Problem 0.0.1. Show that $J_{-n}(x) = (-1)^n J_n(x)$

Solution. We have,

$$J_{-n}(x) = \sum_{r=0}^{\infty} \frac{(-1)^r \left(\frac{x}{2}\right)^{-n+2r}}{r!\,(-n+r)!} \tag{1}$$

Let,

$$r - n = s$$
$$\Rightarrow r = n + s$$

From (1),

$$\begin{aligned} J_{-n}(x) &= \sum \frac{(-1)^{n+s} \left(\frac{x}{2}\right)^{-n+2(n+s)}}{(n+s)!\,(-n+n+s)!} \\ &= (-1)^n \sum \frac{(-1)^s \left(\frac{x}{2}\right)^{n+2s}}{s!\,(n+s)!} \\ &= (-1)^n J_n(x) \end{aligned}$$

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Thus, we obtain

Solution.

$$\begin{aligned} a2 &= -\frac{\alpha(\alpha+1)}{1 \cdot 2} a_0 \\ a_4 &= -\frac{(\alpha-2)(\alpha+3)}{3 \cdot 4} = (-1)^2 \frac{\alpha(\alpha-2)(\alpha+1)(\alpha+3)}{4!} a_0 \\ &\vdots \\ a_{2n} &= (-1)^n \frac{\alpha(\alpha-2)\ldots(\alpha-2n+2)\cdot(\alpha+1)(\alpha+3)\ldots(\alpha+2n-1)}{(2n)!} a_0 \end{aligned}$$

Similarly, we can compute $a_3, a_5, a_7, \ldots,$ in terms of a_1 and obtain hello there how are you asda ada asa

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