

# Nonparametric Bandits Leveraging Informational Externalities to Learn the Demand Curve

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  - How can we incorporate (simple) economic theory?
  - What is the benefit?



# The Problem - Learning Unknown Demand Curves

- In 2016, the Atlanta Falcons dramatically slashed concessions prices to improve brand equity. When asked how they projected the volume of sales to change, the CEO of ownership, Steve Cannon replied, “It could be a **10% bump**, it could be a **30% bump**, who knows.” The next season sales volume for food **rose 50%**.

# But... Companies are reluctant to experiment with price

## Column: Why Businesses Don't Experiment

A debate ensued among the group: Are we willing to sacrifice some customers "just" to learn how the new pricing approaches work?

They hedged. They asked me what *I* thought the best approach was. I told them that I was willing to share my intuition but that intuition is a remarkably bad thing to rely on. Only an experiment gives you the evidence you need. In the end, it wasn't enough to convince them, and they called off the project.

### Issues

- Potential short term losses for uncertain long term gains
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Can incorporating theory improve learning with an **unknown** demand curve?

# Desired Features For Price Experimentation

## Goal

Maximizing earning while learning

- Maximize long run profits while minimizing costs of experimentation

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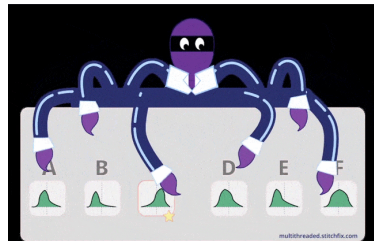
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- No parametric form for demand curve assumed
- **Adaptively set price based on incoming experimentation data**



# Introducing... Multi-Armed Bandits

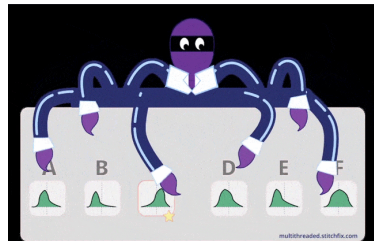
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[Link to Animation](#)

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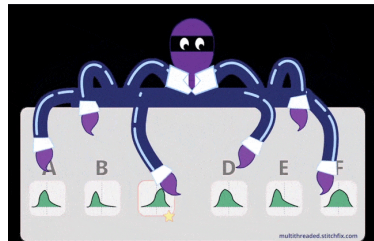
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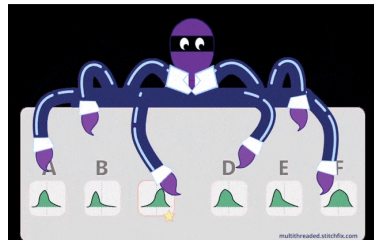
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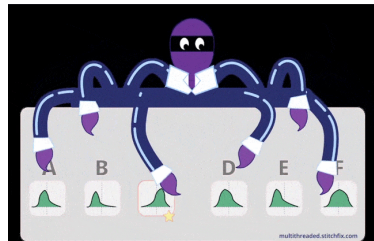
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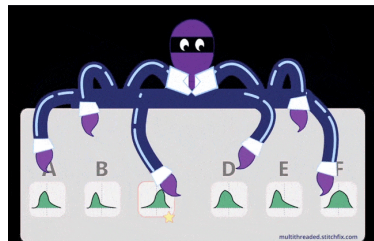
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- Key is to balance **exploration** (finding best arm) and **exploitation** (gaining from best arm)
- **Belongs to class of** reinforcement learning problems



[Link to Animation](#)

# Multi-Armed Bandits in Marketing

Use Case	Objective (maximize)	Examples of Arms	Arm A	Arm B	Arm C
Advertisements	Conversions	Ads	Emotional	Informative	Funny
Recommendation Systems	Purchases	Movies	E.T.	Indiana Jones	Rain Man
<b>Pricing</b>	<b>Profits</b>	<b>Prices</b>	<b>\$1</b>	<b>\$5</b>	<b>\$20</b>



Extra-Terrestrial? Economic Theory?

