Designing Freemium: Strategic Balancing of Growth and Monetization

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Abstract

The Freemium business model, commonly used in digital products, has both a perpetually free but limited version and a premium version with enhanced features that requires a fee. These firms often have a referral program in which current customers get additional benefits (e.g., extra storage space) for referring friends to the firm. While this strategy allows a firm to acquire large user base at low cost, it needs to be balanced against revenue, and we examine how attractive a referral offer should be in order to encourage users to bring in new customers without dissuading them from upgrading. Since the majority of freemium customers are "free," i.e. they do not pay, it is crucial to understand their long run value, which results from their future upgrades or upgrades by those they refer. We develop an empirical model from microfoundations, including both discrete and continuous choices, and investigate the dynamics of consumers' plan choice, usage, and referral behavior using a novel panel data set from a leading cloud-based storage service. Using counterfactual analysis, we determine the optimal balance of growth and monetization, and quantify the value of free customers.

Keywords: *Freemium; Referral Programs; Business Models.*

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1 Introduction

Over the past decade, several companies have increasingly turned to the subscription model for revenue generation. Firms often offer a feature-limited but *perpetually free* version of their product (as opposed to limited period trial versions) in order to rapidly develop a large consumer base, with the expectation that users will upgrade to the paid premium version. This versioning based business model – referred to as "freemium," a hybrid of free and premium products – has been successfully adopted in Silicon Valley and largely popularized among the newer generation of start-ups. According to the New York Times, freemium is one of the most prevalent business models among Web start-ups because relying on advertising as the sole stream of revenue might not be sufficient or sustainable.[1] To date, over 80% of the top-grossing iOS apps have adopted the freemium model, with the largest freemium start-ups having acquired over hundreds of millions in venture funding.[2][3] The success of this business model has been further validated by many of the largest companies across multiple digital sectors, for example, online social networking sites such as LinkedIn, music services such as Pandora and Spotify, and cloud storage companies such as Box and Dropbox. Even media companies such as the New York Times (with its online pay wall) and mobile payment companies like Paypal utilize freemium. In the offline context, some consumer banks can also be characterized as using this model, with free checking accounts along with premium relationship accounts comprising the differentiated product offerings.

Freemium is often used by firms because of its ability to attract a large number of users to the free version, i.e., as a customer acquisition strategy. Start-up companies facing capital constraints often choose this strategy over investing in advertising or using a sales force to obtain new customers. When coupled with a powerful consumer-to-consumer referral program, its effectiveness in acquisition is often magnified since a free product is easier to recommend. As a result, companies using this strategy see that a large percentage, often 95% or more, of their consumer base are free users who do not contribute directly to the firm's revenues. While attracting a large user base is vital for establishing company value, firms must generate revenue for sustainability. Hence, it requires balancing dual tasks of growing the consumer base by offering a free service and maintaining premium services to incentivize upgrades in order to stay profitable (Needleman and Loten, 2012).

The freemium setting we examine features a leading online file synchronization and backup service. It raises several important questions in marketing and customer behavior that we empirically investigate. The

[1]"Ad Revenue on the Web? No Sure Bet," The New York Times, May 25, 2009. http://www.nytimes.com/2009/05/25/technology/start-ups/25startup.html.

[2]"Freemium apps continue to flourish in 2012." IntoMobile, December 22, 2011. http://www.intomobile.com/2011/12/22/freemium-apps-continue-flourishing-2012/.

[3]TechCrunch Crunchbase for Evernote, Pandora, 37 Signals, Spiceworks, and Dropbox, accessed June 5, 2013. http://www.crunchbase.com/company/.

research questions we examine include the following:

1. How do consumers decide whether and when to upgrade? Do they typically wait until they need the premium service before upgrading? Do premium customers demonstrate usage behaviors different from free customers?
2. How do consumers tradeoff upgrading to a premium plan compared to using other strategies like making referrals or actively deleting less useful content?
3. What is the optimal balance of growth and monetization from the firm's perspective? How does this change when the firm's objective is to acquire the largest customer base compared to the typical objective of discounted stream of profits?
4. What is the value of a "free" customer?

First, we aim to understand the drivers of upgrade, referral and deletion decisions made by consumers over time. Second, in our setting, the firm provides incentives for customers to make referrals, which are in the form of additional value in terms of permanent capacity increase, rather than monetary incentives. Such referrals may thus serve as a substitute to upgrade to the premium plan, since they enhance the value of the free product. Thus, there is a crucial tradeoff between growth and monetization. Increasing the incentive might result in a higher degree of incentivized word-of-mouth, but might result in lower long-term monetization due to diminished incentive to upgrade. Finally, we extend the idea of customer lifetime value (Gupta et al., 2006; Chan et al., 2011; Khan et al., 2009) to networked and platform settings (Sriram et al., 2015b), where a free customer may be valuable to a firm because they bring in (or refer) other customers who may become premium customers. Since the vast majority of consumers are "free" consumers, they are important for growth, and we characterize and quantify their value to the firm.

We use a unique panel data set of consumer activities to examine these questions relating to the freemium setting. There are multiple sources of value consumers obtain from the service. First, their files in their accounts are synchronized immediately across all connected devices, including computers, mobile phones, and tablets. Second, the files are backed up in the firm's online storage repositories, accessible from any Internet-connected computer using a Web interface. In the course of using the service, consumers add and delete files and also refer other consumers to the service; however, the primary revenue generating activity is when consumers upgrade from a free to a premium account, allowing for more storage capacity.

We develop a dynamic framework based on microfoundations to characterize the dynamic behavior of consumers in this setting. Consumers using the free version of the product have a *baseline quota capacity* for storage, and in each period they choose whether to upgrade to a paid premium plan that provides them an *enhanced quota capacity* for storage.[4] In addition to this decision, they also have the choice to refer a friend and obtain a referral bonus quota if the friend becomes a customer of the firm by subscribing to either the free or premium service. Consumers also choose to actively manage their storage by deleting files to maintain the limited space in their accounts, freeing up storage for future use. In our model, consumers thus obtain a *flow utility* from the amount of storage currently used, as well as decision or *action utilities* corresponding to the addition, deletion, or referrals. They face a potential disutility related to the decision to upgrade their service by paying a price. Note that the benefits of upgrading accrue over time, since the action increases the constraint on storage from the free quota (2 GB) to the premium quota (50 GB).

In the freemium setting, inter-temporal dynamics and trade-offs play a highly significant role, since the consumer predicts future usage (and available storage) in determining the tradeoff of current decisions on upgrading, deleting or referring friends weighted against the costs of those decisions. Thus, we model users as forward-looking consumers, who trade off the cost of upgrading to a premium plan with the cost of finding and determining older files to delete, when newer content needs to be synchronized over time, as well as the likelihood that they will hit the limit of the free product. Consumers also refer friends to the service, and while they receive a referral bonus when the friend joins, the referring consumer is not able to control the timing of joining and thus forms an expectation over how many of his referred friends might join during each period. Our microfoundations-based model thus jointly incorporates discrete and continuous choices for consumer upgrade, usage, and referral behaviors, allowing consumers to dynamically balance the tradeoffs between these decisions.

The estimation of our dynamic structural model involves several computational challenges, given that the state space has both continuous (amount of usage) and discrete (type of plan, referrals accepted by friends) dimensions. In addition, whereas upgrading and referral behavior are discrete choices, the number of files to delete in order to to create free storage space is a continuous decision, complicating the modeling and estimation process. We find that our likelihood function is highly irregular and jagged, making it important to use a robust method to obtain the global maximum. We use a conjunction of different approaches to overcome these computational and estimation-related challenges. First, we use a Bayesian methodology, using a modified version of the Imai-Jain-Ching (IJC) algorithm (Imai et al., 2009) that helps deal with the complex, highly irregular likelihood function. Second, we make extensive use of quadrature approaches to computing integrals for the likelihood to improve accuracy and computational time.

Below, we summarize our findings. Recall that consumers obtain flow utility from having an amount of storage to synchronize and back up their files, as expected since it is the primary value of the service. They also have a high and convex cost of deleting files, likely from being able to pick appropriate files that are no

[4] The free and premium service plans are identical in all aspects except for the storage quota.

longer needed in order to maintain sufficient free storage capacity for future usage. Consumers also have a negative utility, or a cost of referring friends to the service, and weigh that against the probability that the referral will be accepted as well as the firm's offered amount of increase in baseline quota from the referral bonus incentive.

We next focus on counterfactuals using the parameter estimates to help answer the firm's design questions that were detailed earlier. Recall that the referral bonus corresponds to more storage for each successful referral added to the baseline quota. Examining the impact of changing referral incentives is crucial since higher referral bonuses can directly affect growth, or the speed of product adoption, and therefore help the firm rapidly reach a critical install base. However, the negative impact of providing more storage through referrals is due to the cannibalization effect: the free product augmented with referral bonuses now becomes closer in product space to the premium product, resulting in more intense substitution with the premium product. Achieving a balance of these impacts is critical to the growth and success of firms in the freemium business model.

Even without considering the cost of supporting free consumers, we find a saturation point in the relationship between referral incentives and the overall user referrals. In other words, giving away too many referral incentives may actually decrease the overall number of referrals sent, as compared to the overall number of referrals in a scenario that gives away a moderate amount of referral incentives. This result may initially appear counter to our intuition, but here's the underlying intuition for this effect: if a consumer can receive the same amount of bonus space for one referral, which may be sufficient for his use, then the consumer might have lower incentive to send out additional referrals. This is especially true for a segment of customers who do not value storage as highly as "power users." More generally, we find that the shape of the consumer response of referral incentives either plateau after a certain level of incentive or an inverted-U-shape, implying that the firm should neither offer too small of an incentive (MB), because consumers may not find it worthwhile to refer anyone, nor too large of an incentive, because it may limit consumers' motivation to send out higher numbers of referral invites. We find the optimal static incentive amount to be approximately double the incentive observed during our data time period. Lastly, we find that the real strength of referral programs comes in accelerating the time to referrals, as we observe a reduction as high as 20% in the time to first referral acceptance in our counterfactual.

From a managerial perspective our findings have several implications. The existence of a large proportion of free consumers makes it difficult to assess firm value and future potential for a start-up entrepreneurial firm: the firm observes zero cash flow from free consumers, making it challenging to accurately project the future stream of cash flows from the consumer base. The profitability of these services depends heavily on repeated consumer usage of features. Without a model of consumer behavior, it is difficult to predict how consumers will respond in usage behaviors in order to compensate for the change in incentives for referral. Without an accurate understanding of consumer value and how it dynamically evolves over time, it is difficult to understand the impact on monetization and the tradeoff with growth.

Overall, our contributions are in developing a dynamic structural framework for the freemium business model that helps investigate the crucial tradeoff between growth and monetization by using the referral program as a key lever. Substantively, we uncover the intertemporal tradeoffs in consumer behavior and quantify the benefits to storage, as well as referral and deletion costs. We also conduct counterfactual analyses to determine the optimal design of the referral program. Methodologically, we advance the incorporation of both discrete and continuous choices in this framework and demonstrate how to reduce computational burden.

Our work can be extended along a number of dimensions in future research. First, we currently model consumer behavior conditional on adoption of the service, allowing consumers to only choose between free and premium plans. In other settings, there may be multiple premium product versions to choose from. We note that the data set is from a period when this firm had little competition.

Related Literature

Our work intersects multiple domains of literature from a substantive viewpoint: *consumer-to-consumer referrals*, *product sampling*, *and product line design.* There are a number of studies that examined the role of word-of-mouth, which broadly focus on two different aspects of consumer to consumer communication. Earlier research focused on organic word-of-mouth, e.g. people taking to their friends about a recent movie (Godes and Mayzlin, 2004). Word of mouth was found to play an important role in such settings, and could have a positive impact on sales and could be as important (or more) compared to traditional marketing interventions like advertising (Trusov et al., 2009). The second aspect, to which this paper is more closely tied to has received more attention in theoretical rather than empirical treatment. More specifically, in referral programs, studies have examined the design (Buttle, 1998; Silverman, 1997), and optimizing overall referrals (Biyalogorsky et al., 2001), and more broadly on business that serve as referral infomediaries (Chen et al., 2002). Related research has investigated how brand impacts referral, and has found through experiments, that for strong brands, it is good to reward both the sender and the receiver of the referral in order to maximize referral rates (Ryu and Feick, 2007). Such a design feature is also used by the referral program of our study context.

Another related literature is product sampling. Prior studies on digital goods focused on the fact that they are experience goods and contended that consumers require time to obtain value from these goods and services (Jain et al., 1995; Heiman and Muller, 1996; Lehmann and Esteban-Bravo, 2006; Heiman et al., 2001; Chellappa and Shivendu, 2005). Therefore, firms can influence the propensity of a consumer to adopt a service by providing *free trials* or *free samples (Bawa and Shoemaker, 2004)*. Our context, however, differs temporally. In lieu of offering a limited-time free trial, the freemium model offers a perpetually free product, which can end up serving as a close substitute. Therefore the issue of cannibalization of the premium product is of significant concern. A growing body of literature is emerging that tackles these issues in the form of theoretical models that explore the economics of freemium (Niculescu and Wu, 2013), but given that the dynamic long-term effects are of first-order importance, the paucity of empirical (and even theoretical, with a few exceptions) research is striking. Another related study focusing on fee versus free pricing suggests that content providers may be able to adjust the amount of free and premium content counter-cylically in response to demand conditions (Lambrecht and Misra, 2016).

From a methodological perspective, our work follows the stream of dynamic discrete-choice structural models in the tradition of Rust (1987). To our knowledge, while there are other models of discrete and continuous choice models (Hanemann, 1984; Song and Chintagunta, 2007), we are one of the first studies in marketing to incorporate multiple discrete and continuous actions in a dynamic structural model and to estimate it using a technique that recovers the value function. Bajari et al. (2007) and Ryan (2012) use BBL to estimate a dynamic structural model with both discrete and continuous actions; however, for counterfactuals, the value functions of consumers must be recomputed. Several authors have examined constrained continuous choices, most notably Timmins (2002), who uses discretization in conjunction with grid inversion to obtain the unobservable shock corresponding to the continuous choice.

Next, we detail the institutional context and the relevant features of the service. In section §2, we describe the details of the data set that we use, as well as model-free evidence that supports our initial conjectures of the value of free consumers. In §3, we develop the model from microfoundations, and in §4, we detail the estimation procedure. In §5, we present the estimation results and the findings of various counterfactual analyses. Lastly, we conclude with a discussion of the managerial implications and avenues for further research.

2 Institutional Setting and Data Description

The freemium company that provided the consumer data is a leading online storage company that stores consumer files in the cloud, synchronizing across multiple devices (e.g., laptop, desktop, and mobile phone). The company was founded in the 2000s and currently has hundreds of millions of users world-wide. Later in 2012, major competitors entered the space by introducing similar versions of the service.[5] During the time period of our data, the cloud storage industry was fragmented among smaller providers. However, our focal firm quickly emerged as a dominant player in *synchronization*, while many potential competitors acted primarily as *backup* services. The firm's product and service offerings evolved to include shared usage, where customers could collaborate on projects using shared folders. However, our data comes from an earlier stage in the company's offerings where the primary offering was personal usage.

Usage: The value of the service is for consumers to store and sync files in the cloud and to share files with other users. Consumers do this by installing an application on their desktops. This application appears as a special folder on the consumer's desktop. Consumers can then add files to their accounts by simply dragging files into the folder, as one would do with any normal folder. Once the files are added to this folder, copies of the files are then transferred onto the firm's servers and can be accessed through all of the devices which a consumer has the service software installed or via an online interface (similar to the workings of a web-mail interface). While a consumer can access his files through different means (e.g., desktop, mobile devices, or web interface), the primary method that consumers use to access the service at the time of our observation is via their desktops, and therefore we focus on this point of usage in our analysis.

