Test dataset for unit testing

By Mehran Amiri

For FRsutils library (https://github.com/mehi64/FRsutils)

**Change log**

|  |  |  |
| --- | --- | --- |
| ***Date (dd.mm.YYYY)*** | ***Change*** | ***Person*** |
| 12.05.2025 | Added the document (t-norm, implicator test data) | Mehran Amiri |
| 13.05.2025 | Added similarity test data + LB for ITFRS | Mehran Amiri |
| 14.05.2025 | Added UB for ITFRS | Mehran Amiri |
| 15.05.2025 | Added data for Gaussian similarity | Mehran Amiri |
| 25.05,2025 | Added data for VQRS | Mehran Amiri |
| 08.06.2025 | Drastic product tnorm data | Mehran Amiri |
| 19.06.2025 | Checking tnorms and adding data for yager tnorm for p=5 | Mehran Amiri |
| 21.06.2025 | Add values for implicators goguen, rescher, yager, weber, fodor | Mehran Amiri |
| 23.06.2025 | Add values to ITFRS for implicators (goguen, rescher, yager, weber, fodor) and TNorms () | Mehran Amiri |
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**NOTEs:**

* **When calculating values, calculate them with as many as floating points possible. Then, at the end, you can round it. Otherwise, when testing it will be hard to test correctly.**
* **It is a good idea to have tests with real numbers for A(y). Now, it is just a binary value appropriate for classification tasks but not for regression.**

# Implicators

## Data and outputs

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| a | b | 1-a | ab | 1-a+ab | 1-a+b | max(1-a,b) | b/a | **Gaines** | **Goedel** | **KD** | **reichenbach** | **Luk** | **goguen** | **rescher** | **Yager** | **weber** | **fodor** |
| 2.10 | 4.32 | - | - | - | - | - | - | **-** | - | - | - | - | - | - | - | - | - |
| -0.20 | -0.78 | - | - | - | - | - | - | **-** | - | - | - | - | - | - | - | - | - |
| 0.73 | 0.18 | 0.27 | 0.1314 | 0.4014 | 0.45 | 0.27 | 0.246575 |  | 0.18 | 0.27 | 0.4014 | 0.45 | 0.246575 | 0.00 | 0.285989 | 1.00 | 0.27 |
| 0.18 | 0.73 | 0.82 | 0.1314 | 0.9514 | 1.55 | 0.82 | - |  | 1.0 | 0.82 | 0.9514 | 1.00 | 1.00 | 1.00 | 0.944927 | 1.00 | 1.00 |
| 0.88 | 0.88 | 0.12 | 0.7744 | 0.8914 | 1.00 | 0.88 | - |  | 1.0 | 0.88 | 0.8944 | 1.00 | 1.00 | 1.00 | 0.893603 | 1.00 | 1.00 |
| 0.91 | 0.48 | 0.09 | 0.4368 | 0.5263 | 0.57 | 0.48 | 0.527473 |  | 0.48 | 0.48 | 0.5268 | 0.57 | 0.527473 | 0.00 | 0.512778 | 1.00 | 0.48 |
| 1.00 | 1.00 | 0.00 | 1.00 | 1.00 | 1.00 | 1.00 | - |  | 1.0 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| 0.00 | 0.00 | 1.00 | 0.00 | 1.00 | 1.00 | 1.00 | - |  | 1.0 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| 0.00 | 1.00 | 1.00 | 0.00 | 1.00 | 2.00 | 1.00 | - |  | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |  |  | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 1.00 | 0.65 | 0.00 | 0.65 | 0.65 | 0.65 | 0.65 |  |  | 0.65 | 0.65 | 0.65 | 0.65 | 0.65 | 0.00 | 0.65 | 0.65 | 0.65 |
| 0.65 | 1.00 | 0.35 | 0.65 | 1.00 | 1.35 | 1.00 |  |  | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| 0.55 | 0.00 | 0.45 | 0.00 | 0.45 | 0.45 | 0.45 |  |  | 0.00 | 0.45 | 0.45 | 0.45 | 0.00 | 0.00 | 0.00 | 1.00 | 0.45 |
| 0.00 | 0.55 | 1.00 | 0.00 | 1.00 | 1.55 | 1.00 |  |  | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |

