Test dataset for unit testing

By Mehran Amiri

For FRsutils library (https://github.com/mehi64/FRsutils)

**Change log**

|  |  |  |
| --- | --- | --- |
| ***Date (dd.mm.YYYY)*** | ***Change*** | ***Person*** |
| 12.05.2025 | Added the document (t-norm, implicator test data) | Mehran Amiri |
| 13.05.2025 | Added similarity test data + LB for ITFRS | Mehran Amiri |
| 14.05.2025 | Added UB for ITFRS | Mehran Amiri |
| 15.05.2025 | Added data for Gaussian similarity | Mehran Amiri |
| 25.05,2025 | Added data for VQRS | Mehran Amiri |
| 08.06.2025 | Drastic product tnorm data | Mehran Amiri |
| 19.06.2025 | Checking tnorms and adding data for yager tnorm for p=5 | Mehran Amiri |
| 21.06.2025 | Add values for implicators goguen, rescher, yager, weber, fodor | Mehran Amiri |
| 23.06.2025 | Add values to ITFRS for implicators (goguen, rescher, yager, weber, fodor) and new TNorms | Mehran Amiri |
| 26.06.2025 | Add values to OWAFRS for implicators (goguen, rescher, yager, weber, fodor) and new TNorms | Mehran Amiri |
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|  |  |  |

Contents

[Implicators 8](#_Toc201648208)

[Data and outputs 8](#_Toc201648209)

[OWA\_weights data for test 9](#_Toc201648210)

[Data for testing t-norms in a scalar way 10](#_Toc201648211)

[Yager tnorm results (p=0.835, 1/p =1.1976047904) 10](#_Toc201648212)

[Yager tnorm results (p=5, 1/p =0.2) 10](#_Toc201648213)

[~~Data for testing t-norms in a map/vectorized way~~ 11](#_Toc201648214)

[Similarities 13](#_Toc201648215)

[X matrix (each row is a data Instance) 13](#_Toc201648216)

[Linear similarity 13](#_Toc201648217)

[Element-wise calculations of |v1 - v2|, part of linear similarity 13](#_Toc201648218)

[Element-wise similarity of Instances (linear similarity) 13](#_Toc201648219)

[Final similarity\_matrix\_with\_linear\_similarity\_minimum\_tnorm 13](#_Toc201648220)

[Final similarity\_matrix\_with\_linear\_similarity\_product\_tnorm 14](#_Toc201648221)

[Final similarity\_matrix\_with\_linear\_similarity\_luk\_tnorm 14](#_Toc201648222)

[Gaussian Similarity 15](#_Toc201648223)

[Element-wise calculations of (v1 - v2)2, part of the Gaussian similarity 15](#_Toc201648224)

[Gaussian similarity elementwise (sigma = 0.67) 15](#_Toc201648225)

[Gaussian similarity with product tnorm 15](#_Toc201648226)

[Gaussian similarity with minimum tnorm 15](#_Toc201648227)

[Gaussian similarity with luk tnorm 16](#_Toc201648228)

[Lower and Upper approximations (ITFRS and OWAFRS) 17](#_Toc201648229)

[y (labels) 17](#_Toc201648230)

[similarity\_matrix (a) 17](#_Toc201648231)

[label\_masks (b or A(y)) 17](#_Toc201648232)

[A(y) / sim 17](#_Toc201648233)

[max(1-sim, A(y)) 18](#_Toc201648234)

[Interim 1 - sim 18](#_Toc201648235)

[Interim Sim \* A(y) 18](#_Toc201648236)

[Interim 1-sim + (sim \* A(y)) 18](#_Toc201648237)

[Interim 1 - sim + A(y) 19](#_Toc201648238)

[Luk Implicator results 19](#_Toc201648239)

[KD Implicator results 19](#_Toc201648240)

[Reichenbach Implicator results 20](#_Toc201648241)

[Goedel Implicator results 20](#_Toc201648242)

[Gaines Implicator results 20](#_Toc201648243)

[Lower approximation with all Implicators 22](#_Toc201648244)

[Min and product t-norm(similarity\_matrix, label\_masks) 25](#_Toc201648245)

[Upper approximations 27](#_Toc201648246)

[OWAFRS lower approximation 28](#_Toc201648247)

[KD Implicator results with making main diagonal 0.0 28](#_Toc201648248)

[Descending sorted KD Implicator results after making main diagonal 0.0 28](#_Toc201648249)

[OWAinf calculations for KD Implicator 28](#_Toc201648250)

[Goedel Implicator results with making main diagonal 0.0 29](#_Toc201648251)

[Descending sorted Goedel Implicator results after making main diagonal 0.0 29](#_Toc201648252)

[OWAinf calculations for Goedel Implicator 29](#_Toc201648253)

[OWAFRS upper approximation 30](#_Toc201648254)

[Min and product t-norm(similarity\_matrix, label\_masks) 30](#_Toc201648255)

[Descending sorted min results after making main diagonal 0.0 31](#_Toc201648256)

[OWAsup calculations for min tnorm 31](#_Toc201648257)

[Lower and Upper approximations (VQRS) 31](#_Toc201648258)

[y (labels) 31](#_Toc201648259)

[label\_masks 32](#_Toc201648260)

[similarity\_matrix after setting 0.0 into main diagonal elements 32](#_Toc201648261)

[VQRS nominator step1 (min[R(x,y),A(x)]) 32](#_Toc201648262)

[Nominator {sum(min[R(x,y),A(x)])} 32](#_Toc201648263)

[Denominator (sum [R(x,y)]) 32](#_Toc201648264)

[Division results (nominator/denominator) 32](#_Toc201648265)

[Upper approximation (alpha =0.2, beta = 1.0) with quad fuzzy quantifier 32](#_Toc201648266)

[Lower approximation (alpha =0.1, beta = 0.6) with quad fuzzy quantifier 33](#_Toc201648267)

**NOTEs:**

* **When calculating values, calculate them with as many as floating points possible. Then, at the end, you can round it. Otherwise, when testing it will be hard to test correctly.**
* **It is a good idea to have tests with real numbers for A(y). Now, it is just a binary value appropriate for classification tasks but not for regression.**

# Implicators

## Data and outputs

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| a | b | 1-a | ab | 1-a+ab | 1-a+b | max(1-a,b) | b/a | **Gaines** | **Goedel** | **KD** | **reichenbach** | **Luk** | **goguen** | **rescher** | **Yager** | **weber** | **fodor** |
| 2.10 | 4.32 | - | - | - | - | - | - | **-** | - | - | - | - | - | - | - | - | - |
| -0.20 | -0.78 | - | - | - | - | - | - | **-** | - | - | - | - | - | - | - | - | - |
| 0.73 | 0.18 | 0.27 | 0.1314 | 0.4014 | 0.45 | 0.27 | 0.246575 |  | 0.18 | 0.27 | 0.4014 | 0.45 | 0.246575 | 0.00 | 0.285989 | 1.00 | 0.27 |
| 0.18 | 0.73 | 0.82 | 0.1314 | 0.9514 | 1.55 | 0.82 | - |  | 1.0 | 0.82 | 0.9514 | 1.00 | 1.00 | 1.00 | 0.944927 | 1.00 | 1.00 |
| 0.88 | 0.88 | 0.12 | 0.7744 | 0.8914 | 1.00 | 0.88 | - |  | 1.0 | 0.88 | 0.8944 | 1.00 | 1.00 | 1.00 | 0.893603 | 1.00 | 1.00 |
| 0.91 | 0.48 | 0.09 | 0.4368 | 0.5263 | 0.57 | 0.48 | 0.527473 |  | 0.48 | 0.48 | 0.5268 | 0.57 | 0.527473 | 0.00 | 0.512778 | 1.00 | 0.48 |
| 1.00 | 1.00 | 0.00 | 1.00 | 1.00 | 1.00 | 1.00 | - |  | 1.0 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| 0.00 | 0.00 | 1.00 | 0.00 | 1.00 | 1.00 | 1.00 | - |  | 1.0 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| 0.00 | 1.00 | 1.00 | 0.00 | 1.00 | 2.00 | 1.00 | - |  | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |  |  | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 1.00 | 0.65 | 0.00 | 0.65 | 0.65 | 0.65 | 0.65 |  |  | 0.65 | 0.65 | 0.65 | 0.65 | 0.65 | 0.00 | 0.65 | 0.65 | 0.65 |
| 0.65 | 1.00 | 0.35 | 0.65 | 1.00 | 1.35 | 1.00 |  |  | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| 0.55 | 0.00 | 0.45 | 0.00 | 0.45 | 0.45 | 0.45 |  |  | 0.00 | 0.45 | 0.45 | 0.45 | 0.00 | 0.00 | 0.00 | 1.00 | 0.45 |
| 0.00 | 0.55 | 1.00 | 0.00 | 1.00 | 1.55 | 1.00 |  |  | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |

# OWA\_weights data for test

 owa\_infimum\_weights\_linear\_len\_5 = np.array([0.06666667, 0.13333333, 0.2, 0.26666667, 0.33333333])

        owa\_infimum\_weights\_linear\_len\_10 = np.array([0.01818182, 0.03636364, 0.05454545, 0.07272727, 0.09090909, 0.10909091, 0.12727273, 0.14545455, 0.16363636, 0.18181818])

        owa\_suprimum\_weights\_linear\_len\_8 = np.array([0.22222222, 0.19444444, 0.16666667, 0.13888889, 0.11111111, 0.08333333, 0.05555556, 0.02777778])

        owa\_supriimum\_weights\_linear\_len\_13 = np.array([0.14285714, 0.13186813, 0.12087912, 0.10989011, 0.0989011,  0.08791209, 0.07692308, 0.06593407, 0.05494505, 0.04395604, 0.03296703, 0.02197802, 0.01098901])

# Data for testing t-norms in a scalar way

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| a | b | ab | a+b | a+b-1 | 1-a | 1-b | 1+(1-a)(1-b) | a+b-ab | **Min** | **Product** | **Luk** | **Drastic Prod** | **Hamacher prod** | **Einstein** | **Nilpotent min** |
| 2.10 | 4.32 | - | - | - | - | - | - | - | - | - | - | - | - | - |  |
| -0.20 | -0.78 | - | - | - | - | - | - | - | - | - | - | - | - | - |  |
| 0.73 | 0.18 | 0.1314 | 0.91 | -0.09 | 0.27 | 0.82 | 1.2214 | 0.7786 | 0.18 | 0.1314 | 0.00 | 0.00 | 0.168764 | 0.107581 | 0.00 |
| 0.18 | 0.73 | 0.1314 | 0.91 | -0.09 | 0.82 | 0.27 | 1.2214 | 0.7786 | 0.18 | 0.1314 | 0.00 | 0.00 | 0.168764 | 0.107581 | 0.00 |
| 0.88 | 0.88 | 0.7744 | 1.76 | 0.76 | 0.12 | 0.12 | 1.0144 | 0.9856 | 0.88 | 0.7744 | 0.76 | 0.00 | 0.785714 | 0.763407 | 0.88 |
| 0.91 | 0.48 | 0.4368 | 1.39 | 0.39 | 0.09 | 0.52 | 1.0468 | 0.9532 | 0.48 | 0.4368 | 0.39 | 0.00 | 0.458246 | 0.417271 | 0.48 |
| 1.00 | 1.00 | 1.00 | 2.00 | 1.00 | 0.00 | 0.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| 0.00 | 0.00 | 0.00 | 0.00 | -1.00 | 1.00 | 1.00 | 2.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 1.00 | 0.65 | 0.65 | 1.65 | 0.65 | 0.00 | 0.35 | 1.00 | 1.00 | 0.65 | 0.65 | 0.65 | 0.65 | 0.65 | 0.65 | 0.65 |
| 0.37 | 1.00 | 0.37 | 1.37 | 0.37 | 0.63 | 0.00 | 1.00 | 1.00 | 0.37 | 0.37 | 0.37 | 0.37 | 0.37 | 0.37 | 0.37 |

## Yager tnorm results (p=0.835, 1/p =1.1976047904)

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| a | b | ab | 1-a | 1-b | (1-a)p | (1-b)p | (1-a)p +(1-b)p | [(1-a)p +(1-b)p](1/p) | Min{1, [(1-a)p +(1-b)p](1/p)} | **Yager** |
| 2.10 | 4.32 | - | - | - | - | - | - | - | - | - |
| -0.20 | -0.78 | - | - | - | - | - | - | - | - | - |
| 0.73 | 0.18 | 0.1314 | 0.27 | 0.82 | 0.335111 | 0.847295 | 1.182406 | 1.2222095 | 1.00 | 0.00 |
| 0.18 | 0.73 | 0.1314 | 0.82 | 0.27 | 0.847295 | 0.335111 | 1.182406 | 1.2222095 | 1.00 | 0.00 |
| 0.88 | 0.88 | 0.7744 | 0.12 | 0.12 | 0.170261 | 0.170261 | 0.340522 | 0.275229 | 0.275229 | 0.724771 |
| 0.91 | 0.48 | 0.4368 | 0.09 | 0.52 | 0.133904 | 0.579246 | 0.713150 | 0.667066 | 0.667066 | 0.332934 |
| 1.00 | 1.00 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 1.00 |
| 0.00 | 0.00 | 0.00 | 1.00 | 1.00 | 1.00 | 1.00 | 2.00 | 2.293586 | 1.00 | 0.00 |
| 1.00 | 0.65 | 0.65 | 0.00 | 0.35 | 0.00 | 0.416195 | 0.416195 | 0.35 | 0.35 | 0.65 |
| 0.37 | 1.00 | 0.37 | 0.63 | 0.00 | 0.679907 | 0.00 | 0.679907 | 0.63 | 0.63 | 0.37 |

## Yager tnorm results (p=5, 1/p =0.2)

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| a | b | ab | 1-a | 1-b | (1-a)p | (1-b)p | (1-a)p +(1-b)p | [(1-a)p +(1-b)p](1/p) | Min{1, [(1-a)p +(1-b)p](1/p)} | **Yager** |
| 2.10 | 4.32 | - | - | - | - | - | - | - | - | - |
| -0.20 | -0.78 | - | - | - | - | - | - | - | - | - |
| 0.73 | 0.18 | 0.1314 | 0.27 | 0.82 | 0.0014348907 | 0.3707398432 | 0.3721747339 | 0.820633756 | 0.820633756 | 0.179366244 |
| 0.18 | 0.73 | 0.1314 | 0.82 | 0.27 | 0.3707398432 | 0.0014348907 | 0.3721747339 | 0.820633756 | 0.820633756 | 0.179366244 |
| 0.88 | 0.88 | 0.7744 | 0.12 | 0.12 | 0.0000248832 | 0.0000248832 | 0.0000497664 | 0.1378438026 | 0.1378438026 | 0.8621561974 |
| 0.91 | 0.48 | 0.4368 | 0.09 | 0.52 | 0.0000059049 | 0.0380204032 | 0.0380263081 | 0.5200161511 | 0.5200161511 | 0.4799838489 |
| 1.00 | 1.00 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 1.00 |
| 0.00 | 0.00 | 0.00 | 1.00 | 1.00 | 1.00 | 1.00 | 2.00 | 1.148698355 | 1.00 | 0.00 |
| 1.00 | 0.65 | 0.65 | 0.00 | 0.35 | 0.00 | 0.0052521875 | 0.0052521875 | 0.35 | 0.35 | 0.65 |
| 0.37 | 1.00 | 0.37 | 0.63 | 0.00 | 0.0992436543 | 0.00 | 0.0992436543 | 0.63 | 0.63 | 0.37 |

# ~~Data for testing t-norms in a map/vectorized way~~

This is not needed anymore because we test \_\_call\_\_ and then after making sure it is correct, reduce results compared against \_\_call\_\_ results. But we did not delete calculations in this section. They may come in handy some time.