Plans: Once the files are stored in the account, they take up space that counts towards an account quota. All consumers are presented with the choice of different plans: Free, Premium-Tier-1 and Premium-Tier-2. Free is the basic plan with which the consumer receives 2 GB of quota; Premium-Tier-1 and Premium-Tier-2 are the premium plans with which consumers receive 50 and 100 GB of storage, respectively. We focus on the first two plans and do not consider the third, given the lack of consumers in our sample who choose that plan. We refer to the Premium-Tier-1 plan as the *premium plan* hereafter. These premium plans work on a subscription basis, with the options of monthly or yearly payment plans, and consumers in our sample choose the monthly plan.[6]

Referrals: A consumer can earn additional storage space through the referral invite program. In order to use the referral program, a consumer can send out a unique link to other consumers who have yet to join; this unique link includes an identification number that links the invites back to the original sender. There is no limit on the number of referral invites that one can send out, but for the referral to count towards one's quota bonus, the friends must join using the attached, unique link. In addition, the referral invite works "both ways," in that a consumer joining through the original invite also receives an additional 250 MB of space. Hence, senders have the incentive to always include a link with their word-of-mouth, and receivers

[5] All of these other competing services also use the freemium model.

[6] Hitting the Quota: When a consumer runs out of space, any files that the consumer adds into the service will no longer be uploaded to the server. Most importantly, all file synchronization stops, and since this is the primary value proposition of the service, the software is rendered virtually useless to the consumer. We observe from the data that almost all users tend to leave "cushion space" in their accounts or delete files when they need to add files larger than available space, or upgrade their plan.

have the incentive to join via the links.[7] Therefore, while there may be some cases in which consumers will not be identified as "referred" consumers in our data and bias our results of the effect of WOM and the usage behavior of non-referred consumers, this problem may be at a minimum. A consumer accepts the referral invite by signing up for the service. Once this is done, the original sender receives credit for the invite acceptance and earns an additional 250 MB of space for permanent free use. While this is a very effective way for consumers to gain space, a consumer can only receive credit for a maximum of 32 acceptances.[8]

Data

We detail the characteristics of our data set and describe model-free evidence that will help us motivate the mechanisms for the model described. We obtain a representative random sample of $N = 500$ anonymous consumers from the focal firm who joined during the first two years of the firm's history.

The random sample seed window includes the first two years after the launch of the service. We then obtain all of these consumers' user activities from their join dates until December 31, 2011. Our panel data include the detailed click-stream data of these consumers over the four year period, which we aggregate into a monthly level. We observe a number of consumer behaviors that are relevant to our analysis. These activities include:

- Total number of files stored and the storage amount (MB)
- Amount of files deleted (MB)
- Amount of files added (MB)
- Number of referral invites sent to consumers not already joined the service
- Number of referrals accepted each period
- Plan choice and payment plan when upgrading.

Many freemium companies observe premium-to-total consumer ratios ranging from the single digits to over 10%. Therefore, we observe a total of 202 incidences in which customers have chosen the premium plan. In Table 1, we examine the key features of the data.

[7]One might be concerned that this incentive system may encourage consumers who are already planning to join to actively seek out invites from other consumers. If this were the case, then these "willing" consumers may already have a favorable disposition towards the service and are more likely to behave favorably towards the service (e.g., use the service heavily, send out more invites, more likely to upgrade to a plan later on). We acknowledge that this will bias our results upward and one way to possibly check for the existence of this behavior is to conduct surveys on existing consumer population.

[8]As with any reward system, we have to be aware of of consumers trying to gain more space by "gaming" the system, mainly by creating clone accounts using additional email addresses. The firm is aware of this and spends significant resources exactly to correct these gaming behavior in consumers by not rewarding false referrals. For instance, they can verify the source of two very different emails from the same consumer by verifying whether these clone accounts install the software on the same device using machine footprints such as MAC addresses. Because of the efforts from the company in correcting gaming behavior, we assume that the integrity of our data is not compromised by this behavior.

Table 1: Summary Statistics Across Consumers in Sample

Statistics	Mean	SD Across Population	MIN	MAX
Number of Consumers	500	-	-	-
Total # of Observations	13,438	-	-	-
Time Periods (Months)	26.9	5.1	21	48
Avg. Monthly Storage Level	412 MB	1.01 GB	1.05 MB	9.65 GB
Tot. Addition	2.59 GB	7.89 GB	1.4 MB	94.1 GB
Avg. Monthly Addition	94.7 MB	307 MB	0.2 MB	4.09 GB
Tot. Deletion	1.81 GB	6.69 GB	0 MB	83.5 GB
Avg. Monthly Deletion	65 MB	264 MB	0 MB	3.6 GB
Total Referrals Sent	2.65	6.54	0	32
Tot. Referral Accepted	1.08	3.47	0	32
Referral Acceptance Rate (Accepted/Sent)	46.7%	38%	0 %	100 %

Note that during this time period, the primary value proposition of the service is in its across-device syncing and back-up features. Therefore, most of the customers in our sample are using and referring customers for this purpose primarily. From our interaction with the company, it is noted that only in the later years did a majority of customers' usage and referral behavior shift to a focus on sharing.

2.1 Model-Free Data Patterns

In this section we examine the data patterns of consumer behavior with the goal of clarifying the key data features that our model needs to characterize, which inspire the major design choices of our model. Ultimately, we observe a pattern in the data that justify the value of a free consumer. First, consumers upgrade themselves over time as they become closer to reaching quota. This is the first value of the free customer.

Upgrade and Storage: We observe two patterns from examining Figure 1. First, customers take many weeks before they upgrade to premium. In our sample, the fastest customer to upgrade does so after using the service for 15 weeks. The left panel shows that a majority of premium consumers upgrade within the first year of using the service. The graph on the right shows the growth of the average consumers storage within his first 90 weeks. This graph is from the perspective of the consumers, in that we see the average storage used per consumer grow over time. This suggests that consumers begin using the service as free consumers and later upgrade to a premium plan once they store enough files. **Quota and Upgrade:** Another important pattern in the data is that customers upgrade before their accounts become full. Figure 2 shows the customer upgrade probability at different storage levels. Two things stand out from the graph. The first is that as the storage level approaches the free quota in storage capacity, the upgrade probability steadily rises. Second, there are customers who upgrade when their quota is far away from being full, therefore indicating that they could potentially be upgrading in anticipation of higher future storage levels. From direct communication

Figure 1: Upgrade Patterns

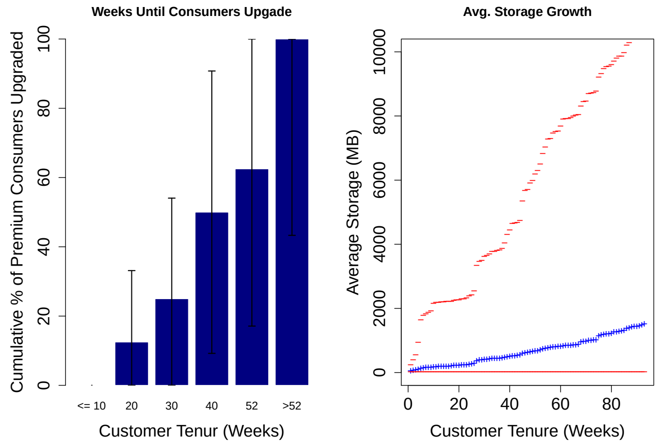
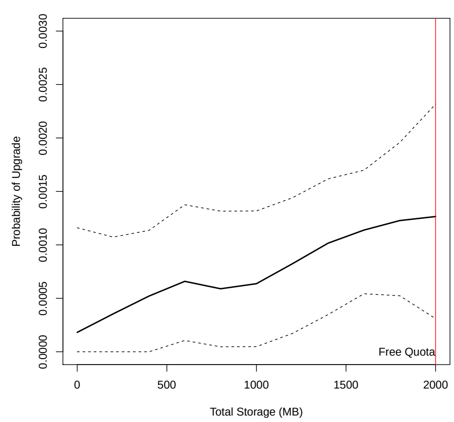


Figure 2: Upgrade Probability by Usage Level



with a set of customers, one of the main reasons why they would upgrade before a full account is because they want to avoid hitting quota when they need to receive files for new or active projects they would have to place and store in the service. **Deletion and Referral:** We are interested in examining whether consumer behavior changes as they approach the quota. For deletion, we might expect that consumers delete more when they are closer to quota, to retain free space for additional new content. However, in examining this pattern, we might have a selection effect, where some consumers might have more activity, impacting both deletion and usage, which might cause these consumers to cluster around the quota. Therefore, we difference out the average deletion across time from each consumers decisions, and examine $\left(d_{it} - \bar{d}_i\right)$ as the demeaned deletion, as measured in GB's.

In Figure 3, we find that within consumers, deletion increases significantly as we approach the quota. In addition, if customers upgrade to the premium plan, we would expect to see the amount of deletion to be

Figure 3: Deletion vs. Storage

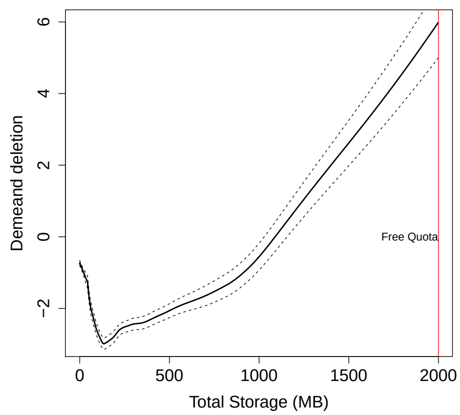


Figure 4: Premium Customers Deletion vs. Storage

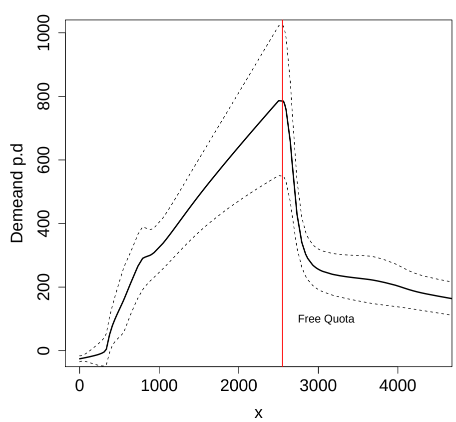


Figure 5: Referrals vs. Storage.

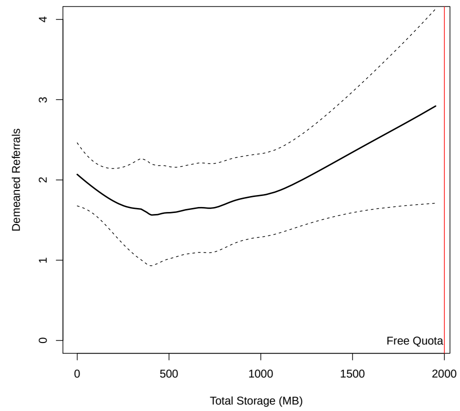
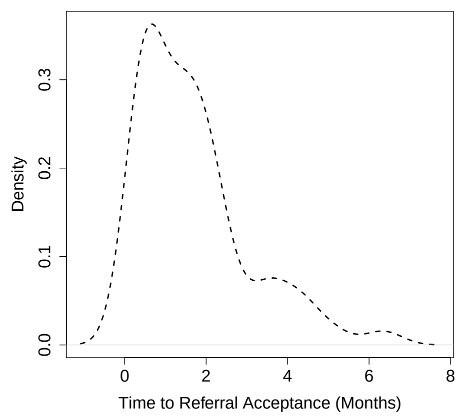
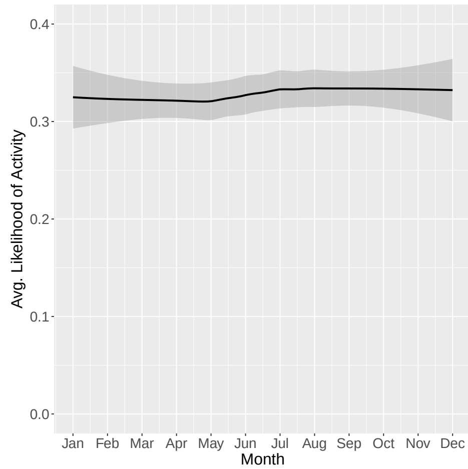


Figure 6: Referrals Acceptance Time.



lower in the premium storage region. In Figure 4, we see that for the premium customers, the average deletion (pre-upgrade) indeed rises as they approach the free quota; however, the deletion activities becomes lower once customers' storage levels are in the premium region. In Figure 5, we examine the within consumer variation in referrals, and find that consumers increase their overall referrals sent as they approach the quota. Whereas with deletion, consumers immediately obtain more available capacity, for referrals, they only receive the permanent increase in quota when the person receiving the referrals installs the software on their system, which typically takes time. Figure 6 illustrates the distribution of referral acceptance times. **Seasonality:** Seasonality is another dynamic aspect of consumer behavior. However, since this is a storage and synchronization service that is used continuously, we would not expect to see strong indicators of seasonality. At an aggregate level, we do not see usage patterns indicative of seasonality. We examine this by examining the aggregate likelihood of monthly customer activity (addition, deletion, referral or upgrade)

Figure 7: Probability of Customer Activity by Month.



in Figure 7.[9] We see that the aggregate activity level does not vary significantly from month to month. Broadly, the data patterns help inform us about the aspects of the model to focus our attention to. We develop a model below that incorporates the endogenous usage, referral and plan choice behavior, accounting for uncertainty along a number of dimensions.

3 Model

The freemium business model is defined as a firm offering at least two differentiated variations of its service: one version with limited features but *perpetually free* and the other versions with enhanced features at the cost of a subscription fee. The free and premium plans are identical in terms of quality, and the products differ only in terms of the additional premium features offered (e.g., increased storage capacity quota).

The most valuable asset of free consumers remains in their potential—the potential to upgrade, the potential to refer friends to join the service. Understanding these factors is key to the success of the firm. A structural model that characterizes the dynamic response of consumer behavior is therefore critical for the following reasons. First, we need to account for multiple dimensions of consumer choice—referral, plan choice, and deletion—in an integrated model of consumer behavior. Customers send out referral invites to friends to earn additional free space. However, their motivation to upgrade or delete can be diminished by the extra space earned from referrals. In addition, consumer deletion behavior inherently differs according

[9]We first construct customer activity as a binary variable coded as a 1 if there is any addition, deletion, referral or upgrade activity in a given month for each customer. Then, we find the empirical probability of the entire population to have activity in a given month by calculating the mean of this variable by months. Then, we constructed the confidence intervals of each month by estimating a simple logistic regression that uses the month variable to predict the activity variable. Since the average customers has close to 2 years of data, half of the periods they are define as "inactive." We also note that when we examine the nonparametric relationship between likelihood of activity and storage level, we observe a slight dip in probability of activity when users are close to the quota, and we speculate that this could be due to some users signing up for alternative services to get storage.

to their chosen plans. Those who have chosen the free plan may have to delete more in order to maintain enough space to store files, and those who have chosen the upgraded plan do not have to delete much due to the wealth of new space. The decisions are endogenous, and we need a methodology that can explicitly model these *highly interdependent* choices of the consumer.

Second, we need a structural model because we wish to conduct counterfactual experiments to observe the effects of changing firm policies on consumer behavior. With atheoretical models, the outcomes of changes in certain product design variables, e.g., referral incentives, cannot be readily characterized as there are often no variations in these variables during the observed data period. A model based on microfoundations of consumer behavior uses theory about consumer behavior as a complement to data to enable us to recover primitives of consumer preferences, which are likely to be invariant to changes in these product design and other policy variables. These preference parameters can then be used to evaluate how consumers would make choices in a counterfactual scenario, enabling us to provide recommendations of managerial interest.

A fundamental process for which we need to account in our model is the inter-temporal tradeoff in upgrade, referral, and deletion behavior. Below, we provide a characterization of how consumers use the service and the tradeoffs in making these choices. The source of dynamic behavior comes from a combination of three factors: a) uncertainty in file addition, b) substantial penalty of a full account, and c) uncertainty in ease of upgrade, deletion and referral.

