

**Question 1****Part (a)**

From the definition of Z-Transform:

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)z^{-n}$$

So we get:

$$X(z) = (3)z^{-3} + (0)z^{-2} + (0)z^{-1} + (6)z^0 + (1)z^1 + (-4)z^2$$

$$X(z) = 3z^{-3} + 6 + z - 4z^2$$

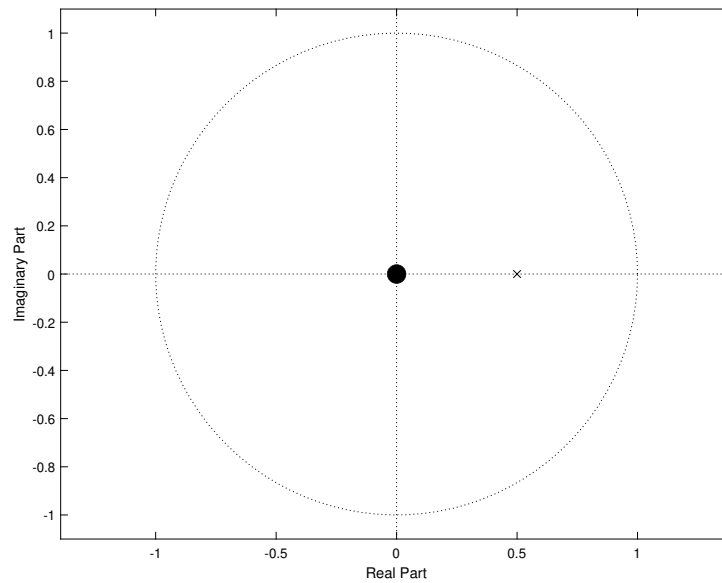
**Part (b)**

Figure 1: PZ plot for Q1

**Proof 1**

We know that if system is FIR, then it is non-recursively and defined as follows:

$$y(n) = \sum_{k=-\infty}^{\infty} b_k x(n - k)$$

## Homework 2

Taking Z-transform, we get:

$$Y(z) = \sum_{k=-\infty}^{\infty} b_k X(z) z^{-k}$$

From above equation, we can observe the following constraint for FIR system:

1. It can contain any number of zeros.
2. It does not contain any poles.
3. Its ROC will be the whole z plane (due to absense of poles).

Since the given system violates the condition 2 and 3, it is not FIR. So it can be safely termed **IIR**.

### Proof 2

We have:

$$H(z) = G z^{N-M} \frac{\prod_{k=1}^M (z - z_k)}{\prod_{k=1}^N (z - p_k)}$$

where  $G = \frac{b_o}{a_o}$ .

In our case:

$$N = M = 1$$

Then:

$$H(z) = G z^0 \frac{z}{z - 0.5}$$

$$H(z) = G \frac{z}{z - 0.5}$$

We know that:

$$H(z) = \frac{Y(z)}{X(z)}$$

So the system function can be written as:

$$\frac{Y(z)}{X(z)} = \frac{b_o}{a_o} \left( \frac{z}{z - 0.5} \right)$$

Multiply and divide by  $z^{-1}$ :

$$\frac{Y(z)}{X(z)} = \frac{b_o}{a_o} \left( \frac{1}{1 - 0.5z^{-1}} \right)$$

Re-arranging:

$$a_o Y(z) - 0.5 a_o Y(z) z^{-1} = b_o X(z)$$

**Homework 2**

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Converting to LCCDE form:

$$a_o y(n) - 0.5a_o y(n-1) = b_o x(n-1)$$

$$\boxed{a_o y(n) = 0.5a_o y(n-1) + b_o x(n-1)}$$

The given LCCDE is recursively defined hence the system is indeed IIR. Verified!

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**Homework 2**

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**Question 2**

Given:

$$y(n) = 0.8y(n-1) - 0.6y(n-2) + x(n) + 2x(n-1)$$

**Part (a)**

Taking Z-Transform on both sides:

$$Y(z) = 0.8z^{-1}Y(z) - 0.6Y(z)z^{-2} + X(z) + 2X(z)z^{-1}$$

$$Y(z)(1 + 0.6z^{-2} - 0.8z^{-1}) = X(z)(1 + 2z^{-1})$$

$$\frac{Y(z)}{X(z)} = \frac{1 + 2z^{-1}}{1 - 0.8z^{-1} + 0.6z^{-2}}$$

$$= \frac{z^2 + 2z}{z^2 - 0.8z + 0.6}$$

$$H(z) = \frac{z(z+2)}{z^2 - 0.8z + 0.6}$$

**Part (b)****Zeros**

Zeros are as follows:

$$z = 0$$

$$z = -2$$

**Poles**

Calculating roots of the equation below:

$$z^2 - 0.8z + 0.6 = 0$$

Poles are as follows:

$$p = 0.4 + 0.6633j$$

$$p = 0.4 - 0.6633j$$

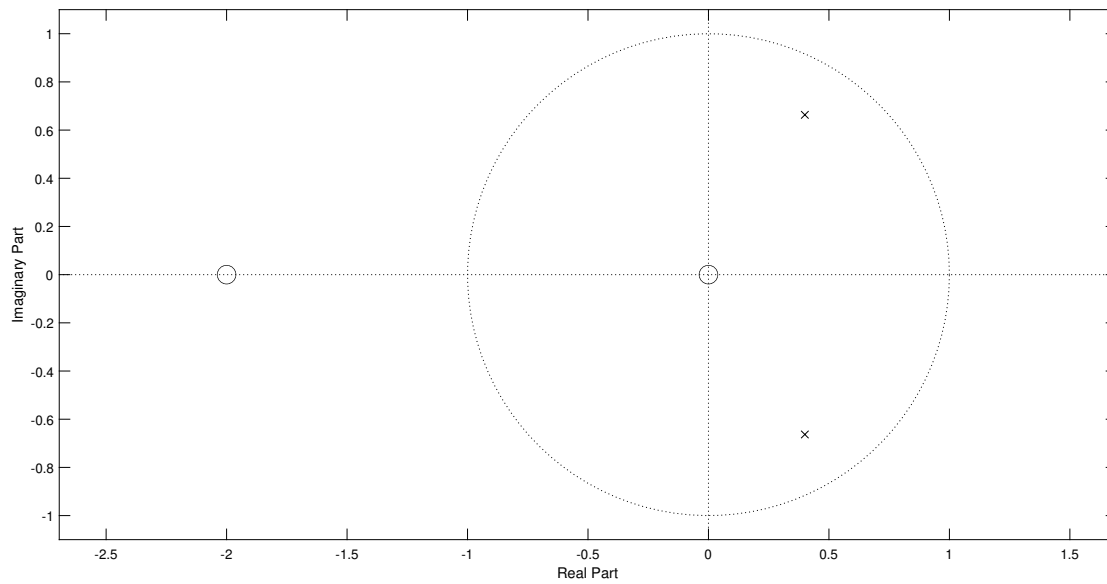


Figure 2: PZ plot

**Part (c)**

The outer most pole is located at:

$$r = \sqrt{0.4^2 + 0.6633^2} = 0.7746$$

Since system is causal, the ROC consists of  $r > 0.7746$ . This implies that the ROC will include the unit circle (we may also observe this from the PZ plot). So system is stable.

**Question 3**

Given:

$$H(z) = \frac{z^2 + z}{z^2 + z - 0.75}$$

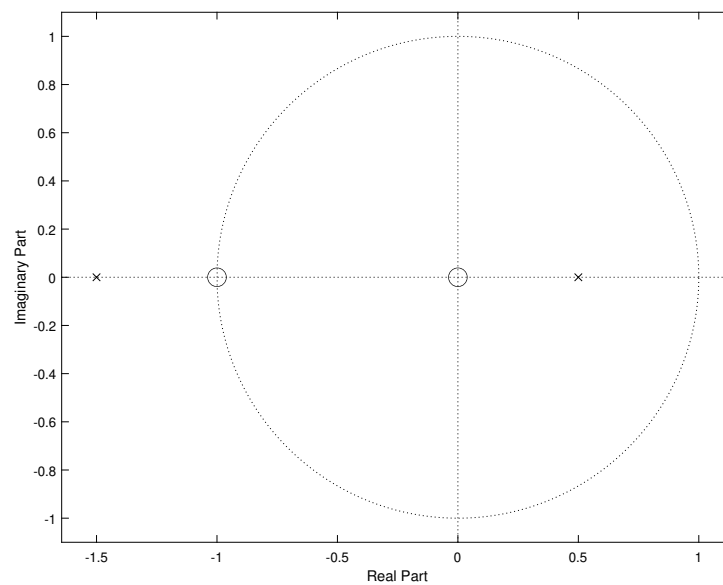
**Part (a)**

Figure 3: PZ plot for Q3

**Zeros**

Zeros are calculated as follows:

$$z(z + 1) = 0$$

Then,

$$z = 0$$

$$z = -1$$

**Poles**

Poles are calculated as follows:

$$z^2 + z - 0.75 = 0$$

## Homework 2

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$$p = -1.5$$

$$p = 0.5$$

### Part (b)

The outer most pole lies at:

$$r = 1.5$$

With the system obeying causality, its ROC will be  $r > 1.5$ . So ROC doesn't include the unit circle. So system is **unstable**

