Test dataset for unit testing

By Mehran Amiri

For FRsutils library (https://github.com/mehi64/FRsutils)

**Change log**

|  |  |  |
| --- | --- | --- |
| ***Date (dd.mm.YYYY)*** | ***Change*** | ***Person*** |
| 12.05.2025 | Added the document (t-norm, implicator test data) | Mehran Amiri |
| 13.05.2025 | Added similarity test data + LB for ITFRS | Mehran Amiri |
| 14.05.2025 | Added UB for ITFRS | Mehran Amiri |
| 15.05.2025 | Added data for Gaussian similarity | Mehran Amiri |
| 25.05,2025 | Added data for VQRS | Mehran Amiri |
| 08.06.2025 | Drastic product tnorm data | Mehran Amiri |
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Contents

[Implicators 6](#_Toc200284762)

[Data and outputs 6](#_Toc200284763)

[OWA\_weights data for test 6](#_Toc200284764)

[Data for testing t-norms in a scalar way 7](#_Toc200284765)

[Data for testing t-norms in a map/vectorized way 8](#_Toc200284766)

[Similarities 9](#_Toc200284767)

[X matrix (each row is a data Instance) 9](#_Toc200284768)

[Linear similarity 9](#_Toc200284769)

[Element-wise calculations of |v1 - v2|, part of linear similarity 9](#_Toc200284770)

[Element-wise similarity of Instances (linear similarity) 9](#_Toc200284771)

[Final similarity\_matrix\_with\_linear\_similarity\_minimum\_tnorm 9](#_Toc200284772)

[Final similarity\_matrix\_with\_linear\_similarity\_product\_tnorm 10](#_Toc200284773)

[Final similarity\_matrix\_with\_linear\_similarity\_luk\_tnorm 10](#_Toc200284774)

[Gaussian Similarity 11](#_Toc200284775)

[Element-wise calculations of (v1 - v2)2, part of the Gaussian similarity 11](#_Toc200284776)

[Gaussian similarity elementwise (sigma = 0.67) 11](#_Toc200284777)

[Gaussian similarity with product tnorm 11](#_Toc200284778)

[Gaussian similarity with minimum tnorm 11](#_Toc200284779)

[Gaussian similarity with luk tnorm 12](#_Toc200284780)

[Lower and Upper approximations (ITFRS and OWAFRS) 13](#_Toc200284781)

[y (labels) 13](#_Toc200284782)

[label\_masks (b) 13](#_Toc200284783)

[similarity\_matrix (a) 13](#_Toc200284784)

[Interim 1 - sim 14](#_Toc200284785)

[Interim Sim \* A(y) 14](#_Toc200284786)

[Interim 1 - sim + A(y) 14](#_Toc200284787)

[Luk Implicator results 14](#_Toc200284788)

[KD Implicator results 15](#_Toc200284789)

[Reichenbach Implicator results 15](#_Toc200284790)

[Goedel Implicator results 15](#_Toc200284791)

[Gaines Implicator results 15](#_Toc200284792)

[Lower approximation with all Implicators 16](#_Toc200284793)

[Min and product t-norm(similarity\_matrix, label\_masks) 16](#_Toc200284794)

[Upper approximations 17](#_Toc200284795)

[OWAFRS lower approximation 18](#_Toc200284796)

[KD Implicator results with making main diagonal 0.0 18](#_Toc200284797)

[Descending sorted KD Implicator results after making main diagonal 0.0 18](#_Toc200284798)

[OWAinf calculations for KD Implicator 18](#_Toc200284799)

[Goedel Implicator results with making main diagonal 0.0 19](#_Toc200284800)

[Descending sorted Goedel Implicator results after making main diagonal 0.0 19](#_Toc200284801)

[OWAinf calculations for Goedel Implicator 19](#_Toc200284802)

[OWAFRS upper approximation 20](#_Toc200284803)

[Min and product t-norm(similarity\_matrix, label\_masks) 20](#_Toc200284804)

[Descending sorted min results after making main diagonal 0.0 20](#_Toc200284805)

[OWAsup calculations for min tnorm 20](#_Toc200284806)

[Lower and Upper approximations (VQRS) 21](#_Toc200284807)

[y (labels) 21](#_Toc200284808)

[label\_masks 21](#_Toc200284809)

[similarity\_matrix after setting 0.0 into main diagonal elements 21](#_Toc200284810)

[VQRS nominator step1 (min[R(x,y),A(x)]) 21](#_Toc200284811)

[Nominator {sum(min[R(x,y),A(x)])} 21](#_Toc200284812)

[Denominator (sum [R(x,y)]) 21](#_Toc200284813)

[Division results (nominator/denominator) 22](#_Toc200284814)

[Upper approximation (alpha =0.2, beta = 1.0) with quad fuzzy quantifier 22](#_Toc200284815)

[Lower approximation (alpha =0.1, beta = 0.6) with quad fuzzy quantifier 23](#_Toc200284816)