# OWA\_weights data for test

 owa\_infimum\_weights\_linear\_len\_5 = np.array([0.06666667, 0.13333333, 0.2, 0.26666667, 0.33333333])

        owa\_infimum\_weights\_linear\_len\_10 = np.array([0.01818182, 0.03636364, 0.05454545, 0.07272727, 0.09090909, 0.10909091, 0.12727273, 0.14545455, 0.16363636, 0.18181818])

        owa\_suprimum\_weights\_linear\_len\_8 = np.array([0.22222222, 0.19444444, 0.16666667, 0.13888889, 0.11111111, 0.08333333, 0.05555556, 0.02777778])

        owa\_supriimum\_weights\_linear\_len\_13 = np.array([0.14285714, 0.13186813, 0.12087912, 0.10989011, 0.0989011,  0.08791209, 0.07692308, 0.06593407, 0.05494505, 0.04395604, 0.03296703, 0.02197802, 0.01098901])

# Data for testing t-norms in a scalar way

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| a | b | ab | a+b | a+b-1 | 1-a | 1-b | 1+(1-a)(1-b) | a+b-ab | **Min** | **Product** | **Luk** | **Drastic Prod** | **Hamacher prod** | **Einstein** | **Nilpotent min** |
| 2.10 | 4.32 | - | - | - | - | - | - | - | - | - | - | - | - | - |  |
| -0.20 | -0.78 | - | - | - | - | - | - | - | - | - | - | - | - | - |  |
| 0.73 | 0.18 | 0.1314 | 0.91 | -0.09 | 0.27 | 0.82 | 1.2214 | 0.7786 | 0.18 | 0.1314 | 0.00 | 0.00 | 0.168764 | 0.107581 | 0.00 |
| 0.18 | 0.73 | 0.1314 | 0.91 | -0.09 | 0.82 | 0.27 | 1.2214 | 0.7786 | 0.18 | 0.1314 | 0.00 | 0.00 | 0.168764 | 0.107581 | 0.00 |
| 0.88 | 0.88 | 0.7744 | 1.76 | 0.76 | 0.12 | 0.12 | 1.0144 | 0.9856 | 0.88 | 0.7744 | 0.76 | 0.00 | 0.785714 | 0.763407 | 0.88 |
| 0.91 | 0.48 | 0.4368 | 1.39 | 0.39 | 0.09 | 0.52 | 1.0468 | 0.9532 | 0.48 | 0.4368 | 0.39 | 0.00 | 0.458246 | 0.417271 | 0.48 |
| 1.00 | 1.00 | 1.00 | 2.00 | 1.00 | 0.00 | 0.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| 0.00 | 0.00 | 0.00 | 0.00 | -1.00 | 1.00 | 1.00 | 2.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 1.00 | 0.65 | 0.65 | 1.65 | 0.65 | 0.00 | 0.35 | 1.00 | 1.00 | 0.65 | 0.65 | 0.65 | 0.65 | 0.65 | 0.65 | 0.65 |
| 0.37 | 1.00 | 0.37 | 1.37 | 0.37 | 0.63 | 0.00 | 1.00 | 1.00 | 0.37 | 0.37 | 0.37 | 0.37 | 0.37 | 0.37 | 0.37 |

## Yager tnorm results (p=0.835, 1/p =1.1976047904)

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| a | b | ab | 1-a | 1-b | (1-a)p | (1-b)p | (1-a)p +(1-b)p | [(1-a)p +(1-b)p](1/p) | Min{1, [(1-a)p +(1-b)p](1/p)} | **Yager** |
| 2.10 | 4.32 | - | - | - | - | - | - | - | - | - |
| -0.20 | -0.78 | - | - | - | - | - | - | - | - | - |
| 0.73 | 0.18 | 0.1314 | 0.27 | 0.82 | 0.335111 | 0.847295 | 1.182406 | 1.2222095 | 1.00 | 0.00 |
| 0.18 | 0.73 | 0.1314 | 0.82 | 0.27 | 0.847295 | 0.335111 | 1.182406 | 1.2222095 | 1.00 | 0.00 |
| 0.88 | 0.88 | 0.7744 | 0.12 | 0.12 | 0.170261 | 0.170261 | 0.340522 | 0.275229 | 0.275229 | 0.724771 |
| 0.91 | 0.48 | 0.4368 | 0.09 | 0.52 | 0.133904 | 0.579246 | 0.713150 | 0.667066 | 0.667066 | 0.332934 |
| 1.00 | 1.00 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 1.00 |
| 0.00 | 0.00 | 0.00 | 1.00 | 1.00 | 1.00 | 1.00 | 2.00 | 2.293586 | 1.00 | 0.00 |
| 1.00 | 0.65 | 0.65 | 0.00 | 0.35 | 0.00 | 0.416195 | 0.416195 | 0.35 | 0.35 | 0.65 |
| 0.37 | 1.00 | 0.37 | 0.63 | 0.00 | 0.679907 | 0.00 | 0.679907 | 0.63 | 0.63 | 0.37 |

