Homework 1

Question 1

Part (a)

The sketch is given as follows:

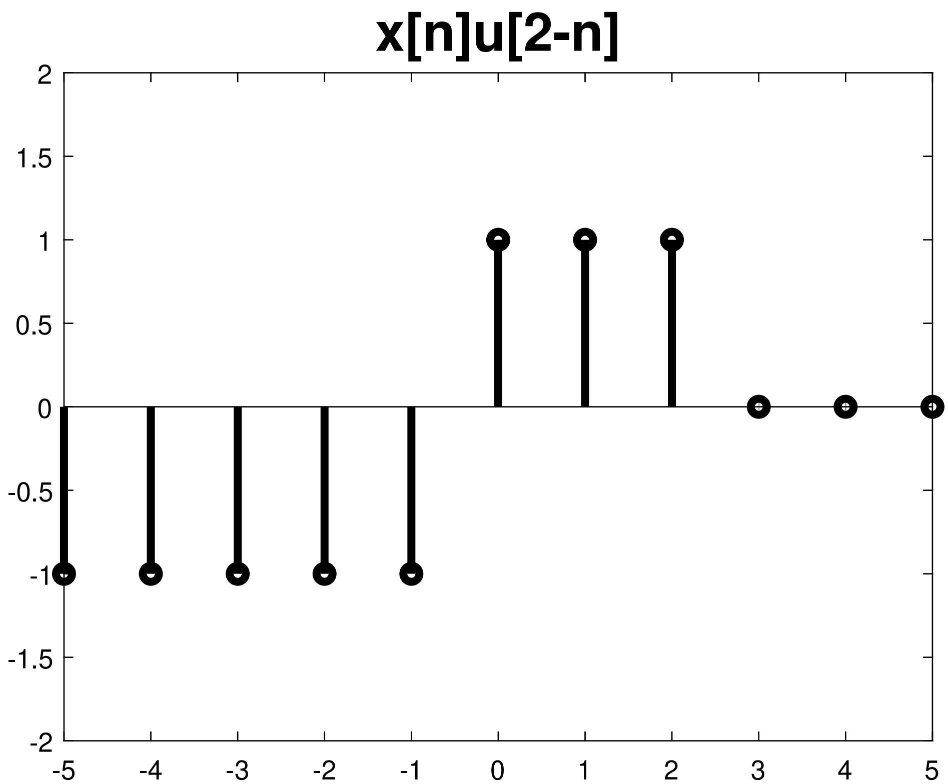

Figure 1: part (a) plot

Part (b)

The sketch is given as follows:

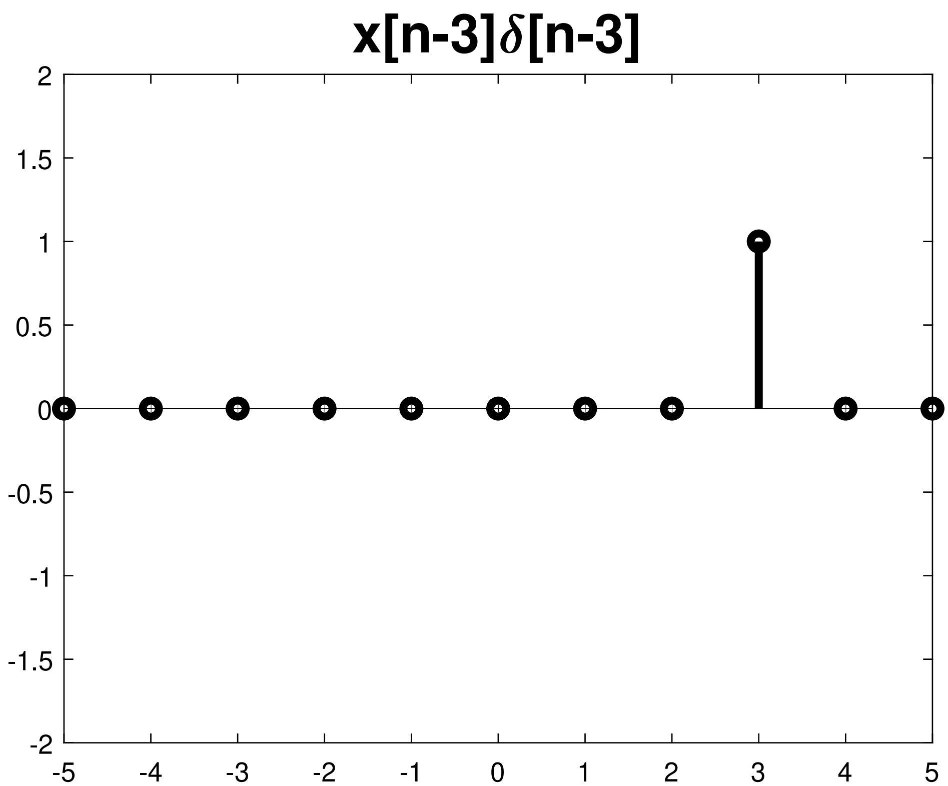

Figure 2: part (b) plot

Question 2

Part (a)