# Implicators

## Data and outputs

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| a | b | 1-a | ab | 1-a+ab | 1-a+b | **Gaines**  **output** | **Goedel**  **output** | **KD output** | **reichenbach**  **output** | **Luk output** |
| 2.10 | 4.32 | - | - | - | - | - | - | - | - | - |
| -0.20 | -0.78 | - | - | - | - | - | - | - | - | - |
| 0.73 | 0.18 | 0.27 | 0.1314 | 0.4014 | 0.45 | 0.246575 | 0.18 | 0.27 | 0.4014 | 0.45 |
| 0.18 | 0.73 | 0.82 | 0.1314 | 0.9514 | 1.55 | 1.0 | 1.0 | 0.82 | 0.9514 | 1.00 |
| 0.88 | 0.88 | 0.12 | 0.7744 | 0.8914 | 1.00 | 1.0 | 1.0 | 0.88 | 0.8944 | 1.00 |
| 0.91 | 0.48 | 0.09 | 0.4368 | 0.5263 | 0.57 | 0.527473 | 0.48 | 0.48 | 0.5268 | 0.57 |
| 1.00 | 1.00 | 0.00 | 1.00 | 1.00 | 1.00 | 1.0 | 1.0 | 1.00 | 1.00 | 1.00 |
| 0.00 | 0.00 | 1.00 | 0.00 | 1.00 | 1.00 | 1.0 | 1.0 | 1.00 | 1.00 | 1.00 |

# OWA\_weights data for test

 owa\_infimum\_weights\_linear\_len\_5 = np.array([0.06666667, 0.13333333, 0.2, 0.26666667, 0.33333333])

        owa\_infimum\_weights\_linear\_len\_10 = np.array([0.01818182, 0.03636364, 0.05454545, 0.07272727, 0.09090909, 0.10909091, 0.12727273, 0.14545455, 0.16363636, 0.18181818])

        owa\_suprimum\_weights\_linear\_len\_8 = np.array([0.22222222, 0.19444444, 0.16666667, 0.13888889, 0.11111111, 0.08333333, 0.05555556, 0.02777778])

        owa\_supriimum\_weights\_linear\_len\_13 = np.array([0.14285714, 0.13186813, 0.12087912, 0.10989011, 0.0989011,  0.08791209, 0.07692308, 0.06593407, 0.05494505, 0.04395604, 0.03296703, 0.02197802, 0.01098901])

# Data for testing t-norms in a scalar way

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| a | b | ab | a+b | a+b-1 | 1-a | 1-b | 1+(1-a)(1-b) | a+b-ab | **Min** | **Product** | **Luk** | **Drastic Prod** | **Hamacher prod** | **Einstein** | **Nilpotent min** |
| 2.10 | 4.32 | - | - | - | - | - | - | - | - | - | - | - | - | - |  |
| -0.20 | -0.78 | - | - | - | - | - | - | - | - | - | - | - | - | - |  |
| 0.73 | 0.18 | 0.1314 | 0.91 | -0.09 | 0.27 | 0.82 | 1.2214 | 0.7786 | 0.18 | 0.1314 | 0.00 | 0.00 | 0.168764 | 0.107581 | 0.00 |
| 0.18 | 0.73 | 0.1314 | 0.91 | -0.09 | 0.82 | 0.27 | 1.2214 | 0.7786 | 0.18 | 0.1314 | 0.00 | 0.00 | 0.168764 | 0.107581 | 0.00 |
| 0.88 | 0.88 | 0.7744 | 1.76 | 0.76 | 0.12 | 0.12 | 1.0144 | 0.9856 | 0.88 | 0.7744 | 0.76 | 0.00 | 0.785714 | 0.763407 | 0.88 |
| 0.91 | 0.48 | 0.4368 | 1.39 | 0.39 | 0.09 | 0.52 | 1.0468 | 0.9532 | 0.48 | 0.4368 | 0.39 | 0.00 | 0.458246 | 0.417271 | 0.48 |
| 1.00 | 1.00 | 1.00 | 2.00 | 1.00 | 0.00 | 0.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| 0.00 | 0.00 | 0.00 | 0.00 | -1.00 | 1.00 | 1.00 | 2.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 1.00 | 0.65 | 0.65 | 1.65 | 0.65 | 0.00 | 0.35 | 1.00 | 1.00 | 0.65 | 0.65 | 0.65 | 0.65 | 0.65 | 0.65 | 0.65 |
| 0.37 | 1.00 | 0.37 | 1.37 | 0.37 | 0.63 | 0.00 | 1.00 | 1.00 | 0.37 | 0.37 | 0.37 | 0.37 | 0.37 | 0.37 | 0.37 |

## Yager tnorm results (p=0.835, 1/p =1.1976047904)

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| a | b | ab | 1-a | 1-b | (1-a)p | (1-b)p | (1-a)p +(1-b)p | [(1-a)p +(1-b)p](1/p) | Min{1, [(1-a)p +(1-b)p](1/p)} | **Yager** |
| 2.10 | 4.32 | - | - | - | - | - | - | - | - | - |
| -0.20 | -0.78 | - | - | - | - | - | - | - | - | - |
| 0.73 | 0.18 | 0.1314 | 0.27 | 0.82 | 0.335111 | 0.847295 | 1.182406 | 1.2222095 | 1.00 | 0.00 |
| 0.18 | 0.73 | 0.1314 | 0.82 | 0.27 | 0.847295 | 0.335111 | 1.182406 | 1.2222095 | 1.00 | 0.00 |
| 0.88 | 0.88 | 0.7744 | 0.12 | 0.12 | 0.170261 | 0.170261 | 0.340522 | 0.275229 | 0.275229 | 0.724771 |
| 0.91 | 0.48 | 0.4368 | 0.09 | 0.52 | 0.133904 | 0.579246 | 0.713150 | 0.667066 | 0.667066 | 0.332934 |
| 1.00 | 1.00 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 1.00 |
| 0.00 | 0.00 | 0.00 | 1.00 | 1.00 | 1.00 | 1.00 | 2.00 | 2.293586 | 1.00 | 0.00 |
| 1.00 | 0.65 | 0.65 | 0.00 | 0.35 | 0.00 | 0.416195 | 0.416195 | 0.35 | 0.35 | 0.65 |
| 0.37 | 1.00 | 0.37 | 0.63 | 0.00 | 0.679907 | 0.00 | 0.679907 | 0.63 | 0.63 | 0.37 |