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## Goal

Leverage **informational externalities** within MAB framework to improve learning

# Contributions and Features

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Develop new principled approach that incorporates informational externalities derived from theory with nonparametric bandits

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- Provides a measure of uncertainty (variance) for every possible price level
- Provides an automatic approach with no human judgement required
- **Runs in real time**

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  - **We propose new sampling method to obtain monotonic draws from the GP**

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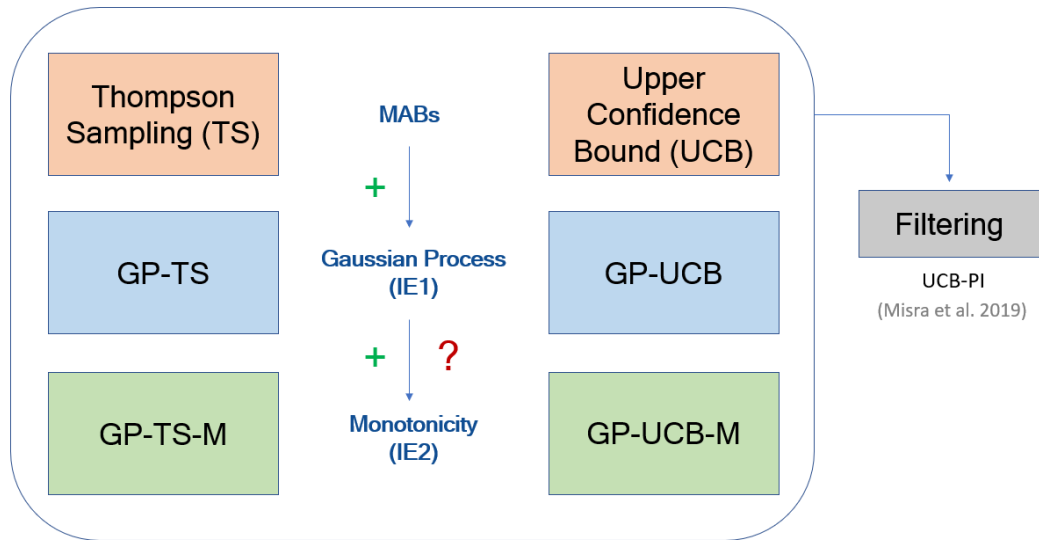
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- Information from posterior GP scaled by price accordingly (GP-UCB / GP-TS)



# Building Blocks

- Decision Rule for Picking Arm to Experiment: UCB, TS
- Gaussian Process can be used to flexibly model any nonparametric demand curve
  - Monotonicity restricts the set of functions

# Overview of MABs and Informational Externalities

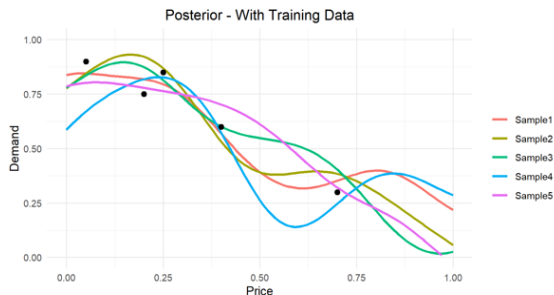
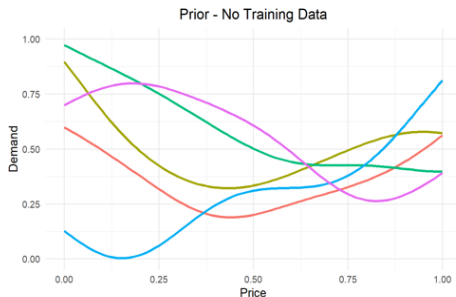


# Overview of GPs

Intuitively, a Gaussian process is a probability distribution over possible functions