Learning is often an important aspect of consumer dynamics in many settings (see the excellent review by Ching et al. (2013)). However, we abstract away from modeling learning in this study. In environments where learning is important, we might expect to see limited-time offers to facilitate consumer learning. In fact, one of the important design features in Freemium is that the free product is not a limited-time (or trial period) offer, rather it is designed to be available perpetually. First, unlike in typical choice contexts where consumers are modeled as obtaining quality signals with each purchase, we don't see switching over time between choices, which are indicative of learning (Sriram et al., 2015a). Learning is typically identified by reduced variability in choice switching, as consumers become less uncertain over time. Second, in our setting, consumers use the product continually (often several hours per day), and thus in a short time, they typically would have experienced a significant reduction in their uncertainty regarding product quality. We focus therefore on long-run dynamics, where learning is unlikely to be a first-order modeling consideration.

Uncertainty in File Addition

First, consumers face uncertainty when anticipating additional storage needed for files each period, as the need for addition is often stochastic. In order to store files, a consumer requires free space and therefore must select a plan that fits the amount of data he will receive in the current period. When faced with the space constraint, a consumer can either upgrade to a higher quota premium plan, gain additional space from referring others to join (and having them join), or delete files to make space immediately.

Uncertainty in Ease of Upgrade, Deletion, and Referral Decisions

The upgrade decision is affected by dynamic factors seen in examples of time period specific unobserved factors from our discussion with current consumers. Please see Section A.3 in the Appendix for detailed discussion.

If a consumer chooses not to upgrade, he has the choice to gain more space from deleting files. However, similar to the case of upgrades, consumers also faces an uncertainty in the ease (or cost) of deletion. Users of the service typically express that it is easier to delete during certain months while harder during others, (i.e., deadlines at work or exams at school), and we observe this in the data as lumpy deletion patterns. This forces the user to continually consider the tradeoff of whether to upgrade in the current period in order to reduce the deletions that he may have to make in the future. Therefore at a certain point it may be optimal for a consumer to upgrade in order to outweigh the cost of continually deleting in the future.

A consumer faces two forms of uncertainty with regard to referrals. The first is the ease of referral from month to month. Second is the uncertainty of when the invitation will be accepted, given that there is a substantial lag before acceptance. Therefore, a consumer cannot simply send out invites the month that he runs out of space. He must consider well in advance how likely his invites will be accepted and if his usage will be sufficiently below quota by the time the additional free space is acquired.

3.1 Sequence of Events

The primary value of the service is to help consumers store and sync files in cloud storage. A consumer's total storage capacity quota depends on the plan as follows: 1) *Free*: Plan with 2 GB of storage and 2) *Premium*: 50 GB for $10/month.

In each time period t (month), a consumer $i \in \{1, ..., N\}$ chooses three decisions to maximize his utility: 1) whether to upgrade to a premium plan or remain a free consumer, 2) how many consumers to send referral invites, 3) storage amount of files (i.e., how many MBs) to delete.

At the beginning of the period t, a consumer i observes all state variables, and then he observes a_{it}, the amount of data (MB) that need to be added to his synchronized folders, and then proceeds to make plan choice and referral decisions, and then deletion decisions. We define the state space to include current used storage (x_{it}), plan status or current plan periods remaining (z_{it}), cumulative referrals sent (R_{it}) and cumulative referrals accepted (R_{it}^a):

$$s_{it} = (x_{it}, z_{it}, R_{it}, R_{it}^a)$$

1. Observe all state variables $(x_{it}, z_{it}, R_{it}, R_{it}^a)$, and exogenous a_{it}, the *amount of data* which need to be added to a consumer's synchronized folders.

2. Realize the number of friends who have accepted i's referrals, r_{it}^a, based on the total number of outstanding invitations $r_{it}^a \leq (R_{it} - R_{it}^a)$.

3. Observe discrete-choice shock for upgrade/referral choices, $\epsilon_{it}(y_{it}, r_{it})$ and then choose decisions y_{it} and r_{it} simultaneously.

4. State variable z_{it} updates based on the upgrade decisions y_{it}. State variables R_{it} updates based on referral decision r_{it}, i.e., $R_{it+1} = R_{it} + r_{it}$.

5. Take a draw of $r^a \sim binomial(R_{it} - R_{it}^a, p^a)$, where $p^a = f(z_{it}, R, R_{it}^a)$.

6. State variable R_{it}^a updates based on realized r^a, i.e., $R_{it+1}^a = R_{it}^a + r_{it}^a$.

7. Observes continuous-choice deletion shock ν_{it} and choose deletion decisions d_{it}.

8. Update state variable x_{it}, where x_{it} is the used storage (e.g., total amount of data stored in the folders) combined at the end of the period. $x_{it+1} = x_{it} + a_{it} - d_{it}$

We begin the period with the exogenous addition, since addition of files is the fundamental value proposition of the service, and all other consumer decisions depend on the amount of data that need to be added. The consumer then observes the value of r^a, which is stochastic and follows a binomial distribution, parametrized by p^a, the acceptance probability in the population and $(R_{it} - R_{it}^a)$, the number of outstanding referral invites, where R_{it} is the cumulative number of referral invites sent, and R_{it}^a is the cumulative accepted referral invites.

Then, the consumer makes the joint decision of plan choice (y_{it}) and the number of referral invitations to send (r_{it}). For the upgrade option, we specifically look at two options:

$$y_{it} = \begin{cases} 1, & \text{if customer upgrades with the monthly payment option,} \\ 0, & \text{if customer does not upgrade.} \end{cases}$$

For option $y_{it} = 1$, a consumer upgrades from a free to a premium plan and pays a price P^m.Simultaneously, a consumer determines the number of referral invitations to send to other consumers who have yet to sign up. This is modeled as a discrete count variable bounded by R^{max}:

$$r_{it} \in \{0, 1, ..., R^{max}\}.$$

The state variables z_{it} and R_{it} update according to the chosen y_{it} and r_{it} choices. z_{it} is the number of premium plan months left in one's account, and $z = 0$ for consumers who are in the free plan. R_{it} is the total number of referrals sent. The state variable R^a_{it}, the total number of accepted referrals, also updates according to the realized value of r^a. Then, the consumer observes the continuous-choice specific deletion shock ν_{it}. This is interpreted as the idiosyncratic factors that make deletion easier or harder. For instance, one period, a consumer may find it harder to delete from his folders because he is traveling, so ν_{it} would be a low value. Another period could represent cleaning, which makes it easier for a consumer to delete unnecessary files, rendering the shock high. Lastly, we update the state variable x_{it}, which is the cumulative amount of *storage used* in a consumer's account.[10]

3.2 Period Utility Function

We now describe how each part of the timeline component contributes to a consumer's period utility, separating out the flow utility from action utility. Note that the consumer's decision-making process is not solely based on this utility, but is based on inter-temporal trade-offs as described previously. We suppress the i subscript for all variables and homogeneous parameters for expositional clarity, even though the model is at an individual level. At each time period t, a consumer gains utility from having files stored in his account and from having folders that are free of files he no longer needs. In addition, he incurs a cost for the effort to delete files, to pay to upgrade to a premium account, and to send out referral invites to his friends.

We express the period utility in Equation 1 as:

$$u(x_t, d_t, r_t, y_t) = \underbrace{\theta_i x_t}_{\text{Storage Benefit}} + \underbrace{\alpha d_t^2 + \alpha_0 d_t \nu_t}_{\text{Deletion Utility}} + \underbrace{\rho_1 r_t + \rho_2 r_t^2}_{\text{Referral Utility}} + \underbrace{\gamma P^m \mathbf{1}\left[y_t = 1\right]}_{\text{Upgrade Cost}} + \epsilon_t\left(y_t, r_t\right). \tag{1}$$

Let $\theta_i x_t$ be the *storage benefit*, or the utility contribution of storage, where θ_i is the file storage benefit coefficient. We employ a quadratic cost (utility) function for the deletion action. Let $\alpha d_t^2 + \alpha_0 d_t \nu_t$ be the deletion utility, where (α, α_0) is the vector of coefficients of deletion cost. Similarly, we also employ a linear and quadratic cost functions for the number of referrals, specified as $\rho_1 r_t$ and $\rho_2 r_t^2$, respectively; ρ_1 are ρ_2

[10] *Model Timing Considerations:* While decision timing is unlikely to significantly alter the results of a dynamic infinite horizon model, we still discuss the implications of other possible orders of the consumer decisions. First we might consider the sequencing of the deletion decisions. There are two possibilities: 1) the deletion decisions occur before the upgrade/referral decisions or 2) all decisions are made simultaneously. First, note that if we were to place deletions first, we would be assuming that customers have the same deletion behaviors regardless of whether they opt for the status quo or they gain more space from upgrading/referring. Given the institutional context, this is inconsistent with their understanding of actual consumer deletion behavior—a key reason why customers refer or upgrade is because they want more storage space. Therefore, customers do not need to delete as much in a premium plan than in a free plan.

are the coefficients of referral costs. Note that since r_t is a discrete decision, we do not have a multiplicative shock ν_t as we do with the deletion utility.

In addition, we specify $\mathbf{D}_t$ as the vector of decision variables such that $\mathbf{D_t} = (y_t, r_t, d_t)$. $\mathbf{S_t}$ is the vector of observable state variables such that $\mathbf{S_t} = (x_t, z_t, R_t, R_t^a)$. ϵ_t is the vector of discrete-choice private shocks related to the joint upgrade and referral decision, such that $\epsilon_t = (\epsilon_t(y=0, r=0), ..., \epsilon_t(y=1, r=r^{max}))$. Θ is the vector of structural parameters to be estimated such that $\Theta = (\{\theta_i\}_1^N, \alpha, \alpha_0, \gamma, \rho)$. The storage utility term, a *flow utility*, contributes to a consumer's utility each period that files are stored in the account.[11] The deletion, referral, and upgrade utility terms, all *action utilities*, only contribute to a consumer's utility when the actions are taken each period. Below we examine each of these components of the consumer's utility.

- **Storage**: At each period, a consumer receives utility from using the service. x_t denotes the current used storage.

- **Deletion**: The utility specification for deletion allows a parsimonius but flexible form to allow consumers to either bunch their deletion activity or intertemporally smooth such activity (there may be a potential convex cost to deletion) depending on the signs of the parameters.[12] The constraints on how much a consumer can delete is enforced implicitly as being in a range with minimum deletion required to maintain the storage within quota, and the maximum restricted to the currently used storage. Please see Section A.1 of the Appendix for a detailed discussion on the derivation of the deletion bounds.

- **Referral**: The benefit and cost from referring other consumers similarly has a number of considerations. Inviting might incur costs, ρ_1 and ρ_2 are the coefficients of the convex transaction and reputation cost that a customer incurs for sending out an additional invite. The convex cost reflects the fact that it becomes increasingly difficult for customers to consider an additional friend to invite in any given month. Accepted referrals result in a *permanent* additional storage quota capacity of m. r_t is the number of referral invites that a consumer sends at time t. The benefit of an accepted referral is reflected directly in the current quota capacity Q_{it}. The uncertainty that customer faces with regard to how many referrals will be accepted in each period is captured via the binomial distributed shock $r^a \sim Binomial(n, p)$, with parameters $n = R_t - R_t^a$ and $p = p^a$, which are realized at the end of each period.

[11] The flow utility here is similar to a consumer receiving flow utility in each period after purchasing a durable good (e.g., a car or television).

[12] A potential reason why deletion is an increasing function of amount to be deleted is that when the amount of storage to be deleted increases, is it likely that a user may take more time to look for files to delete. This is a common scenario when we had discussed with actual users of the service that when they delete files, they would have to sort through old photos, videos, or work documents and this takes time and effort. We thank an anonymous reviewer for reminding us of this reason.

3.3 State Evolution and Law of Motion

The state variable x_t tracks the total amount of data stored in a consumer's account. This is updated via the linear law of motion $x_{t+1} = x_t + a_t - d_t$, which is simply the sum of the amount of data observed at the beginning of the period and the observed addition amount, subtracted by the amount deleted in the particular period.

The state variable z_t keeps track of the number of periods until the consumer's next payment.[13] z_t is set to 1 if he chooses plan 1. z_t decreases by 1 each period. The state evolution for z_t is specified as:

$$z_t = \begin{cases} 0, & z_{t-1} = 0 \; and \; y_t = 0 \\ 1, & z_{t-1} = 0 \; and \; y_t = 1 \end{cases}$$

A proportion of the referral invites are actually accepted, and we keep track of the total number of successful invites as R_t^a. R^{max} is the empirical number of maximum per-period invitations in the data. Whenever an invite is accepted, a consumer gains an additional m MBs of space to his current quota capacity. More specifically, we define the variable Q_{it} as a function of the total number of successful invites (R_{it}^a), the baseline quota capacity (Q^{free}), referral bonus capacity (m), and incremental amount of space provided by the premium plan quota capacity ($Q^{premium}$):

$$Q_{it} = Q^{free} + mR_{it}^a + \mathbf{1}[z_{it} \geq 1]Q^{premium}$$

Table 2 below summarizes the state variables and the corresponding laws of motion.

Table 2: State Variables and Law of Motion

State / Shock	**Description**	**Type**	**Law of Motion / Distribution**
Q_{it}	Current Quota Capacity	Observed	$Q_{it} = Q^{free} + mR_{it}^a + \mathbf{1}[z_{it} \geq 1]Q^{premium}$
x_{it}	Storage used	Observed	$x_{it+1} = x_{it} + a_{it} - d_{it}$
R_{it}	Total referrals sent	Observed	$R_{it+1} = R_{it} + r_{it}$
R_{it}^a	Total referrals accepted	Observed	$R_{it+1}^a = R_{it}^a + r_{it}^a$
z_{it}	Months of premium plan remaining	Observed	$z_{it+1} = z_{it} - 1$
$\epsilon_{it}(y, r)$	Upgrade / referral shock	Unobserved	Type 1 Extreme-Value
ν_{it}	Deletion shock	Unobserved	Log-Normal
a_{it}	Addition amount	Observed	Non-Parametric Distribution

Next, we describe the dynamic considerations impacting consumer utility and the construction of the value function.

[13] We observe that 35% of the customers in the sample downgrade from the premium to the free, so therefore it is important to model this state variable.