## Yager tnorm results (p=5, 1/p =0.2)

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| a | b | ab | 1-a | 1-b | (1-a)p | (1-b)p | (1-a)p +(1-b)p | [(1-a)p +(1-b)p](1/p) | Min{1, [(1-a)p +(1-b)p](1/p)} | **Yager** |
| 2.10 | 4.32 | - | - | - | - | - | - | - | - | - |
| -0.20 | -0.78 | - | - | - | - | - | - | - | - | - |
| 0.73 | 0.18 | 0.1314 | 0.27 | 0.82 |  |  |  |  |  |  |
| 0.18 | 0.73 | 0.1314 | 0.82 | 0.27 |  |  |  |  |  |  |
| 0.88 | 0.88 | 0.7744 | 0.12 | 0.12 |  |  |  |  |  |  |
| 0.91 | 0.48 | 0.4368 | 0.09 | 0.52 |  |  |  |  |  |  |
| 1.00 | 1.00 | 1.00 | 0.00 | 0.00 |  |  |  |  |  |  |
| 0.00 | 0.00 | 0.00 | 1.00 | 1.00 |  |  |  |  |  |  |
| 1.00 | 0.65 | 0.65 | 0.00 | 0.35 |  |  |  |  |  |  |
| 0.37 | 1.00 | 0.37 | 0.63 | 0.00 |  |  |  |  |  |  |

# Data for testing t-norms in a map/vectorized way

Similarity map

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 1.0 | 0.2673 | 0.25456 | 0.1197 | 0.09504 |
| 0.2673 | 1.0 | 0.0658 | 0.1624 | 0.054 |
| 0.25456 | 0.0658 | 1.0 | 0.3157 | 0.53217 |
| 0.1197 | 0.1624 | 0.3157 | 1.0 | 0.53872 |
| 0.09504 | 0.054 | 0.53217 | 0.53872 | 1.0 |

Mask map

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 1.0 | 1.0 | 0.0 | 1.0 | 0.0 |
| 1.0 | 1.0 | 0.0 | 1.0 | 0.0 |
| 0.0 | 0.0 | 1.0. | 0.0 | 1.0 |
| 1.0 | 1.0 | 0.0 | 1.0 | 0.0 |
| 0.0 | 0.0 | 1.0 | 0.0 | 1.0 |

Output of product tnorm

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 1.0 | 0.2673 | 0.0 | 0.1197 | 0.0 |
| 0.2673 | 1.0 | 0.0 | 0.1624 | 0.0 |
| 0.0 | 0.0 | 1.0 | 0.0 | 0.53217 |
| 0.1197 | 0.1624 | 0.0 | 1.0 | 0.0 |
| 0.0 | 0.0 | 0.53217 | 0.0 | 1.0 |

Output of minimum tnorm

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 1.0 | 0.2673 | 0.0 | 0.1197 | 0.0 |
| 0.2673 | 1.0 | 0.0 | 0.1624 | 0.0 |
| 0.0 | 0.0 | 1.0 | 0.0 | 0.53217 |
| 0.1197 | 0.1624 | 0.0 | 1.0 | 0.0 |
| 0.0 | 0.0 | 0.53217 | 0.0 | 1.0 |

Output of Luk tnorm

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 1.0 | 0.2673 | 0.0 | 0.1197 | 0.0 |
| 0.2673 | 1.0 | 0.0 | 0.1624 | 0.0 |
| 0.0 | 0.0 | 1.0 | 0.0 | 0.53217 |
| 0.1197 | 0.1624 | 0.0 | 1.0 | 0.0 |
| 0.0 | 0.0 | 0.53217 | 0.0 | 1.0 |

# Similarities

## X matrix (each row is a data Instance)

|  |  |  |  |
| --- | --- | --- | --- |
| **Inst 1** | 0.10 | 0.32 | 0.48 |
| **Inst 2** | 0.20 | 0.78 | 0.93 |
| **Inst 3** | 0.73 | 0.18 | 0.28 |
| **Inst 4** | 0.91 | 0.48 | 0.73 |
| **Inst 5** | 1.00 | 0.28 | 0.47 |