### Part (c)

The system function can be written as:

$$\frac{Y(z)}{X(z)} = \frac{z^2 + z}{z^2 + z - 0.75}$$

Multiply and divide by  $z^{-2}$

$$\frac{Y(z)}{X(z)} = \frac{1 + z^{-1}}{1 + z^{-1} - 0.75z^{-2}}$$

$$Y(z) + Y(z)z^{-1} - 0.75Y(z)z^{-2} = X(z) + X(z)z^{-1}$$

Above equation can now be converted into LCCDE form:

$$y(n) + y(n-1) - 0.75y(n-2) = x(n) + x(n-1)$$

$$y(n) = -y(n-1) + 0.75y(n-2) + x(n) + x(n-1)$$

**Homework 2****Question 4**

Given:

$$X(z) = \frac{z}{z^2 + z - 0.75}$$

$$\frac{X(z)}{z} = \frac{1}{z^2 + z - 0.75}$$

Creating partial fractions:

$$\frac{1}{(z + 1.5)(z - 0.5)} = \frac{A_1}{(z + 1.5)} + \frac{A_2}{(z - 0.5)}$$

$$1 = A_1(z - 0.5) + A_2(z + 1.5)$$

**For  $A_1$**

Put  $z = -1.5$

$$1 = -2A_1 + 0$$

$$\boxed{A_1 = -0.5}$$

**For  $A_2$**

Put  $z = 0.5$

$$0.5 = 2A_2 + 0$$

$$\boxed{A_2 = 0.25}$$

Then Partial fraction expansion becomes:

$$\boxed{\frac{X(z)}{z} = \frac{-0.5}{(z + 1.5)} + \frac{0.25}{(z - 0.5)}}$$

$$X(z) = \frac{-0.5z}{(z + 1.5)} + \frac{0.25z}{(z - 0.5)}$$

Multiply and divide by  $z^{-1}$ :

$$X(z) = \frac{-0.5}{(1 + 1.5z^{-1})} + \frac{0.25}{(1 - 0.5z^{-1})}$$

From the lookup table, the inverse transform is given as:

$$\boxed{x(n) = [-0.5(-1.5)^n + 0.25(0.5)^n]u(n)}$$



## Question 5

## Part (a)

Since the filter is high pass, we may place pole at  $\omega = \pi$  and zero at  $\omega = 0$ . One possible arrangement is shown in the PZ plot:

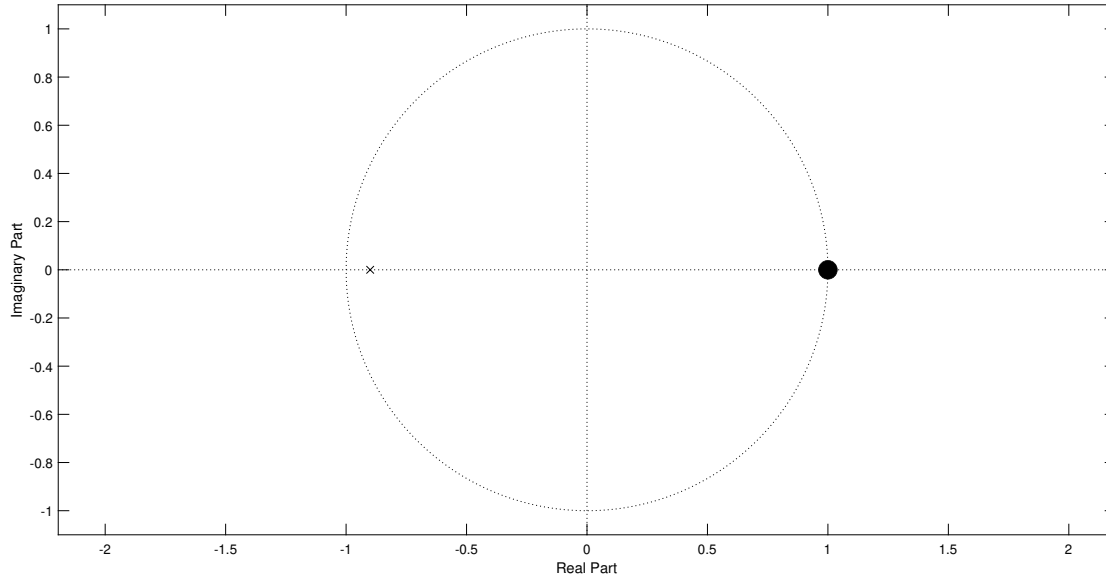


Figure 4: PZ plot for Q5

## Part (b)

1. This is a band-pass filter for frequencies near  $\omega = \pm\pi/2$ . It is so because we see poles at frequency  $\pm\pi/2$  (Poles are placed near frequencies to be emphasized). Similarly, we see zeros at  $\omega = \pi$  and  $\omega = 0$  (zeros are placed near frequencies to be de-emphasized). Hence this filter will block low and high frequencies ( $\omega = 0$  and  $\omega = \pi$ ) and pass frequencies around  $\omega = \pm\pi/2$ .
2. At  $\omega = 0$ , the response is:

$$|H(\omega)| = b_o \frac{\text{product of length of vector: zero to unit circle}}{\text{product of length of vector: pole to unit circle}}$$

Hence the response in this case will be (Assuming poles at  $r = 0.9$ ):

$$|H(\omega)| = b_o \frac{1 \times 1}{0.9 \times 0.9}$$

$$|H(\omega)| = 1.234b_o$$

## Index

The complete source code can be found at: [GitHub/mehhdiii/](https://github.com/mehhdiii/)