## Yager tnorm results (p=5, 1/p =0.2)

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| a | b | ab | 1-a | 1-b | (1-a)p | (1-b)p | (1-a)p +(1-b)p | [(1-a)p +(1-b)p](1/p) | Min{1, [(1-a)p +(1-b)p](1/p)} | **Yager** |
| 2.10 | 4.32 | - | - | - | - | - | - | - | - | - |
| -0.20 | -0.78 | - | - | - | - | - | - | - | - | - |
| 0.73 | 0.18 | 0.1314 | 0.27 | 0.82 | 0.0014348907 | 0.3707398432 | 0.3721747339 | 0.820633756 | 0.820633756 | 0.179366244 |
| 0.18 | 0.73 | 0.1314 | 0.82 | 0.27 | 0.3707398432 | 0.0014348907 | 0.3721747339 | 0.820633756 | 0.820633756 | 0.179366244 |
| 0.88 | 0.88 | 0.7744 | 0.12 | 0.12 | 0.0000248832 | 0.0000248832 | 0.0000497664 | 0.1378438026 | 0.1378438026 | 0.8621561974 |
| 0.91 | 0.48 | 0.4368 | 0.09 | 0.52 | 0.0000059049 | 0.0380204032 | 0.0380263081 | 0.5200161511 | 0.5200161511 | 0.4799838489 |
| 1.00 | 1.00 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 1.00 |
| 0.00 | 0.00 | 0.00 | 1.00 | 1.00 | 1.00 | 1.00 | 2.00 | 1.148698355 | 1.00 | 0.00 |
| 1.00 | 0.65 | 0.65 | 0.00 | 0.35 | 0.00 | 0.0052521875 | 0.0052521875 | 0.35 | 0.35 | 0.65 |
| 0.37 | 1.00 | 0.37 | 0.63 | 0.00 | 0.0992436543 | 0.00 | 0.0992436543 | 0.63 | 0.63 | 0.37 |

# ~~Data for testing t-norms in a map/vectorized way~~

This is not needed anymore because we test \_\_call\_\_ and then after making sure it is correct, reduce results compared against \_\_call\_\_ results. But we did not delete calculations in this section. They may come in handy some time.

Similarity map

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 1.0 | 0.2673 | 0.25456 | 0.1197 | 0.09504 |
| 0.2673 | 1.0 | 0.0658 | 0.1624 | 0.054 |
| 0.25456 | 0.0658 | 1.0 | 0.3157 | 0.53217 |
| 0.1197 | 0.1624 | 0.3157 | 1.0 | 0.53872 |
| 0.09504 | 0.054 | 0.53217 | 0.53872 | 1.0 |

Mask map

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 1.0 | 1.0 | 0.0 | 1.0 | 0.0 |
| 1.0 | 1.0 | 0.0 | 1.0 | 0.0 |
| 0.0 | 0.0 | 1.0. | 0.0 | 1.0 |
| 1.0 | 1.0 | 0.0 | 1.0 | 0.0 |
| 0.0 | 0.0 | 1.0 | 0.0 | 1.0 |

Output of product tnorm

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 1.0 | 0.2673 | 0.0 | 0.1197 | 0.0 |
| 0.2673 | 1.0 | 0.0 | 0.1624 | 0.0 |
| 0.0 | 0.0 | 1.0 | 0.0 | 0.53217 |
| 0.1197 | 0.1624 | 0.0 | 1.0 | 0.0 |
| 0.0 | 0.0 | 0.53217 | 0.0 | 1.0 |

Output of minimum tnorm

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 1.0 | 0.2673 | 0.0 | 0.1197 | 0.0 |
| 0.2673 | 1.0 | 0.0 | 0.1624 | 0.0 |
| 0.0 | 0.0 | 1.0 | 0.0 | 0.53217 |
| 0.1197 | 0.1624 | 0.0 | 1.0 | 0.0 |
| 0.0 | 0.0 | 0.53217 | 0.0 | 1.0 |

Output of Luk tnorm

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 1.0 | 0.2673 | 0.0 | 0.1197 | 0.0 |
| 0.2673 | 1.0 | 0.0 | 0.1624 | 0.0 |
| 0.0 | 0.0 | 1.0 | 0.0 | 0.53217 |
| 0.1197 | 0.1624 | 0.0 | 1.0 | 0.0 |
| 0.0 | 0.0 | 0.53217 | 0.0 | 1.0 |