3.4 Dynamics in Consumer Decisions

The dynamics in the consumer's decisions stem from the inter-temporal tradeoff of the consumer's current benefit versus the future benefits of upgrading to a premium account, deleting files to gain free space, and referring other consumers due to social and practical benefits. We model the consumer's problem as maximizing the sum of discounted period utilities:

$$\sum_{\mathbf{t=0}}^{\infty} \beta^{\mathbf{t}} \mathbf{E}_{a,r^a,\epsilon} \left[\max_{(y,r)} \mathbf{E}_{\nu} \left[\max_{d} u(\mathbf{D}_{it}; \mathbf{S_{it}}, \epsilon_{it}, \nu_{it}; \Theta) \right] \middle| \mathbf{S}_{it} \right] \tag{2}$$

where β is the assumed discount factor for all consumers and the period utility function is specified in Equation 1 above. The functional solution to the above dynamic programming problem, i.e., the value function satisfies the Bellman equation:

$$V(\mathbf{S}, \epsilon, a; \Theta) = \max_{y,r \in \Gamma} \mathbf{E}_{\nu} \left[\max_{d \in H} u(\mathbf{D}, \mathbf{S}, \epsilon, \nu; \Theta) + \beta \mathbf{E} \left[V(\mathbf{S}', \epsilon', a'; \Theta) \right] \right] \tag{3}$$

The integrated Bellman equation, EV, expresses the fixed-point that we solve to derive the solution of the expected value function with the discrete-choice shocks ϵ as well as a and r^a integrated out:

$$EV(\mathbf{S}; \Theta) = \mathbf{E}_{a,r^a,\epsilon} \left[\max_{y,r \in \Gamma} \mathbf{E}_{\nu} \left[\max_{d \in H} u(\mathbf{D}, \mathbf{S}, \epsilon, \nu; \Theta) + \beta \mathbf{E} \left[V(\mathbf{S}', \epsilon', a'; \Theta) \right] \right] \right] \tag{4}$$

The difference in interpretation for the expected value function is that it is the value function prior to consumers observing all shocks and therefore is expressed only as a function of the state variables $\mathbf{S}_{it}$. For expositional clarity, we decompose the EV into multiple stages according to the sequence of decisions: a) upgrade/referral discrete choice and b) deletion - continuous choice. Equation 4 can then be expressed as follows:

$$EV(x, z, R, R^a; \Theta) = \mathbf{E}_{a,r^a,\epsilon} \left[\max_{y,r \in \Gamma(z)} \epsilon(\mathbf{y}, \mathbf{r}) + \mathbf{V_{jk}} \left(\mathbf{x}', \mathbf{z}', \mathbf{R}', \mathbf{R^{a'}}; \mathbf{\Theta} \right) \right], \tag{5}$$

where z', R', and $R^{a\prime}$ are the updated state variables after the decisions y and r, such that $R' = R + r$ and $R^{a\prime} = R^a + r^a$. In turn, a discrete-choice specific value functions for choices $y = j$ and $r = k$ can be specified as:

$$V_{jk} \left(x', z', R', R^{a'}; \Theta \right) = \mathbf{E}_{\nu} \left[\max_{d \in H(x', z', R^{a\prime})} u + \beta \ \mathbf{EV} \left(x', z', R' R^{a'}; \Theta \right) \right]. \tag{6}$$

x' is the intermediate state of x and is updated as $x' = x + a$. $\Gamma(z)$ is the the choice set for the discrete decisions y and r, specified as $(y, r) \in \Gamma(z)$ where:

$$\Gamma(z) = \begin{cases} \{0,1\} \times \{0,...,R^{max}\}, & \text{z=0} \\ \{0\} \times \{0,...,R^{max}\}, & \text{otherwise} \end{cases},$$

again reflecting the fact that when a consumer is already on the premium plan ($z > 0$), he has no choice to make regarding the (free versus premium) product, and so the choice is trivially set to 0. For the referral choice a customer can choose to send out $r = \{0, ..., R^{max}\}$ number of referrals.[14] u is the component of the utility that excludes the discrete-choice shock $\epsilon(y, r)$. The choice sets for the continuous deletion decision d is specified by the correspondence constraint $H(\cdot)$:

$$H(x', z', R^{a\prime}) = [\max(0, x' - Q(z', R^{a\prime})),\ x'],$$

reflecting the fact that consumers cannot delete more than the total amount stored and must delete a sufficient amount so that they do not exceed their quotas.

Consider an example, where the consumer has stored $x = 1.5$ GB, and has a quota capacity of $Q(z' = 0, R^{a'} = 0) = 2$ GB. Suppose he wants to add $a = 1$ GB, then, $x' = x + a = (1.5 + 1)\, GB$, so the consumer must delete at least 0.5 GB, i.e., $(x' - Q(0,0))$ of data in order to stay within the limit. Observe that if the consumer had chosen to upgrade earlier in the period to a monthly premium plan, he could have chosen $y = 1$, resulting in $z = 1$ and a corresponding quota of $Q(1, 0) = 50$ GB, allowing for a much higher degree of flexibility.

3.5 Identification

In a dynamic structural model with nonlinearities, the identification of parameters requires careful consideration. We aim to provide intuition for the separate identification of each of the parameters in the period utility function of the consumer. We also follow with Monte Carlo data simulations to confirm the mapping from parameter to data. First, we note that we have normalized α_0, and do not identify it separately in any of the model specifications we consider.

We now consider how storage θ and deletion α parameters would be separately identified, i.e. what data patterns allow us to distinguish these effects? Our identification is based on using the deletion distribution within a consumer, specifically the mean and variance. Below we identify how the data patterns can indicate high or low magnitudes of θ and α. Denote the deletion distribution *within a consumer* as characterized by $\left(\mu_d, \sigma_d^2\right)$. The mean and variance correspond to the panel nature of data rather than the cross-section.

[14] We empircally set $R^{max} = 32$ since that is the maximum referral in the data. In addition, the company has capped the referral incentive policy so that customers cannot receive any referral benefits beyond this point.

Then, the mapping that follows is derived from the idea that consumers with a high value of storage utility θ will have a lower preference for deletion (all else being the same). Consumers with more negative α will have a preference for both lower levels of deletion as well as lower variance in deletion across time. Thus, such consumers will be averse to bunching their deletion significantly, whereas consumers with less negative values of α would display more bunching. The idea is that mean of the deletion distribution impacts the storage utility term, whereas the variance does not directly impact it. With $\theta > 0$ and $\alpha < 0$, the correspondence of parameter magnitudes to the deletion distribution can be mapped as follows:

- Low θ and Low $|\alpha|$: high μ_d and high σ_d^2
- Low θ and High $|\alpha|$: high μ_d and low σ_d^2
- High θ and Low $|\alpha|$: low μ_d and high σ_d^2
- High θ and High $|\alpha|$: low μ_d and low σ_d^2

Note that while we have a specific parametric structure for the deletion and storage utility, and the argument holds in part due to this structure. However, in principle, this argument would carry over to any structure with convex deletion cost. Also, we are not relying on the cross-sectional nature of the data for the above identification argument. However, the cross-sectional variation across consumers helps to identify the mean and variance of the hyper parameters.

For the price parameter γ, the timing of upgrade and more specifically, the capacity used (x_{it}) and the distance to quota ($Q_{it} - x_{it}$) at the time of upgrade aid in identification. If consumers are very price sensitive, they will be more inclined to wait until they reach quota before they upgrade (small distance to quota), whereas if consumers on average are less price sensitive, then they might upgrade earlier (large distance to quota). Note that Q_{it} independently varies from x_{it} because the current quota can change due to the number of accepted referrals. In addition, if consumers are more price sensitive, also we will see more deletion before upgrade than after upgrade. Note that since deletion decisions happen in both free and premium regions, we can separately identify the deletion parameter from the price coefficient.

ρ_1 and ρ_2 represent the referral cost, and this could be a linear or quadratic specification. Somewhat similar to deletion, the parameter is identified from the time variation in r_{it}, the consumers' referral behavior. Consider a customer who sends a total of ten referrals in a tenure of six months. The way a consumer disperses his referrals over time tells us a lot about his referral cost, specifically whether referral patterns display *bunching* or *intertemporal smoothing*. For instance, if the customer sends only one referral a day, essentially dispersing his referrals across time, this would suggest a costly quadratic term in the cost of referral, and therefore the parameter would be expected to be negative. On the other hand, if the customer lumps all referrals in one day, this would suggest the opposite, and the quadratic cost would be low in magnitude. The specific temporal shape and average levels of referrals help us identify the linear or the quadratic specification of the referral costs.

There is however, a distinction between deletion and referral in terms of intertemporal dynamics. If a consumers makes a deletion during a period, then he is guaranteed to have that space free at the beginning of the next period. However, with referrals, there is uncertainty in whether a referral sent will be accepted as well as when it might be accepted (it might be accepted several periods later). This time sequence of referral acceptances also provides additional variation for separate identification.

In order to verify that true parameter values can be recovered in our estimation procedure, we conduct a Monte Carlo experiment where we estimate our model with synthetic data generated from known parameter values. Please see Section A.2.1 in the Appendix for details.

3.6 Heterogeneity

In this section, we detail our unobserved heterogeneity specification, following on the utility description from Equation 1. We further define a distribution over θ_i such that:

$$\theta_i \sim N\left(\kappa, \sigma^2\right),$$

where κ is the population-level mean for θ_i, and σ^2 is the population-level variance. We assume independent, diffuse normal priors on the population parameter and a diffuse Inverse-Gamma prior on σ^2. We use this specification of the individual-level parameter θ_i in order to capture any observed heterogeneity in how much customers value file storage.

Recall that the parameters are identified from the individual-level variations across time in storage activity (x_{it}) in our panel data. In our panel data, we observe individual-level consumer behavior over a four-year period, and therefore when aggregated at the monthly level, all consumers have at least 21 months of data.

4 Estimation

The structural parameters $\Theta = (\{\theta_i\}_1^N, \alpha, \alpha_0, \gamma, \rho)$ represent the benefit to storage, deletion costs, price coefficient, and the referral cost. Since our data is aggregated at the monthly level, we set at $\beta = 0.99$ to reflect a annual discount factor of 0.9. Our model and setting present several challenges in estimation: a) discrete-continuous state space, b) discrete-continuous decisions, and c) jagged likelihood. We considered possible estimation approaches; we explain why we decided to use the Bayesian Imai-Jain-Ching (Imai et al.,

2009) method (IJC), and we discuss how IJC alleviates the aforementioned challenges.

Two common classes of estimation approaches include iteration-based methods in the tradition of Rust (1987) and two-step methods that follow the tradition of Hotz and Miller (1993) and Hotz et al. (1994), and include simulation based methods.[15] The advantage of the first approach is that we obtain an estimate of the value function at the end of the estimation process, but this comes at a higher computational cost than the simulation-based methods. While the simulation-based methods are computationally light, such as BBL (Bajari et al., 2007) and POB (Pakes et al., 2007), their accuracy depends on being able to correctly recover the primitives of the agent's policy function in the first step, as any errors in the first step will propagate into the second step and potentially become magnified through the simulation process.

Our setting presents a few immediate estimation challenges. First, the NFXP estimator is computationally demanding because it fully solves the Bellman equation at every guess of the parameter value. In addition, the fixed-point iteration must be solved across all states, and therefore the computational time for each iteration of the NFXP estimator increases as the size of the state space grows. The IJC algorithm alleviates this computational challenge. It does so by 1) evaluating the fixed-point iteration once per guess of the parameters and stores a collection of these values and 2) approximating the value function by using a history of past stored value functions weighted by kernels. One of the primary benefits is the ability to account for individual-level unobserved heterogeneity in a flexible manner. This is another advantage over BBL, with which we would have been restricted to an observable heterogeneity specification, or finite mixtures with Arcidiacono and Miller (2011).

Furthermore, we gain additional computational savings by avoiding the calculation of the numerical integral over the entire support of the deletion shock ν. To do this, we use Gaussian quadrature to approximate the integral over a subset of the support. This idea is similar to that of using splines, where the Gaussian quadrature makes a polynomial approximation of the function over a small number of nodes, a subset of the entire support.

Another challenge to tackle is the discrete-continuous choice aspect of the model. IJC, like NFXP, is designed to estimate dynamic models with discrete-choice controls. In order to handle the continuous-choice control in our problem, we combine the IJC algorithm with a likelihood modification derived from the Euler equation in the spirit of the continuous-choice dynamic models described in the macroeconomics literature.

[15] The fundamental idea of the iteration-based estimators is to nest a fixed-point iteration step within the maximization step of MLE. First, one solves the value function of the consumer dynamic programming problem via a fixed-point iteration of the Bellman equation for a given parameter guess. The solution to the fixed-point is a contraction-mapping, and therefore, under regularity conditions, we are guaranteed to find a unique solution to the value function. In the second step of the procedure, conditional on solving the value function, the problem is a traditional maximum-likelihood estimation problem, and one can proceed using traditional optimization routines to obtain a consistent estimate of the structural parameters. The algorithm iterates through these two steps for every guess of the parameter value. This procedure, referred to as the Nested Fixed Point (NFXP) estimator, is the workhorse in estimating many dynamic discrete-choice structural models.

We obtain log-likelihood values of the continuous-choice shocks in a grid-inversion fashion as Timmins (2002).

Lastly, a consequence of using the grid inversion technique is that the likelihood can be jagged and multi-modal. Therefore, not only does the likelihood not have a analytic derivative, but the jagged likelihood can cause traditional gradient-based optimization methods to become fixed at local maxima. Therefore, in practice, the model from Timmins (2002) is best estimated using comparison methods with multiple starting values. Once the optimizer is in the locality of a globally optimal region, only then can one trust gradient-based methods to find the global optimum. This can be computationally demanding and can take a bit of a coordination effort in order to ensure one finds the global optimum. In addition, if the number of parameters of one's discrete-continuous model were high, then estimating this model using traditional gradient methods would be practically infeasible. The IJC method can handle this challenge since Markov Chain Monte Carlo's (MCMC) stochastic optimization nature is robust to complex likelihood shapes that are highly non-monotonic and can handle parameter space with high dimensionality (Imai et al., 2009). We have verified this in our own context with extensive Monte Carlo simulations and find that global optimum is achieved regardless how jagged the likelihood is, with a number of different initial parameter values.

To tackle all of the estimation challenges, we use a modified version of the Bayesian Imai-Jain-Ching (IJC) algorithm in conjunction with several numerical computation techniques, such as Gaussian quadratures, to estimate a discrete-continuous choices dynamic structural model in a Bayesian fashion. The IJC algorithm is a variant of MCMC. It builds upon MCMC methods, based on full likelihood estimation, in that it uses Gaussian kernels and stored histories of stored pseudo-value functions to approximate the true value function. It provides the benefits of MCMC while alleviating the heavy computational burden of solving the Bellman equation at each MCMC iteration when estimating a full-solution Bayesian dynamic discrete choice model with forward-looking agents.

The IJC algorithm is a modified version of MCMC, and it follows these four steps at every MCMC iteration k:

1. Draw proposed parameter values, Θ^{*k}.

2. Evaluate pseudo-Expected Value Functions (pseudo-EVF) at currently proposed parameters and the last accepted parameters, $\tilde{EW}(D, \cdot, \Theta^{*k})$,$\tilde{EW}(D, \cdot, \Theta^{*k-1})$. These pseudo-EVFs are approximations to the Expected Value Function in Equation 4, and they are constructed using previously stored pseudo-Value Functions, from $H^k = \{\Theta^{*l}, \tilde{W}(\cdot, \cdot, \Theta^{*l})\}_{l=1}^{l=k-1}$, via kernel methods.

3. Calculate pseudo-likelihood values at the currently proposed parameters and the last accepted parameters, $\tilde{L}(\Theta^{*k}, \tilde{EW}(\cdot, \cdot, \Theta^{*k}))$ and $\tilde{L}(\Theta^{*k-1}, \tilde{EW}(\cdot, \cdot, \Theta^{*k-1}))$, using the pseudo-EVFs calculated in the previous step. These likelihood values are used in a traditional Metropolis-Hastings step to decide

whether to accept or reject Θ^{*k}. Since the prior and the pseudo-likelihood are not conjugate, we cannot obtain a closed-form distribution on the posterior, and therefore we cannot use a Gibbs sampler.

4. Create a new pseudo-Value Function $\tilde{W}(\cdot,\cdot,\Theta^{*k})$ by evaluating the Bellman operator on Equation 3. This is then added to the history of past proposal parameters and pseudo-Value Functions, H^k. In the specific context of a dynamic discrete-choice problem from Ching et al. (2012), the pseudo-Value Function is referred to as the pseudo-Emax function.

4.1 Likelihood Specification

Now we explain the formation of the likelihood specification. With the standard conditional independence assumption, the individual likelihood for the model can be specified as:

$$
\begin{aligned}
L_i(\Theta) &= \prod_{t=1}^{T} \mathbf{P}\left(y_t, r_t, d_t \middle| x_t, z_t, R_t, R_t^a, a, r^a; \Theta\right) \\
&= \prod_{t=1}^{T} \mathbf{P}\left(y_t, r_t \middle| x_t, z_t, R_t, R_t^a, a, r^a; \Theta\right) \mathbf{f}\left(d_t \middle| x_t, z_t, R_t, R_t^a, a, r^a; \Theta\right)
\end{aligned}
$$

where the first term is the likelihood contribution from the consumer's discrete choices, i.e., from the plan-choice (y) and number of referral invites (r) the consumer sends out. We can factor the joint likelihood into the products of the discrete choice (y, r) and the continuous choice d due to the timing assumption: consumers make the y and r decisions simultaneously, before the decision on d. Assuming $\epsilon(y, r)$ to be distributed type 1 extreme value, the functional form of the likelihood can be expressed in terms of the discrete-choice specific value functions as follows:

$$
\mathbf{P}\left(y_t, r_t \middle| x_t, z_t, R_t, R_t^a, a, r^a\right) = \left[\frac{\exp\left(V_{jk}\left(x_t, z_t, R_t, R_t^a; \Theta\right)\right)}{\sum_m \sum_n \exp\left(V_{mn}\left(x_t, z_t, R_t, R_t^a; \Theta\right)\right)}\right]^{1[y_t=j, r_t=k]}
$$

We must next specify $f\left(d_t \middle| x_t, z_t, R_t, R_t^a, a, r^a; \Theta\right)$, which is the likelihood contribution from the continuous deletion choice d_t.