## Linear similarity

### Element-wise calculations of |v1 - v2|, part of linear similarity

The matrix is symmetric.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **SIM** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | [0.0, 0.0, 0.0] | [0.1, 0.46, 0.45] | [0.63, 0.14, 0.20] | [0.81, 0.16, 0.25] | [0.90, 0.04, 0.01] |
| **Inst 2** | [0.1, 0.46, 0.45] | [0.0, 0.0, 0.0] | [0.53, 0.60, 0.65] | [0.71, 0.30, 0.20] | [0.80, 0.50, 0.46] |
| **Inst 3** | [0.63, 0.14, 0.20] | [0.53, 0.60, 0.65] | [0.0, 0.0, 0.0] | [0.18, 0.30, 0.45] | [0.27, 0.10, 0.19] |
| **Inst 4** | [0.81, 0.16, 0.25] | [0.71, 0.30, 0.20] | [0.18, 0.30, 0.45] | [0.0, 0.0, 0.0] | [0.09, 0.20, 0.26] |
| **Inst 5** | [0.90, 0.04, 0.01] | [0.80, 0.50, 0.46] | [0.27, 0.10, 0.19] | [0.09, 0.20, 0.26] | [0.0, 0.0, 0.0] |

### Element-wise similarity of Instances (linear similarity)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **SIM** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | [1.0, 1.0,1.0] | [0.9, 0.54, 0.55] | [0.37, 0.86, 0.8 ] | [0.19, 0.84, 0.75] | [0.1 , 0.96, 0.99] |
| **Inst 2** | [0.90 , 0.54, 0.55] | [1.0, 1.0,1.0] | [0.47, 0.4 , 0.35] | [0.29, 0.7 , 0.8 ] | [0.2 , 0.5 , 0.54] |
| **Inst 3** | [0.37, 0.86, 0.80 ] | [0.47, 0.4 , 0.35] | [1.0, 1.0,1.0] | [0.82, 0.7 , 0.55] | [0.73, 0.9 , 0.81] |
| **Inst 4** | [0.19, 0.84, 0.75] | [0.29, 0.70 , 0.8 ] | [0.82, 0.7 , 0.55] | [1.0, 1.0,1.0] | [0.91, 0.8 , 0.74] |
| **Inst 5** | [0.10 , 0.96, 0.99] | [0.20 , 0.5 , 0.54] | [0.73, 0.9 , 0.81] | [0.91, 0.8 , 0.74] | [1.0, 1.0,1.0] |

### Final similarity\_matrix\_with\_linear\_similarity\_minimum\_tnorm

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **SIMILARITIES** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.0 | 0.54 | 0.37 | 0.19 | 0.1 |
| **Inst 2** | 0.54 | 1.0 | 0.35 | 0.29 | 0.2 |
| **Inst 3** | 0.37 | 0.35 | 1.0 | 0.55 | 0.73 |
| **Inst 4** | 0.19 | 0.29 | 0.55 | 1.0 | 0.74 |
| **Inst 5** | 0.10 | 0.20 | 0.73 | 0.74 | 1.0 |

### Final similarity\_matrix\_with\_linear\_similarity\_product\_tnorm

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **SIMILARITIES** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.00000 | 0.26730 | 0.25456 | 0.11970 | 0.09504 |
| **Inst 2** | 0.26730 | 1.00000 | 0.06580 | 0.16240 | 0.05400 |
| **Inst 3** | 0.25456 | 0.06580 | 1.00000 | 0.31570 | 0.53217 |
| **Inst 4** | 0.11970 | 0.16240 | 0.31570 | 1.00000 | 0.53872 |
| **Inst 5** | 0.09504 | 0.05400 | 0.53217 | 0.53872 | 1.00000 |

### Final similarity\_matrix\_with\_linear\_similarity\_luk\_tnorm

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **SIMILARITIES** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.0000 | 0.0000 | 0.0300 | 0.0000 | 0.0500 |
| **Inst 2** | 0.0000 | 1.0000 | 0.0000 | 0.0000 | 0.0000 |
| **Inst 3** | 0.0300 | 0.0000 | 1.0000 | 0.0700 | 0.4400 |
| **Inst 4** | 0.0000 | 0.0000 | 0.0700 | 1.0000 | 0.4500 |
| **Inst 5** | 0.0500 | 0.0000 | 0.4400 | 0.4500 | 1.0000 |

## Gaussian Similarity

### Element-wise calculations of (v1 - v2)2, part of the Gaussian similarity

The matrix is symmetric.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **SIM** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | [0.0, 0.0, 0.0] | [0.0100, 0.2116, 0.2025] | [0.3969, 0.0196, 0.0400] | [0.6561, 0.0256, 0.0625] | [0.8100, 0.0016, 0.0001] |
| **Inst 2** | [0.0100, 0.2116, 0.2025] | [0.0, 0.0, 0.0] | [0.2809, 0.3600, 0.4225] | [0.5041, 0.0900, 0.0400] | [0.6400, 0.2500, 0.2116] |
| **Inst 3** | [0.3969, 0.0196, 0.0400] | [0.2809, 0.3600, 0.4225] | [0.0, 0.0, 0.0] | [0.0324, 0.0900, 0.2025] | [0.0729, 0.0100, 0.0361] |
| **Inst 4** | [0.6561, 0.0256, 0.0625] | [0.5041, 0.0900, 0.0400] | [0.0324, 0.0900, 0.2025] | [0.0, 0.0, 0.0] | [0.0081, 0.0400, 0.0676] |
| **Inst 5** | [0.81, 0.0016, 0.0001] | [0.6400, 0.2500, 0.2116] | [0.0729, 0.0100, 0.0361] | [0.0081, 0.0400, 0.0676] | [0.0, 0.0, 0.0] |