# Similarities

## X matrix (each row is a data Instance)

|  |  |  |  |
| --- | --- | --- | --- |
| **Inst 1** | 0.10 | 0.32 | 0.48 |
| **Inst 2** | 0.20 | 0.78 | 0.93 |
| **Inst 3** | 0.73 | 0.18 | 0.28 |
| **Inst 4** | 0.91 | 0.48 | 0.73 |
| **Inst 5** | 1.00 | 0.28 | 0.47 |

## Linear similarity

### Element-wise calculations of |v1 - v2|, part of linear similarity

The matrix is symmetric.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **SIM** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | [0.0, 0.0, 0.0] | [0.1, 0.46, 0.45] | [0.63, 0.14, 0.20] | [0.81, 0.16, 0.25] | [0.90, 0.04, 0.01] |
| **Inst 2** | [0.1, 0.46, 0.45] | [0.0, 0.0, 0.0] | [0.53, 0.60, 0.65] | [0.71, 0.30, 0.20] | [0.80, 0.50, 0.46] |
| **Inst 3** | [0.63, 0.14, 0.20] | [0.53, 0.60, 0.65] | [0.0, 0.0, 0.0] | [0.18, 0.30, 0.45] | [0.27, 0.10, 0.19] |
| **Inst 4** | [0.81, 0.16, 0.25] | [0.71, 0.30, 0.20] | [0.18, 0.30, 0.45] | [0.0, 0.0, 0.0] | [0.09, 0.20, 0.26] |
| **Inst 5** | [0.90, 0.04, 0.01] | [0.80, 0.50, 0.46] | [0.27, 0.10, 0.19] | [0.09, 0.20, 0.26] | [0.0, 0.0, 0.0] |

### Element-wise similarity of Instances (linear similarity)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **SIM** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | [1.0, 1.0,1.0] | [0.9, 0.54, 0.55] | [0.37, 0.86, 0.8 ] | [0.19, 0.84, 0.75] | [0.1 , 0.96, 0.99] |
| **Inst 2** | [0.90 , 0.54, 0.55] | [1.0, 1.0,1.0] | [0.47, 0.4 , 0.35] | [0.29, 0.7 , 0.8 ] | [0.2 , 0.5 , 0.54] |
| **Inst 3** | [0.37, 0.86, 0.80 ] | [0.47, 0.4 , 0.35] | [1.0, 1.0,1.0] | [0.82, 0.7 , 0.55] | [0.73, 0.9 , 0.81] |
| **Inst 4** | [0.19, 0.84, 0.75] | [0.29, 0.70 , 0.8 ] | [0.82, 0.7 , 0.55] | [1.0, 1.0,1.0] | [0.91, 0.8 , 0.74] |
| **Inst 5** | [0.10 , 0.96, 0.99] | [0.20 , 0.5 , 0.54] | [0.73, 0.9 , 0.81] | [0.91, 0.8 , 0.74] | [1.0, 1.0,1.0] |

### Final similarity\_matrix\_with\_linear\_similarity\_minimum\_tnorm

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **SIMILARITIES** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.0 | 0.54 | 0.37 | 0.19 | 0.1 |
| **Inst 2** | 0.54 | 1.0 | 0.35 | 0.29 | 0.2 |
| **Inst 3** | 0.37 | 0.35 | 1.0 | 0.55 | 0.73 |
| **Inst 4** | 0.19 | 0.29 | 0.55 | 1.0 | 0.74 |
| **Inst 5** | 0.10 | 0.20 | 0.73 | 0.74 | 1.0 |

### Final similarity\_matrix\_with\_linear\_similarity\_product\_tnorm

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **SIMILARITIES** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.00000 | 0.26730 | 0.25456 | 0.11970 | 0.09504 |
| **Inst 2** | 0.26730 | 1.00000 | 0.06580 | 0.16240 | 0.05400 |
| **Inst 3** | 0.25456 | 0.06580 | 1.00000 | 0.31570 | 0.53217 |
| **Inst 4** | 0.11970 | 0.16240 | 0.31570 | 1.00000 | 0.53872 |
| **Inst 5** | 0.09504 | 0.05400 | 0.53217 | 0.53872 | 1.00000 |

### Final similarity\_matrix\_with\_linear\_similarity\_luk\_tnorm

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **SIMILARITIES** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.0000 | 0.0000 | 0.0300 | 0.0000 | 0.0500 |
| **Inst 2** | 0.0000 | 1.0000 | 0.0000 | 0.0000 | 0.0000 |
| **Inst 3** | 0.0300 | 0.0000 | 1.0000 | 0.0700 | 0.4400 |
| **Inst 4** | 0.0000 | 0.0000 | 0.0700 | 1.0000 | 0.4500 |
| **Inst 5** | 0.0500 | 0.0000 | 0.4400 | 0.4500 | 1.0000 |