Similarity map

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 1.0 | 0.2673 | 0.25456 | 0.1197 | 0.09504 |
| 0.2673 | 1.0 | 0.0658 | 0.1624 | 0.054 |
| 0.25456 | 0.0658 | 1.0 | 0.3157 | 0.53217 |
| 0.1197 | 0.1624 | 0.3157 | 1.0 | 0.53872 |
| 0.09504 | 0.054 | 0.53217 | 0.53872 | 1.0 |

Mask map

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 1.0 | 1.0 | 0.0 | 1.0 | 0.0 |
| 1.0 | 1.0 | 0.0 | 1.0 | 0.0 |
| 0.0 | 0.0 | 1.0. | 0.0 | 1.0 |
| 1.0 | 1.0 | 0.0 | 1.0 | 0.0 |
| 0.0 | 0.0 | 1.0 | 0.0 | 1.0 |

Output of product tnorm

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 1.0 | 0.2673 | 0.0 | 0.1197 | 0.0 |
| 0.2673 | 1.0 | 0.0 | 0.1624 | 0.0 |
| 0.0 | 0.0 | 1.0 | 0.0 | 0.53217 |
| 0.1197 | 0.1624 | 0.0 | 1.0 | 0.0 |
| 0.0 | 0.0 | 0.53217 | 0.0 | 1.0 |

Output of minimum tnorm

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 1.0 | 0.2673 | 0.0 | 0.1197 | 0.0 |
| 0.2673 | 1.0 | 0.0 | 0.1624 | 0.0 |
| 0.0 | 0.0 | 1.0 | 0.0 | 0.53217 |
| 0.1197 | 0.1624 | 0.0 | 1.0 | 0.0 |
| 0.0 | 0.0 | 0.53217 | 0.0 | 1.0 |

Output of Luk tnorm

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 1.0 | 0.2673 | 0.0 | 0.1197 | 0.0 |
| 0.2673 | 1.0 | 0.0 | 0.1624 | 0.0 |
| 0.0 | 0.0 | 1.0 | 0.0 | 0.53217 |
| 0.1197 | 0.1624 | 0.0 | 1.0 | 0.0 |
| 0.0 | 0.0 | 0.53217 | 0.0 | 1.0 |

# Similarities

## X matrix (each row is a data Instance)

|  |  |  |  |
| --- | --- | --- | --- |
| **Inst 1** | 0.10 | 0.32 | 0.48 |
| **Inst 2** | 0.20 | 0.78 | 0.93 |
| **Inst 3** | 0.73 | 0.18 | 0.28 |
| **Inst 4** | 0.91 | 0.48 | 0.73 |
| **Inst 5** | 1.00 | 0.28 | 0.47 |

## Linear similarity

### Element-wise calculations of |v1 - v2|, part of linear similarity

The matrix is symmetric.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **SIM** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | [0.0, 0.0, 0.0] | [0.1, 0.46, 0.45] | [0.63, 0.14, 0.20] | [0.81, 0.16, 0.25] | [0.90, 0.04, 0.01] |
| **Inst 2** | [0.1, 0.46, 0.45] | [0.0, 0.0, 0.0] | [0.53, 0.60, 0.65] | [0.71, 0.30, 0.20] | [0.80, 0.50, 0.46] |
| **Inst 3** | [0.63, 0.14, 0.20] | [0.53, 0.60, 0.65] | [0.0, 0.0, 0.0] | [0.18, 0.30, 0.45] | [0.27, 0.10, 0.19] |
| **Inst 4** | [0.81, 0.16, 0.25] | [0.71, 0.30, 0.20] | [0.18, 0.30, 0.45] | [0.0, 0.0, 0.0] | [0.09, 0.20, 0.26] |
| **Inst 5** | [0.90, 0.04, 0.01] | [0.80, 0.50, 0.46] | [0.27, 0.10, 0.19] | [0.09, 0.20, 0.26] | [0.0, 0.0, 0.0] |

### Element-wise similarity of Instances (linear similarity)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **SIM** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | [1.0, 1.0,1.0] | [0.9, 0.54, 0.55] | [0.37, 0.86, 0.8 ] | [0.19, 0.84, 0.75] | [0.1 , 0.96, 0.99] |
| **Inst 2** | [0.90 , 0.54, 0.55] | [1.0, 1.0,1.0] | [0.47, 0.4 , 0.35] | [0.29, 0.7 , 0.8 ] | [0.2 , 0.5 , 0.54] |
| **Inst 3** | [0.37, 0.86, 0.80 ] | [0.47, 0.4 , 0.35] | [1.0, 1.0,1.0] | [0.82, 0.7 , 0.55] | [0.73, 0.9 , 0.81] |
| **Inst 4** | [0.19, 0.84, 0.75] | [0.29, 0.70 , 0.8 ] | [0.82, 0.7 , 0.55] | [1.0, 1.0,1.0] | [0.91, 0.8 , 0.74] |
| **Inst 5** | [0.10 , 0.96, 0.99] | [0.20 , 0.5 , 0.54] | [0.73, 0.9 , 0.81] | [0.91, 0.8 , 0.74] | [1.0, 1.0,1.0] |

### Final similarity\_matrix\_with\_linear\_similarity\_minimum\_tnorm

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **SIMILARITIES** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.0 | 0.54 | 0.37 | 0.19 | 0.1 |
| **Inst 2** | 0.54 | 1.0 | 0.35 | 0.29 | 0.2 |
| **Inst 3** | 0.37 | 0.35 | 1.0 | 0.55 | 0.73 |
| **Inst 4** | 0.19 | 0.29 | 0.55 | 1.0 | 0.74 |
| **Inst 5** | 0.10 | 0.20 | 0.73 | 0.74 | 1.0 |

### Final similarity\_matrix\_with\_linear\_similarity\_product\_tnorm

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **SIMILARITIES** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.00000 | 0.26730 | 0.25456 | 0.11970 | 0.09504 |
| **Inst 2** | 0.26730 | 1.00000 | 0.06580 | 0.16240 | 0.05400 |
| **Inst 3** | 0.25456 | 0.06580 | 1.00000 | 0.31570 | 0.53217 |
| **Inst 4** | 0.11970 | 0.16240 | 0.31570 | 1.00000 | 0.53872 |
| **Inst 5** | 0.09504 | 0.05400 | 0.53217 | 0.53872 | 1.00000 |

### Final similarity\_matrix\_with\_linear\_similarity\_luk\_tnorm

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **SIMILARITIES** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.0000 | 0.0000 | 0.0300 | 0.0000 | 0.0500 |
| **Inst 2** | 0.0000 | 1.0000 | 0.0000 | 0.0000 | 0.0000 |
| **Inst 3** | 0.0300 | 0.0000 | 1.0000 | 0.0700 | 0.4400 |
| **Inst 4** | 0.0000 | 0.0000 | 0.0700 | 1.0000 | 0.4500 |
| **Inst 5** | 0.0500 | 0.0000 | 0.4400 | 0.4500 | 1.0000 |

## Gaussian Similarity

### Element-wise calculations of (v1 - v2)2, part of the Gaussian similarity

The matrix is symmetric.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **SIM** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | [0.0, 0.0, 0.0] | [0.0100, 0.2116, 0.2025] | [0.3969, 0.0196, 0.0400] | [0.6561, 0.0256, 0.0625] | [0.8100, 0.0016, 0.0001] |
| **Inst 2** | [0.0100, 0.2116, 0.2025] | [0.0, 0.0, 0.0] | [0.2809, 0.3600, 0.4225] | [0.5041, 0.0900, 0.0400] | [0.6400, 0.2500, 0.2116] |
| **Inst 3** | [0.3969, 0.0196, 0.0400] | [0.2809, 0.3600, 0.4225] | [0.0, 0.0, 0.0] | [0.0324, 0.0900, 0.2025] | [0.0729, 0.0100, 0.0361] |
| **Inst 4** | [0.6561, 0.0256, 0.0625] | [0.5041, 0.0900, 0.0400] | [0.0324, 0.0900, 0.2025] | [0.0, 0.0, 0.0] | [0.0081, 0.0400, 0.0676] |
| **Inst 5** | [0.81, 0.0016, 0.0001] | [0.6400, 0.2500, 0.2116] | [0.0729, 0.0100, 0.0361] | [0.0081, 0.0400, 0.0676] | [0.0, 0.0, 0.0] |