Given:

$$y[n] = x[-(n-2)]$$

1. **Memorylessness:** The system is not memoryless as $y[n]$ depends on $x[2-n]$.

2. **Causality:** We present a counter example to prove system is non-causal. For $n < 0$ we see that the system depends on future values i.e.:

$$n < 0 \implies n - 2 < -2 \implies -(n-2) > 2$$

   That is to say, for all negative values of n, the system depends on $n > 2$. Hence it is non-causal.

3. **Linear:** Assume 2 arbitrary inputs $ax_1[n]$ and $bx_2[n]$ (a, b are constants). Their individual responses are as:

$$y_1[n] = ax_1[2-n]$$

and

$$y_2[n] = bx_2[2-n]$$

On the other hand, summing the inputs first and then calculating their responses:

$$x[n] = ax_1[n] + bx_2[n]$$

Since time shifting and scaling are distributive (the system is only performing these two operations), we have:

$$y[n] = ay_1[n] + by_2[n]$$

System is linear.

4. **Time Invariant:** let us define an input $x_1[n-n_o]$. Applying it to the system, we get:

$$y_1[n] = x[2-(n-n_o)]$$

Now, we shift $y[n]$ by n_o:

$$y[n-n_o] = x[2-(n-n_o)]$$

Since both are same, the system is time invariant.

5. **Stable:** For a bounded input $-B_x < x[n] < B_x$, the system is just shifting and time reversing the input signal. Hence:

$$-B_y < y[n] = x[2-n] < B_y$$

System is stable.

Part (b)

Given:

$$y[n] = nx[-n]$$

1. **Memorylessness:** The system is not memoryless as $y[n]$ depends on $x[-n]$.

2. **Causality:** System is non-causal as it depends on future values for $n < 0$

3. **Linear:** Assume 2 arbitrary inputs $ax_1[n]$ and $bx_2[n]$ (a, b are constants). Their individual responses are as:

$$y_1[n] = anx_1[-n]$$

and

$$y_2[n] = bnx_2[-n]$$

On the other hand, summing the inputs first and then calculating their responses:

$$x[n] = ax_1[n] + bx_2[n]$$

$$y[n] = n(ax_1[-n] + bx_2[-n])$$
$$y[n] = anx_1[-n] + bnx_2[-n]$$

Since:

$$y[n] = y_1[n] + y_2[n]$$

System is linear.

4. **Time Invariant:** let us define an input $x_1[n - n_o]$. Applying it to the system, we get:

$$y_1[n] = nx[-(n - n_o)]$$

Now, we shift $y[n]$ by n_o:

$$y[n - n_o] = (n - n_o)x[-(n - n_o)]$$

Since both are **not** same, the system is time variant.

5. **Stable:** For a bounded input $-B_x < x[n] < B_x$:

$$y[n] = nx[2 - n]$$

The output is not bounded, hence system is not stable.

Question 3

Reversing and shifting the impulse response h by n and plotting as a function of k in figure 3.