## Gaussian Similarity

### Element-wise calculations of (v1 - v2)2, part of the Gaussian similarity

The matrix is symmetric.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **SIM** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | [0.0, 0.0, 0.0] | [0.0100, 0.2116, 0.2025] | [0.3969, 0.0196, 0.0400] | [0.6561, 0.0256, 0.0625] | [0.8100, 0.0016, 0.0001] |
| **Inst 2** | [0.0100, 0.2116, 0.2025] | [0.0, 0.0, 0.0] | [0.2809, 0.3600, 0.4225] | [0.5041, 0.0900, 0.0400] | [0.6400, 0.2500, 0.2116] |
| **Inst 3** | [0.3969, 0.0196, 0.0400] | [0.2809, 0.3600, 0.4225] | [0.0, 0.0, 0.0] | [0.0324, 0.0900, 0.2025] | [0.0729, 0.0100, 0.0361] |
| **Inst 4** | [0.6561, 0.0256, 0.0625] | [0.5041, 0.0900, 0.0400] | [0.0324, 0.0900, 0.2025] | [0.0, 0.0, 0.0] | [0.0081, 0.0400, 0.0676] |
| **Inst 5** | [0.81, 0.0016, 0.0001] | [0.6400, 0.2500, 0.2116] | [0.0729, 0.0100, 0.0361] | [0.0081, 0.0400, 0.0676] | [0.0, 0.0, 0.0] |

### Gaussian similarity elementwise (sigma = 0.67)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **SIM** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | [1.0000, 1.0000, 1.0000] | [0.9889, 0.7900, 0.7981] | [0.6427, 0.9784, 0.9564] | [0.4815, 0.9719, 0.9328] | [0.4057, 0.9982, 0.9999] |
| **Inst 2** | [0.9889, 0.7900, 0.7981] | [1.0000, 1.0000, 1.0000] | [0.7313, 0.6697, 0.6246] | [0.5704, 0.9046, 0.9564] | [0.4902, 0.7569, 0.7900] |
| **Inst 3** | [0.6427, 0.9784, 0.9564] | [0.7313, 0.6697, 0.6246] | [1.0000, 1.0000, 1.0000] | [0.9646, 0.9046, 0.7981] | [0.9220, 0.9889, 0.9606] |
| **Inst 4** | [0.4815, 0.9719, 0.9328] | [0.5704, 0.9046, 0.9564] | [0.9646, 0.9046, 0.7981] | [1.0000, 1.0000, 1.0000] | [0.9910, 0.9564, 0.9275] |
| **Inst 5** | [0.4057, 0.9982, 0.9999] | [0.4902, 0.7569, 0.7900] | [0.9220, 0.9889, 0.9606] | [0.9910, 0.9564, 0.9275] | [1.0000, 1.0000, 1.0000] |

### Gaussian similarity with product tnorm

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **SIM** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.0000 | 0.6235 | 0.6014 | 0.4365 | 0.4049 |
| **Inst 2** | 0.6235 | 1.0 | 0.3059 | 0.4935 | 0.2932 |
| **Inst 3** | 0.6014 | 0.3059 | 1.0 | 0.6964 | 0.8759 |
| **Inst 4** | 0.4365 | 0.4935 | 0.6964 | 1.0 | 0.8791 |
| **Inst 5** | 0.4049 | 0.2932 | 0.8759 | 0.8791 | 1.0 |

### Gaussian similarity with minimum tnorm

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **SIM** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.0 | 0.7900 | 0.6427 | 0.4815 | 0.4057 |
| **Inst 2** | 0.7900 | 1.0 | 0.6246 | 0.5704 | 0.4902 |
| **Inst 3** | 0.6427 | 0.6246 | 1.0 | 0.7981 | 0.9220 |
| **Inst 4** | 0.4815 | 0.5704 | 0.7981 | 1.0 | 0.9275 |
| **Inst 5** | 0.4057 | 0.4902 | 0.9220 | 0.9275 | 1.0 |

### Gaussian similarity with luk tnorm

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **SIM** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | [1.0000 | 0.5770 | 0.5775 | 0.3862 | 0.4038] |
| **Inst 2** | [0.5770 | 1.0000 | 0.0256 | 0.4314 | 0.0371] |
| **Inst 3** | [0.5775 | 0.0256 | 1.0000 | 0.6673 | 0.8715] |
| **Inst 4** | [0.3862 | 0.4314 | 0.6673 | 1.0000 | 0.8749] |
| **Inst 5** | [0.4038 | 0.0371 | 0.8715 | 0.8749 | 1.0000] |