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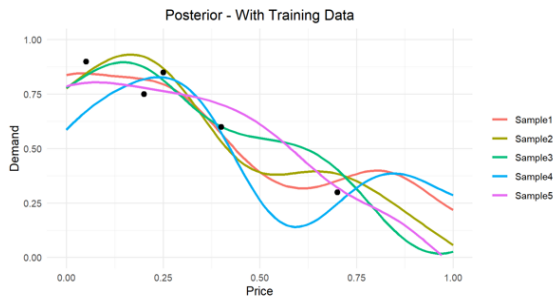
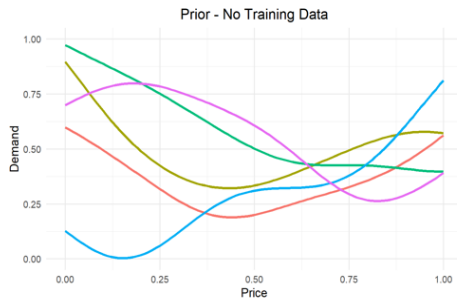
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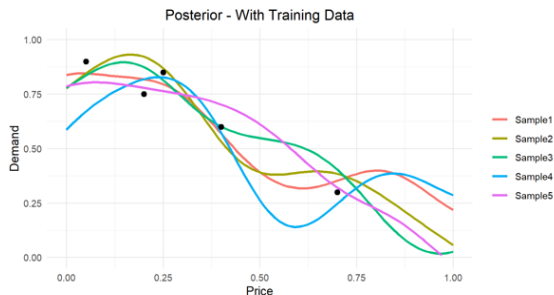
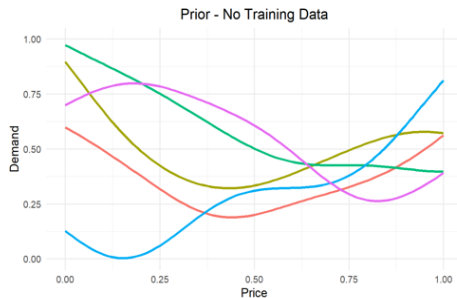
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- Nonparametric method (accommodates almost any possible demand curve)
- **Can sample an entire demand curve at once from GP**



# Incorporating Second Informational Externality

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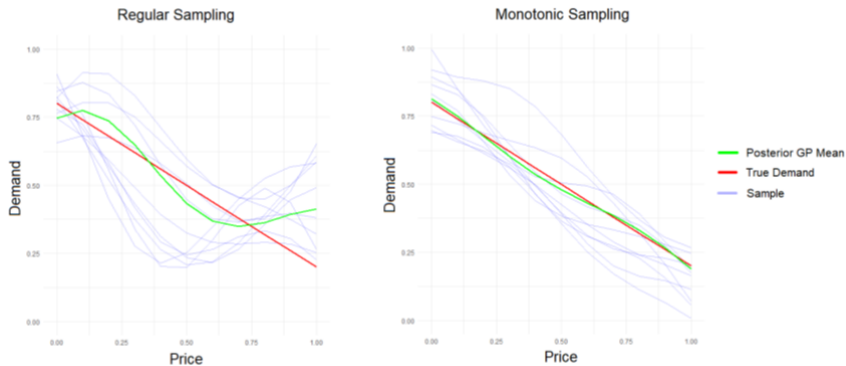
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- Need way to obtain a random monotonically decreasing draw from the posterior GP
- Not entirely clear how to do so

# Monotonic Sampling



**How do we obtain a random monotonic draw from the GP in a principled way?**

# Proposed Sampling Approach (I)

## Proposed Approach

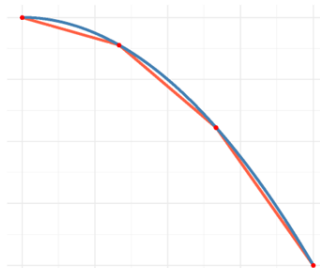
Any function can be estimated by linearly interpolating between knot points

- Converges to true curve as number of knots approaches infinity

$$D(\cdot) \approx \sum_{j=0}^N D(\mu_j) h_j(\cdot)$$

Similarly, any function can be estimated by its **intercept** and **first derivatives** at knots

$$D(p) \approx \sum_{j=0}^N D'(\mu_j) \int_0^p h_j(t) dt$$



# Proposed Sampling Approach (II)

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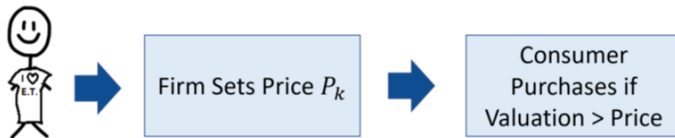
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- Monotonically decreasing = all negative derivatives
- Easier sampling problem (that of sampling from a Truncated Normal<sup>1</sup>)

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# Algorithm Evaluation – Setup

