Demand Estimation for Subscription Models

Identifying Willingness to Pay without Price Variation

Vineet Kumar

Yale University, USA

Yale Faculty Seminar

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Research Overview – Substantive

Designing Freemium: Strategic Balancing of Growth and Monetization,

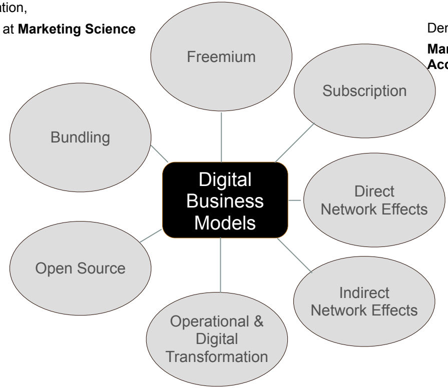
Revision at **Marketing Science**

Designing Plans on Digital Platforms, *Working Paper*

Demand Estimation for Subscriptions

Marketing Science (Cond. Accepted)

The Dynamic Effects of Bundling as a Product Strategy, **Marketing Science**



Can Random Friends Seed More Buzz and Adoption? Leveraging the Friendship Paradox,

Management Science (Accepted)

Competitive Product Strategy for Open Source Software, **Marketing Science**

Other:

Match Your Own Price? Self-Matching as a Multichannel Retailer's Pricing Strategy,

Marketing Science

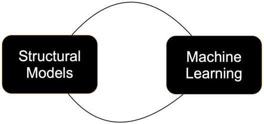
Designing Pricing Strategy for Operational and Technological Transformation,

Management Science

Research Overview – Methodological

- Structural Models:
  - Linear Estimation of Aggregate Dynamic Discrete Demand for Durable Goods without the Curse of Dimensionality, with C. Chou and T. Derdenger **Marketing Science**
  - Estimating Dynamic Discrete Choice Models with Aggregate Data: Properties of the Inclusive Value Approximation, with T. Derdenger, **Quantitative Marketing and Economics**

Research Overview – Methodological



- Machine Learning:
  - Nonparametric Bandits Leveraging Informational Externalities to Learn the Demand Curve, with I. Weaver, Major Revision at **Marketing Science**
  - A Theory-Based Interpretable Deep Learning Architecture for Music Emotion, with H. Fong and K. Sudhir, Major Revision at **Marketing Science**
  - Automatically Discovering Unknown Product Attributes Impacting Consumer Preferences, with A. Sisodia and A. Burnap, Revision at **Journal of Marketing Research**

Subscription business

- Subscription market is fast growing and potentially huge
  - Growth rate $>100\%$ each year in the past 5 years
  - Multibillion revenue per year
  - Across a wide range of product categories (digital $+$ physical)
  - **Pay upfront and consume over time**

Subscription Services

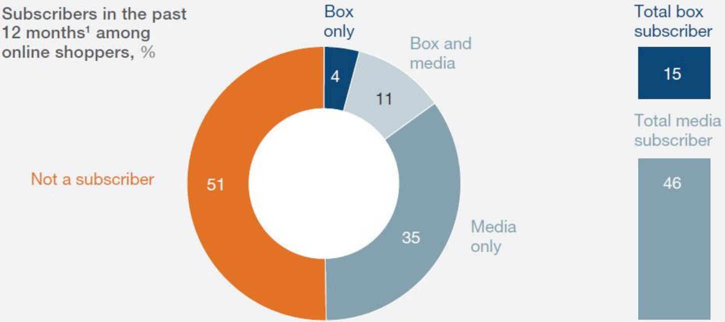
Frontier Airlines Now Has an Unlimited Pass for Summer — Here's How to Score One

“For people with flexible schedules, this is a terrific opportunity to have a truly epic summer and then some, soaking up rays on the beach, exploring national parks and visiting new cities.”

By **Alison Fox** | Updated on February 1, 2023

Subscription Services

Subscriptions are an increasingly common way to buy products and services online.



