Homework 2

Digital Signal Processing
CE 352 / EE 453 (L1)

Fall 2021

Note: You are only allowed to refer to the *roots(p)* command from MATLAB while writing up the solution for this homework assignment.

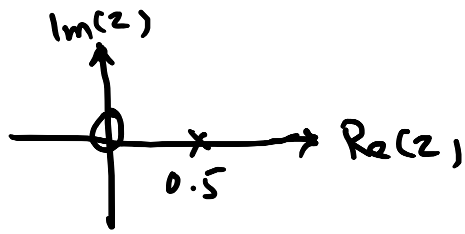
Question 1
(20 points)
(a)

Determine the z-Transform of the following signal (including ROC):

$$x(n) = \{3, 0, 0, \underset{\uparrow}{6}, 1, -4\}$$

(b)

Given the following pole-zero plot for the system function [H(z)] of an LTI system,



Is the LTI system an FIR system or an IIR system? Justify your answer based on z-Transform and its ROC.

Question 2
(20 points)

Given a causal system:

$$y(n) = 0.8\, y(n-1) - 0.6\, y(n-2) + x(n) + 2x(n-1)$$

a) Determine the system function H(z)
b) Sketch pole-zero plot of H(z)
c) Is this system stable?

Question 3
(20 points)

Given that

$$H(z) = \frac{z^2 + z}{z^2 + z - 0.75}$$

is a causal system.

a) Sketch its pole-zero plot.
b) Is this system stable?
c) Find its difference equation representation.

Question 4

(20 points)

Given the z-Transform of x(n)

$$X(z) = \frac{z}{z^2 + z - 0.75}$$

Determine x(n) through inverse z-Transform by using the partial fraction expansion method. (Assume ROC is $|z| > \frac{3}{2}$). For your reference, a table of common z-transform pairs is provided below:

	Signal, $x(n)$	z-Transform, $X(z)$	ROC
1	$\delta(n)$	1	All z
2	$u(n)$	$\frac{1}{1-z^{-1}}$	$\|z\| > 1$
3	$a^n u(n)$	$\frac{1}{1-az^{-1}}$	$\|z\| > \|a\|$
4	$na^n u(n)$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\|z\| > \|a\|$
5	$-a^n u(-n-1)$	$\frac{1}{1-az^{-1}}$	$\|z\| < \|a\|$
6	$-na^n u(-n-1)$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\|z\| < \|a\|$
7	$\cos(\omega_0 n)u(n)$	$\frac{1-z^{-1}\cos\omega_0}{1-2z^{-1}\cos\omega_0+z^{-2}}$	$\|z\| > 1$
8	$\sin(\omega_0 n)u(n)$	$\frac{z^{-1}\sin\omega_0}{1-2z^{-1}\cos\omega_0+z^{-2}}$	$\|z\| > 1$
9	$a^n\cos(\omega_0 n)u(n)$	$\frac{1-az^{-1}\cos\omega_0}{1-2az^{-1}\cos\omega_0+a^2z^{-2}}$	$\|z\| > \|a\|$
10	$a^n\sin(\omega_0 n)u(n)$	$\frac{1-az^{-1}\sin\omega_0}{1-2az^{-1}\cos\omega_0+a^2z^{-2}}$	$\|z\| > \|a\|$

Question 5

(20 points)

(a)

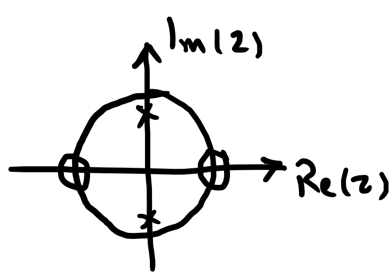
A digital filter is characterized by the following properties:

1. It is highpass and has one pole and one zero.
2. The pole is at a distance of r=0.9 from the origin of the z-plane.
3. Constant signals do not pass through the system.

Plot the pole-zero pattern of the system function H(z) for the digital filter.

(b)

The pole-zero plot (along with unit circle) for the system function H(z) of a digital filter is provided below:



1. Is this a lowpass, highpass, or bandpass filter. Justify your answer based on the geometrical interpretation of the relationship between pole-zero locations of system function and the magnitude response of a digital filter.
2. What is the value of magnitude response at $\omega = 0$.