### Gaussian similarity elementwise (sigma = 0.67)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **SIM** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | [1.0000, 1.0000, 1.0000] | [0.9889, 0.7900, 0.7981] | [0.6427, 0.9784, 0.9564] | [0.4815, 0.9719, 0.9328] | [0.4057, 0.9982, 0.9999] |
| **Inst 2** | [0.9889, 0.7900, 0.7981] | [1.0000, 1.0000, 1.0000] | [0.7313, 0.6697, 0.6246] | [0.5704, 0.9046, 0.9564] | [0.4902, 0.7569, 0.7900] |
| **Inst 3** | [0.6427, 0.9784, 0.9564] | [0.7313, 0.6697, 0.6246] | [1.0000, 1.0000, 1.0000] | [0.9646, 0.9046, 0.7981] | [0.9220, 0.9889, 0.9606] |
| **Inst 4** | [0.4815, 0.9719, 0.9328] | [0.5704, 0.9046, 0.9564] | [0.9646, 0.9046, 0.7981] | [1.0000, 1.0000, 1.0000] | [0.9910, 0.9564, 0.9275] |
| **Inst 5** | [0.4057, 0.9982, 0.9999] | [0.4902, 0.7569, 0.7900] | [0.9220, 0.9889, 0.9606] | [0.9910, 0.9564, 0.9275] | [1.0000, 1.0000, 1.0000] |

### Gaussian similarity with product tnorm

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **SIM** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.0000 | 0.6235 | 0.6014 | 0.4365 | 0.4049 |
| **Inst 2** | 0.6235 | 1.0 | 0.3059 | 0.4935 | 0.2932 |
| **Inst 3** | 0.6014 | 0.3059 | 1.0 | 0.6964 | 0.8759 |
| **Inst 4** | 0.4365 | 0.4935 | 0.6964 | 1.0 | 0.8791 |
| **Inst 5** | 0.4049 | 0.2932 | 0.8759 | 0.8791 | 1.0 |

### Gaussian similarity with minimum tnorm

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **SIM** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.0 | 0.7900 | 0.6427 | 0.4815 | 0.4057 |
| **Inst 2** | 0.7900 | 1.0 | 0.6246 | 0.5704 | 0.4902 |
| **Inst 3** | 0.6427 | 0.6246 | 1.0 | 0.7981 | 0.9220 |
| **Inst 4** | 0.4815 | 0.5704 | 0.7981 | 1.0 | 0.9275 |
| **Inst 5** | 0.4057 | 0.4902 | 0.9220 | 0.9275 | 1.0 |

### Gaussian similarity with luk tnorm

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **SIM** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | [1.0000 | 0.5770 | 0.5775 | 0.3862 | 0.4038] |
| **Inst 2** | [0.5770 | 1.0000 | 0.0256 | 0.4314 | 0.0371] |
| **Inst 3** | [0.5775 | 0.0256 | 1.0000 | 0.6673 | 0.8715] |
| **Inst 4** | [0.3862 | 0.4314 | 0.6673 | 1.0000 | 0.8749] |
| **Inst 5** | [0.4038 | 0.0371 | 0.8715 | 0.8749 | 1.0000] |

# Lower and Upper approximations (ITFRS and OWAFRS)

## y (labels)

|  |  |
| --- | --- |
| **Inst 1** | 1.0 |
| **Inst 2** | 1.0 |
| **Inst 3** | 0.0 |
| **Inst 4** | 1.0 |
| **Inst 5** | 0.0 |

## similarity\_matrix (a)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **SIMILARITIES** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.00 | 0.54 | 0.37 | 0.19 | 0.10 |
| **Inst 2** | 0.54 | 1.00 | 0.35 | 0.29 | 0.20 |
| **Inst 3** | 0.37 | 0.35 | 1.00 | 0.55 | 0.73 |
| **Inst 4** | 0.19 | 0.29 | 0.55 | 1.00 | 0.74 |
| **Inst 5** | 0.10 | 0.20 | 0.73 | 0.74 | 1.00 |

## label\_masks (b or A(y))

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Label masks** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.00 | 1.0 | 0.0 | 1.0 | 0.0 |
| **Inst 2** | 1.0 | 1.0 | 0.0 | 1.0 | 0.0 |
| **Inst 3** | 0.0 | 0.0 | 1.0 | 0.0 | 1.0 |
| **Inst 4** | 1.0 | 1.0 | 0.0 | 1.0 | 0.0 |
| **Inst 5** | 0.0 | 0.0 | 1.0 | 0.0 | 1.0 |

## A(y) / sim = (b/a)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Label masks** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.00 | 1.851852 | 0.00 | 5.263158 | 0.00 |
| **Inst 2** | 1.851852 | 1.00 | 0.00 | 3.448276 | 0.00 |
| **Inst 3** | 0.00 | 0.00 | 1.00 | 0.00 | 1.369863 |
| **Inst 4** | 5.263158 | 3.448276 | 0.00 | 1.00 | 0.00 |
| **Inst 5** | 0.00 | 0.00 | 1.369863 | 0.00 | 1.00 |

## max(1-sim, A(y)) = max(1-a, b)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Label masks** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.00 | 1.00 | 0.63 | 1.00 | 0.90 |
| **Inst 2** | 1.00 | 1.00 | 0.65 | 1.00 | 0.80 |
| **Inst 3** | 0.63 | 0.65 | 1.00 | 0.45 | 1.00 |
| **Inst 4** | 1.00 | 1.00 | 0.45 | 1.00 | 0.26 |
| **Inst 5** | 0.90 | 0.80 | 1.00 | 0.26 | 1.00 |

## Interim 1 – sim = (1-a)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 0.00 | 0.46 | 0.63 | 0.81 | 0.90 |
| **Inst 2** | 0.46 | 0.00 | 0.65 | 0.71 | 0.80 |
| **Inst 3** | 0.63 | 0.65 | 0.00 | 0.45 | 0.27 |
| **Inst 4** | 0.81 | 0.71 | 0.45 | 0.00 | 0.26 |
| **Inst 5** | 0.90 | 0.80 | 0.27 | 0.26 | 0.00 |

## Interim Sim \* A(y) = a\*b

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.0 | 0.54 | 0.0 | 0.19 | 0.0 |
| **Inst 2** | 0.54 | 1.0 | 0.0 | 0.29 | 0.0 |
| **Inst 3** | 0.0 | 0.0 | 1.0 | 0.0 | 0.73 |
| **Inst 4** | 0.19 | 0.29 | 0.0 | 1.0 | 0.0 |
| **Inst 5** | 0.0 | 0.0 | 0.73 | 0.0 | 1.0 |

## Interim 1-sim + (sim \* A(y)) = 1- a + (ab)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.0 | 1.0 | 0.63 | 1.0 | 0.9 |
| **Inst 2** | 1.0 | 1.0 | 0.65 | 1.0 | 0.80 |
| **Inst 3** | 0.63 | 0.65 | 1.00 | 0.45 | 1.00 |
| **Inst 4** | 1.00 | 1.00 | 0.45 | 1.00 | 0.26 |
| **Inst 5** | 0.90 | 0.80 | 1.00 | 0.26 | 1.00 |

## Interim 1 - sim + A(y) = 1 – a + b

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.0 | 1.46 | 0.63 | 1.81 | 0.9 |
| **Inst 2** | 1.46 | 1.0 | 0.65 | 1.71 | 0.8 |
| **Inst 3** | 0.63 | 0.65 | 1.0 | 0.45 | 1.27 |
| **Inst 4** | 1.81 | 1.71 | 0.45 | 1.0 | 0.26 |
| **Inst 5** | 0.9 | 0.8 | 1.27 | 0.26 | 1.0 |