### Gaussian similarity elementwise (sigma = 0.67)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **SIM** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | [1.0000, 1.0000, 1.0000] | [0.9889, 0.7900, 0.7981] | [0.6427, 0.9784, 0.9564] | [0.4815, 0.9719, 0.9328] | [0.4057, 0.9982, 0.9999] |
| **Inst 2** | [0.9889, 0.7900, 0.7981] | [1.0000, 1.0000, 1.0000] | [0.7313, 0.6697, 0.6246] | [0.5704, 0.9046, 0.9564] | [0.4902, 0.7569, 0.7900] |
| **Inst 3** | [0.6427, 0.9784, 0.9564] | [0.7313, 0.6697, 0.6246] | [1.0000, 1.0000, 1.0000] | [0.9646, 0.9046, 0.7981] | [0.9220, 0.9889, 0.9606] |
| **Inst 4** | [0.4815, 0.9719, 0.9328] | [0.5704, 0.9046, 0.9564] | [0.9646, 0.9046, 0.7981] | [1.0000, 1.0000, 1.0000] | [0.9910, 0.9564, 0.9275] |
| **Inst 5** | [0.4057, 0.9982, 0.9999] | [0.4902, 0.7569, 0.7900] | [0.9220, 0.9889, 0.9606] | [0.9910, 0.9564, 0.9275] | [1.0000, 1.0000, 1.0000] |

### Gaussian similarity with product tnorm

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **SIM** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.0000 | 0.6235 | 0.6014 | 0.4365 | 0.4049 |
| **Inst 2** | 0.6235 | 1.0 | 0.3059 | 0.4935 | 0.2932 |
| **Inst 3** | 0.6014 | 0.3059 | 1.0 | 0.6964 | 0.8759 |
| **Inst 4** | 0.4365 | 0.4935 | 0.6964 | 1.0 | 0.8791 |
| **Inst 5** | 0.4049 | 0.2932 | 0.8759 | 0.8791 | 1.0 |

### Gaussian similarity with minimum tnorm

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **SIM** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.0 | 0.7900 | 0.6427 | 0.4815 | 0.4057 |
| **Inst 2** | 0.7900 | 1.0 | 0.6246 | 0.5704 | 0.4902 |
| **Inst 3** | 0.6427 | 0.6246 | 1.0 | 0.7981 | 0.9220 |
| **Inst 4** | 0.4815 | 0.5704 | 0.7981 | 1.0 | 0.9275 |
| **Inst 5** | 0.4057 | 0.4902 | 0.9220 | 0.9275 | 1.0 |

### Gaussian similarity with luk tnorm

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **SIM** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | [1.0000 | 0.5770 | 0.5775 | 0.3862 | 0.4038] |
| **Inst 2** | [0.5770 | 1.0000 | 0.0256 | 0.4314 | 0.0371] |
| **Inst 3** | [0.5775 | 0.0256 | 1.0000 | 0.6673 | 0.8715] |
| **Inst 4** | [0.3862 | 0.4314 | 0.6673 | 1.0000 | 0.8749] |
| **Inst 5** | [0.4038 | 0.0371 | 0.8715 | 0.8749 | 1.0000] |

# Lower and Upper approximations (ITFRS and OWAFRS)

## y (labels)

|  |  |
| --- | --- |
| **Inst 1** | 1.0 |
| **Inst 2** | 1.0 |
| **Inst 3** | 0.0 |
| **Inst 4** | 1.0 |
| **Inst 5** | 0.0 |

## label\_masks (b)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Label masks** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.0 | 1.0 | 0.0 | 1.0 | 0.0 |
| **Inst 2** | 1.0 | 1.0 | 0.0 | 1.0 | 0.0 |
| **Inst 3** | 0.0 | 0.0 | 1.0 | 0.0 | 1.0 |
| **Inst 4** | 1.0 | 1.0 | 0.0 | 1.0 | 0.0 |
| **Inst 5** | 0.0 | 0.0 | 1.0 | 0.0 | 1.0 |

## similarity\_matrix (a)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **SIMILARITIES** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.00 | 0.54 | 0.37 | 0.19 | 0.10 |
| **Inst 2** | 0.54 | 1.00 | 0.35 | 0.29 | 0.20 |
| **Inst 3** | 0.37 | 0.35 | 1.00 | 0.55 | 0.73 |
| **Inst 4** | 0.19 | 0.29 | 0.55 | 1.00 | 0.74 |
| **Inst 5** | 0.10 | 0.20 | 0.73 | 0.74 | 1.00 |

## Interim 1 - sim

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 0.00 | 0.46 | 0.63 | 0.81 | 0.90 |
| **Inst 2** | 0.46 | 0.00 | 0.65 | 0.71 | 0.80 |
| **Inst 3** | 0.63 | 0.65 | 0.00 | 0.45 | 0.27 |
| **Inst 4** | 0.81 | 0.71 | 0.45 | 0.00 | 0.26 |
| **Inst 5** | 0.90 | 0.80 | 0.27 | 0.26 | 0.00 |

## Interim Sim \* A(y)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.0 | 0.54 | 0.0 | 0.19 | 0.0 |
| **Inst 2** | 0.54 | 1.0 | 0.0 | 0.29 | 0.0 |
| **Inst 3** | 0.0 | 0.0 | 1.0 | 0.0 | 0.73 |
| **Inst 4** | 0.19 | 0.29 | 0.0 | 1.0 | 0.0 |
| **Inst 5** | 0.0 | 0.0 | 0.73 | 0.0 | 1.0 |