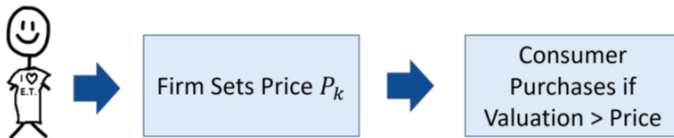


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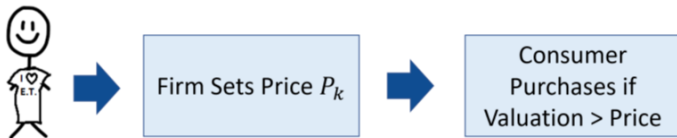
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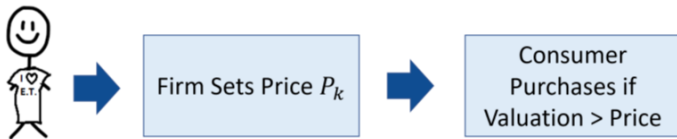
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  - 100 arms:  $\{0.01, 0.02, \dots, 1\}$

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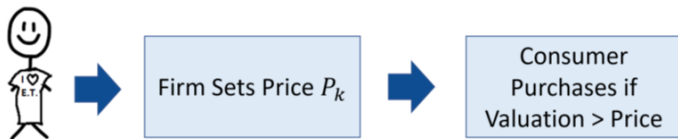
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- Algorithm is updated with purchase decision every 10 customers<sup>2</sup>

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# Algorithm Evaluation – Setup

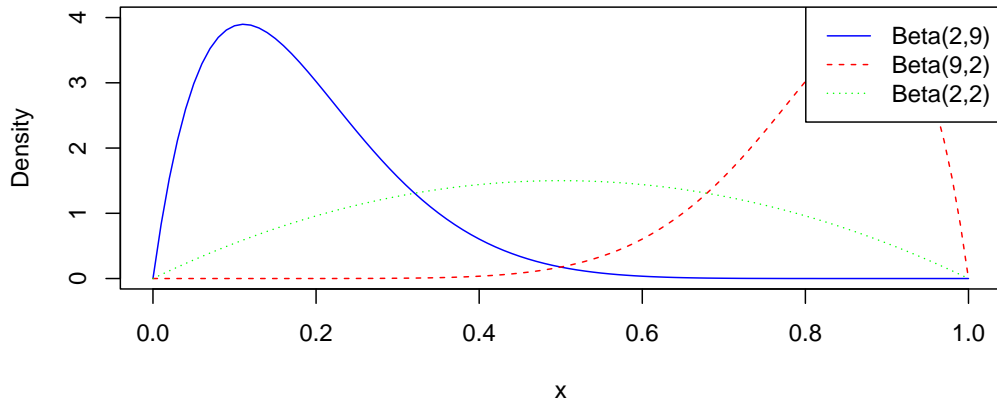


- Underlying WTP distribution is chosen
- Prices are normalized between 0 and 1
- 3 different price sets are tried
  - 5 arms:  $\{0.1, 0.3, 0.5, 0.7, 0.9\}$
  - 10 arms:  $\{0.1, 0.2, \dots, 1\}$
  - 100 arms:  $\{0.01, 0.02, \dots, 1\}$
- Algorithm is updated with purchase decision every 10 customers<sup>2</sup>
- Results are averaged across 1000 separate runs