Following Timmins (2002), the continuous-choice shock ν_t can be inverted from the policy function $d_t = g\left(\nu_t; x_t, z_t, R_t, R_t^a\right)$ for given values of all the other actions and state variables. Once the ν_t is recovered, it can then be evaluated at the density function of the specified distribution of $P(\nu_t|\cdot)$ in order to get the likelihood contribution. Observe that we have a monotonic relationship between d_t and ν_t, therefore it is possible to invert values of ν_t from observed values of d_t through the function $g(\cdot)$, the first order condition from the deletion sub-problem. The likelihood can then be formed as:

$$f\left(d_t \middle| x_t, z_t, R_t, R_t^a; \Theta\right) = P\left(\nu_t = g^{-1}\left(x_t, z_t, R_t, R_t^a\right) \middle| x_t, z_t, R_t, R_t^a; \Theta\right) \left|\frac{\partial g^{-1}(\cdot)}{\partial d_t}\right|,$$

In addition, we account for the boundary constraints on the optimal deletion amount. The minimum simply ensures that a consumer cannot delete more than the current amount in the consumer's account, and the maximum ensures that consumers are forced to delete excess files that put the account usage over current quota capacity. We discuss how the bounds of the deletions can be handled in Section A.1 of the Appendix.

5 Results

In this section, we present the results of our estimation. We obtained these values through 50,000 iterations of the IJC algorithm using random initial values for each individual customer. Convergence is assessed via inspection on three independent chains with the Gelman-Rubin statistic approaching 1, and we use the last 10,000 iterations for inference. To summarize the results of the individual-level posterior distribution, we present in Table 3 the mean and HPD of the population distribution of individual-level parameters. [16]

Table 3: Summary of Bayesian IJC Estimates & Model Comparison

Parameter	(I) Myopic Model	(II) Linear Referral Model	(III)† Quadratic Referral Model	(IV) Linear Quadratic Referral Model
E[θ_i]: Storage Benefit††		0.047	0.021	0.022
		(0.002, 0.069)	(0.003, 0.045)	(0.003,0.046)
α: Deletion Cost	-0.0471	-0.037	-0.0316	-0.03
	(-0.0477, -0.0465)	(-0.056, -0.017)	(-0.038, -0.025)	(-0.033,-0.027)
γ: Price Coefficient	-0.051	-0.163	-0.170	-0.180
	(-0.053, -0.049)	(-0.207, -0.115)	(-0.327, -0.018)	(-0.208,-0.157)
ρ_1: Linear Ref. Cost Spec.		-2.55		-2.22
		(-3.45, -1.66)		(-3.90,-1.08)
ρ_2 : Quad. Ref. Cost Spec.	-0.231		-0.249	0.197
	(-0.237, -0.225)		(-0.322,-0.180)	(-0.133,0.391)
Log-Likelihood	-274,882	-87,811	-66,218	-75,913
LMD	-274,884	-88,486	-66,689	-77,367
-DIC	-1,649,288	-595,339	-384,210	-620,516

† Indicates best fit and proposed model

††Paratheses indicate the 95% quantiles of the population distribution of individual-level E[θ_i]

All of the signs of the parameters are as expected. We now explain the intuitive implications of each

[16] For stability in estimation, we set the storage utility x to be on the scale of 0.5GB and we normalize $\alpha_0 = 1$.

parameter, with the first parameter contributing as a flow utility and the last three contributing as action utilities.

First, we examine the parameter θ_i, which is the benefit to storage. This parameter is the linear benefit to a consumer having files stored in his account folder. The positive coefficient indicates that the typical consumer, on average, receives positive flow utility for having a larger amount of data stored in his folders over time. Consumers value having files stored over time as opposed to simply adding files into the folder *temporarily* and then using the service purely to transfer files between different computers and mobile devices. Simply put, if a customer has higher value in this parameter than the population mean, then he values the service higher than the average customer, and if lower, he "needs" the storage space less than the typical customer.

α denotes the cost of deletion. In our estimation, we allow this parameters to freely vary between positive and negative support, allowing us to capture the flexible deletion behavior for each customer. The negative coefficient indicates that the typical consumer has a convex cost to deletion. This means that it becomes incrementally more costly for consumers to delete files as the amount of files needed to be deleted increases. In other words, consumers prefer many months in which they delete a modest amount as opposed to a few months of a large amount of deletion. This type of "smoothing" behavior indicates that the firm may wish to think about ways to profit from a different storage accounting scheme in which, in lieu of establishing a quota for the total amount of storage per month, the firm can adjust an upload/download bandwidth scheme. Other existing freemium companies such as Evernote use such an approach.

Next, γ is the price coefficient. This parameter denotes how price sensitive the typical consumers may be. The negative value of this estimate is as expected and indicates the magnitude of the costs that consumers must bear when upgrading to a premium plan. Lastly, ρ_1 and ρ_2 are the costs of referral for consumers. This cost could be attributed to the cognitive, social, and other costs of actually sending out invitations to friends. We found the best fit model to include only ρ_2, and the quadratic nature of this term reflects the fact that, as each consumer considers sending more referrals per period, it has higher marginal costs. One interpretation might be that it becomes harder to identify friends who do not already have invitations.[17] The benefit for each referral is accounted by an expectation of referral bonus quota included in the correspondence constraints ($H(\cdot)$) in the dynamic problem of Equation 4. The quota increases according to a specified referral incentive amount (250 MB).

[17]We conducted multiple specifications and found that the model with a purely quadratic referral cost (ρ_2) has the best fit. Please see Table 3.

5.1 Counterfactuals: Growth versus Monetiziation.

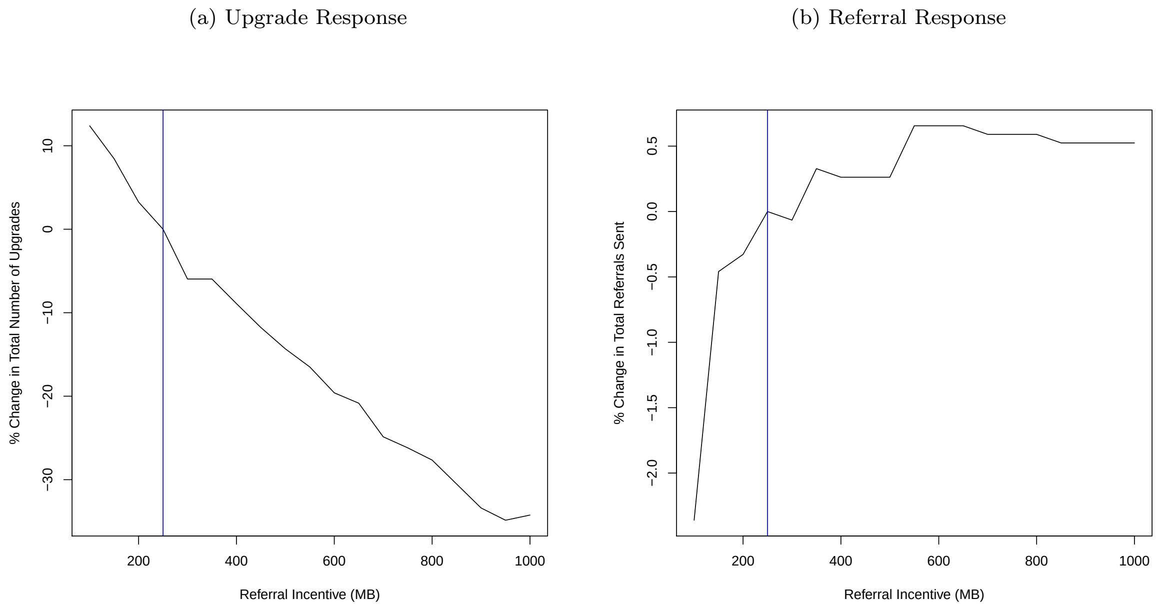
In this section, we present the results of the counterfactual simulations generated from the estimated parameters. The goal of these "what-if" analyses is to examine the growth versus monetization trade-offs that a freemium firm would have to face. We examine "growth" in the context of how our focal firm sees it, which essential entails of two parts: (1) acquiring new customers and (2) to keep them engaged so they do not churn. For the first part of acquiring customers, their main channel of customer acquisition is through the design of the referral program, so we first conduct a series of counterfactuals to see the effects of changing the magnitude of referral incentives on consumer upgrade and referral behavior. Then, in Section 5.1.2, we examine the second aspect of growth by examining how the firm can retain customers and to improve total revenue by changing quota and referral incentives decisions. Lastly, we examine the monetization aspect in Section 5.1.3, where we explore how can the firm monetize by charging higher prices, and to attentuate its negative effects by making the product better, through the levers of increasing quota or referral incentives.

5.1.1 Growth: Acquisition

Impact of Referral Incentives

Most firms are interested in the freemium business model since, when paired with referral incentives, it has the potential to help the firm rapidly grow its consumer base. Since the most important stated goal of any early stage company is to gain traction by obtaining a large user base, it is of interest for firms to understand how to make this process more effective. First, we explore what would happen if we were to generate counterfactuals for the representative customer using the population mean of all parameters by changing the referral incentives offered for each accepted referral invite. In order to generate these simulations, we mirror the exact conditions of the exogenous variables (e.g., addition shock, referral acceptance rates) that each of the 500 customers experience, and we only change the referral incentive they are rewarded. The default incentive in the data observation period is 250 MB per invite accepted, as shown in the blue vertical lines. We vary the incentive across a wide range, from as little as 100 MB per referral to as large as 1,000 MB per referral. The result of this is shown in Figure 8.

Figure 8: Referral Counterfactuals

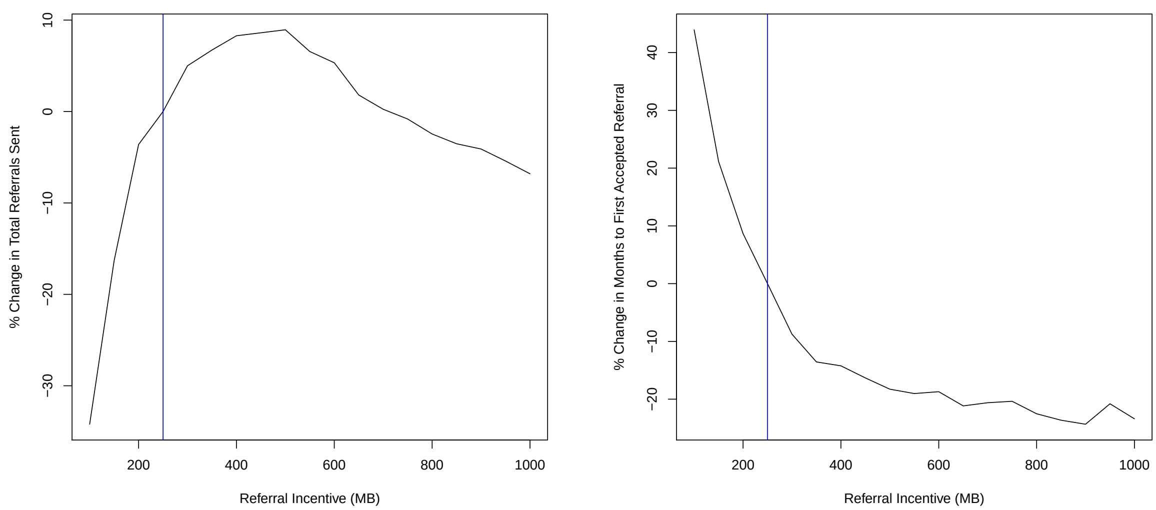


First, in the right panel, as the referral increases, we notice a non-linear response in the total number of referrals sent by all customers. Total referrals decrease as the referral incentives decrease, as expected, and total referrals increase as the incentive increases. However, we can see that for the average customer, the increase in total referrals levels off after 600 MB. On the other hand, in the left panel we show the decrease in upgrade rates as referral incentive increases in Figure 8a. Note that the decrease in upgrades can reach as large as 30%; therefore, it calls to question whether the cost of increasing the incentive to as high as 1 GB is worth the trouble.

Furthermore, note that since customers may value storage differently, this heterogeneity may affect their referral behavior. In our model we capture the heterogeneity in customers' valuation of storage with the parameter θ_i, and therefore, we could generate a representative "low-segment" customer for the customers with modest storage needs (i.e., lower 50% quantile) by seeing if these customers have different referral behavior than the average representative customers. This is shown in the left panel of Figure 9.

Figure 9: Referral Counterfactuals - Low Segment

(a) Total Number of Referrals (b) Speed of Referrals



What is interesting is that these customers do exhibit a different pattern than the average customers—an inverted-U behavior with a higher magnitude of change. First, similar to the average customer, as we increase referral incentives, we see an increase in the number of referrals sent. This change has a much higher magnitude of change than in Figure 8b. However after 550 MB of incentive, the average number of referrals sent actually decreases. This indicates that the maximal amount of referral incentive lies around 550 MB if our goal is to increase the average number of referrals sent. If the firm gives too much space for the referral incentive, it is not encouraging more, but rather fewer, referrals. We speculate this is because if a consumer can gain enough free space with only one referral, the marginal value of additional referrals is not as high, so why go through all the trouble of inviting more? This is perhaps especially true for this low-segment, since its valuation of MBs is lower than average, therefore "needing" storage less than the average customer. We further explore this storage valuation difference in various segments of customers in Section 5.1.1.

Speed of Referrals Accepted

If changing referral incentives is limited in affecting the total number of referrals sent by customers, then are there any referral objectives that it is effective at influencing? In this section, we explore the effect of referral incentives on changing the speed of referrals. In Figure 9b, we examine the effect of the change in referral incentive on how soon the customer sends the first referral—and therefore subsequently the first referral accepted. In the simulation, we hold the referral acceptance rate to be constant and the same as we observed in the data, and therefore the change across different regimes purely comes from how soon customers on average are sending out referral invites.

We see in Figure 9b that by increasing the referral incentive to even the amount of 500 MB, we can see a decrease of almost 20% in the time to first referral accepted. In the simulation, the average time to first referral acceptance is nine months (36 weeks) for the 250 MB regime, and the 20% decrease translates to a seven-week decrease. As a sanity check, this figure of 36 weeks is for the customer segment who values storage less than the average customer (27 weeks), and thus this segment takes longer than the average segment to send out referrals—leading to an overall longer time to first referral accepted. In addition, note that in the simulation, we chose a conservative approach in simulating referrals by not generating newly joined referred customers. However, in real life, these referred customers could send out referrals as well, leading to a snowball effect. Therefore, we do not include the potential exponential growth of referrals, and the above effects may be interpreted as a lower-bound. For start-ups, where the average investment round is expected to last 12 to 18 months, an acceleration of one or two months in customer growth via the referral program could make a sizable difference.[18]

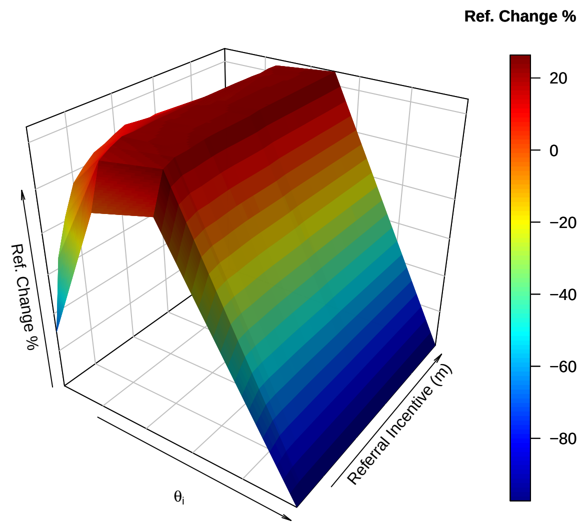
Heterogeneity in Customer Referral Response

In the counterfactuals found in Section 5.1.1, we stated that the potential explanation for the inverted-U referral behavior is due to referral bonuses exceeding how much customers "need the service." If this pattern were true, we would see the peak in the inverted-U vary by customers' estimated θ_i values. In Figure 10, we show that this pattern is indeed true, as we segment the customers into four quartiles of their estimated θ_i values. Note that as θ_i increases, the total baseline referrals increases, as expected.