## Interim 1 - sim + A(y)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.0 | 1.46 | 0.63 | 1.81 | 0.9 |
| **Inst 2** | 1.46 | 1.0 | 0.65 | 1.71 | 0.8 |
| **Inst 3** | 0.63 | 0.65 | 1.0 | 0.45 | 1.27 |
| **Inst 4** | 1.81 | 1.71 | 0.45 | 1.0 | 0.26 |
| **Inst 5** | 0.9 | 0.8 | 1.27 | 0.26 | 1.0 |

## Luk Implicator results

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.0 | 1.0 | 0.63 | 1.0 | 0.9 |
| **Inst 2** | 1.0 | 1.0 | 0.65 | 1.0 | 0.8 |
| **Inst 3** | 0.63 | 0.65 | 1.0 | 0.45 | 1.0 |
| **Inst 4** | 1.0 | 1.0 | 0.45 | 1.0 | 0.26 |
| **Inst 5** | 0.9 | 0.8 | 1.0 | 0.26 | 1.0 |

## KD Implicator results

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.0 | 1.0 | 0.63 | 1.0 | 0.9 |
| **Inst 2** | 1.0 | 1.0 | 0.65 | 1.0 | 0.8 |
| **Inst 3** | 0.63 | 0.65 | 1.0 | 0.45 | 1.0 |
| **Inst 4** | 1.0 | 1.0 | 0.45 | 1.0 | 0.26 |
| **Inst 5** | 0.9 | 0.8 | 1.0 | 0.26 | 1.0 |

## Reichenbach Implicator results

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.0 | 1.0 | 0.63 | 1.0 | 0.9 |
| **Inst 2** | 1.0 | 1.0 | 0.65 | 1.0 | 0.8 |
| **Inst 3** | 0.63 | 0.65 | 1.0 | 0.45 | 1.0 |
| **Inst 4** | 1.0 | 1.0 | 0.45 | 1.0 | 0.26 |
| **Inst 5** | 0.9 | 0.8 | 1.0 | 0.26 | 1.0 |

## Goedel Implicator results

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.0 | 1.0 | 0.0 | 1.0 | 0.0 |
| **Inst 2** | 1.0 | 1.0 | 0.0 | 1.0 | 0.0 |
| **Inst 3** | 0.0 | 0.0 | 1.0 | 0.0 | 1.0 |
| **Inst 4** | 1.0 | 1.0 | 0.0 | 1.0 | 0.0 |
| **Inst 5** | 0.0 | 0.0 | 1.0 | 0.0 | 1.0 |

## Gaines Implicator results

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.0 | 1.0 | 0.0 | 1.0 | 0.0 |
| **Inst 2** | 1.0 | 1.0 | 0.0 | 1.0 | 0.0 |
| **Inst 3** | 0.0 | 0.0 | 1.0 | 0.0 | 1.0 |
| **Inst 4** | 1.0 | 1.0 | 0.0 | 1.0 | 0.0 |
| **Inst 5** | 0.0 | 0.0 | 1.0 | 0.0 | 1.0 |

NOTES:

1. KD, luk and Reichenbach results are the same for this example.

Since for the calculations of lower approximation, we calculate *Inf* which is basically a minimum, to exclude the same instance from calculations we don’t need anything because the diagonal is set to 1.0 which is ignored by min operator. To be sure all is correct, inside code, we set main diagonal to 1.0

Since for the calculations of upper approximation, we calculate sup which is basically a maximum, to exclude the same instance from calculations we need to set the main diagonal to 0.0 which is ignored by max operator. Otherwise all upper approxamations will be 1.0.

## Lower approximation with all Implicators

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Richenbach** | 0.63 | 0.65 | 0.45 | 0.26 | 0.26 |
| **KD** | 0.63 | 0.65 | 0.45 | 0.26 | 0.26 |
| **Luk** | 0.63 | 0.65 | 0.45 | 0.26 | 0.26 |
| **Goedel** | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| **Gaines** | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |

## Min and product t-norm(similarity\_matrix, label\_masks)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **t-norm** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 0.0 | 0.54 | 0.0 | 0.19 | 0.0 |
| **Inst 2** | 0.54 | 0.0 | 0.0 | 0.29 | 0.0 |
| **Inst 3** | 0.0 | 0.0 | 0.0 | 0.0 | 0.73 |
| **Inst 4** | 0.19 | 0.29 | 0.0 | 0.0 | 0.0 |
| **Inst 5** | 0.0 | 0.0 | 0.73 | 0.0 | 0.0 |

**NOTE: min and product tnorm give the same results**

**Main diagonal is set to 0.0 so that sup operator ignores the information of the same instance**

## Upper approximations

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Min t-norm** | 0.54 | 0.54 | 0.73 | 0.29 | 0.73 |
| **Prod t-norm** | 0.54 | 0.54 | 0.73 | 0.29 | 0.73 |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |

# OWAFRS lower approximation

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| lower\_approximation\_weights | 0.1 | 0.2 | 0.3 | 0.4 |