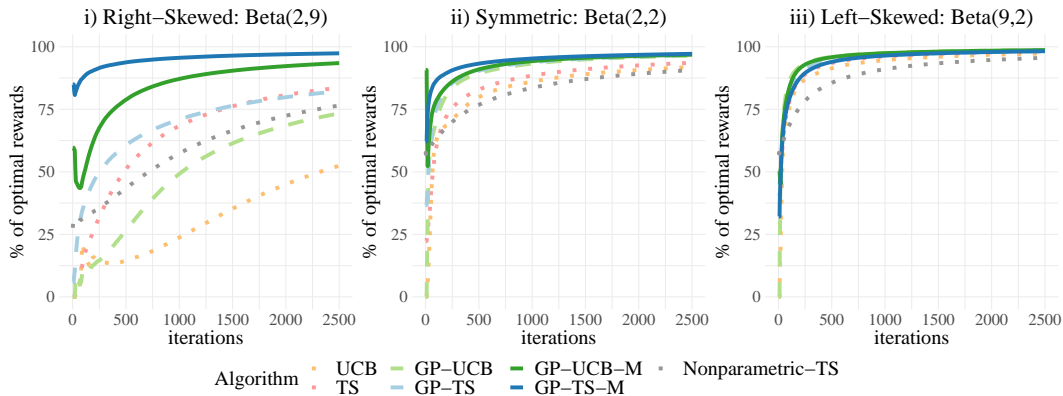
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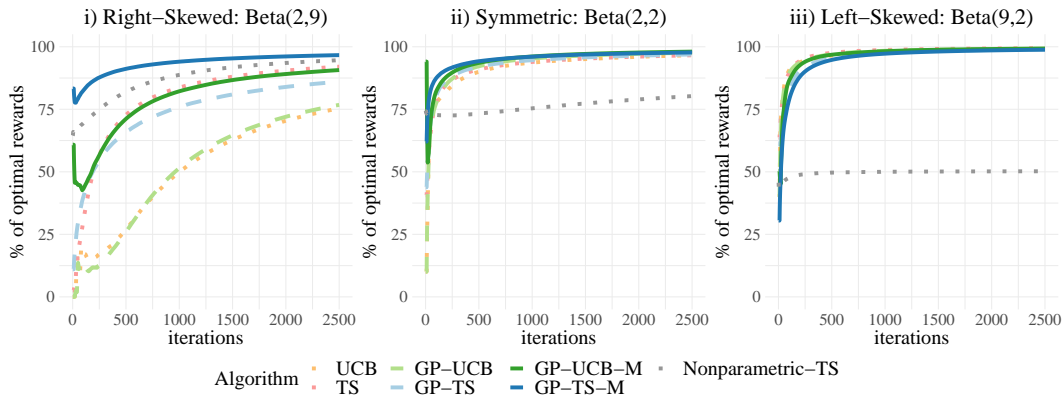
## Beta(2,9), Beta(9,2), and Beta(2,2) Distributions