Note: Figures may not sum to 100%, because of rounding.

[1]Which of the following have you purchased or subscribed to in the past 12 months? % of those selecting online subscription-box service that delivers products regularly (eg, Blue Apron, Dollar Shave Club, Ipsy, Stitch Fix), subscription-based media (eg, ClassPass, Hulu, Netflix, Spotify), both, or neither.

McKinsey&Company | **Source:** McKinsey analysis

Subscription Services

E-commerce subscriptions generally fall into one of three categories.

E-commerce subscriptions, %		Key consumer value	Description	Example companies
Subscribe for replenishment	32	Save time and money	Replenish the same or similar items Primary categories are commodity items such as razors, vitamins	Amazon Subscribe & Save, Dollar Shave Club, and Ritual
Subscribe for curation	55	Be surprised by product variety	Receive a curated selection of different items, with varying levels of consumer decision making required Primary categories are apparel, food, beauty products	Birchbox, Blue Apron, and Stitch Fix
Subscribe for access	13	Gain exclusive access	Membership provides access and can convey additional "VIP" perks Primary categories are apparel, food,	JustFab, NatureBox, and Thrive Market
	100%			

Subscription Services

Industry	Product or Service	Price ($)	Period	Total subscribers
Media & Entertainment	Netflix	9.99	Monthly	23 million (US)
	Spotify	9.99	Monthly	70 million (World)
	New York Times	3.75	Weekly	4 million (US)
	MoviePass	19.95	Monthly	2 million
	Kindle Unlimited	9.99	Monthly	–
	Apple News	9.99	Monthly	36 million
Software-as-a-Service	Microsoft Office 365	9.99	Monthly	120 million
	Adobe Creative Cloud (One App)	20.99	Monthly	15 million
	Dropbox Premium	9.99	Monthly	>11 million
Membership Clubs	Costco (Basic)*	60	Annual	94 million
	Amazon Prime	119	Annual	90 million
	24 hour fitness (Gym)	40	Monthly	4 million
eCommerce	Harry's	35	Monthly	–
	Birchbox	15	Monthly	2 million
	Rent the Runway	159	Monthly	6 million
Transportation	Public Transit Pass (MTA)	121	30-days	–
	Uber Ride Pass*	14.99	Monthly	–
	Jetblue "All You can Jet" Pass	699	Monthly	–

Subscription business

- Design product + pricing in subscription markets:
  - Which plans to offer?
  - What feature or value dimensions to offer in each plan?
  - How to price the plans?
  - How to design plans for specific demographic segments (e.g. students).
- Everything relies on knowing the distribution of willingness to pay (WTP) for subscription service.
  - Demand curve
  - Elasticities

Related Research - WTP elicitation

- WTP has been a topic of interest in marketing and economics
- Conjoint typically helps in figuring out valuation or part-worths for attributes (Green and Rao, 1971)
- Revealed preference stream uses transaction data for demand estimation, with individual data (Guadagni and Little 1983) or aggregate data (Berry 1994, BLP 1995)
- Comprehensive Survey: Breidert (2007)

What's common to above?

All these cases have price variation!

Related Research - Consumption Data

- Models in marketing and economics typically focus on **Purchase**
- **Consumption** data typically an afterthought — often unobserved
- Vast majority of applications in consumer packaged goods where usage is not observed by the researcher
  - Gupta (1988), Sun (2005), Hendel and Nevo (2006a, 2006b, 2013)
- Consumption in above is inferred, treated like nuisance
  - Limited exceptions with consumption data: Nevo, Turner and Williams (2016), Huang, Khwaja and Sudhir (2015)

Big Picture Idea:

Leverage high frequency usage data for identification.

