evaluated when you perform equation (2).
If $w_4^{(2)}$ is not achieved desired accuracy, then

$$w_4^{(3)} = w_3 + \frac{h}{24}\left[9f(t_4, w_4^{(2)}) + 19f(t_3, w_3) - 5f(t_2, w_2) + f(t_1, w_1)\right] \tag{5}$$

If still not get required accuracy, we will have to repeat these processes.
If required accuracy will achieve (say $w_4^{(0)}$ or $w_4^{(1)}$ or $w_4^{(2)}$ or $w_4^{(3)}$), then

$$w_5^{(1)} = w_4 + \frac{h}{24}\left[9f(t_5, w_5^{(0)}) + 19f(t_4, w_4) - 5f(t_3, w_3) + f(t_2, w_2)\right] \tag{6}$$

where,

$$w_5^{(0)} = w_4 + \frac{h}{24}\left[55f(t_4, w_4) - 59f(t_3, w_3) + 37f(t_2, w_2) - 9f(t_1, w_1)\right] \tag{7}$$

[Here w_4 is either $w_4^{(0)}$ or $w_4^{(1)}$ or $w_4^{(2)}$ or $w_4^{(3)}$]
Equation (6) is repeated until get required accuracy.

Example. IVP: $y' = y - t^2 + 1$; $0 \le t \le 2$, $y(0) = 0.5$ and $N = 10$, so $h = 0.2$.

$w_0 = 0.5$	$w_1 = \alpha_1$	$w_2 = \alpha_2$	$w_3 = \alpha_3$	w_4	w_5	w_4	w_4	w_4	w_4	w_4
$t = 0$	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0
t_0	t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_9	t_{10}

For Predictor-corrector process; here $w_0 = \alpha = 0.5$ is known, so we have to evaluate w_1, w_2, w_3 with RK4, then using these values, we will get $w_4^{(0)}$ from (2). After getting $w_4^{(0)}$, we will obtain $w_4^{(1)}$ using (3).

$w_4^{(1)}$ is the approximate value at $t = t_4 = 0.8$. Such process is needed to repeat until $t = t_{10} = 2$.

Note.

$$w_{i+1}^{(k+1)} = w_i + \frac{h}{24}\left[9f(t_{i+1}, w_{i+1}^{(k)}) + 19f(t_i, w_i) - 5f(t_{i-1}, w_{i-1}) + f(t_{i-2}, w_{i-2})\right]$$

Here 'k' is the iteration index and i is the grid/node index.
Let $k = 0$; then,

$$w_{i+1}^{(1)} = w_i + \frac{h}{24}\left[9f(t_{i+1}, w_{i+1}^{(0)}) + 19f(t_i, w_i) - 5f(t_{i-1}, w_{i-1}) + f(t_{i-2}, w_{i-2})\right]; \quad i = 3, 4, 5, \ldots$$

So,

$$w_4^{(1)} = w_3 + \frac{h}{24}\left[9f(t_4, w_4^{(0)}) + 19f(t_3, w_3) - 5f(t_2, w_2) + f(t_1, w_1)\right]$$

which is equation (3).
If $i = 4$, then we will get equation (6) and so on.
If we put $k = 1$ and $i = 3$, then we will get equation (4) and so on.