Demand Estimation for Subscription Models

Identifying Willingness to Pay without Price Variation

Cheng Chou [1]   Vineet Kumar [2]

[1]University of Leicester, UK

[2]Yale University, USA

Summer Institute in Competitive Strategy
UC Berkeley, CA
June 2023

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  - **Pay upfront and consume over time**

Subscription Services

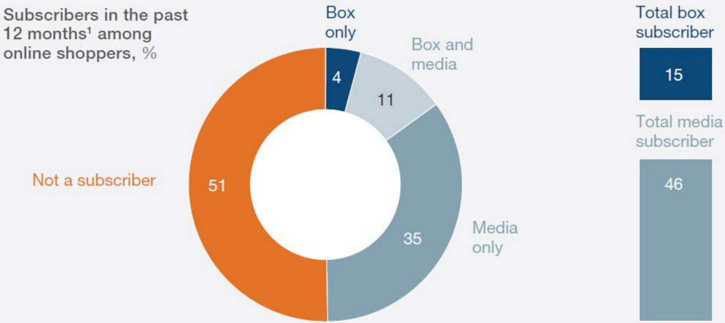
Frontier Airlines Now Has an Unlimited Pass for Summer — Here's How to Score One

“For people with flexible schedules, this is a terrific opportunity to have a truly epic summer and then some, soaking up rays on the beach, exploring national parks and visiting new cities.”

By **Alison Fox** | Updated on February 1, 2023

Subscription Services

Subscriptions are an increasingly common way to buy products and services online.

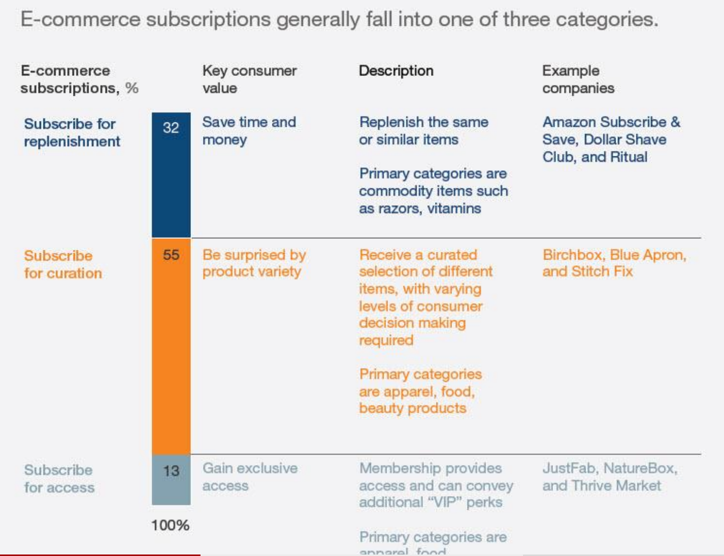


Note: Figures may not sum to 100%, because of rounding.

[1]Which of the following have you purchased or subscribed to in the past 12 months? % of those selecting online subscription-box service that delivers products regularly (eg, Blue Apron, Dollar Shave Club, Ipsy, Stitch Fix), subscription-based media (eg, ClassPass, Hulu, Netflix, Spotify), both, or neither.

McKinsey&Company | **Source:** McKinsey analysis

Subscription Services


E-commerce subscriptions generally fall into one of three categories.

E-commerce subscriptions, %		Key consumer value	Description	Example companies
Subscribe for replenishment	32	Save time and money	Replenish the same or similar items Primary categories are commodity items such as razors, vitamins	Amazon Subscribe & Save, Dollar Shave Club, and Ritual
Subscribe for curation	55	Be surprised by product variety	Receive a curated selection of different items, with varying levels of consumer decision making required Primary categories are apparel, food, beauty products	Birchbox, Blue Apron, and Stitch Fix
Subscribe for access	13	Gain exclusive access	Membership provides access and can convey additional "VIP" perks Primary categories are	JustFab, NatureBox, and Thrive Market
	100%			