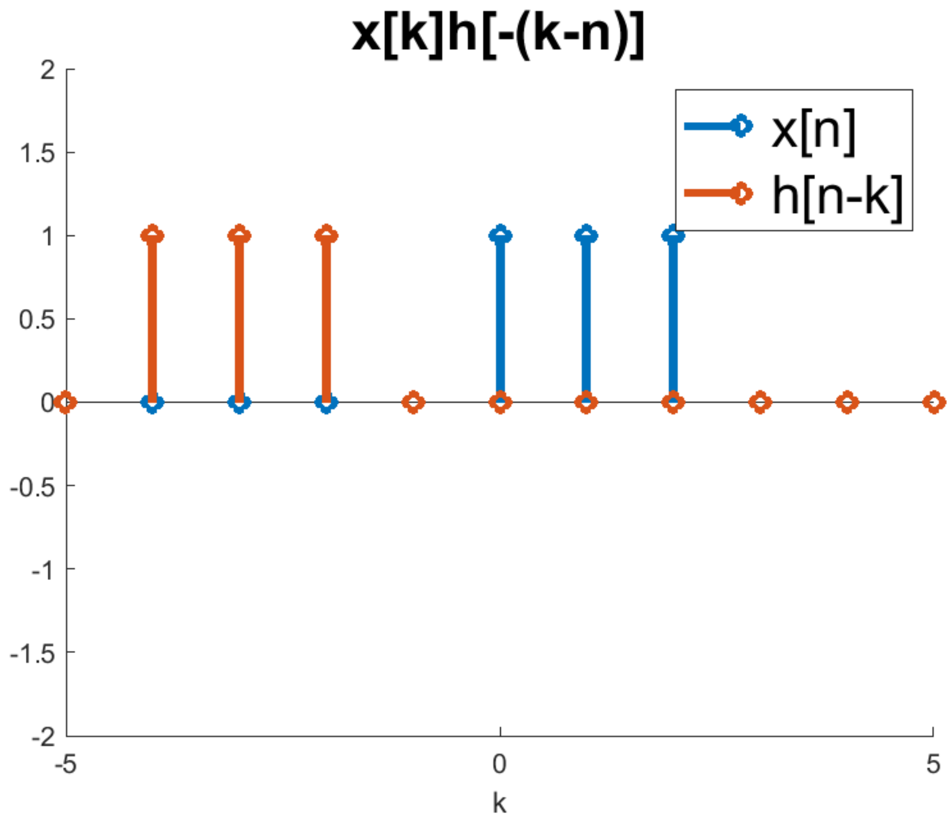

Figure 3: convolution sum

The convolution is then calculated as follows:

- For $n < 0$:
  Since there is no overlap,
  $$y[n] = 0$$
- For $0 \leq n \leq 4$:
  Overlap exists,
  $$\begin{bmatrix} y[0] \\ y[1] \\ y[2] \\ y[3] \\ y[4] \end{bmatrix} = \begin{bmatrix} 1 \\ -1+1 \\ -2+1 \\ -2 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -1 \\ -2 \\ -1 \end{bmatrix}$$

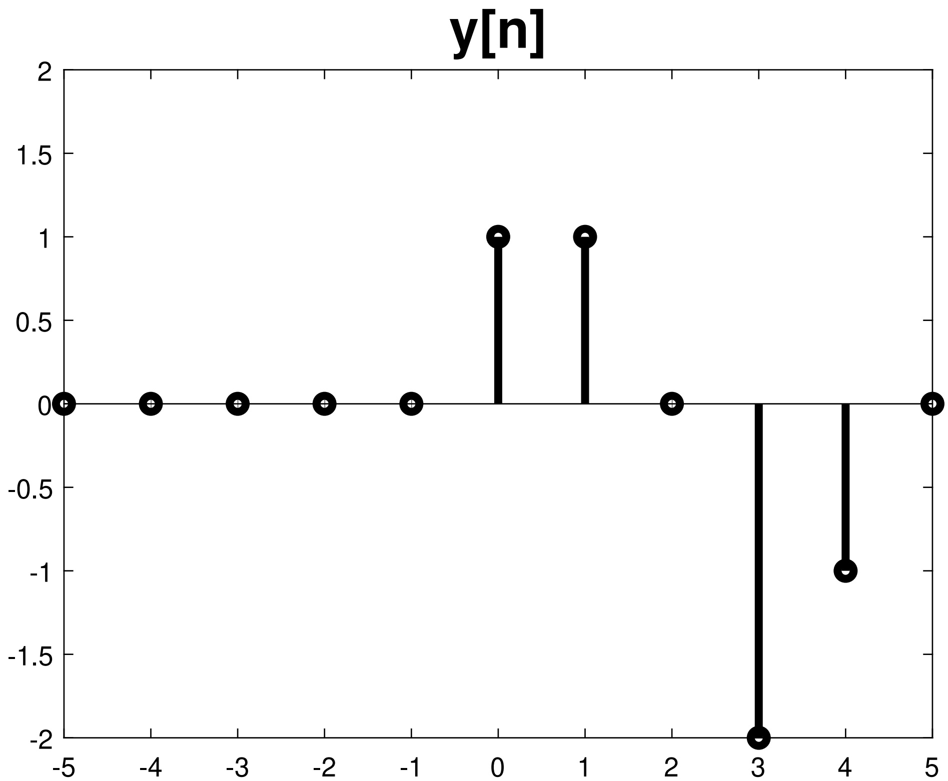

Figure 4: Convolution output

- For $n > 4$:
  No overlap,
  $$y[n] = 0$$

Figure 4 summarizes the convolution results.