Contribution

- Main contribution: a novel method to identify & estimate the distribution of WTP given customer characteristics and product features when only usage variation is present.
- We also obtain the conditional WTP distribution (so, we can get WTP based on observables like gender / age / student etc.)
- No existing research that demonstrates how to obtain the WTP distribution in the absence of price variation.
    - Nevo, Turner and Williams (ECTA, 2016) leverages an "overage charge"

Research questions

Focus:

Obtain WTP estimates for a subscription service with high frequency usage data

More specifically:

1. In absence of price variation, under what conditions on usage is it possible to identify distribution of WTP?
2. Is price variation the same as usage variation or is there *additional* value?
3. What demand responses and profits to counterfactual product and pricing choices by the firm can be determined?

With Price Variation – Notation

Cross section data with price variation.

Notation

- i indicates a consumer
- Subscription decision: $S_i = 1$ (sub) and $= 0$ (not).
- WTP: W_i
- Price: P_i

- Decision rule:

$$\underbrace{W_i - P_i}_{\substack{\text{money-metric} \\ \text{utility of service}}} \quad \text{vs} \quad \underbrace{\mu = 0}_{\substack{\text{money-metric utility} \\ \text{of outside option}}} \quad \Rightarrow$$

$$S_i = \begin{cases} 1, & W_i > P_i \\ 0, & W_i \leq P_i. \end{cases}$$

or $S_i = \mathbb{I}(W_i > P_i)$.

- When $W_i \perp\!\!\!\perp P_i$, for any w in the support of P_i

$$\underbrace{\Pr(W_i > w)}_{\substack{\text{Parameter: prob WTP} \\ > w \text{ in the entire pop}}} = \underbrace{\Pr(S_i = 1 \mid P_i = w)}_{\substack{\text{Data: Mkt shr in the pop} \\ \text{facing price } w}}.$$

Model Overview and Elements

Model is based on microfoundations of usage based on leisure, aggregated over time

- **High frequency Usage:** Consumer has a daily leisure budget, allocated between focal good and everything else
  - Exogenous shifters impact leisure budget
  - Form expectations over the daily leisure process, conditional on observables
  - Rational expectations (or perfect foresight)
- **Low frequency Purchase:** Consumer makes purchase (subscription) decisions every T periods at constant price P
  - Form expectations about future usage in making purchase decision

Microfoundations of Usage

Consider the consumer allocating leisure time:

- consumer leisure time spent in focal activity, e.g. listening to streaming music q_{it},
- Other "leisure" activities (e.g. playing outdoors) q_{0it}.
- Specify a money-metric utility function:

$$u_{it}(q_{it}, q_{0it}) = D_{it} u^{(1)}(q_{it}, q_{0it}; \theta_{im(t)}) + (1 - D_{it}) u^{(0)}(q_{0it}; \theta_{im(t)})$$

- $D_{it} \in \{0, 1\}$ is an indicator for whether the focal activity is present or absent $\implies$ rationalizes zero usage in many periods

Microfoundations of Usage

We need to characterize usage at the daily level and relate to the monthly level WTP

- Daily leisure is modeled as depending on exogenous factors Z_{it}:

$$\ell_{it} = \mu_i + \gamma' Z_{it} + \varepsilon_{it},$$

- μ_i is heterogeneous across individuals
- Z_{it} includes example variables like weekend or holiday dummy variables or weather
- Leisure shocks ε_{it} can be serially correlated (ignore for now)
- Monthly expected leisure $\boxed{L_{im} \equiv \sum_{t:m(t)=m} (\mu_i + \gamma' Z_{it})}$

Characterization of Value of Subscription

Connecting daily usage of focal service to monthly indirect utility:

Theorem (Usage to Indirect Utility)

For any utility function homogeneous of degree 1, the difference between the expected monthly indirect utilities with and without a subscription, W_{im}, satisfies

$$W_{im} = \alpha_{im} L_{im} \qquad \text{or} \qquad \ln W_{im} = \ln \alpha_{im} + \ln L_{im},$$