## Interim [A(y)] ^ sim = b^a

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Label masks** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | c | 1.0 | 0.0 | 1.0 | 0.0 |
| **Inst 2** | 1.0 | 1.0 | 0.0 | 1.0 | 0.0 |
| **Inst 3** | 0.0 | 0.0 | 1.0 | 0.0 | 1.0 |
| **Inst 4** | 1.0 | 1.0 | 0.0 | 1.0 | 0.0 |
| **Inst 5** | 0.0 | 0.0 | 1.0 | 0.0 | 1.0 |

## Luk Implicator results

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.0 | 1.0 | 0.63 | 1.0 | 0.9 |
| **Inst 2** | 1.0 | 1.0 | 0.65 | 1.0 | 0.8 |
| **Inst 3** | 0.63 | 0.65 | 1.0 | 0.45 | 1.0 |
| **Inst 4** | 1.0 | 1.0 | 0.45 | 1.0 | 0.26 |
| **Inst 5** | 0.9 | 0.8 | 1.0 | 0.26 | 1.0 |

## KD Implicator results

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.0 | 1.0 | 0.63 | 1.0 | 0.9 |
| **Inst 2** | 1.0 | 1.0 | 0.65 | 1.0 | 0.8 |
| **Inst 3** | 0.63 | 0.65 | 1.0 | 0.45 | 1.0 |
| **Inst 4** | 1.0 | 1.0 | 0.45 | 1.0 | 0.26 |
| **Inst 5** | 0.9 | 0.8 | 1.0 | 0.26 | 1.0 |

## Reichenbach Implicator results

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.0 | 1.0 | 0.63 | 1.0 | 0.9 |
| **Inst 2** | 1.0 | 1.0 | 0.65 | 1.0 | 0.8 |
| **Inst 3** | 0.63 | 0.65 | 1.0 | 0.45 | 1.0 |
| **Inst 4** | 1.0 | 1.0 | 0.45 | 1.0 | 0.26 |
| **Inst 5** | 0.9 | 0.8 | 1.0 | 0.26 | 1.0 |

## Goedel Implicator results

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.0 | 1.0 | 0.0 | 1.0 | 0.0 |
| **Inst 2** | 1.0 | 1.0 | 0.0 | 1.0 | 0.0 |
| **Inst 3** | 0.0 | 0.0 | 1.0 | 0.0 | 1.0 |
| **Inst 4** | 1.0 | 1.0 | 0.0 | 1.0 | 0.0 |
| **Inst 5** | 0.0 | 0.0 | 1.0 | 0.0 | 1.0 |

## ~~Gaines Implicator results~~

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **~~Inst 1~~** | **~~Inst 2~~** | **~~Inst 3~~** | **~~Inst 4~~** | **~~Inst 5~~** |
| **~~Inst 1~~** | ~~1.0~~ | ~~1.0~~ | ~~0.0~~ | ~~1.0~~ | ~~0.0~~ |
| **~~Inst 2~~** | ~~1.0~~ | ~~1.0~~ | ~~0.0~~ | ~~1.0~~ | ~~0.0~~ |
| **~~Inst 3~~** | ~~0.0~~ | ~~0.0~~ | ~~1.0~~ | ~~0.0~~ | ~~1.0~~ |
| **~~Inst 4~~** | ~~1.0~~ | ~~1.0~~ | ~~0.0~~ | ~~1.0~~ | ~~0.0~~ |
| **~~Inst 5~~** | ~~0.0~~ | ~~0.0~~ | ~~1.0~~ | ~~0.0~~ | ~~1.0~~ |

## Goguen Implicator results

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.00 | 1.00 | 0.00 | 1.00 | 0.00 |
| **Inst 2** | 1.00 | 1.00 | 0.00 | 1.00 | 0.0 |
| **Inst 3** | 0.00 | 0.00 | 1.00 | 0.00 | 1.00 |
| **Inst 4** | 1.00 | 1.00 | 0.00 | 1.00 | 0.00 |
| **Inst 5** | 0.00 | 0.00 | 1.00 | 0.00 | 1.00 |

## Rescher Implicator results

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.00 | 1.00 | 0.00 | 1.00 | 0.00 |
| **Inst 2** | 1.00 | 1.00 | 0.00 | 1.00 | 0.0 |
| **Inst 3** | 0.00 | 0.00 | 1.00 | 0.00 | 1.00 |
| **Inst 4** | 1.00 | 1.00 | 0.00 | 1.00 | 0.00 |
| **Inst 5** | 0.00 | 0.00 | 1.00 | 0.00 | 1.00 |

## Weber Implicator results

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| **Inst 2** | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| **Inst 3** | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| **Inst 4** | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| **Inst 5** | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |

## Fodor Implicator results

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.00 | 1.00 | 0.63 | 1.00 | 0.90 |
| **Inst 2** | 1.00 | 1.00 | 0.65 | 1.00 | 0.80 |
| **Inst 3** | 0.63 | 0.65 | 1.00 | 0.45 | 1.00 |
| **Inst 4** | 1.00 | 1.00 | 0.45 | 1.00 | 0.26 |
| **Inst 5** | 0.90 | 0.80 | 1.00 | 0.26 | 1.00 |

## Yager Implicator results

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Label masks** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.00 | 1.0 | 0.0 | 1.0 | 0.0 |
| **Inst 2** | 1.0 | 1.0 | 0.0 | 1.0 | 0.0 |
| **Inst 3** | 0.0 | 0.0 | 1.0 | 0.0 | 1.0 |
| **Inst 4** | 1.0 | 1.0 | 0.0 | 1.0 | 0.0 |
| **Inst 5** | 0.0 | 0.0 | 1.0 | 0.0 | 1.0 |

NOTES:

1. KD, luk and Reichenbach results are the same for this example.
2. Gougen. Goedel. Rescher, yager results are the same for this example.
3. Since for the calculations of lower approximation, we calculate *Inf* which is basically a minimum, to exclude the same instance from calculations we don’t need anything because the diagonal is set to 1.0 which is ignored by min operator. To be sure all is correct, inside code, we set main diagonal to 1.0
4. Since for the calculations of upper approximation, we calculate sup which is basically a maximum, to exclude the same instance from calculations we need to set the main diagonal to 0.0 which is ignored by max operator. Otherwise all upper approxamations will be 1.0.

## Lower approximation with all Implicators

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Richenbach** | 0.63 | 0.65 | 0.45 | 0.26 | 0.26 |
| **KD** | 0.63 | 0.65 | 0.45 | 0.26 | 0.26 |
| **Luk** | 0.63 | 0.65 | 0.45 | 0.26 | 0.26 |
| **Goedel** | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| **~~Gaines~~** | ~~0.0~~ | ~~0.0~~ | ~~0.0~~ | ~~0.0~~ | ~~0.0~~ |
| **Goguen** | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| **Rescher** | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| **Weber** | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| **Fodor** | 0.63 | 0.65 | 0.45 | 0.26 | 0.26 |
| **Yager** | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |

## Interim [sim+A(y) ] = (a+b)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **t-norm** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 2.00 | 1.54 | 0.37 | 1.19 | 0.10 |
| **Inst 2** | 1.54 | 2.00 | 0.35 | 1.29 | 0.20 |
| **Inst 3** | 0.37 | 0.35 | 2.00 | 0.55 | 1.73 |
| **Inst 4** | 1.19 | 1.29 | 0.55 | 2.00 | 0.74 |
| **Inst 5** | 0.10 | 0.20 | 1.73 | 0.74 | 2.00 |

## Interim [sim+A(y) -1.00] = (a+b-1)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **t-norm** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.00 | 0.54 | -0.63 | 0.19 | -0.90 |
| **Inst 2** | 0.54 | 1.00 | -0.65 | 0.29 | -0.80 |
| **Inst 3** | -0.63 | -0.65 | 1.00 | -0.45 | 0.73 |
| **Inst 4** | 0.19 | 0.29 | -0.45 | 1.00 | -0.26 |
| **Inst 5** | -0.90 | -0.80 | 0.73 | -0.26 | 1.00 |