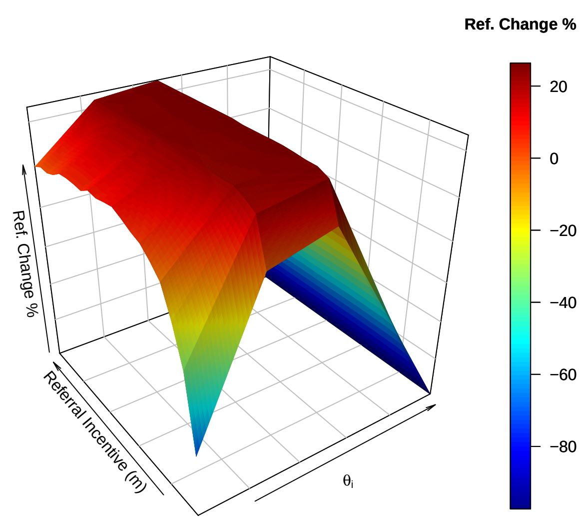
[18] Roth, David. "Launching a Startup - How Much Money Do You Really Need? And From Who?" Forbes, May 8, 2013. http://www.forbes.com/sites/davidroth/2013/05/08/launching-a-startup-how-much-money-do-you-really-need-and-from-who/. Accessed August 9, 2015.

Zwilling, Martin. "Startup Runway Length Depends on Your Burn Rate." Start-Up Professionals, February 21, 2012. http://blog.startupprofessionals.com/2012/02/startup-runway-length-depends-on-your.html. Accessed August 9, 2015.

Figure 10: Referral Counterfactuals and Customer Heterogeneity



(a) High Segment Perspective



(b) Low Segment Perspective (Rotated)

Note that intuition also suggests that, if customers highly value storage, at some point they will not bother with gaining storage space from sending out referrals, but instead substitute to upgrading. This is the pattern that we see in the highest customer quantile of θ_i, hence the dip in baseline referrals in Figure

10. This finding highlights the importance of capturing the heterogeneity in how much customers value the firm's product, as different segments of customers may yield vastly different referral behavior.

Dynamic Trajectories of Referral Incentive

In the previous section, we assumed that the trajectory of referral incentives is static. This means that once the firm has decided a particular referral incentive, it remains unchanged to the consumer forever. For the following simulations, we explore the case in which the firm offers incentives in different orders and gives higher incentives during a promotional period. We design the counterfactual along two dimensions: 1) limited time nature of bonus and 2) sequential order of bonus. To conduct the simulation, we generate 500 consumers for one year worth of data, all observing the same exogenous shocks in order to minimize simulation error. The firm announces the referral policy in the first period, so we assume that customers have perfect knowledge on when and what amount the referral incentive change will occur. We simulate the scenarios in which the firms start out with the default referral incentive of 250 MB per referral accepted and then ramp up to 500 MB per referral accepted at the six-month interval (Promo-Later) and the reverse (Promo-First). In addition, we also conduct a six-month "limited-time" promotion three months into the simulation period (Promo-Middle). Table 4 summarizes revenue (from the number of consumers who choose a premium plan in the year), the total number of referrals sent, and the time to first referrals accepted across all people.

Table 4: Dynamic Trajectory Results

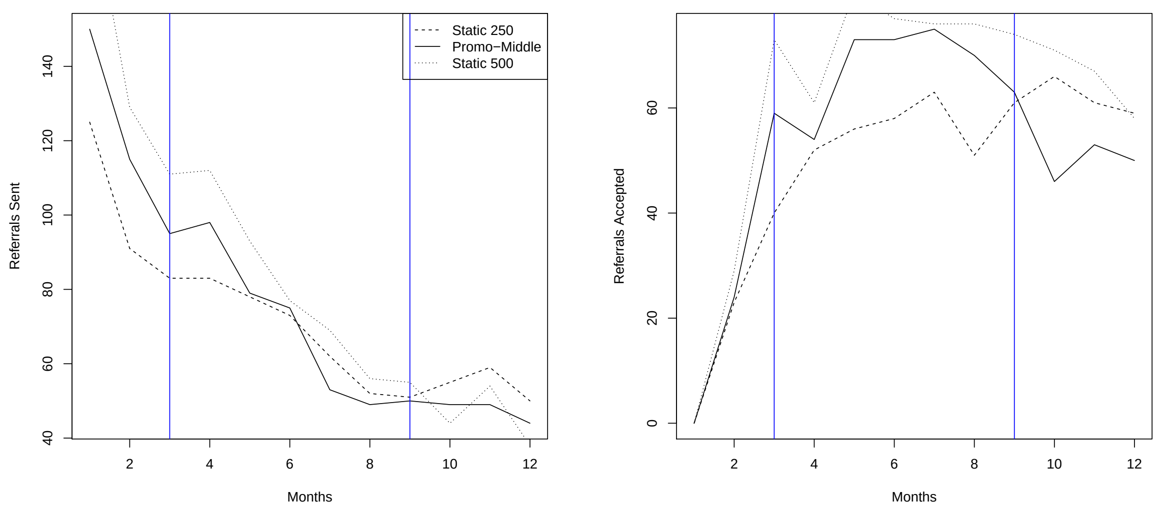
	Revenue ($)	**% Change Rev. Obj.**	**Amount of Ref.**	**% Change Ref. Obj.**	**Time to First Ref. Acc. (Weeks)**	**% Change Speed Obj.**
Static 250 (Benchmark)	1,748.25	-	862	-	27	-
Promo-First	**1,658.34**	**-5.1%**	877	1.7%	26	-3.7%
Promo-Middle	1,558.44	-11%	906	5.1%	**24**	**-11.1%**
Promo-Later	1,258.74	-28%	**1,008**	**17%**	25	-7.4%
Static 500	1,178.82	-33%	1,026	19%	23	-14.8%

An examination of the various counterfactuals from Table 4 suggests that there are different ways to tradeoff growth and monetization. For instance, giving the referral promotion in the latter half-year yields the most total number of referrals with a 17% increase in referrals over the Static-250 benchmark. The reverse plan, giving promotion first (Promo-First), yields the closest revenue to the benchmark plan. Lastly, the Promo-Middle plan actually yields the quickest referrals accepted among the three (at an 11% reduction in time). This plan achieves this result by shifting many of the referrals that would have been sent later in the year, as compared to the benchmark, to an earlier time. Therefore, there is a higher number of referrals accepted in the first nine months, but the referrals accepted would fall below the benchmark after when the promotion is over. This behavior is shown in the right panel of Figure 11b.

Figure 11: Dynamic Trajectory Referral Response

(a) Referrals Sent

(b) Referral Sent



Impact of Referrals on Value of a Free Consumer

In this section, we run a counterfactual to gain a better understanding of the contribution of the referral program on the value of a free consumer. In order to calculate this, we conduct a counterfactual in which the referral program does not exist.[19] We generate 500 identical free consumers, each starting out by choosing the free plan, and then we simulate their behavior for 12 months. We chose this length in order to simulate what would happen if the customers were to use the service for a year. We then compare the number of consumers who upgrade in this setting with the number of customers who upgrade in the scenario in which the referral program does exist.[20] The difference in the fraction of consumers who upgrade is characterized as the average value of a free consumer based on referrals, and it accrues even if these consumers never upgrade.

[19] We operationalize this by making the referral bonus quota per referral to be 0 and setting the $R^{max} = 0$

[20] We simulate the baseline setting by using the estimated primitives and setting all incentives to mirror the conditions in the actual data. We do this instead of subtracting the counterfactual from the actual data in order to minimize errors from simulation and model fit. In an additional effort to minimize simulation error, we use the the same sequence of draws for a, ν, $\epsilon(y, r)$ for both the baseline counterfactual and the no-referral counterfactual. To calculate the baseline counterfactual, we do the following three steps:

1. Start 500 organic consumers on the free plan, then simulate for 12 periods for each consumer.
2. For each period, we sum the total number of referrals accepted.
3. For each incremental referral accepted each week, we simulate an additional consumer for the remainder of the periods. We do this for only one pass to capture only the first-degree referral adoption as a conservative measure of value of the referral program.

We can then interpret this value as being a lower bound since some of these consumers will upgrade over time. We repeat this simulation procedure 100 times, and we report the means of each customer measures, as well as the 2.5% and 97.5% critical values in parentheses.

Table 5: Comparison of Referral and No-Referral Program Counterfactuals

	With Referral Program (m=250)	**Without Referral Program**
Value of a Free Customer	$0.52	$0.29
	($0.40, $0.65)	($0.19, $0.39)
Total Number of Consumers	1,717	500
(Organic + Referred)	(1687, 1747)	
Total Revenue	$3,132.66	$1,742.36
	($2,378.02, $3,887.31)	($1,120.64, $2,364.08)
Organic Conversion Rate	4.6%	7.4%
	(2.6%, 6.7%)	(5.2%, 10.0%)
Organic Premium Consumers	23	37
	(12, 34)	(24, 50)
Referred Premium Consumers	65	0
	(52, 84)	
Total Premium Consumers	88	37
	(67, 117)	(24, 50)
Avg. Monthly Addition	74 MB	74 MB
	(54 MB, 94 MB)	(55 MB, 93 MB)
Avg. Monthly Deletion	26.68 MB	35.16 MB
	(19.40 MB, 33.95 MB)	(27.29 MB, 43.02 MB)
Avg. Monthly Storage	370.68 MB	327.23 MB
	(238.06 MB, 503.31 MB)	(204.43 MB, 450.03 MB)

† **Parentheses indicate the 95% quantiles of the counterfactual results from 100 simulations.**

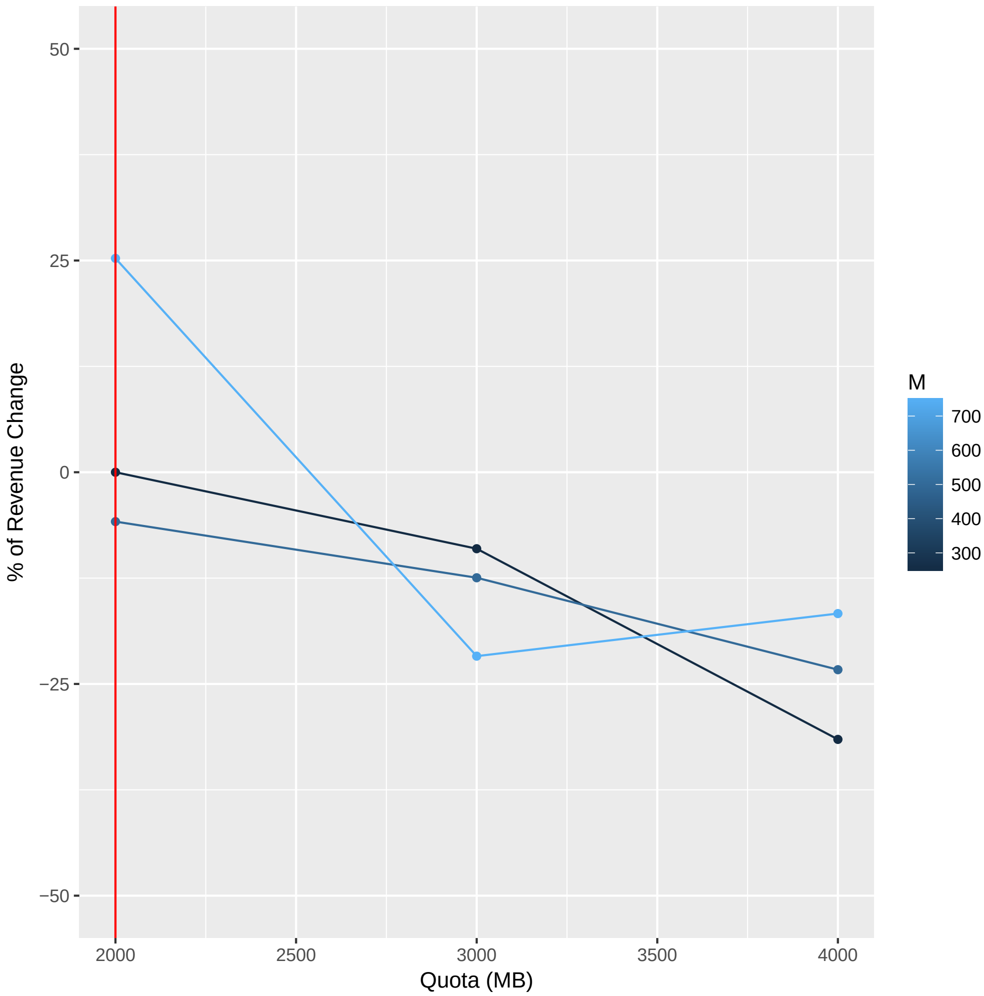
We now examine the results of this counterfactual in Table 5. There are a few interesting observations. First, the number of organic premium consumers is 38% lower in the referral scenario than the no-referral scenario. This makes sense, since with the referral program, customers can gain space from sending out referrals, therefore reducing part of the need to upgrade. Second, we observe that in order to compensate for the amount of space that would have been gained from the referrals, consumers in the no-referral scenario delete 31% more each month. This is as expected, since consumers need to make space, and they do not have the means to gain space from referrals; their only other option is to increase the monthly deletion. The implication of this is that consumers have a lower amount of storage per period, and therefore, each consumer will have a lower probability of choosing the premium plan per period.

In addition, we estimate the range on the value of a free consumer *per month* to be from $0.40 to $0.65, with an average of $0.52 ($3,132/500 Original Consumers/12 Months). This value comes from generating 500 free consumers and simulating their various consumer behavior over 12 months of usage.

Lastly, we find that while the referral program reduces the number of premium customers in the organic batch, it more than makes up for this loss by bringing in more than three times the number of customers through referrals, indicating the significant value in the referral program. Another way to think about this would be to decompse the value of the customers into two parts: a) customers who would upgrade by themselves in absence of the referral program and b) customers who bring in other customers who eventually upgrade. By dividing the value of free consumers in the Without Referral scenario by the corresponding figure in the With Referral scenario, we can see that part (a) accounts for 56% of the total value of the free consumers, while 44% of the value of the free consumers comes from the referral program.

5.1.2 Growth: Retention

In this section we examine the second aspect of growth, which is how to keep the newly acquired customers engaged so they continue to use the service and do not leave. From our discussion with the focal firm, one of the main reasons why customers leave is due to the fact that they run out of storage stage, and they get tired of deleting from their accounts. Therefore, in order to keep customers happy, the firm has two options at their disposal, they can either increase the free account quota (Q), or they can increase the referral incentive (m), which essentially increases the free quota through a smaller amount of increments. This is not an easy choice for the firm to make since both decisions could reduce revenue, since less number of customers would be upgrading, and customers may be heterogeneousin terms of how much they value each MB of storage. Complicating this fact is the fact that there is heterogeneity in customers' referral costs as well. Therefore, the firm is interested in a combination of not only changing the referral incentive, but also the quota design.