- The daily usage of the subscription satisfies

$$Q_{it} = D_{it} r_{im(t)} \ell_{it},$$

- What class of utility functions are included?
  - Cobb-Douglas, CES, perfect substitutes, perfect complements, Leontief

Subscription Decisions

We know that WTP is: $W_{im} = \alpha_{im} L_{im}$

- account of consumer heterogeneity, both observed X_{im} and unobserved U_{im}. Consider a linear projection of $\ln \alpha_{im}$ onto X_{im} as:

$$\ln \alpha_{im} = \beta_0 + \beta_1' X_{1im} + U_{im},$$

where $\beta' = (\beta_0, \beta_1')$ and $X_{im}' = (1, X_{1im}')$.

- Subscription choice $S_{im} = \mathbb{I}(\ln W_{im} > \ln P)$ becomes

$$S_{im} = \mathbb{I}(\ln L_{im} + \beta' X_{im} - \ln P + U_{im} > 0).$$

Exogenous Variation

What exogenous variations are required for identification?

Assumption (Exogenous Variation)

$\mathbf{Z}_{im} \perp\!\!\!\perp U_{im} \mid (X_{im}, \mu_i)$,

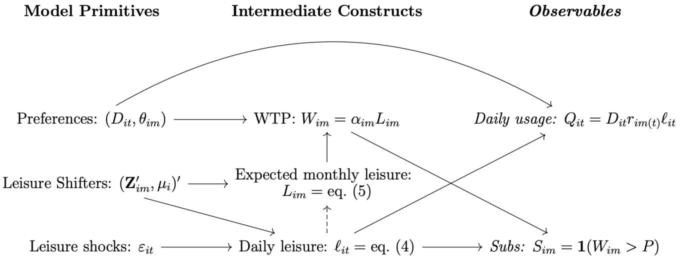
- Above implies monthly expected leisure $L_{im} \perp\!\!\!\perp U_{im} \mid (X_{im}, \mu_i)$ because the randomness of L_{im} only comes from $\mathbf{Z}_{im}$ and μ_i.

Model Components – Overview

Model Primitives | **Intermediate Constructs** | ***Observables***

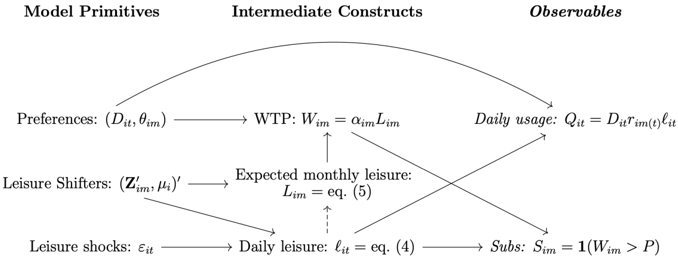
$$
\begin{tikzcd}
\text{Preferences: } (D_{it}, \theta_{im}) \arrow[r] \arrow[rr, bend left] & \text{WTP: } W_{im} = \alpha_{im} L_{im} \arrow[ddr] & \textit{Daily usage: } Q_{it} = D_{it} r_{im(t)} \ell_{it} \\
\text{Leisure Shifters: } (\mathbf{Z}'_{im}, \mu_i)' \arrow[r] \arrow[dr] & \begin{array}{c}\text{Expected monthly leisure:}\\ L_{im} = \text{eq. } (5)\end{array} \arrow[u] & \\
\text{Leisure shocks: } \varepsilon_{it} \arrow[r] & \text{Daily leisure: } \ell_{it} = \text{eq. } (4) \arrow[r] \arrow[u, dashed] \arrow[uur] & \textit{Subs: } S_{im} = \mathbf{1}(W_{im} > P)
\end{tikzcd}
$$

Model Components – Overview



- Usage model [High frequency]:
  - Leisure shifters $\Longrightarrow$ Daily leisure $\Longrightarrow$ Usage
  - Usage / Leisure parameters are separately identified.
- Purchase model [Low frequency]:
  - Expectation of Leisure shifters $\Longrightarrow$ Aggregate Leisure
  - Aggregate Leisure + Consumer-level variables (vary across consumer and time) $\Longrightarrow$ Purchase