# Lower and Upper approximations (ITFRS and OWAFRS)

## y (labels)

|  |  |
| --- | --- |
| **Inst 1** | 1.0 |
| **Inst 2** | 1.0 |
| **Inst 3** | 0.0 |
| **Inst 4** | 1.0 |
| **Inst 5** | 0.0 |

## similarity\_matrix (a)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **SIMILARITIES** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.00 | 0.54 | 0.37 | 0.19 | 0.10 |
| **Inst 2** | 0.54 | 1.00 | 0.35 | 0.29 | 0.20 |
| **Inst 3** | 0.37 | 0.35 | 1.00 | 0.55 | 0.73 |
| **Inst 4** | 0.19 | 0.29 | 0.55 | 1.00 | 0.74 |
| **Inst 5** | 0.10 | 0.20 | 0.73 | 0.74 | 1.00 |

## label\_masks (b or A(y))

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Label masks** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.00 | 1.0 | 0.0 | 1.0 | 0.0 |
| **Inst 2** | 1.0 | 1.0 | 0.0 | 1.0 | 0.0 |
| **Inst 3** | 0.0 | 0.0 | 1.0 | 0.0 | 1.0 |
| **Inst 4** | 1.0 | 1.0 | 0.0 | 1.0 | 0.0 |
| **Inst 5** | 0.0 | 0.0 | 1.0 | 0.0 | 1.0 |

## A(y) / sim = (b/a)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Label masks** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.00 | 1.851852 | 0.00 | 5.263158 | 0.00 |
| **Inst 2** | 1.851852 | 1.00 | 0.00 | 3.448276 | 0.00 |
| **Inst 3** | 0.00 | 0.00 | 1.00 | 0.00 | 1.369863 |
| **Inst 4** | 5.263158 | 3.448276 | 0.00 | 1.00 | 0.00 |
| **Inst 5** | 0.00 | 0.00 | 1.369863 | 0.00 | 1.00 |

## max(1-sim, A(y)) = max(1-a, b)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Label masks** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.00 | 1.00 | 0.63 | 1.00 | 0.90 |
| **Inst 2** | 1.00 | 1.00 | 0.65 | 1.00 | 0.80 |
| **Inst 3** | 0.63 | 0.65 | 1.00 | 0.45 | 1.00 |
| **Inst 4** | 1.00 | 1.00 | 0.45 | 1.00 | 0.26 |
| **Inst 5** | 0.90 | 0.80 | 1.00 | 0.26 | 1.00 |

## Interim 1 – sim = (1-a)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 0.00 | 0.46 | 0.63 | 0.81 | 0.90 |
| **Inst 2** | 0.46 | 0.00 | 0.65 | 0.71 | 0.80 |
| **Inst 3** | 0.63 | 0.65 | 0.00 | 0.45 | 0.27 |
| **Inst 4** | 0.81 | 0.71 | 0.45 | 0.00 | 0.26 |
| **Inst 5** | 0.90 | 0.80 | 0.27 | 0.26 | 0.00 |

## Interim Sim \* A(y) = a\*b

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.0 | 0.54 | 0.0 | 0.19 | 0.0 |
| **Inst 2** | 0.54 | 1.0 | 0.0 | 0.29 | 0.0 |
| **Inst 3** | 0.0 | 0.0 | 1.0 | 0.0 | 0.73 |
| **Inst 4** | 0.19 | 0.29 | 0.0 | 1.0 | 0.0 |
| **Inst 5** | 0.0 | 0.0 | 0.73 | 0.0 | 1.0 |

## Interim 1-sim + (sim \* A(y)) = 1- a + (ab)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.0 | 1.0 | 0.63 | 1.0 | 0.9 |
| **Inst 2** | 1.0 | 1.0 | 0.65 | 1.0 | 0.80 |
| **Inst 3** | 0.63 | 0.65 | 1.00 | 0.45 | 1.00 |
| **Inst 4** | 1.00 | 1.00 | 0.45 | 1.00 | 0.26 |
| **Inst 5** | 0.90 | 0.80 | 1.00 | 0.26 | 1.00 |

## Interim 1 - sim + A(y) = 1 – a + b

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.0 | 1.46 | 0.63 | 1.81 | 0.9 |
| **Inst 2** | 1.46 | 1.0 | 0.65 | 1.71 | 0.8 |
| **Inst 3** | 0.63 | 0.65 | 1.0 | 0.45 | 1.27 |
| **Inst 4** | 1.81 | 1.71 | 0.45 | 1.0 | 0.26 |
| **Inst 5** | 0.9 | 0.8 | 1.27 | 0.26 | 1.0 |