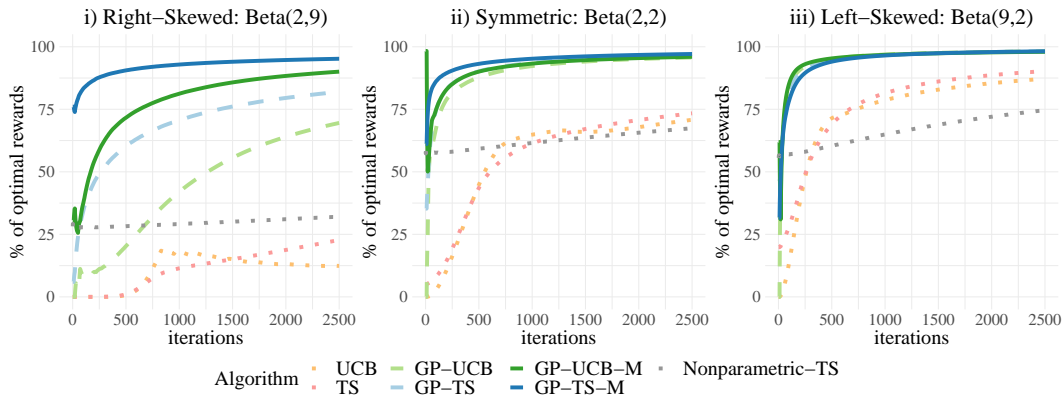
# Rewards - 10 Arms



# Rewards - 5 Arms



# Rewards - 100 Arms



# Performance Uplift from Informational Externalities

		UCB			TS		
		5 Arms	10 Arms	100 Arms	5 Arms	10 Arms	100 Arms
Uplift from 1st Externality (GP compared to base algos)	Beta(2,9)	1.8%	39.9%	461.9%	-6.5%	-1.6%	260.6%
	Beta(2,2)	1.1%	4.9%	35.2%	0.3%	3.2%	31.3%
	Beta(9,2)	0.1%	1.4%	12.6%	-0.5%	0.1%	8.8%
	Pooled	<b>0.9%</b>	<b>11.1%</b>	<b>54.7%</b>	<b>-2.1%</b>	<b>0.6%</b>	<b>48.4%</b>
Uplift from 2nd Externality (GP-M compared to GP algos)	Beta(2,9)	18.1%	27.4%	29.5%	12.3%	18.4%	16.1%
	Beta(2,2)	0.5%	0.4%	0.3%	0.9%	0.6%	0.8%
	Beta(9,2)	0.0%	0.0%	-0.2%	-0.1%	0.1%	0.0%
	Pooled	<b>5.3%</b>	<b>7.6%</b>	<b>7.8%</b>	<b>4.0%</b>	<b>5.7%</b>	<b>5.0%</b>
Uplift from both Externalities (GP-M compared to base algos)	Beta(2,9)	20.3%	78.3%	627.5%	5.0%	16.4%	318.7%
	Beta(2,2)	1.5%	5.3%	35.6%	1.2%	3.8%	32.3%
	Beta(9,2)	0.2%	1.4%	12.4%	-0.6%	0.1%	8.8%
	Pooled	<b>6.2%</b>	<b>19.6%</b>	<b>66.7%</b>	<b>1.8%</b>	<b>6.3%</b>	<b>55.9%</b>

## 1st Externality

- Uplift increases with number of arms
- Magnitude of uplifts largest for right-skewed

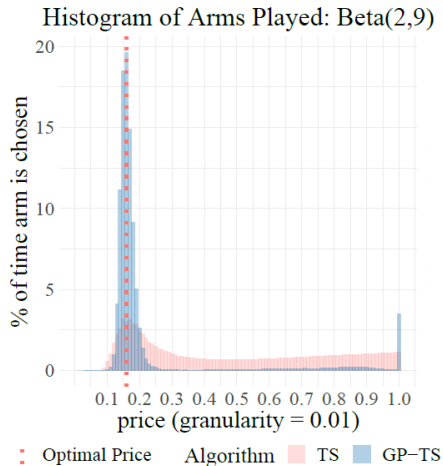
## 2nd Externality

- Consistent uplifts regardless of number of arms
- Large uplift for right-skewed, small for symmetric and left-skewed

# Explanation - 1st Externality

The first externality becomes more important as the number of arms increases

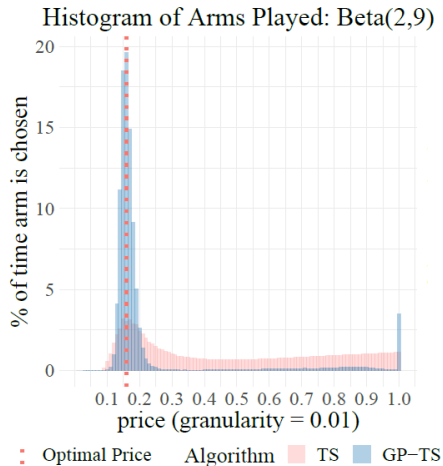
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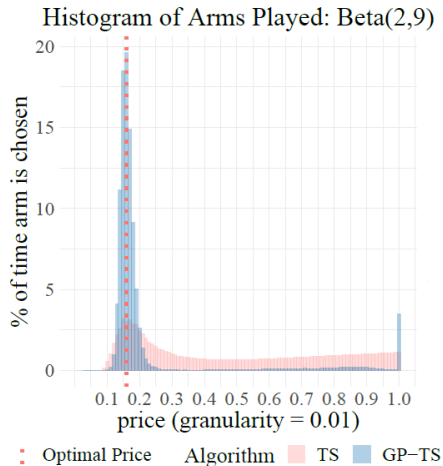
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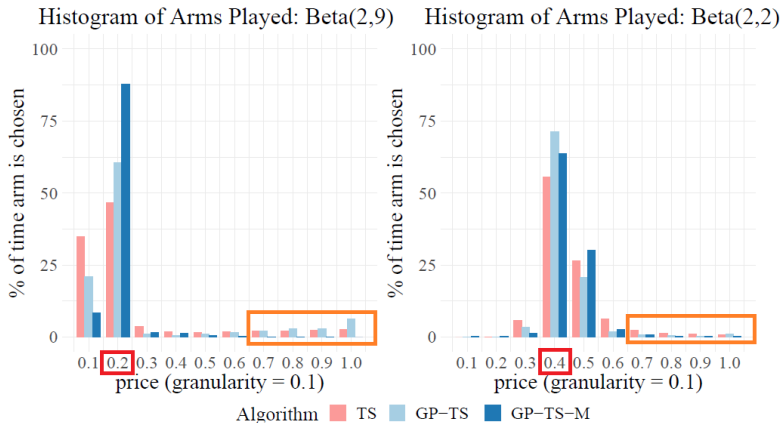
- Without considering correlation, an algorithm has to test each arm individually
- As the number of arms increases, learning is spread too thin
- Learning across arms ameliorates this problem allowing algorithm to narrow in on best arms more quickly