Subscription Services

Industry	Product or Service	Price ($)	Period	Total subscribers
Media & Entertainment	Netflix	9.99	Monthly	23 million (US)
	Spotify	9.99	Monthly	70 million (World)
	New York Times	3.75	Weekly	4 million (US)
	MoviePass	19.95	Monthly	2 million
	Kindle Unlimited	9.99	Monthly	–
	Apple News	9.99	Monthly	36 million
Software-as-a-Service	Microsoft Office 365	9.99	Monthly	120 million
	Adobe Creative Cloud (One App)	20.99	Monthly	15 million
	Dropbox Premium	9.99	Monthly	>11 million
Membership Clubs	Costco (Basic)*	60	Annual	94 million
	Amazon Prime	119	Annual	90 million
	24 hour fitness (Gym)	40	Monthly	4 million
eCommerce	Harry's	35	Monthly	–
	Birchbox	15	Monthly	2 million
	Rent the Runway	159	Monthly	6 million
Transportation	Public Transit Pass (MTA)	121	30-days	–
	Uber Ride Pass*	14.99	Monthly	–
	Jetblue "All You can Jet" Pass	699	Monthly	–

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    - Elasticities

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What's common to above?

All these cases have price variation!

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Big Picture Idea:

Leverage high frequency usage data for identification.

Usage is captured at higher frequency than purchase.

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    - Gentzkow (2007)...

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Obtain WTP estimates for a subscription service with high frequency usage data

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More specifically:

1. In absence of price variation, under what conditions on usage is it possible to identify distribution of WTP?
2. Is price variation the same as usage variation or is there *additional* value?
3. What demand responses and profits to counterfactual product and pricing choices by the firm can be determined?

With Price Variation – Notation

Cross section data with price variation.

Notation

- i indicates a consumer
- Subscription decision: $S_i = 1$ (sub) and $= 0$ (not).
- WTP: W_i
- Price: P_i

- Decision rule:

$$\underbrace{W_i - P_i}_{\substack{\text{money-metric}\\ \text{utility of service}}} \quad \text{vs} \quad \underbrace{\mu = 0}_{\substack{\text{money-metric utility}\\ \text{of outside option}}} \quad \Rightarrow$$

$$S_i = \begin{cases} 1, & W_i > P_i \\ 0, & W_i \leq P_i. \end{cases}$$

or $S_i = \mathbb{I}(W_i > P_i)$.

- When $W_i \perp\!\!\!\perp P_i$, for any w in the support of P_i

$$\underbrace{\Pr(W_i > w)}_{\substack{\text{Parameter: prob WTP}\\ > w \text{ in the entire pop}}} = \underbrace{\Pr(S_i = 1 \mid P_i = w)}_{\substack{\text{Data: Mkt shr in the pop}\\ \text{facing price } w}}.$$

Model Overview and Elements

Should we model greater utility and WTP with greater usage?

Model is based on microfoundations of usage based on leisure, aggregated over time

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- $D_{it} \in \{0, 1\}$ is an indicator for whether the focal activity is present or absent $\implies$ rationalizes zero usage in many periods

Microfoundations of Usage

We need to characterize usage at the daily level and relate to the monthly level WTP

- Daily leisure is modeled as depending on exogenous factors Z_{it}:

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- Monthly expected leisure $\boxed{L_{im} \equiv \sum_{t:m(t)=m} (\mu_i + \gamma' Z_{it})}$

Characterization of Value of Subscription

Connecting daily usage of focal service to monthly indirect utility:

Theorem (Usage to Indirect Utility)

For any utility function homogeneous of degree 1, the difference between the expected monthly indirect utilities with and without a subscription, W_{im}, satisfies

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- What class of utility functions are included?
  - Cobb-Douglas, CES, perfect substitutes, perfect complements, Leontief

Subscription Decisions

We know that WTP is: $W_{im} = \alpha_{im} L_{im}$

- account of consumer heterogeneity, both observed X_{im} and unobserved U_{im}. Consider a linear projection of $\ln \alpha_{im}$ onto X_{im} as:

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- Subscription choice $S_{im} = \mathbb{I}(\ln W_{im} > \ln P)$ becomes

$$S_{im} = \mathbb{I}(\ln L_{im} + \beta' X_{im} - \ln P + U_{im} > 0).$$

Exogenous Variation

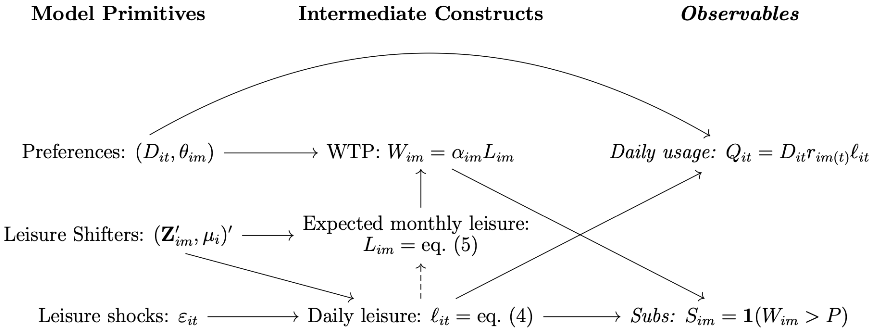
What exogenous variations are required for identification?

Assumption (Exogenous Variation)

$\mathbf{Z}_{im} \perp\!\!\!\perp U_{im} \mid (X_{im}, \mu_i)$,

- Above implies monthly expected leisure $L_{im} \perp\!\!\!\perp U_{im} \mid (X_{im}, \mu_i)$ because the randomness of L_{im} only comes from $\mathbf{Z}_{im}$ and μ_i.

Model Components – Overview



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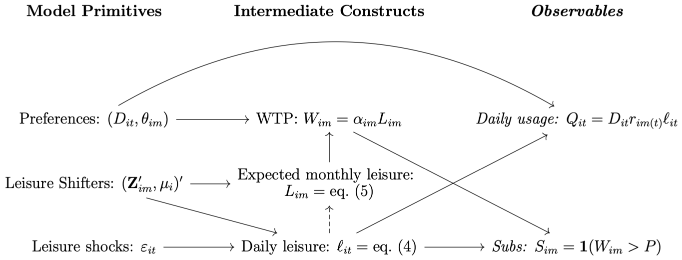

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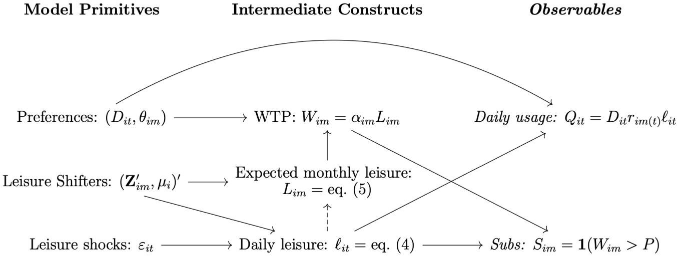

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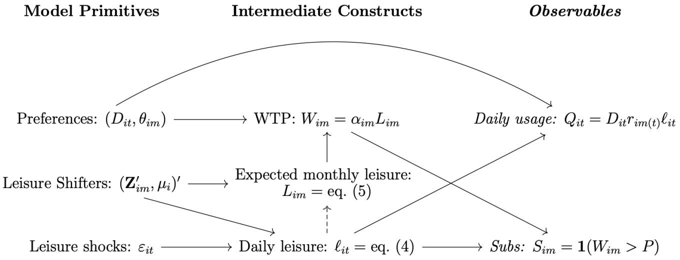

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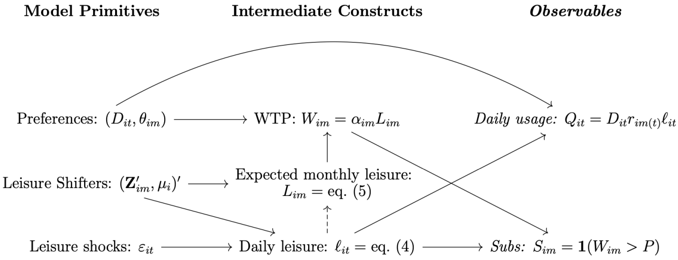

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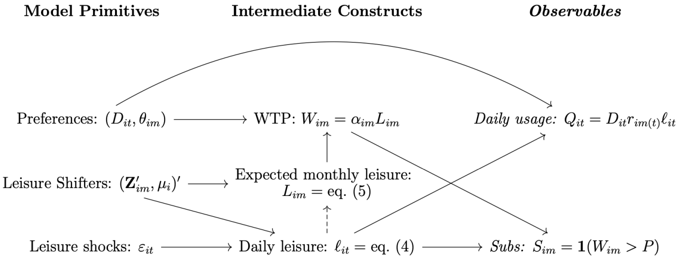