## Interim [A(y)] ^ sim = b^a

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Label masks** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | c | 1.0 | 0.0 | 1.0 | 0.0 |
| **Inst 2** | 1.0 | 1.0 | 0.0 | 1.0 | 0.0 |
| **Inst 3** | 0.0 | 0.0 | 1.0 | 0.0 | 1.0 |
| **Inst 4** | 1.0 | 1.0 | 0.0 | 1.0 | 0.0 |
| **Inst 5** | 0.0 | 0.0 | 1.0 | 0.0 | 1.0 |

## Luk Implicator results

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.0 | 1.0 | 0.63 | 1.0 | 0.9 |
| **Inst 2** | 1.0 | 1.0 | 0.65 | 1.0 | 0.8 |
| **Inst 3** | 0.63 | 0.65 | 1.0 | 0.45 | 1.0 |
| **Inst 4** | 1.0 | 1.0 | 0.45 | 1.0 | 0.26 |
| **Inst 5** | 0.9 | 0.8 | 1.0 | 0.26 | 1.0 |

## KD Implicator results

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.0 | 1.0 | 0.63 | 1.0 | 0.9 |
| **Inst 2** | 1.0 | 1.0 | 0.65 | 1.0 | 0.8 |
| **Inst 3** | 0.63 | 0.65 | 1.0 | 0.45 | 1.0 |
| **Inst 4** | 1.0 | 1.0 | 0.45 | 1.0 | 0.26 |
| **Inst 5** | 0.9 | 0.8 | 1.0 | 0.26 | 1.0 |

## Reichenbach Implicator results

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.0 | 1.0 | 0.63 | 1.0 | 0.9 |
| **Inst 2** | 1.0 | 1.0 | 0.65 | 1.0 | 0.8 |
| **Inst 3** | 0.63 | 0.65 | 1.0 | 0.45 | 1.0 |
| **Inst 4** | 1.0 | 1.0 | 0.45 | 1.0 | 0.26 |
| **Inst 5** | 0.9 | 0.8 | 1.0 | 0.26 | 1.0 |

## Goedel Implicator results

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.0 | 1.0 | 0.0 | 1.0 | 0.0 |
| **Inst 2** | 1.0 | 1.0 | 0.0 | 1.0 | 0.0 |
| **Inst 3** | 0.0 | 0.0 | 1.0 | 0.0 | 1.0 |
| **Inst 4** | 1.0 | 1.0 | 0.0 | 1.0 | 0.0 |
| **Inst 5** | 0.0 | 0.0 | 1.0 | 0.0 | 1.0 |

## ~~Gaines Implicator results~~

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **~~Inst 1~~** | **~~Inst 2~~** | **~~Inst 3~~** | **~~Inst 4~~** | **~~Inst 5~~** |
| **~~Inst 1~~** | ~~1.0~~ | ~~1.0~~ | ~~0.0~~ | ~~1.0~~ | ~~0.0~~ |
| **~~Inst 2~~** | ~~1.0~~ | ~~1.0~~ | ~~0.0~~ | ~~1.0~~ | ~~0.0~~ |
| **~~Inst 3~~** | ~~0.0~~ | ~~0.0~~ | ~~1.0~~ | ~~0.0~~ | ~~1.0~~ |
| **~~Inst 4~~** | ~~1.0~~ | ~~1.0~~ | ~~0.0~~ | ~~1.0~~ | ~~0.0~~ |
| **~~Inst 5~~** | ~~0.0~~ | ~~0.0~~ | ~~1.0~~ | ~~0.0~~ | ~~1.0~~ |

## Goguen Implicator results

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.00 | 1.00 | 0.00 | 1.00 | 0.00 |
| **Inst 2** | 1.00 | 1.00 | 0.00 | 1.00 | 0.0 |
| **Inst 3** | 0.00 | 0.00 | 1.00 | 0.00 | 1.00 |
| **Inst 4** | 1.00 | 1.00 | 0.00 | 1.00 | 0.00 |
| **Inst 5** | 0.00 | 0.00 | 1.00 | 0.00 | 1.00 |

## Rescher Implicator results

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.00 | 1.00 | 0.00 | 1.00 | 0.00 |
| **Inst 2** | 1.00 | 1.00 | 0.00 | 1.00 | 0.0 |
| **Inst 3** | 0.00 | 0.00 | 1.00 | 0.00 | 1.00 |
| **Inst 4** | 1.00 | 1.00 | 0.00 | 1.00 | 0.00 |
| **Inst 5** | 0.00 | 0.00 | 1.00 | 0.00 | 1.00 |