# Explanation - 2nd Externality

- Generally, bandit algorithms tend to over-explore higher prices

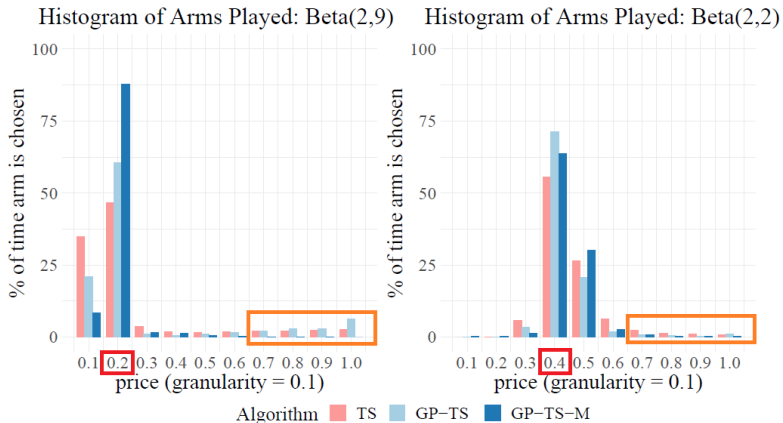


- For Beta(2,9), including monotonicity helps remove the choice of many low performing high prices (orange box)

- For Beta(2,2), the effect is largely mitigated because other algorithms are already learning quickly to move on from these low performing high prices (orange box)

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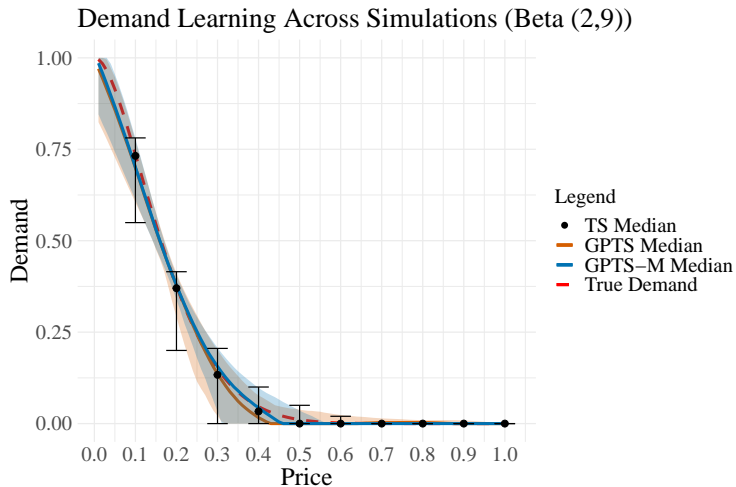
- Generally, bandit algorithms tend to over-explore higher prices
- When the optimal price is low, including monotonicity allows many 'exploitations' from erroneous upward sloping curves to be eliminated