## Interim [sim \* A(y) ] = (a\*b)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **t-norm** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.00 | 0.54 | 0.00 | 0.19 | 0.00 |
| **Inst 2** | 0.54 | 1.00 | 0.00 | 0.29 | 0.00 |
| **Inst 3** | 0.00 | 0.00 | 1.00 | 0.00 | 0.73 |
| **Inst 4** | 0.19 | 0.29 | 0.00 | 1.00 | 0.00 |
| **Inst 5** | 0.00 | 0.00 | 0.73 | 0.00 | 1.00 |

## Interim sim + A(y) - [sim \* A(y) ] = a+b-(a\*b)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **t-norm** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.00 | 1.00 | 0.37 | 1.00 | 0.10 |
| **Inst 2** | 1.00 | 1.00 | 0.35 | 1.00 | 0.20 |
| **Inst 3** | 0.37 | 0.35 | 1.00 | 0.55 | 1.00 |
| **Inst 4** | 1.00 | 1.00 | 0.55 | 1.00 | 0.74 |
| **Inst 5** | 0.10 | 0.20 | 1.00 | 0.74 | 1.00 |

## Interim 2- [sim + A(y) - [sim \* A(y) ]] = 2-(a+b-(a\*b))

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **t-norm** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.00 | 1.00 | 1.63 | 1.00 | 1.90 |
| **Inst 2** | 1.00 | 1.00 | 1.65 | 1.00 | 1.80 |
| **Inst 3** | 1.63 | 1.65 | 1.00 | 1.45 | 1.00 |
| **Inst 4** | 1.00 | 1.00 | 1.45 | 1.00 | 1.26 |
| **Inst 5** | 1.90 | 1.80 | 1.00 | 1.26 | 1.00 |

## Interim (ab)/(a+b-(a\*b))

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **t-norm** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.0 | 0.54 | 0.0 | 0.19 | 0.0 |
| **Inst 2** | 0.54 | 1.0 | 0.0 | 0.29 | 0.0 |
| **Inst 3** | 0.0 | 0.0 | 1.0 | 0.0 | 0.73 |
| **Inst 4** | 0.19 | 0.29 | 0.0 | 1.0 | 0.0 |
| **Inst 5** | 0.0 | 0.0 | 0.73 | 0.0 | 1.0 |

## Interim ((1-a)^p + (1-b)^p)^(1/p) for p=0.83

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **t-norm** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 0.000000 | 0.460000 | 1.870327 | 0.810000 | 2.189314 |
| **Inst 2** | 0.460000 | 0.000000 | 1.894353 | 0.710000 | 2.072393 |
| **Inst 3** | 1.870327 | 1.894353 | 0.000000 | 1.650106 | 0.270000 |
| **Inst 4** | 0.810000 | 0.710000 | 1.650106 | 0.000000 | 1.406053 |
| **Inst 5** | 2.189314 | 2.072393 | 0.270000 | 1.406053 | 0.000000 |

## Interim min[1,((1-a)^p + (1-b)^p)^(1/p)] for p=0.83

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **t-norm** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 0.000000 | 0.460000 | 1.00 | 0.810000 | 1.00 |
| **Inst 2** | 0.460000 | 0.000000 | 1.00 | 0.710000 | 1.00 |
| **Inst 3** | 1.0 | 1.00 | 0.000000 | 1.00 | 0.270000 |
| **Inst 4** | 0.810000 | 0.710000 | 1.00 | 0.000000 | 1.00 |
| **Inst 5** | 1.00 | 1.00 | 0.270000 | 1.00 | 0.000000 |

## yager for p=0.83

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **t-norm** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 0.000000 | 0.54 | 0.00 | 0.19 | 0.00 |
| **Inst 2** | 0.54 | 0.000000 | 0.00 | 0.29 | 0.00 |
| **Inst 3** | 0.0 | 0.00 | 0.000000 | 0.00 | 0.73 |
| **Inst 4** | 0.19 | 0.29 | 0.00 | 0.000000 | 0.00 |
| **Inst 5** | 0.00 | 0.00 | 0.73 | 0.00 | 0.000000 |

## Min and product t-norm(similarity\_matrix, label\_masks)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **t-norm** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 0.0 | 0.54 | 0.0 | 0.19 | 0.0 |
| **Inst 2** | 0.54 | 0.0 | 0.0 | 0.29 | 0.0 |
| **Inst 3** | 0.0 | 0.0 | 0.0 | 0.0 | 0.73 |
| **Inst 4** | 0.19 | 0.29 | 0.0 | 0.0 | 0.0 |
| **Inst 5** | 0.0 | 0.0 | 0.73 | 0.0 | 0.0 |

**NOTE: min and product tnorm give the same results**

**Main diagonal is set to 0.0 so that sup operator ignores the information of the same instance**

## Einstein t-norm

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **t-norm** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 0.00 | 0.54 | 0.00 | 0.19 | 0.00 |
| **Inst 2** | 0.54 | 0.00 | 0.00 | 0.29 | 0.00 |
| **Inst 3** | 0.00 | 0.00 | 0.00 | 0.00 | 0.73 |
| **Inst 4** | 0.19 | 0.29 | 0.00 | 0.00 | 0.00 |
| **Inst 5** | 0.00 | 0.00 | 0.73 | 0.00 | 0.00 |

## luk

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **t-norm** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 0.00 | 0.54 | 0.00 | 0.19 | 0.00 |
| **Inst 2** | 0.54 | 0.00 | 0.00 | 0.29 | 0.00 |
| **Inst 3** | 0.00 | 0.00 | 0.00 | 0.00 | 0.73 |
| **Inst 4** | 0.19 | 0.29 | 0.00 | 0.00 | 0.00 |
| **Inst 5** | 0.00 | 0.00 | 0.73 | 0.00 | 0.00 |

## Drastic

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **t-norm** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 0.00 | 0.54 | 0.00 | 0.19 | 0.00 |
| **Inst 2** | 0.54 | 0.00 | 0.00 | 0.29 | 0.00 |
| **Inst 3** | 0.00 | 0.00 | 0.00 | 0.00 | 0.73 |
| **Inst 4** | 0.19 | 0.29 | 0.00 | 0.00 | 0.00 |
| **Inst 5** | 0.00 | 0.00 | 0.73 | 0.00 | 0.00 |

## Nilpotent

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **t-norm** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 0.00 | 0.54 | 0.00 | 0.19 | 0.00 |
| **Inst 2** | 0.54 | 0.00 | 0.00 | 0.29 | 0.00 |
| **Inst 3** | 0.00 | 0.00 | 0.00 | 0.00 | 0.73 |
| **Inst 4** | 0.19 | 0.29 | 0.00 | 0.00 | 0.00 |
| **Inst 5** | 0.00 | 0.00 | 0.73 | 0.00 | 0.00 |

## Hamacher

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **t-norm** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 0.00 | 0.54 | 0.0 | 0.19 | 0.0 |
| **Inst 2** | 0.54 | 0.00 | 0.0 | 0.29 | 0.0 |
| **Inst 3** | 0.0 | 0.0 | 0.00 | 0.0 | 0.73 |
| **Inst 4** | 0.19 | 0.29 | 0.0 | 0.00 | 0.0 |
| **Inst 5** | 0.0 | 0.0 | 0.73 | 0.0 | 0.00 |

## Upper approximations

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Min t-norm** | 0.54 | 0.54 | 0.73 | 0.29 | 0.73 |
| **Prod t-norm** | 0.54 | 0.54 | 0.73 | 0.29 | 0.73 |
| **Einstein** | 0.54 | 0.54 | 0.73 | 0.29 | 0.73 |
| **luk** | 0.54 | 0.54 | 0.73 | 0.29 | 0.73 |
| **Drastic** | 0.54 | 0.54 | 0.73 | 0.29 | 0.73 |
| **Nilpotent** | 0.54 | 0.54 | 0.73 | 0.29 | 0.73 |
| **Hamacher** | 0.54 | 0.54 | 0.73 | 0.29 | 0.73 |
| **yager** | 0.54 | 0.54 | 0.73 | 0.29 | 0.73 |