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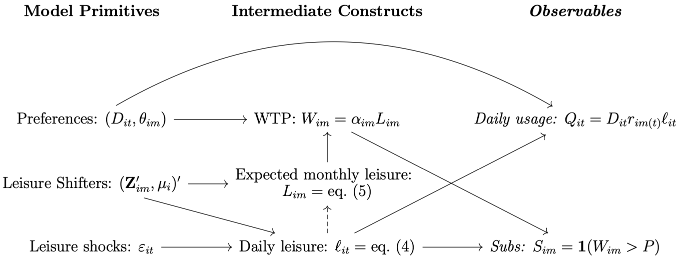

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  - Aggregate Leisure + Consumer-level variables (vary across consumer and time) $\Longrightarrow$ Purchase

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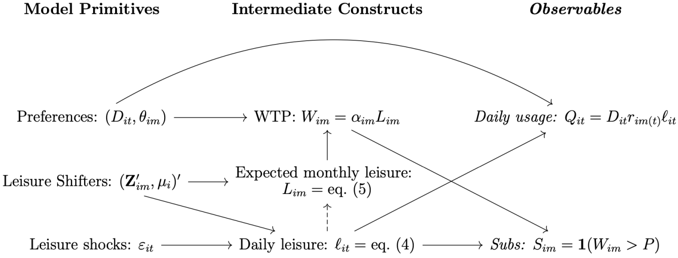

- We capture across consumer heterogeneity in a couple of ways:

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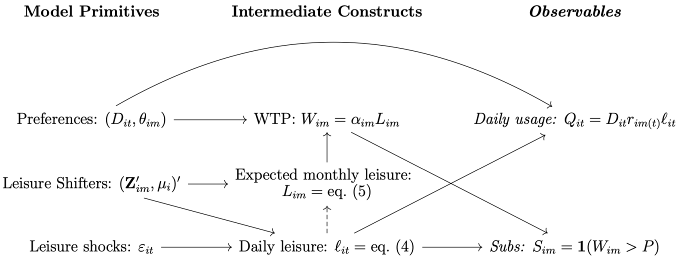

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Model Components – Overview

$$
\begin{tikzcd}
\textbf{Model Primitives} & \textbf{Intermediate Constructs} & \textbf{\textit{Observables}} \\
\text{Preferences: } (D_{it}, \theta_{im}) \arrow[r] \arrow[rr, bend left] & \text{WTP: } W_{im} = \alpha_{im} L_{im} \arrow[ddr] & \textit{Daily usage: } Q_{it} = D_{it} r_{im(t)} \ell_{it} \\
\text{Leisure Shifters: } (\mathbf{Z}'_{im}, \mu_i)' \arrow[r] \arrow[dr] & \begin{array}{c}\text{Expected monthly leisure:}\\ L_{im} = \text{eq. (5)}\end{array} \arrow[u] & \\
\text{Leisure shocks: } \varepsilon_{it} \arrow[r] & \text{Daily leisure: } \ell_{it} = \text{eq. (4)} \arrow[r] \arrow[u, dashed] \arrow[uur] & \textit{Subs: } S_{im} = \mathbf{1}(W_{im} > P)
\end{tikzcd}
$$

- We capture across consumer heterogeneity in a couple of ways:
  - Usage: Unobservable heterogeneity captured by μ_i
  - Purchase: Observed heterogeneity captured by X_{im} and Unobserved heterogeneity by U_{im}

Main Result

Theorem (Parametric Identification of WTP)

We have the following results when $U_{im} \mid (X_{im}, \mu_i) \sim \mathcal{N}(\sigma_{u,\mu}\mu^*_{im}, \sigma^2_u)$

1. *The unknown parameters* $(\beta, \sigma_u, \sigma_{u,\mu})$ *are identified.*
2. *The distribution of WTP is identified, and*

$$F_W(w \mid X_{im}, \mu_i, L_{im}) = \Phi\left[\frac{1}{\sigma_u}\left(\ln w - \ln L_{im} - \beta' X_{im} - \sigma_{u,\mu}\mu^*_{im}\right)\right].$$

We do not need this parametric assumption above.