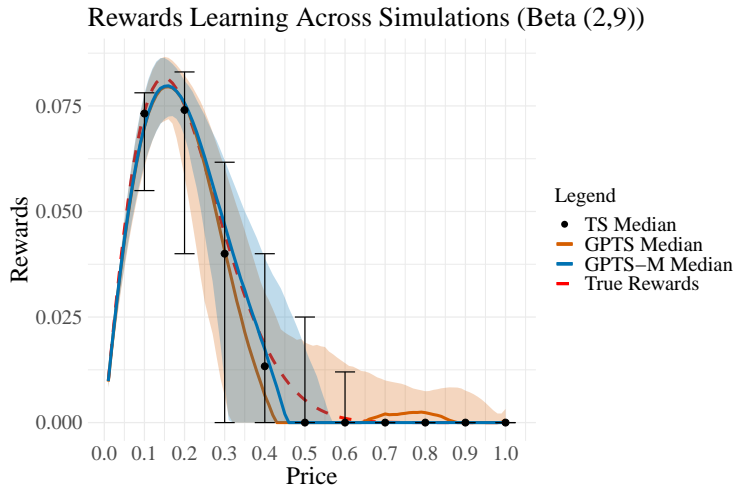
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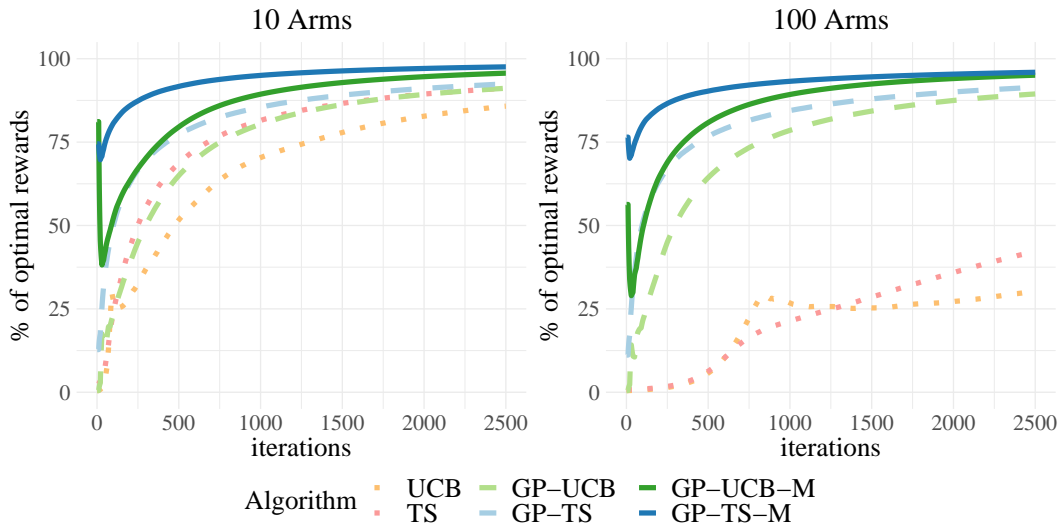
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- Incorporating Theory into ML models requires careful modeling but has significant benefits in improving ML
- Learns much more *efficiently*  $\implies$  Experimentation ( $\Downarrow$ )  $\implies$  Practical value

Thank You!

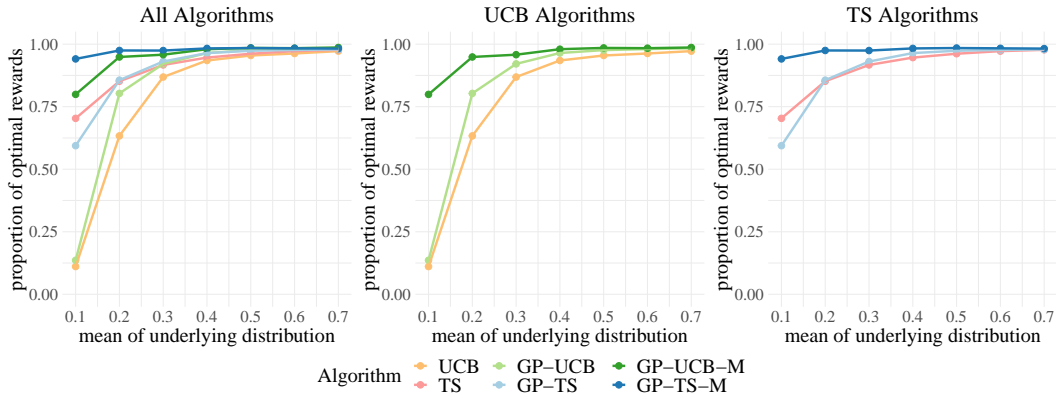
[vineet.kumar@yale.edu](mailto:vineet.kumar@yale.edu)

# ADDITIONAL

# Field Data - Replication



# Fixed Variance - Altering the Mean of Underlying Distribution



The difference in performance shrinks as the mean gets higher