Model Components – Overview



- We capture across consumer heterogeneity in a couple of ways:
    - Usage: Unobservable heterogeneity captured by μ_i
    - Purchase: Observed heterogeneity captured by X_{im} and Unobserved heterogeneity by U_{im}

Main Result

Theorem (Parametric Identification of WTP)

We have the following results when $U_{im} \mid (X_{im}, \mu_i) \sim \mathcal{N}(\sigma_{u,\mu}\mu^*_{im}, \sigma^2_u)$

1. *The unknown parameters* $(\beta, \sigma_u, \sigma_{u,\mu})$ *are identified.*
2. *The distribution of WTP is identified, and*

$$F_W(w \mid X_{im}, \mu_i, L_{im}) = \Phi\left[\frac{1}{\sigma_u}\left(\ln w - \ln L_{im} - \beta' X_{im} - \sigma_{u,\mu}\mu^*_{im}\right)\right].$$

We do not need this parametric assumption above.

Boundary conditions of method

- What happens without usage data? Subscription equation

$$\begin{aligned} S_{im} &= \mathbb{I}(\ln L_{im} - \ln P + \beta' X_{im} + U_{im} > 0) \\ &= \mathbb{I}[(\beta_0 - \ln P) + \beta_1' X_{1im} + (\ln L_{im} + U_{im}) > 0] \end{aligned}$$

  Cannot distinguish between L_{im} and U_{im}

- Without exogenous shifters Z_{it}, again this approach will not work
- Need both usage data and exogenous shifters

Is Usage Variation the Same as Price Variation

- If we want to identify switching costs, no amount of usage variation is sufficient..
  - Why?
- Consider a more general subscription choice with δ:

$$S_{im} = \mathbb{I}(\ln L_{im} - \ln(P_{im} - \delta' X_{2im}) + \beta_0 + \beta_1' X_{1im} + U_{im} > 0).$$

Switching Cost

Need at least 2 price levels to identify switching cost.

Big usage data of YBOX, a music streaming service

- YBOX is a music streaming service targeting Southeast Asia.
- 1 million users data (Jan 2015–Feb 2017):
  - subscription history
  - daily # of seconds listening music with the service
  - basic demographics (age and gender)
- No price variation for monthly music streaming service over time
- Average daily listening hours range from 45 mins to $>$ 6 hours
- Average monthly listening hours range from less than 1 hour to more than 150 hours.

Estimation – Usage

- Leisure: $\ell_{it} = \mu_i + \gamma' Z_{it} + \varepsilon_{it}$ and Usage $Q_{it} = D_{it} r_{im(t)} \ell_{it}$
- **Step 1:** Estimate the usage model using finite mixture heterogeneity. Let $(\hat{\mu}_i, \hat{r}_{im}, \hat{\gamma}')$ be the estimates of (μ_i, r_{im}, γ')
- **Step 2:** Estimate monthly expected leisure L_{im} by substituting the unknown parameters (μ_i, γ') with the estimates $(\hat{\mu}_i, \hat{\gamma}')$. Denote this estimator by $\hat{L}_{im}$.

Estimation – Subcription

WTP for the service: $W_{im} = \alpha_{im} L_{im}$

$$\ln \alpha_{im} = \beta_0 + \beta_1' X_{1im} + U_{im}$$

$$S_{im} = \mathbb{I}(\ln L_{im} + \beta' X_{im} - \ln P + U_{im} > 0)$$

- **Step 3:** For each month m, implement a linear regression of $\hat{\mu}_i$ on X_{im} and obtain the residuals $\hat{\mu}^*_{im}$. These residuals are the estimates of μ^*_{im}.
- **Step 4:** Run the probit regression of S_{im} on $\ln(\hat{L}_{im}/P)$, X_{im}, and $\hat{\mu}^*_{im}$. The probit regression provides estimates of σ_u^{-1}, β/σ_u, $\sigma_{u,\mu}/\sigma_u$. Then the estimates of β and $\sigma_{u,\mu}$ are obtained easily.