Question 4

Part (a)

Given:

$$h[n] = \delta[n] - \delta[n-1] + \delta[n+1]$$

Causality

Inspecting the given impulse response, we see that it is non zero when $n < 0$ due to presence of an impulse i.e.:

$$h[n] = \ldots \delta[n+1]$$

Hence system is non-causal.

Stability

For stability, check for absolute summability:

$$\sum_{k=-\infty}^{k=\infty} |h[k]| < \infty$$

Since $h[n]$ is non zero at 3 points, the summation reduces to:

$$\sum_{k=-\infty}^{k=\infty} |h[k]| = 1 - 1 + 1 = 1 < \infty$$

Hence system is stable.

Part (b)

Given

$$h[n] = (0.2)^n u[n]$$

Causality

Since $u[n]$ is zero for $n < 0$, h is goes to zero for $n < 0$. The system is causal.

Stability

Checking for absolute summability:

$$\sum_{n=0}^{n=+\infty} |(0.2)^n u[n]| < \infty$$

$$\sum_{n=1}^{n=+\infty} |(0.2)^{n-1}| < \infty$$

Using infinite geometric series formula:

$$\sum_{n=1}^{n=+\infty} ar^{n-1} = \frac{a}{1-r}$$

With $a = 1$ and $r = 0.2$, we get:

$$\frac{1}{1-0.2} = 1.25 < \infty$$

Hence system is stable.

Question 5

Part (a)

1. IIR, as system is recursively defined.
2. FIR, as system is non-recursive
3. IIR, as system is recursively defined.

Part (b) and (c)

The block diagrams are as follows:

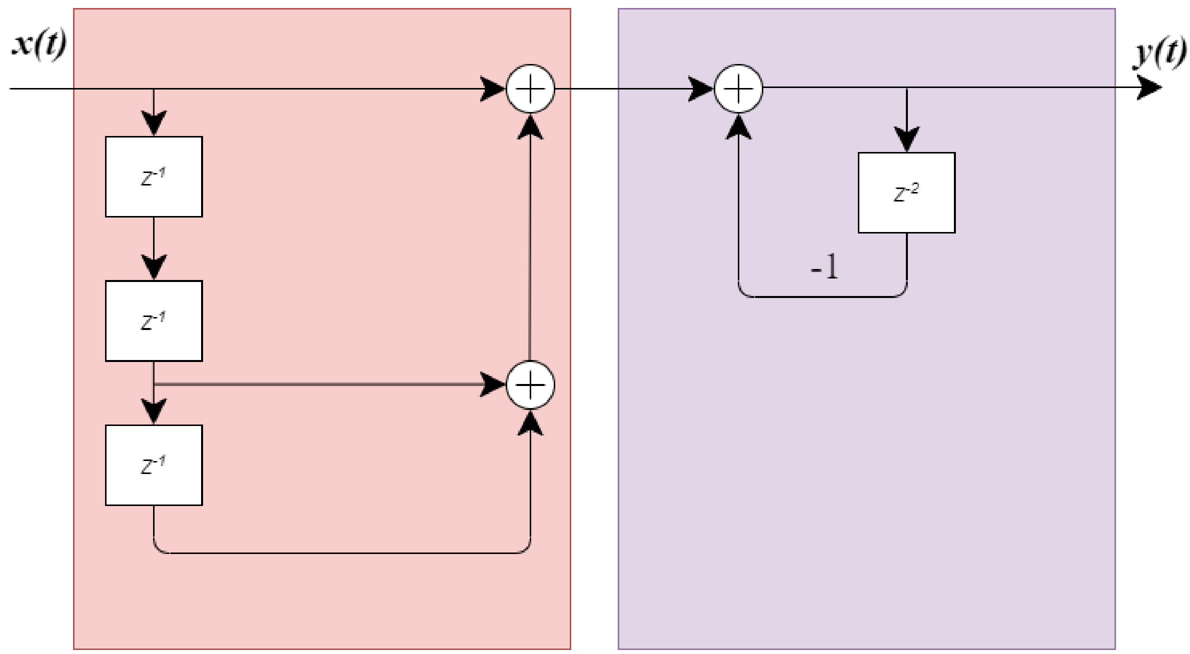

Figure 5: Direct Form I for system #1

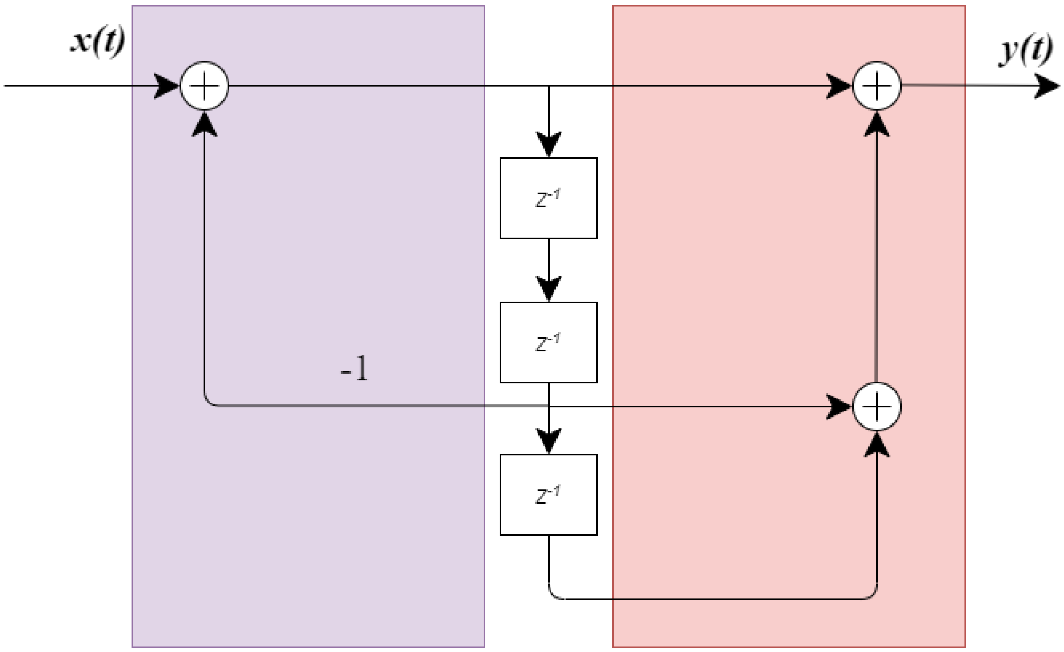

Figure 6: Direct Form II for system #1

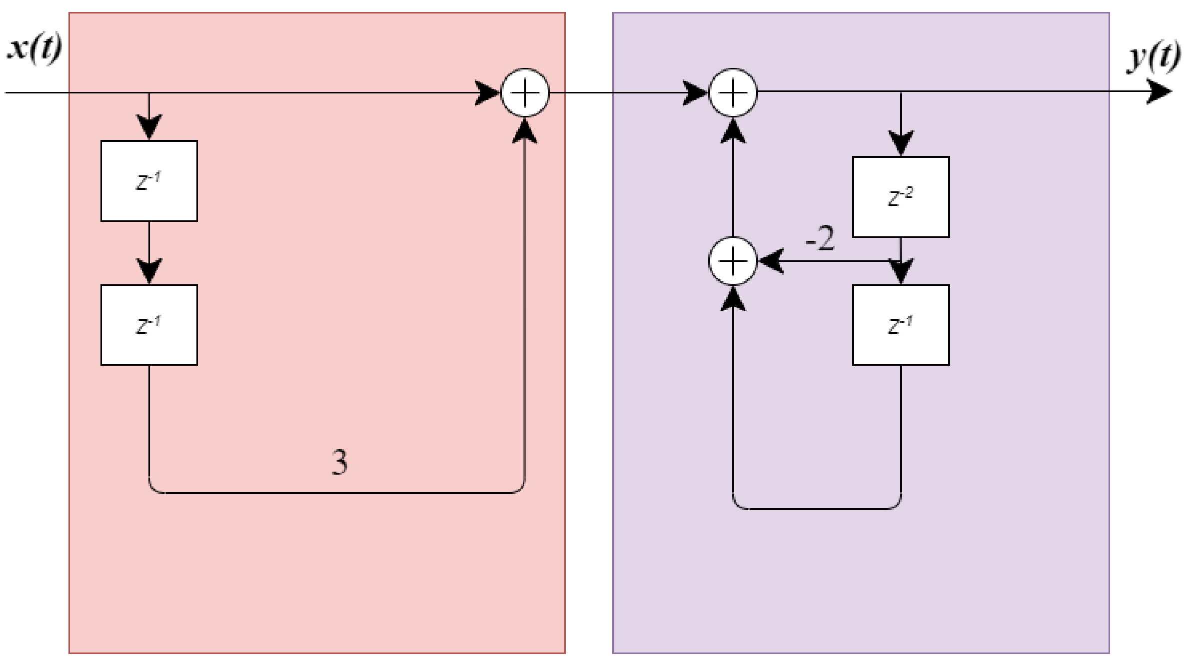

Figure 7: Direct Form I for system #3

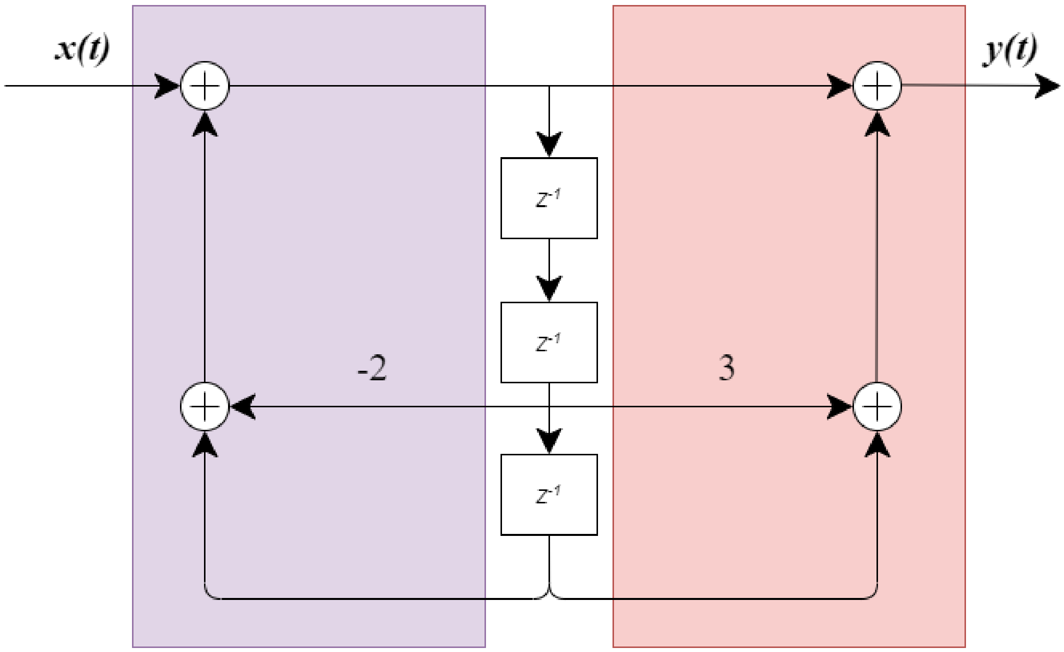

Figure 8: Direct Form II for system #3

Notes

1. The figures used were generated in MATLAB using the following script:

```
%% q1a
close all;
n = -5:5;


x = (n<0)*(-1) + (n>=0 & n<=2);

stem(n, x, 'Linewidth', 3)
title('x[n]u[2-n]', 'fontsize', 20)
xlim([-5 5])
ylim([-2 2])
% legend('x[n]', 'u[2-n]')
print -deps figures/1a.eps

%% q1b

n = -5:5;

x = (n==3);

stem(n, x, 'Linewidth', 3)
title('x[n-3]\delta[n-3]', 'fontsize', 20)

xlim([-5 5])
ylim([-2 2])

print -deps figures/1b.eps

%% q3


n = -5:6;

x1 = (n>=0 & n<=2);
h = [n>=-4 & n<=-2];
figure()
hold on
stem(n, x1, 'Linewidth', 3)
title('x[n]', 'fontsize', 20)
xlabel('k')
xlim([-5 5])
ylim([-2 2])
stem(n, h, '-','Linewidth', 3)
title('x[k]h[-(k-n)]', 'fontsize', 20)
legend('x[n]', 'h[n-k]','fontsize', 20)
```

```
hold off
print -dpng figures/3in.png

figure()
x = (n<0)*(0) + (n==0)+ (n==1)*0 ...
+ (n==2)*(-1)+ (n==3)*(-2)+ (n==4)*(-1)+(n>0 & n<=2);

stem(n, x, 'Linewidth', 3)
title('y[n]', 'fontsize', 20)
xlim([-5 5])
ylim([-2 2])

print -deps figures/3.eps
```

2. Complete folder with TeX source can be found at: github.com/mehhdiii/DSP-Basics