# OWAFRS lower approximation

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| lower\_approximation\_weights | 0.1 | 0.2 | 0.3 | 0.4 |

## KD Implicator results with making main diagonal 0.0

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 0.0 | 1.0 | 0.63 | 1.0 | 0.9 |
| **Inst 2** | 1.0 | 0.0 | 0.65 | 1.0 | 0.8 |
| **Inst 3** | 0.63 | 0.65 | 0.0 | 0.45 | 1.0 |
| **Inst 4** | 1.0 | 1.0 | 0.45 | 0.0 | 0.26 |
| **Inst 5** | 0.9 | 0.8 | 1.0 | 0.26 | 0.0 |

## Descending sorted KD Implicator results after making main diagonal 0.0

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.0 | 1.0 | 0.9 | 0.63 | 0.0 |
| **Inst 2** | 1.0 | 1.0 | 0.8 | 0.65 | 0.0 |
| **Inst 3** | 1.0 | 0.65 | 0.63 | 0.45 | 0.0 |
| **Inst 4** | 1.0 | 1.0 | 0.45 | 0.26 | 0.0 |
| **Inst 5** | 1.0 | 0.9 | 0.8 | 0.26 | 0.0 |

## OWAinf calculations for KD Implicator

|  |
| --- |
| 0.1+0.2+0.27+0.252=0.822 |
| 0.1+0.2+0.24+0.26=0.8 |
| 0.1+0.13+0.189+0.18=0.599 |
| 0.1+0.2+0.135+0.104=0.539 |
| 0.1+0.18+0.24+0.104=0.624 |

|  |
| --- |
| 0.1 |
| 0.2 |
| 0.3 |
| 0.4 |

x

=

|  |  |  |  |
| --- | --- | --- | --- |
| 1.0 | 1.0 | 0.9 | 0.63 |
| 1.0 | 1.0 | 0.8 | 0.65 |
| 1.0 | 0.65 | 0.63 | 0.45 |
| 1.0 | 1.0 | 0.45 | 0.26 |
| 1.0 | 0.9 | 0.8 | 0.26 |

The results of Reichenbach, luk and KD and fodor are the same

## Goedel Implicator results with making main diagonal 0.0

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 0.0 | 1.0 | 0.0 | 1.0 | 0.0 |
| **Inst 2** | 1.0 | 0.0 | 0.0 | 1.0 | 0.0 |
| **Inst 3** | 0.0 | 0.0 | 0.0 | 0.0 | 1.0 |
| **Inst 4** | 1.0 | 1.0 | 0.0 | 0.0 | 0.0 |
| **Inst 5** | 0.0 | 0.0 | 1.0 | 0.0 | 0.0 |

## Descending sorted Goedel Implicator results after making main diagonal 0.0

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.0 | 1.0 | 0.0 | 0.0 | 0.0 |
| **Inst 2** | 1.0 | 1.0 | 0.0 | 0.0 | 0.0 |
| **Inst 3** | 1.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| **Inst 4** | 1.0 | 1.0 | 0.0 | 0.0 | 0.0 |
| **Inst 5** | 1.0 | 0.0 | 0.0 | 0.0 | 0.0 |

## OWAinf calculations for Goedel Implicator

|  |
| --- |
| 0.1+0.2=0.3 |
| 0.1+0.2=0.3 |
| 0.1=0.1 |
| 0.1+0.2=0.3 |
| 0.1=0.1 |

|  |
| --- |
| 0.1 |
| 0.2 |
| 0.3 |
| 0.4 |

x

=

|  |  |  |  |
| --- | --- | --- | --- |
| 1.0 | 1.0 | 0.0 | 0.0 |
| 1.0 | 1.0 | 0.0 | 0.0 |
| 1.0 | 0.0 | 0.0 | 0.0 |
| 1.0 | 1.0 | 0.0 | 0.0 |
| 1.0 | 0.0 | 0.0 | 0.0 |

The results of goedel, Goguen, Rescher, yager and ~~gaines~~ implicators are the same.

## Weber Implicator results after making main diagonal 0.0

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 0.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| **Inst 2** | 1.00 | 0.00 | 1.00 | 1.00 | 1.00 |
| **Inst 3** | 1.00 | 1.00 | 0.00 | 1.00 | 1.00 |
| **Inst 4** | 1.00 | 1.00 | 1.00 | 0.00 | 1.00 |
| **Inst 5** | 1.00 | 1.00 | 1.00 | 1.00 | 0.00 |

## OWAinf calculations for weber Implicator

|  |
| --- |
| 0.1+0.2+0.3+0.4=1.0 |
| 0.1+0.2+0.3+0.4=1.0 |
| 0.1+0.2+0.3+0.4=1.0 |
| 0.1+0.2+0.3+0.4=1.0 |
| 0.1+0.2+0.3+0.4=1.0 |

x

|  |
| --- |
| 0.1 |
| 0.2 |
| 0.3 |
| 0.4 |

|  |  |  |  |
| --- | --- | --- | --- |
| 1.0 | 1.0 | 1.0 | 1.0 |
| 1.0 | 1.0 | 1.0 | 1.0 |
| 1.0 | 1.0 | 1.0 | 1.0 |
| 1.0 | 1.0 | 1.0 | 1.0 |
| 1.0 | 1.0 | 1.0 | 1.0 |

## Fodor Implicator results after making main diagonal 0.0

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 0.00 | 1.00 | 0.63 | 1.00 | 0.90 |
| **Inst 2** | 1.00 | 0.00 | 0.65 | 1.00 | 0.80 |
| **Inst 3** | 0.63 | 0.65 | 0.00 | 0.45 | 1.00 |
| **Inst 4** | 1.00 | 1.00 | 0.45 | 0.00 | 0.26 |
| **Inst 5** | 0.90 | 0.80 | 1.00 | 0.26 | 0.00 |

## OWAinf calculations for fodor Implicator

|  |
| --- |
| 0.1+0.2+0.27+0.252=0.822 |
| 0.1+0.2+0.24+0.26=0.80 |
| 0.1+0.13+0.189+0.18=0.599 |
| 0.1+0.2+0.135+0.104=0.539 |
| 0.1+0.18+0.24+0.104=0.624 |

x

|  |
| --- |
| 0.1 |
| 0.2 |
| 0.3 |
| 0.4 |

|  |  |  |  |
| --- | --- | --- | --- |
| 1.0 | 1.0 | 0.90 | 0.63 |
| 1.0 | 1.0 | 0.80 | 0.65 |
| 1.0 | 0.65 | 0.63 | 0.45 |
| 1.0 | 1.0 | 0.45 | 0.26 |
| 1.0 | 0.90 | 0.80 | 0.26 |

# OWAFRS upper approximation

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| upper\_approximation\_weights | 0.4 | 0.3 | 0.2 | 0.1 |

# Min and product t-norm(similarity\_matrix, label\_masks)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **t-norm** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 0.0 | 0.54 | 0.0 | 0.19 | 0.0 |
| **Inst 2** | 0.54 | 0.0 | 0.0 | 0.29 | 0.0 |
| **Inst 3** | 0.0 | 0.0 | 0.0 | 0.0 | 0.73 |
| **Inst 4** | 0.19 | 0.29 | 0.0 | 0.0 | 0.0 |
| **Inst 5** | 0.0 | 0.0 | 0.73 | 0.0 | 0.0 |

## Descending sorted min results after making main diagonal 0.0

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **t-norm** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 0.54 | 0.19 | 0.0 | 0.0 | 0.0 |
| **Inst 2** | 0.54 | 0.29 | 0.0 | 0.0 | 0.0 |
| **Inst 3** | 0.73 | 0.0 | 0.0 | 0.0 | 0.0 |
| **Inst 4** | 0.29 | 0.19 | 0.0 | 0.0 | 0.0 |
| **Inst 5** | 0.73 | 0.0 | 0.0 | 0.0 | 0.0 |

## OWAsup calculations for min tnorm

|  |
| --- |
| 0.273 |
| 0.303 |
| 0.292 |
| 0.173 |
| 0.292 |

|  |
| --- |
| 0.4 |
| 0.3 |
| 0.2 |
| 0.1 |

x

=

|  |  |  |  |
| --- | --- | --- | --- |
| 0.54 | 0.19 | 0.0 | 0.0 |
| 0.54 | 0.29 | 0.0 | 0.0 |
| 0.73 | 0.0 | 0.0 | 0.0 |
| 0.29 | 0.19 | 0.0 | 0.0 |
| 0.73 | 0.0 | 0.0 | 0.0 |

The results of all tnorms are the same.