## KD Implicator results with making main diagonal 0.0

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 0.0 | 1.0 | 0.63 | 1.0 | 0.9 |
| **Inst 2** | 1.0 | 0.0 | 0.65 | 1.0 | 0.8 |
| **Inst 3** | 0.63 | 0.65 | 0.0 | 0.45 | 1.0 |
| **Inst 4** | 1.0 | 1.0 | 0.45 | 0.0 | 0.26 |
| **Inst 5** | 0.9 | 0.8 | 1.0 | 0.26 | 0.0 |

## Descending sorted KD Implicator results after making main diagonal 0.0

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.0 | 1.0 | 0.9 | 0.63 | 0.0 |
| **Inst 2** | 1.0 | 1.0 | 0.8 | 0.65 | 0.0 |
| **Inst 3** | 1.0 | 0.65 | 0.63 | 0.45 | 0.0 |
| **Inst 4** | 1.0 | 1.0 | 0.45 | 0.26 | 0.0 |
| **Inst 5** | 1.0 | 0.9 | 0.8 | 0.26 | 0.0 |

## OWAinf calculations for KD Implicator

|  |
| --- |
| 0.1+0.2+0.27+0.252=0.822 |
| 0.1+0.2+0.24+0.26=0.8 |
| 0.1+0.13+0.189+0.18=0.599 |
| 0.1+0.2+0.135+0.104=0.539 |
| 0.1+0.18+0.24+0.104=0.624 |

=

|  |
| --- |
| 0.1 |
| 0.2 |
| 0.3 |
| 0.4 |

x

|  |  |  |  |
| --- | --- | --- | --- |
| 1.0 | 1.0 | 0.9 | 0.63 |
| 1.0 | 1.0 | 0.8 | 0.65 |
| 1.0 | 0.65 | 0.63 | 0.45 |
| 1.0 | 1.0 | 0.45 | 0.26 |
| 1.0 | 0.9 | 0.8 | 0.26 |

The results of Reichenbach, luk and KD are the same

## Goedel Implicator results with making main diagonal 0.0

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 0.0 | 1.0 | 0.0 | 1.0 | 0.0 |
| **Inst 2** | 1.0 | 0.0 | 0.0 | 1.0 | 0.0 |
| **Inst 3** | 0.0 | 0.0 | 0.0 | 0.0 | 1.0 |
| **Inst 4** | 1.0 | 1.0 | 0.0 | 0.0 | 0.0 |
| **Inst 5** | 0.0 | 0.0 | 1.0 | 0.0 | 0.0 |

## Descending sorted Goedel Implicator results after making main diagonal 0.0

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.0 | 1.0 | 0.0 | 0.0 | 0.0 |
| **Inst 2** | 1.0 | 1.0 | 0.0 | 0.0 | 0.0 |
| **Inst 3** | 1.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| **Inst 4** | 1.0 | 1.0 | 0.0 | 0.0 | 0.0 |
| **Inst 5** | 1.0 | 0.0 | 0.0 | 0.0 | 0.0 |

## OWAinf calculations for Goedel Implicator

|  |
| --- |
| 0.1+0.2=0.3 |
| 0.1+0.2=0.3 |
| 0.1=0.1 |
| 0.1+0.2=0.3 |
| 0.1=0.1 |

=

|  |
| --- |
| 0.1 |
| 0.2 |
| 0.3 |
| 0.4 |

x

|  |  |  |  |
| --- | --- | --- | --- |
| 1.0 | 1.0 | 0.0 | 0.0 |
| 1.0 | 1.0 | 0.0 | 0.0 |
| 1.0 | 0.0 | 0.0 | 0.0 |
| 1.0 | 1.0 | 0.0 | 0.0 |
| 1.0 | 0.0 | 0.0 | 0.0 |

The results of goedel and gaines implicators are the same.

# OWAFRS upper approximation

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| upper\_approximation\_weights | 0.4 | 0.3 | 0.2 | 0.1 |

# Min and product t-norm(similarity\_matrix, label\_masks)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **t-norm** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 0.0 | 0.54 | 0.0 | 0.19 | 0.0 |
| **Inst 2** | 0.54 | 0.0 | 0.0 | 0.29 | 0.0 |
| **Inst 3** | 0.0 | 0.0 | 0.0 | 0.0 | 0.73 |
| **Inst 4** | 0.19 | 0.29 | 0.0 | 0.0 | 0.0 |
| **Inst 5** | 0.0 | 0.0 | 0.73 | 0.0 | 0.0 |

## Descending sorted min results after making main diagonal 0.0

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **t-norm** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 0.54 | 0.19 | 0.0 | 0.0 | 0.0 |
| **Inst 2** | 0.54 | 0.29 | 0.0 | 0.0 | 0.0 |
| **Inst 3** | 0.73 | 0.0 | 0.0 | 0.0 | 0.0 |
| **Inst 4** | 0.29 | 0.19 | 0.0 | 0.0 | 0.0 |
| **Inst 5** | 0.73 | 0.0 | 0.0 | 0.0 | 0.0 |

## OWAsup calculations for min tnorm

|  |
| --- |
| 0.273 |
| 0.303 |
| 0.292 |
| 0.173 |
| 0.292 |

=

|  |
| --- |
| 0.4 |
| 0.3 |
| 0.2 |
| 0.1 |

x

|  |  |  |  |
| --- | --- | --- | --- |
| 0.54 | 0.19 | 0.0 | 0.0 |
| 0.54 | 0.29 | 0.0 | 0.0 |
| 0.73 | 0.0 | 0.0 | 0.0 |
| 0.29 | 0.19 | 0.0 | 0.0 |
| 0.73 | 0.0 | 0.0 | 0.0 |

The results of minimum and product tnorm are the same.