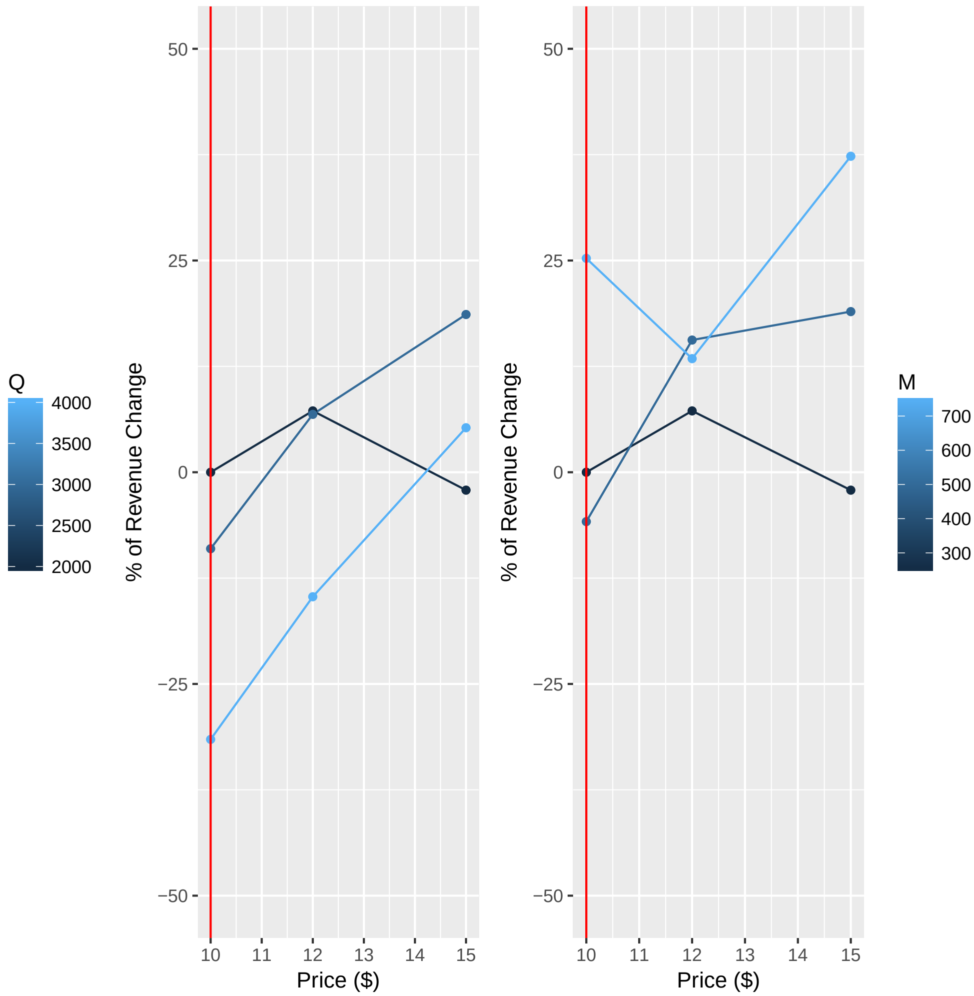
We run the counterfactual across three (high, medium and default) settings of quota (Q), referral incentive (M) and set the price (P) to the default monthly price of \$10. We examine the results in the above figure. First, we see the red vertical line at quota=2,000 MB as the default setting. The darkest blue is also the default referral incentive of 250 MB per referral accepted. We then calculate the percent change in revenues relative to this particular setting. We find that given the darkest blue line, as we increase the quota by 50% to 3,000 MB, as expected we also see a decrease in revenue due to the lower number of customers upgrading. This trend continues as we double the quota to \$4,000\$ MB, yielding the lowest revenue across all of the counterfactuals. Next, we double the referral incentive to 500MB, and we see at the quota=2,000MB, this effect has a slight decrease in the revenue as well. This is as expected due to more space, and as we increase the quota under this setting to 3,000 MB, we see the revenue continue to decrease. However, what's interesting is that as the quota increases to 4,000 MB, we see that the revenue is actually higher than that of the case when the referral incentive was at the default of \$250\$ MB. This is suggesting that there is a gain to having more space where less people exit due to the annoyance cost of deletion, but at the same time in the long run upgrade.

We see this nonlinear effect come in full swing when we examine the referral incentive at 750MB, as the default quota=2,000 MB case has roughly a 25% gain in revenue. While we see that this decreases dramatically to almost -25% for the quota=3,000 MB case, see that for the 4,000 MB case this is the case where it is the best revenue out of the three potential referral schemes. Therefore, from this counterfactual we learned that the answer, to the question of whether it is better to give out referral incentives or quota, depends on the level of each that you would like to set at each other. This validates that its helpful for management to go through an exercise like this in order to capture the nonlinear effects that referral incentives and quota may have on revenue.

5.1.3 Monetization

In this section we examine the monetization aspect, which is essentially is done through raising the price of the product. The biggest concern from raising the price is that the number of upgrades will decrease, and therefore decreasing revenue. Therefore, in order to compensate this, the firm would have to make the product better, either by raising the storage quota (Q) or the referral incentive (m).

We run this counterfactual across a three settings of price (P), quota (Q), and referral incentives (M). We examine the results in the above figure. First we examine the left figure for the effect of price changes on revenue change. We start with the darkest blue line, and see see that when we raise the price by 20% to \$12, we see a gain in revenue. However, when we increase it by 50% to \$15, we see the revenue declines. Next, we examine the lighter shade of blue, which is where we increase the quota to 3,000 MB. While at the initial price we see a decrease in the total revenue due to lower number of people upgrading, we find that this actually allows the firm to get more revenue at prices of \$12 and \$15. This means that improving the product, the firm would be able to charge more for it too. However, what it is interesting is that in the lightest blue, where we increase the quota to 4,000 MB, the total revenue is not as good as the case of quota=3,000 MB. As we have suggested before, this suggests that while firms can improve the product to raise the price, there is such a thing as improving the product too much for a given price increase.



Comparing across to the right figure, we see a similar pattern as we increase price for the default referral incentive setting as well, with the highest price at \$12 before the revenue declines. This was similar to the default setting of quota. What is interesting is that, similar to quota increases, increasing referral incentive can allow the firm to raise a higher price, although that effect is nonlinear, as the jump in revenue is much higher at from \$10 to \$12 as compared to \$12 to \$15. Lastly, what's more interesting is that as the referral incentives gets to be 750MB, we see that across the board the revenues are higher for all three price levels, as compared to the original referral incentive of 250MB.

6 Conclusion

In this study, we develop a dynamic microfoundations-based framework to investigate consumer behavior in the freemium business model. Firms in this fast-growing model often use the free product as a customer acquisition strategy and introduce a referral strategy in which each successful referral provides augmented free product to the consumer. This dynamic, while very helpful from a growth perspective, has the potential to introduce cannibalization of the premium product.

Methodologically, our framework allows us to model multiple consumer choices, including upgrade, usage, and referral behaviors, to understand the tradeoff between growth and monetization. We develop a dynamic structural model incorporating both discrete and continuous actions into an integrated model.

We use data from a leading storage and synchronization service to estimate our model of consumer behavior. We find that consumers are forward-looking and do typically adopt by beginning with the free product and make upgrade decisions significantly in advance of reaching the baseline quota of the free product. Consumers make referrals to friends, which can result in future increases in augmented baseline quota from referral bonus when their friends join and adopt the service. In addition, consumers actively manage their usage by deleting files so that they have an option value for keeping free storage space available on their accounts. They do this despite incurring a cost to delete, which they trade off with future value from additions of new content. We find that deletion and referral actions are often taken in conjunction when a consumer does not wish to upgrade, but wants to obtain more storage space.

Our findings can inform managers in several ways. First, we uncover the critical importance and nature of intertemporal tradeoffs faced by consumers in their decision making process. The referral program contributes not only to the growth of the user base (via more referrals), but also to the speed in which referrals are gained. Second, our results help us understand the benefit of the referral program and also recognize that higher referral incentives may cannibalize the premium product.

The present study could be extended along a few useful avenues. The first is that we do not focus on the acquisition of new, non-referred customers, but rather on characterizing the effects on the current base of users. However, we note that if our proposed recommendation of increasing the referral bonus is implemented, it is likely that customer acquisition will be increased, implying that we would be understating the benefits. Second, since we do not model consumers switching to another synchronization service due to data and the institutional setting, our results hold for a single firm with a captive user base. Since the acquisition of this data set, the industry has become an oligopoly, and it would be useful to model the competitive interactions between firms. However, interestingly, the customers with our focal service provider demonstrate a very low churn rate even under competition. Third, from a modeling perspective, we do not specifically characterize how consumers choose which content to add to the synchronized folder, and we effectively model that as being exogenous. If we did observe the set of possible content from which the consumer chooses what content (or types of content, e.g., documents versus photos) to store in the service, it would lead to a more detailed analysis of how consumers make such decisions. Finally, in this study, we do not focus on shared usage of the services, since the present data from the earlier stages in the company's growth features product features corresponding to personal usage.

Finally, the class of freemium business models features a number of other dynamics. For example, in freemium games like Angry Birds, consumers can purchase virtual goods. In other settings, certain specific features are only offered in premium versions. These contexts might feature interesting freemium dynamics, and we expect our framework broadly featuring usage and purchase can be modified to include the institutional details of those specific settings. In sum, we believe our dynamic structural framework for the freemium business model is the first to model consumer behavior from microfoundations in this fast-growing digital setting. It helps not only understand the drivers of consumer behavior, but also helps make managerial recommendations for a range of counterfactual scenarios.

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A Appendix

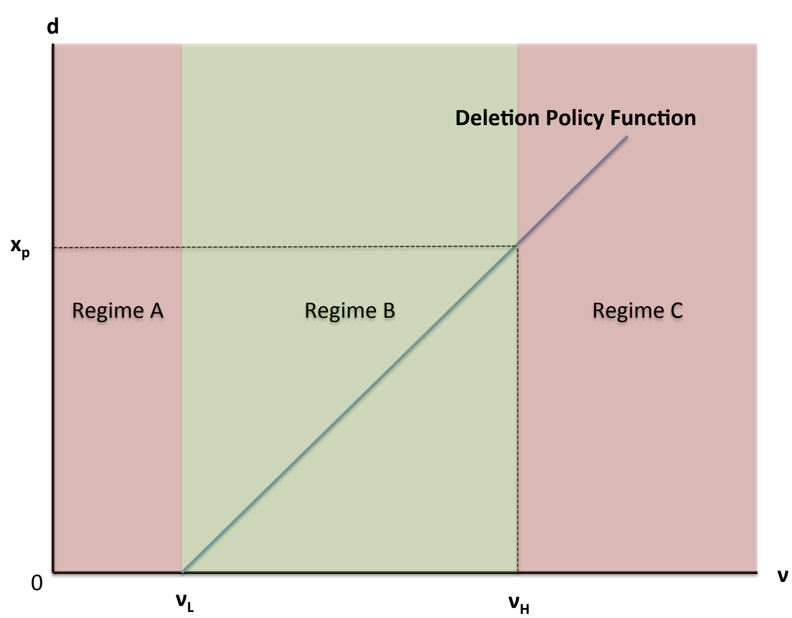
A.1 Deletion Bounds and Corresponding Likelihood Contributions

In this section, we derive the various cases of censorship in the observed deletion behavior. We suppress all i and t subscripts for expositional clarity, Because the parameters θ_i, α are estimated from the recovered ν's, these estimates would be biased if the observations that are censored were not properly accounted for in the log-likelihood. We proceed to discuss how the different ways of ν could be censored.

We first discuss how the deletion behavior itself is bounded, and then we discuss how this is related to the corresponding ν. At any given period, the deletion behavior d is bounded by two factors: 1) x_p - the amount of files in a user's account ($x_p = x+a$), 2) $Q(z, R^a)$ - current total quota capacity. Simply, a customer cannot delete more files than what is currently in the account, and a customer cannot make negative deletion. This bounds the deletion from above and below, as illustrated in the correspondence constraint $H(\cdot)$. The second case is if $x_p > Q(z, R^a)$, and in this special case d is bounded by below by $x_p - Q(z, R^a)$.

A.1.1 Case 1: $x_p < Q(z, R^a)$

In this case, d is bounded below by 0 and above by x_p. The corresponding ν could be visualized in the following figure:



As the figure shows, there are three potential regimes, with regimes A and C where the observed deletion is bounded by 0 and x_p. We now discuss the likelihood contribution to each regime:

Regime A: Observed $d = 0$

In this case, the true ν that was observed by the agent should be in the interval $[0, \nu_L]$, and we are not able to recover the actual ν observed. Therefore, for all observations that fall under this regime, we use the following expression for the likelihood contribution:

$$\mathrm{P}(d = 0 | x, z, R^a) = F(\nu = \nu_L),$$

where $F(\cdot)$ is the CDF of ν, and ν_L is derived empirically from the grid-inversion technique.

Note that we need to be aware of a special case where $x_p = 0$. In this case, since the d is both bounded above and below to zero, there is no information in the data because the potential value for ν could span its entire support. In this special case, we set the likelihood contribution to 1 (therefore log-likelihood is 0), since ν could be any value.

Regime B: Observed $d \in (0, x_p)$

In this case, the observed d is not censored by the bounds. Therefore, we use the likelihood contribution evaluated at the specific recovered value of ν.

Regime C: Observed $d = x_p$

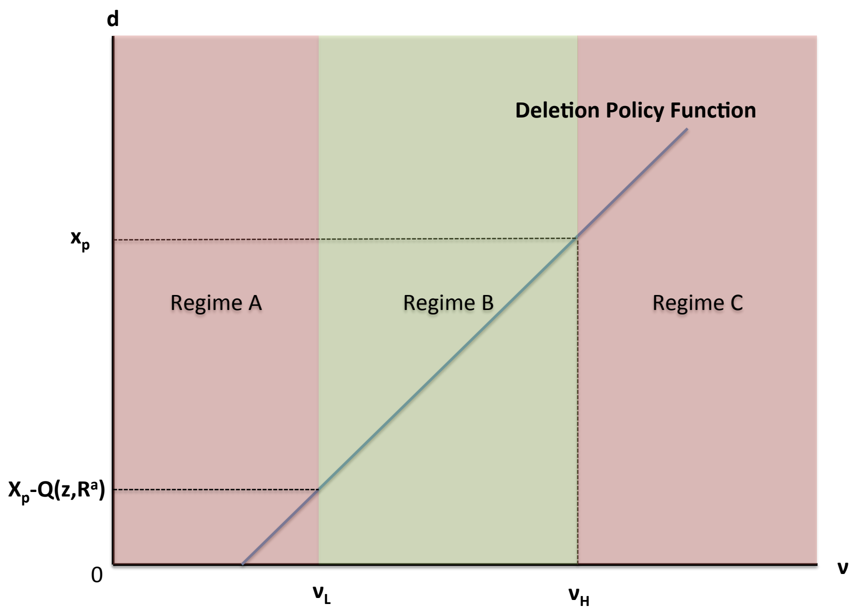
In this case, the potential ν that was observed by the customer could be $\nu \geq \nu_H$, therefore, for all observations that fall under this regime, we use the following expression for the likelihood contribution:

$$\mathrm{P}(d = x_p | x, z, R^a) = 1 - F(\nu = \nu_H),$$

where ν_H is derived from empirically from the grid-inversion technique.

A.1.2 Case 2: $x_p \geq Q(z, R^a)$

In this case, d is bounded below by $x_p - Q(z, R^a)$, and above by x_p. The corresponding ν could be visualized the following figure:



As the figure shows, there are three potential regimes, with regimes A and C where the observed deletion is bounded by 0 and x_p. We now discuss the likelihood contribution to each regime:

Regime A: Observed $d = 0$

In this case, the true ν that was observed by the agent should be in the interval $[0, \nu_L]$, and we are not able to recover the actual ν observed. Therefore, for all observations that fall under this regime, we use the following expression for the likelihood contribution:

$$\mathrm{P}(d = 0 | x, z, R^a) = F(\nu = \nu_L),$$

where $F(\cdot)$ is the CDF of ν, and ν_L is derived empirically from the grid-inversion technique.

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Regime B: Observed $d \in (0, x_p)$

In this case, the observed d is not censored by the bounds. Therefore, we use the likelihood contribution evaluated at the specific recovered value of ν.

Regime C: Observed $d = x_p$

In this case, the potential ν that was observed by the customer could be $\nu \geq \nu_H$, therefore, for all observations that fall under this regime, we use the following expression for the likelihood contribution:

$$\mathrm{P}(d = x_p | x, z, R^a) = 1 - F(\nu = \nu_H),$$

where ν_H is derived empirically from the grid-inversion technique.