# Lower and Upper approximations (VQRS)

## y (labels)

|  |  |
| --- | --- |
| **Inst 1** | 1.0 |
| **Inst 2** | 1.0 |
| **Inst 3** | 0.0 |
| **Inst 4** | 1.0 |
| **Inst 5** | 0.0 |

## label\_masks

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Label masks** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.0 | 1.0 | 0.0 | 1.0 | 0.0 |
| **Inst 2** | 1.0 | 1.0 | 0.0 | 1.0 | 0.0 |
| **Inst 3** | 0.0 | 0.0 | 1.0 | 0.0 | 1.0 |
| **Inst 4** | 1.0 | 1.0 | 0.0 | 1.0 | 0.0 |
| **Inst 5** | 0.0 | 0.0 | 1.0 | 0.0 | 1.0 |

## similarity\_matrix after setting 0.0 into main diagonal elements

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **SIMILARITIES** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 0.00 | 0.54 | 0.37 | 0.19 | 0.10 |
| **Inst 2** | 0.54 | 0.00 | 0.35 | 0.29 | 0.20 |
| **Inst 3** | 0.37 | 0.35 | 0.00 | 0.55 | 0.73 |
| **Inst 4** | 0.19 | 0.29 | 0.55 | 0.00 | 0.74 |
| **Inst 5** | 0.10 | 0.20 | 0.73 | 0.74 | 0.00 |

## VQRS nominator step1 (min[R(x,y),A(x)])

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 0.00 | 0.54 | 0.0 | 0.19 | 0. 0 |
| **Inst 2** | 0.54 | 0.00 | 0.0 | 0.29 | 0. 0 |
| **Inst 3** | 0.0 | 0.0 | 0.00 | 0.0 | 0.73 |
| **Inst 4** | 0.19 | 0.29 | 0.0 | 0.00 | 0.0 |
| **Inst 5** | 0.00 | 0.00 | 0.73 | 0.0 | 0.00 |

## Nominator {sum(min[R(x,y),A(x)])}

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 0.73 | 0.83 | 0.73 | 0.48 | 0.73 |

## Denominator (sum [R(x,y)])

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 1.2 | 1.38 | 2.0 | 1.77 | 1.77 |

## Division results (nominator/denominator)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 0.6083333333333333 | 0.60144928 | 0.365 | 0.27118644 | 0.412429379 |

## Upper approximation (alpha =0.2, beta = 1.0) with quad fuzzy quantifier

For 0.608333:

1- 2 \* (0.608333 – 1.0)2 / (1 - 0.2)2 = 1 - 2\*(0.1534027778 / 0.64) =1 - 2 \* 0.2396918403= 0.5206163194

-------------------------------------------------------------------------------------------------------

For 0.60144928:

1- 2 \* (0.60144928- 1.0)2 / (1 - 0.2)2 = 1 - 2\*(0.1588426764/ 0.64) =1 - 2 \* 0.2481916819 =0.5036166362

-------------------------------------------------------------------------------------------------------

For 0.365:

2 \* (0.365- 0.2)2 / (1 - 0.2)2 = 2\*(0.027225/ 0.64) = 2 \* 0.027225=0.085078125

-------------------------------------------------------------------------------------------------------

For 0.27118644:

2 \* (0.27118644- 0.2)2 / (1 - 0.2)2 = 2\*(0.0050675092398736/ 0.64) = 2 \* 0.0079179831873025 =0.015835966374605

-------------------------------------------------------------------------------------------------------

For 0.412429379:

2 \* (0.412429379- 0.2)2 / (1 - 0.2)2 = 2\*(0.045126241062325641/ 0.64) = 2 \* 0.0705097516598838140625 =0.141019503319767628125

-------------------------------------------------------------------------------------------------------

Final results for upper approximation:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 0.5206163194 | 0.5036166362 | 0.085078125 | 0.015835966374605 | 0.141019503319767628125 |

## Lower approximation (alpha =0.1, beta = 0.6) with quad fuzzy quantifier

For 0.608333:

=1.0

-------------------------------------------------------------------------------------------------------

For 0.60144928:

=1.0

-------------------------------------------------------------------------------------------------------

For 0.365:

1 - 2 \* (0.365- 0.6)2 / (0.6 - 0.1)2 = 1 - 2\*(0.055225/ 0.25) = 1 - 2 \* 0.2209 =1 - 0.4418 =0.5582

-------------------------------------------------------------------------------------------------------

For 0.27118644:

2 \* (0.27118644- 0.1)2 / (0.6 - 0.1)2 = 2\*(0.0293047972398736/ 0.25) = 2 \* 0.1172191889594944=0.2344383779189888

-------------------------------------------------------------------------------------------------------

For 0.412429379:

1 - 2 \* (0.412429379- 0.6)2 / (0.6 - 0.1)2 =1 - 2\*(0.035182737862325641/ 0.25) = 1 - 2 \* 0.140730951449302564 =0.718538097101394872

Final results for lower approximation:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 1.0 | 1.0 | 0.5582 | 0.2344383779189888 | 0.718538097101394872 |

## Upper approximation (alpha =0.2, beta = 1.0) with linear fuzzy quantifier

For 0.608333:

(0.608333 – 0.2) / (1 - 0.2)= 0.408333 / 0.8 = 0.51041625

-------------------------------------------------------------------------------------------------------

For 0.60144928:

(0.60144928 – 0.2) / (1 - 0.2)= 0.40144928/0.8 = 0.5018116

-------------------------------------------------------------------------------------------------------

For 0.365:

(0.365 – 0.2) / (1 - 0.2)= 0.165 / 0.8 = 0.20625

-------------------------------------------------------------------------------------------------------

For 0.27118644:

( 0.27118644 – 0.2) / (1 - 0.2)= 0.07118644 / 0.8 = 0.08898305

-------------------------------------------------------------------------------------------------------

For 0.412429379:

( 0.412429379 – 0.2) / (1 - 0.2)= 0.212429379 / 0.8 = 0.2655367238

-------------------------------------------------------------------------------------------------------

Final results for upper approximation:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 0.51041625 | 0.5018116 | 0.20625 | 0.08898305 | 0.2655367238 |

## Lower approximation (alpha =0.1, beta = 0.6) with quad fuzzy quantifier

For 0.608333:

=1.0

-------------------------------------------------------------------------------------------------------

For 0.60144928:

=1.0

-------------------------------------------------------------------------------------------------------

For 0.365:

(0.365 – 0.1) / (0.6 - 0.1)= 0.265 / 0.5 = 0.53

-------------------------------------------------------------------------------------------------------

For 0.27118644:

(0.27118644 – 0.1) / (0.6 - 0.1)= 0.17118644 / 0.5 = 0.34237288

-------------------------------------------------------------------------------------------------------

For 0.412429379:

( 0.412429379 – 0.1) / (0.6 - 0.1)= 0.312429379 / 0.5 = 0.624858758

Final results for lower approximation:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 1.0 | 1.0 | 0.53 | 0.34237288 | 0.624858758 |