Boundary conditions of method

- What happens without usage data? Subscription equation

$$\begin{aligned} S_{im} &= \mathbb{I}(\ln L_{im} - \ln P + \beta' X_{im} + U_{im} > 0) \\ &= \mathbb{I}[(\beta_0 - \ln P) + \beta_1' X_{1im} + (\ln L_{im} + U_{im}) > 0] \end{aligned}$$

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  Cannot distinguish between L_{im} and U_{im}
- Without exogenous shifters Z_{it}, again this approach will not work
- Need both usage data and exogenous shifters

Is Usage Variation the Same as Price Variation

- If we want to identify switching costs, no amount of usage variation is sufficient..

Switching Cost

Need at least 2 price levels to identify switching cost.

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Is Usage Variation the Same as Price Variation

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  - Why?
- Consider a more general subscription choice with δ:

$$S_{im} = \mathbb{I}(\ln L_{im} - \ln(P_{im} - \delta' X_{2im}) + \beta_0 + \beta_1' X_{1im} + U_{im} > 0).$$

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Estimation – Usage

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- **Step 2:** Estimate monthly expected leisure L_{im} by substituting the unknown parameters (μ_i, γ') with the estimates $(\hat{\mu}_i, \hat{\gamma}')$. Denote this estimator by $\hat{L}_{im}$.

Estimation – Subcription

WTP for the service: $W_{im} = \alpha_{im} L_{im}$

$$\ln \alpha_{im} = \beta_0 + \beta_1' X_{1im} + U_{im}$$

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- **Step 4:** Run the probit regression of S_{im} on $\ln(\hat{L}_{im}/P)$, X_{im}, and $\hat{\mu}^*_{im}$. The probit regression provides estimates of σ_u^{-1}, β/σ_u, $\sigma_{u,\mu}/\sigma_u$. Then the estimates of β and $\sigma_{u,\mu}$ are obtained easily.

Empirical Application with Music Streaming

	All Users	Never Cancelled	Ever Cancelled
Monthly Usage (Hours)	41.73	44.25	18.48
	(50.65)	(52.07)	(24.76)
Daily Usage (Hours): Weekend	1.31	1.39	0.57
	(2.21)	(2.27)	(1.41)
Daily Usage (Hours): Weekdays	1.39	1.47	0.62
	(2.28)	(2.35)	(1.30)
Age	30.91	31.12	29.69
	(9.09)	(9.32)	(7.56)
Female (%)	42.00	42.35	40.00
Number of Users	300	255	45

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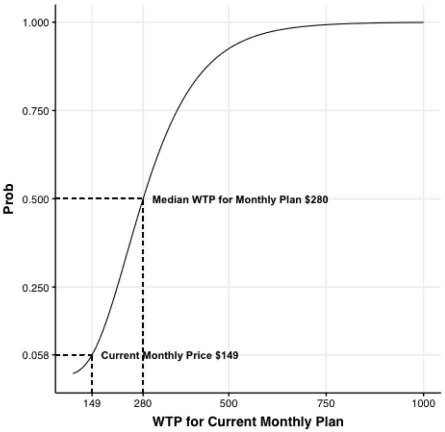

Figure: Estimates of the Distribution of WTP for the Monthly Plan

Empirical Application with Music Streaming

	Parameters	Estimates	Std Err
Usage eq.	$\mu_{Type\,1}$	0.8279	(0.0471)
	$r_{Type\,1}$	2.1130	(0.1566)
	$\gamma_{Holiday,Type\,1}$	0.0297	(0.0157)
	$\gamma_{Weekend,Type\,1}$	0.0257	(0.0142)
	$\mu_{Type\,2}$	0.8339	(0.0539)
	$r_{Type\,2}$	5.3138	(0.9502)
	$\gamma_{Holiday,Type\,2}$	−0.0365	(0.0223)
	$\gamma_{Weekend,Type\,2}$	−0.0369	(0.0251)
	$\gamma_{Humidity}$	−0.0010	(0.0005)
	$\gamma_{Precipitation}$	0.0004	(0.0002)
Subscription eq.	β_0/σ_u	5.9226	(1.4853)
	$1/\sigma_u$	2.5261	(0.7895)
	β_{Age}/σ_u	0.0115	(0.0039)
	β_{Female}/σ_u	0.1095	(0.0698)
	$\sigma_{u,\mu}/\sigma_u$	−6.2721	(14.0592)