A.2 Model Behavior

A.2.1 Monte Carlo Parameter Recovery

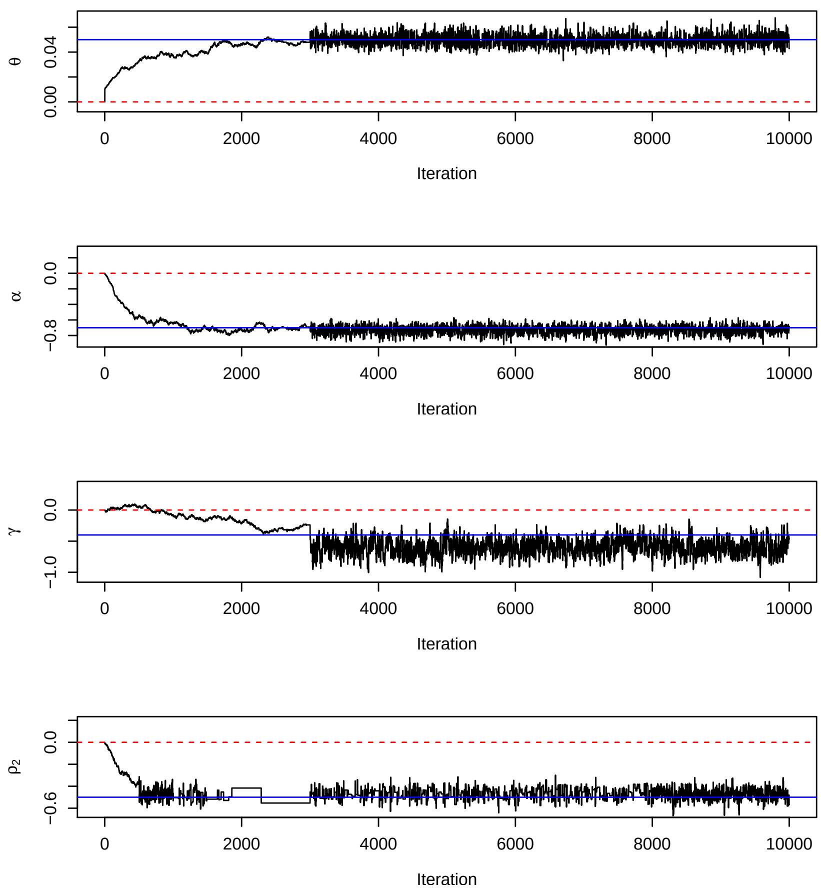
We generate 100 periods of synthetic addition, deletion, referral, and upgrade data for one customer with known parameter values. Then, we estimate our proposed model using the IJC algorithm to see if we can recover the true parameters. The MCMC ran for 10,000 iterations with initial values set at zeros, and we use the last 5,000 iterations for inference. The true values and mean of the estimated parameter posterior distributions are in Table 6, and the 95% HPD of the parameter posterior distributions are in reported in parentheses. We see that the true values for all four parameters fall within the 95% HPD.

Table 6: Monte Carlo: Homogeneous Bayesian IJC Estimates

Parameter	**True Values**	**Monte Carlo Estimates**
θ: Storage Benefit	0.05	0.0495
		(0.0407, 0.0590)
α: Deletion Cost	-0.7	-0.726
		(-0.840, -0.626)
γ: Price Coefficient	-0.4	-0.592
		(-0.847, -0.323)
ρ_2 : Quad. Ref. Cost Spec.	-0.5	-0.469
		(-0.567,-0.377)

We also report the MCMC traceplots for all parameters in Figure 12. The red horizontal dashed lines indicate the zero initial starting values, and the blue lines indicate the true values for each parameters.

Figure 12: Monte Carlo Recovery

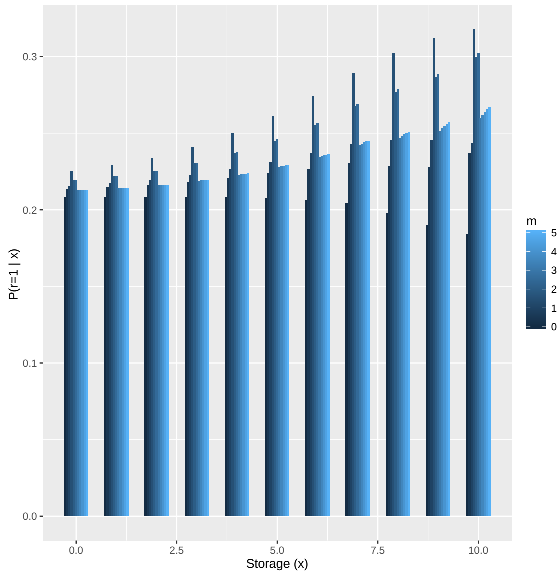


A.2.2 Referral Saturation Behavior

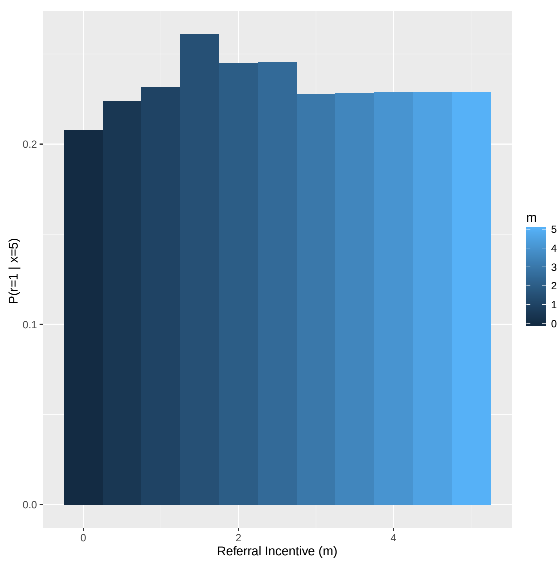
In this section, we generate Monte Carlo customer behavior by varying the magnitude of the referral and storage utility parameters to better understand the referral saturation relationship observed from Sections 5.1.1 and 5.1.1. For this, we calculate the referral probabilities of a free customer across varying ranges of free storage levels by varying different levels of referral incentives (m). For ease of interpretation, we rescale the storage levels at deciles of the free account quota. Furthermore, we set the storage benefit to a known, recoverable, value, and then we calculate the customer's probability of sending out a referral, as we vary the referral incentive from 0% to 500%.

Figure 13: Probability of Sending a Referral

(a) Referral Response Across Various Storage Levels



(b) Referral Response at Storage Level x=5



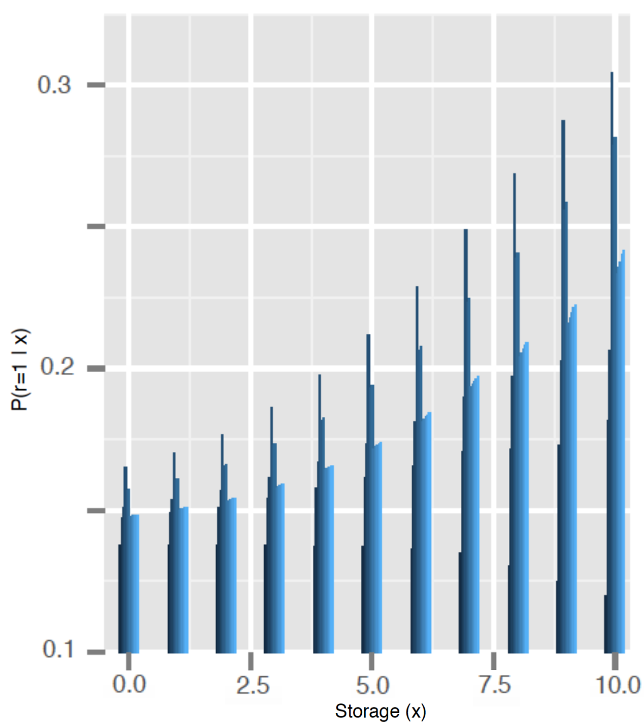
In Figure 13, we examine the results of the referral probability at various levels of storage and referral incentives. In Figure 13a, we see the referral response at various deciles of the free quota, and Figure 13b demonstrates the referral response at storage level equal to the fifth decile.

We notice several data patterns. First, as storage level increases by each decile, the overall probability of referral increases. In addition, as the referral incentive increases, we can see that the referral probability increases as well (seen more clearly in Figure 13b). These are sanity checks, and we observe both of these patterns in the counterfactuals. In both Figures, we observe the inverted-U "referral saturation" behavior as we have seen in Sections 5.1.1 and 5.1.1.

A.2.3 Effect of Changing Referral Cost

Now, we explore the effect of increasing the referral cost, thereby making it more difficult for a customer to send out a referral. We operationalize this by making ρ four times as negative as in Section A.2.2. If it is more difficult for customers to send out referrals, we would expect that the referral probabilities for customers at all levels of storage to decrease. This is another sanity check, and in Figure 14, we indeed observe that the overall referral probability to decrease.

Figure 14: Referral Response of Customer with High Referral Cost



A.2.4 Effect of Changing Storage Utility

In Section 5.1.1, we speculate that one of the reasons why the referral saturation point exists is because the user would "only need so much" storage. In order to explore this concept further, one way to simulate this "need" behavior is to change the storage benefit in the customer's utility, thereby directly changing how much a customer values storage. Thus, we increase the magnitude of the θ_i (storage benefit) by four, therefore making the customer value each MB of storage four times as much as the customers in the previous section. If this behavior were true, we would expect the saturation point to increase to a greater referral incentive, leading to a right-shifted inverted-U relationship. In addition, if the saturation point is greater than the largest simulated referral incentive, we may expect a monotonically increasing relationship.

In Figure 15, we observe that referral probability to have a strictly increasing relationship as related to the referral incentive. This confirms that at a high level of storage benefit, the saturation point of the referral incentive indeed has increased, as now the customer "needs more" storage. The implication of this is that only through estimation can we identify whether a customer is of the type from Figure 13 or Figure 15. If the customer belongs to the former, there exists an optimal referral incentive that can be given. As we show in Figure 10, this saturation behavior is observed for all but the highest quartile of customers in the data. Therefore, when firms are considering the policy implications of changing referral incentives, our proposed model serves as a helpful guide in understanding the potential referral behaviors for the various segments in their customer base.

Another way to simulate a customer with low storage benefit is to decrease the deletion cost, thereby making it easier for a customer to delete files (since he does not value it as much as the other). We have conducted similar Monte Carlo experiments by varying the deletion cost in a similar direction and we see a similar result as well.

A.2.5 Robustness of Referral Saturation

We also explore whether the referral saturation is an artificat of the quadratic specification of the referral cost. To rule this out, we specify a linear referral cost specification (ρr) in the consumer utility, and we generate the consumer referral response. If the referral saturation is an artifact of the quadratic cost specific, we would expect this behavior to disappear with the linear cost specification.

Figure 15: Referral Response of Customer with High Storage Utility

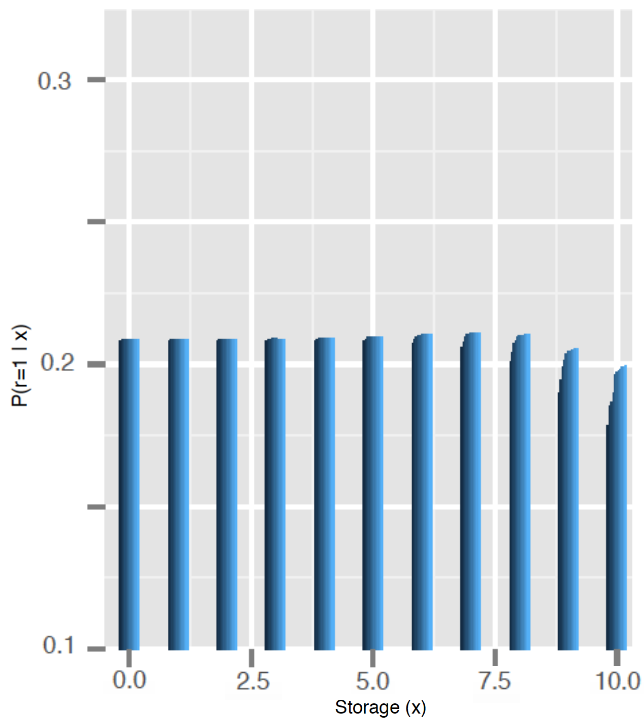
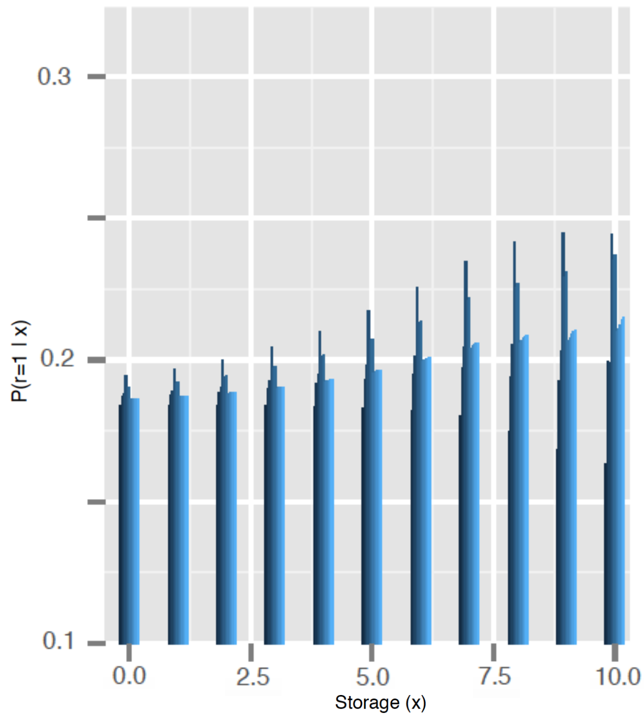


Figure 16: Referral Response of Customer with Linear Referral Cost Specification



We observe in Figure 16 that the referral saturation continues to exist, and therefore we do not believe the referral saturation is a result of the quadratic specification.

A.3 Anecdotal Evidence

While we are not allowed to release the identity of the cloud storage company, we were able to find anecdotal evidence explaining motivation for various types of customer actions for Dropbox, a cloud storage service in the space. We directly pull the following customer quotes from the Dropbox Blog (http://blog.dropbox.com/).

Reason for Usage: Backup

First, we can see that many users use a cloud storage service to backup their files. One can think of someone who has a many mission critical files that needs to be backed-up has high storage value utility.

- "Thanks Dropbox you saved my presentation."
- "I'm a student and I'd plan to use it to backup all of my documents."
- "Good job team! this is the best sync and back up service i see it ever!!!!! and no one will beat it"

Reason for Usage : Syncing Across Devices

In addition to using the service as back-up, we see that customers also use it to sync files across multiple devices. For customers who has many devices, this may also translate to a high storage value utility.

- "I have a home office with three computers at home and 3 at work including one Mac... Dropbox is helping me to tame the beast and get all of these little monsters on the same page."
- "Dropbox is the winner. I'm constantly running between 3 different machines every day. The hassle factor is gone. My files are with me where ever I go."
- "I am a high school science teacher in Wisconsin and I LOVE Dropbox! I use it to sync my computers at school and at home."

Reason for Upgrading

One of the main reasons why customers upgrade is because they run out of space, or eventually will run out of space:

- "I got by on the free version of Dropbox for a while, but ran out of space and switched to being a paying customer."

Reason for Referrals

Note we see two observations with the motivation for referral. The first is that customers send out referrals to get more free space. The second is that they are anticipating they will be "out of space soon", therefore they need to "start referring friends." This is anecdotal evidence that customers send out referrals in anticipation of hitting the storage capacity quota. One of the reasons why "referral people weren't counted" is due to the fact that referrers only get the referral incentive when customers actually join, but referrals don't exactly whether or when their invited friends will actually join.

- "I see that I really shall try to call over my friens in order to gain some more free space :)"
- "I love dropbox, and I reckon I'll be filling up my space soon, better start referring my friends..."
- "Just got 20gb for my dropbox now. I would like yo hit 50gb free like my friend. I still have ten referral people weren't counted."