# Lower and Upper approximations (VQRS)

## y (labels)

|  |  |
| --- | --- |
| **Inst 1** | 1.0 |
| **Inst 2** | 1.0 |
| **Inst 3** | 0.0 |
| **Inst 4** | 1.0 |
| **Inst 5** | 0.0 |

## label\_masks

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Label masks** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.0 | 1.0 | 0.0 | 1.0 | 0.0 |
| **Inst 2** | 1.0 | 1.0 | 0.0 | 1.0 | 0.0 |
| **Inst 3** | 0.0 | 0.0 | 1.0 | 0.0 | 1.0 |
| **Inst 4** | 1.0 | 1.0 | 0.0 | 1.0 | 0.0 |
| **Inst 5** | 0.0 | 0.0 | 1.0 | 0.0 | 1.0 |

## similarity\_matrix after setting 0.0 into main diagonal elements

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **SIMILARITIES** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 0.00 | 0.54 | 0.37 | 0.19 | 0.10 |
| **Inst 2** | 0.54 | 0.00 | 0.35 | 0.29 | 0.20 |
| **Inst 3** | 0.37 | 0.35 | 0.00 | 0.55 | 0.73 |
| **Inst 4** | 0.19 | 0.29 | 0.55 | 0.00 | 0.74 |
| **Inst 5** | 0.10 | 0.20 | 0.73 | 0.74 | 0.00 |

## VQRS nominator step1 (min[R(x,y),A(x)])

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 0.00 | 0.54 | 0.0 | 0.19 | 0. 0 |
| **Inst 2** | 0.54 | 0.00 | 0.0 | 0.29 | 0. 0 |
| **Inst 3** | 0.0 | 0.0 | 0.00 | 0.0 | 0.73 |
| **Inst 4** | 0.19 | 0.29 | 0.0 | 0.00 | 0.0 |
| **Inst 5** | 0.00 | 0.00 | 0.73 | 0.0 | 0.00 |

## Nominator {sum(min[R(x,y),A(x)])}

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 0.73 | 0.83 | 0.73 | 0.48 | 0.73 |

## Denominator (sum [R(x,y)])

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 1.2 | 1.38 | 2.0 | 1.77 | 1.77 |

## Division results (nominator/denominator)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 0.608333 | 0.60144928 | 0.365 | 0.27118644 | 0.412429379 |

## Upper approximation (alpha =0.2, beta = 1.0) with quad fuzzy quantifier

For 0.608333:

2 \* (0.608333 - 0.2)2 / (1 - 0.2)2 = 2\*(0.166735838889 / 0.64) = 2 \* 0.2605247482640625 =0.521049496528125

-------------------------------------------------------------------------------------------------------

For 0.60144928:

2 \* (0.60144928- 0.2)2 / (1 - 0.2)2 = 2\*(0.1611615244125184/ 0.64) = 2 \* 0.25181488189456 =0.50362976378912

-------------------------------------------------------------------------------------------------------

For 0.365:

2 \* (0.365- 0.2)2 / (1 - 0.2)2 = 2\*(0.027225/ 0.64) = 2 \* 0.027225=0.085078125

-------------------------------------------------------------------------------------------------------

For 0.27118644:

2 \* (0.27118644- 0.2)2 / (1 - 0.2)2 = 2\*(0.0050675092398736/ 0.64) = 2 \* 0.0079179831873025 =0.015835966374605

-------------------------------------------------------------------------------------------------------

For 0.412429379:

2 \* (0.412429379- 0.2)2 / (1 - 0.2)2 = 2\*(0.045126241062325641/ 0.64) = 2 \* 0.0705097516598838140625 =0.141019503319767628125

-------------------------------------------------------------------------------------------------------

Final results for upper approximation:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 0.521049496528125 | 0.50362976378912 | 0.085078125 | 0.015835966374605 | 0.141019503319767628125 |

## Lower approximation (alpha =0.1, beta = 0.6) with quad fuzzy quantifier

For 0.608333:

=1.0

-------------------------------------------------------------------------------------------------------

For 0.60144928:

=1.0

-------------------------------------------------------------------------------------------------------

For 0.365:

1 - 2 \* (0.365- 0.6)2 / (0.6 - 0.1)2 = 1 - 2\*(0.055225/ 0.25) = 1 - 2 \* 0.2209 =1 - 0.4418 =0.5582

-------------------------------------------------------------------------------------------------------

For 0.27118644:

2 \* (0.27118644- 0.1)2 / (0.6 - 0.1)2 = 2\*(0.0293047972398736/ 0.25) = 2 \* 0.1172191889594944=0.2344383779189888

-------------------------------------------------------------------------------------------------------

For 0.412429379:

1 - 2 \* (0.412429379- 0.6)2 / (0.6 - 0.1)2 =1 - 2\*(0.035182737862325641/ 0.25) = 1 - 2 \* 0.140730951449302564 =0.718538097101394872

Final results for lower approximation:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 1.0 | 1.0 | 0.5582 | 0.2344383779189888 | 0.718538097101394872 |