Empirical Application with Music Streaming

	All Users	Never Cancelled	Ever Cancelled
Monthly Usage (Hours)	41.73	44.25	18.48
	(50.65)	(52.07)	(24.76)
Daily Usage (Hours): Weekend	1.31	1.39	0.57
	(2.21)	(2.27)	(1.41)
Daily Usage (Hours): Weekdays	1.39	1.47	0.62
	(2.28)	(2.35)	(1.30)
Age	30.91	31.12	29.69
	(9.09)	(9.32)	(7.56)
Female (%)	42.00	42.35	40.00
Number of Users	300	255	45

Empirical Application with Music Streaming

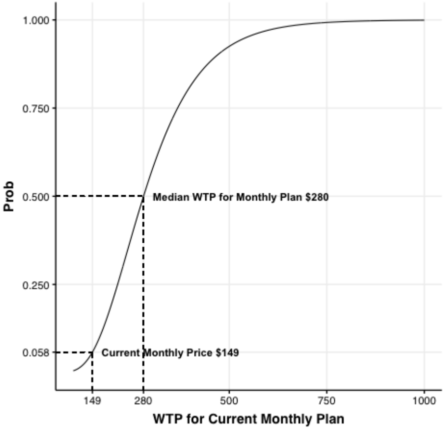


Figure: Estimates of the Distribution of WTP for the Monthly Plan

Empirical Application with Music Streaming

	Parameters	Estimates	Std Err
Usage eq.	$\mu_{Type\,1}$	0.8279	(0.0471)
	$r_{Type\,1}$	2.1130	(0.1566)
	$\gamma_{Holiday,Type\,1}$	0.0297	(0.0157)
	$\gamma_{Weekend,Type\,1}$	0.0257	(0.0142)
	$\mu_{Type\,2}$	0.8339	(0.0539)
	$r_{Type\,2}$	5.3138	(0.9502)
	$\gamma_{Holiday,Type\,2}$	-0.0365	(0.0223)
	$\gamma_{Weekend,Type\,2}$	-0.0369	(0.0251)
	$\gamma_{Humidity}$	-0.0010	(0.0005)
	$\gamma_{Precipitation}$	0.0004	(0.0002)
Subscription eq.	β_0/σ_u	5.9226	(1.4853)
	$1/\sigma_u$	2.5261	(0.7895)
	β_{Age}/σ_u	0.0115	(0.0039)
	β_{Female}/σ_u	0.1095	(0.0698)
	$\sigma_{u,\mu}/\sigma_u$	-6.2721	(14.0592)

Empirical Application with Music Streaming

Segment	Price Elasticity		Revenue Max Price	Mean Usage	Median WTP ($)
All Users	−0.31	(0.10)	206	1.37	280.00
Male	−0.33	(0.11)	202	1.43	275.00
Female	−0.27	(0.08)	212	1.29	288.00
Age $\leq$ 22	−0.37	(0.13)	197	1.45	268.00
Age 23–30	−0.34	(0.11)	201	1.55	273.00
Age $>$ 30	−0.26	(0.08)	214	1.22	290.00

Empirical Application with Music Streaming

User Groups	Humidity Only	Precipitation Only	Both
All Users	−0.307	−0.367	−0.366
	(0.098)	(0.106)	(0.105)
Male	−0.332	−0.397	−0.396
	(0.111)	(0.122)	(0.121)
Female	−0.273	−0.326	−0.325
	(0.083)	(0.090)	(0.089)
Age $\leq$ 22	−0.368	−0.439	−0.437
	(0.129)	(0.142)	(0.141)
Age 23–30	−0.339	−0.405	−0.403
	(0.114)	(0.125)	(0.124)
Age $>$ 30	−0.261	−0.313	−0.312
	(0.078)	(0.083)	(0.083)