Empirical Application with Music Streaming

Segment	Price Elasticity		Revenue Max Price	Mean Usage	Median WTP ($)
All Users	−0.31	(0.10)	206	1.37	280.00
Male	−0.33	(0.11)	202	1.43	275.00
Female	−0.27	(0.08)	212	1.29	288.00
Age $\leq$ 22	−0.37	(0.13)	197	1.45	268.00
Age 23–30	−0.34	(0.11)	201	1.55	273.00
Age > 30	−0.26	(0.08)	214	1.22	290.00

Empirical Application with Music Streaming

User Groups	Humidity Only	Precipitation Only	Both
All Users	−0.307	−0.367	−0.366
	(0.098)	(0.106)	(0.105)
Male	−0.332	−0.397	−0.396
	(0.111)	(0.122)	(0.121)
Female	−0.273	−0.326	−0.325
	(0.083)	(0.090)	(0.089)
Age $\leq$ 22	−0.368	−0.439	−0.437
	(0.129)	(0.142)	(0.141)
Age 23–30	−0.339	−0.405	−0.403
	(0.114)	(0.125)	(0.124)
Age $>$ 30	−0.261	−0.313	−0.312
	(0.078)	(0.083)	(0.083)

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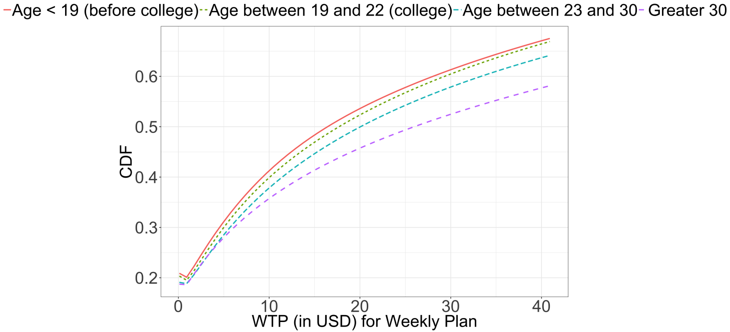
WTP variation with age / college status



Conclusions

Without Price variation, can we obtain WTP?

- A: Qualified Yes.
- What big data on usage tracking can tell us?
  - The distribution of WTP under some restrictions
- Can design counterfactual products and pricing strategies
- Cannot replace the role of price variation, even limited, in identifying switching costs

Strategic Plan Duration

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- When does it work well and when does it not?

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Identify interesting mechanisms based on plan duration

- Shorter plans allow flexibility and could increase consumer WTP

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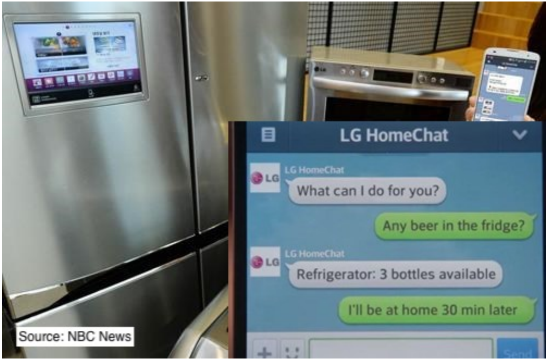
  - But, wait ... including 3rd month $\Longrightarrow$ heterogeneity $\uparrow$
- Can we characterize the optimal duration as a function of heterogeneity distribution?

A bigger picture (of a fridge)



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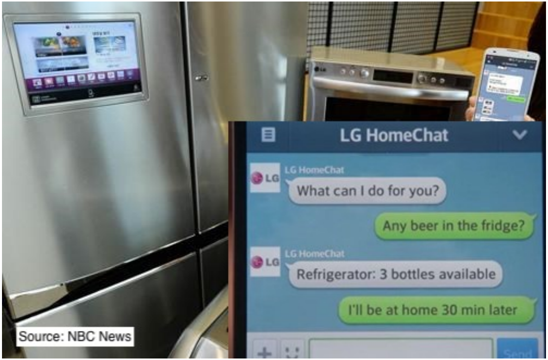

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- Such separation also holds in packaged goods (beer)—but we did not track the usage.