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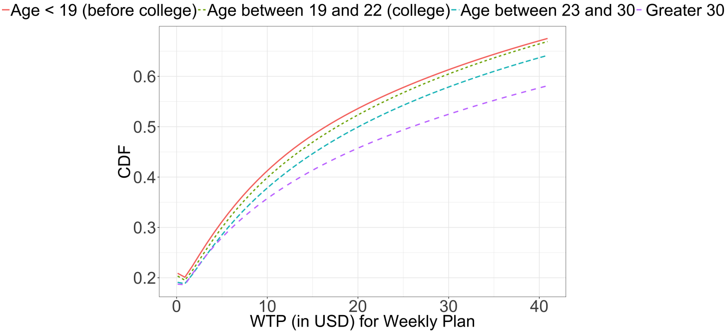
Empirical Application with Music Streaming

WTP variation with age / college status



Conclusions

Without Price variation, can we obtain WTP?

- A: Qualified Yes.
- What big data on usage tracking can tell us?
  - The distribution of WTP under some restrictions
- Can design counterfactual products and pricing strategies
- Cannot replace the role of price variation, even limited, in identifying switching costs

Duration as a segmentation device

- Firms offer plans of different durations, e.g. Amazon offers Prime monthly and annual plans
- What's the distribution of the WTP for the shorter plan?
- One idea is to examine whether we can use duration effectively as a segmentation device
- When does it work well and when does it not?

Segmentation by duration

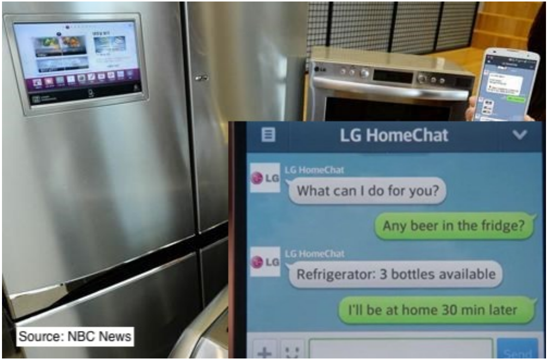
Identify interesting mechanisms based on plan duration

- Shorter plans allow flexibility and could increase consumer WTP
  - WTP for Prime might be higher during holiday season, maybe I just buy then?
- Longer plans:
  - Can lock in consumers in the presence of switching costs, firms have to discount
  - (Bundling-like) Pool over time periods and can help reduce across consumer heterogeneity

Segmentation by duration

- Longer plans:
  - (Bundling-like) Pool over time periods and can help reduce across consumer heterogeneity
  - Firm is deciding between 1 month plan and 2 month plans
  - Consumer A has high utility v_H in month 1 and low utility v_L in month 2, so is HL type
  - If we offer a 2 month plan, then both consumers should have WTP: $(v_L + v_H)$
  - Might make it easier to extract surplus
  - But, wait ... including 3rd month $\implies$ heterogeneity $\uparrow$
- Duration can be a strategic design decision so the firm needs to figure out what duration plans to offer.

A bigger picture (of a fridge)



- Essentially, we need the separation of purchase (subscription) and consumption (usage).
- Such separation also holds in packaged goods (beer)—but we did not track the usage.

My Research Overview

- Substantive: Digital Business Models
- Methodological: Structural Models $\Longleftrightarrow$ Machine Learning
  - Different approaches to ML

Some projects:

- Nonparametric Bandits Leveraging Informational Externalities to Learn the Demand Curve, with Ian Weaver (Major Revision at Marketing Science)
- Automatically Discovering Visual Product Characteristics, with Ankit Sisodia and Alex Burnap (Revision at Journal of Marketing Research)

ADDITIONAL SLIDES