## Weber Implicator results

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| **Inst 2** | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| **Inst 3** | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| **Inst 4** | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| **Inst 5** | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |

## Fodor Implicator results

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.00 | 1.00 | 0.63 | 1.00 | 0.90 |
| **Inst 2** | 1.00 | 1.00 | 0.65 | 1.00 | 0.80 |
| **Inst 3** | 0.63 | 0.65 | 1.00 | 0.45 | 1.00 |
| **Inst 4** | 1.00 | 1.00 | 0.45 | 1.00 | 0.26 |
| **Inst 5** | 0.90 | 0.80 | 1.00 | 0.26 | 1.00 |

## Yager Implicator results

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Label masks** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.00 | 1.0 | 0.0 | 1.0 | 0.0 |
| **Inst 2** | 1.0 | 1.0 | 0.0 | 1.0 | 0.0 |
| **Inst 3** | 0.0 | 0.0 | 1.0 | 0.0 | 1.0 |
| **Inst 4** | 1.0 | 1.0 | 0.0 | 1.0 | 0.0 |
| **Inst 5** | 0.0 | 0.0 | 1.0 | 0.0 | 1.0 |

NOTES:

1. KD, luk and Reichenbach results are the same for this example.
2. Gougen. Goedel. Rescher, yager results are the same for this example.
3. Since for the calculations of lower approximation, we calculate *Inf* which is basically a minimum, to exclude the same instance from calculations we don’t need anything because the diagonal is set to 1.0 which is ignored by min operator. To be sure all is correct, inside code, we set main diagonal to 1.0
4. Since for the calculations of upper approximation, we calculate sup which is basically a maximum, to exclude the same instance from calculations we need to set the main diagonal to 0.0 which is ignored by max operator. Otherwise all upper approxamations will be 1.0.

## Lower approximation with all Implicators

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Richenbach** | 0.63 | 0.65 | 0.45 | 0.26 | 0.26 |
| **KD** | 0.63 | 0.65 | 0.45 | 0.26 | 0.26 |
| **Luk** | 0.63 | 0.65 | 0.45 | 0.26 | 0.26 |
| **Goedel** | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| **~~Gaines~~** | ~~0.0~~ | ~~0.0~~ | ~~0.0~~ | ~~0.0~~ | ~~0.0~~ |
| **Goguen** | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| **Rescher** | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| **Weber** | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| **Fodor** | 0.63 | 0.65 | 0.45 | 0.26 | 0.26 |
| **Yager** | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |

## Interim [sim+A(y) ] = (a+b)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **t-norm** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 2.00 | 1.54 | 0.37 | 1.19 | 0.10 |
| **Inst 2** | 1.54 | 2.00 | 0.35 | 1.29 | 0.20 |
| **Inst 3** | 0.37 | 0.35 | 2.00 | 0.55 | 1.73 |
| **Inst 4** | 1.19 | 1.29 | 0.55 | 2.00 | 0.74 |
| **Inst 5** | 0.10 | 0.20 | 1.73 | 0.74 | 2.00 |

## Interim [sim+A(y) -1.00] = (a+b-1)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **t-norm** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.00 | 0.54 | -0.63 | 0.19 | -0.90 |
| **Inst 2** | 0.54 | 1.00 | -0.65 | 0.29 | -0.80 |
| **Inst 3** | -0.63 | -0.65 | 1.00 | -0.45 | 0.73 |
| **Inst 4** | 0.19 | 0.29 | -0.45 | 1.00 | -0.26 |
| **Inst 5** | -0.90 | -0.80 | 0.73 | -0.26 | 1.00 |

## Interim [sim \* A(y) ] = (a\*b)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **t-norm** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.00 | 0.54 | 0.00 | 0.19 | 0.00 |
| **Inst 2** | 0.54 | 1.00 | 0.00 | 0.29 | 0.00 |
| **Inst 3** | 0.00 | 0.00 | 1.00 | 0.00 | 0.73 |
| **Inst 4** | 0.19 | 0.29 | 0.00 | 1.00 | 0.00 |
| **Inst 5** | 0.00 | 0.00 | 0.73 | 0.00 | 1.00 |

## Interim sim + A(y) - [sim \* A(y) ] = a+b-(a\*b)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **t-norm** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.00 | 1.00 | 0.37 | 1.00 | 0.10 |
| **Inst 2** | 1.00 | 1.00 | 0.35 | 1.00 | 0.20 |
| **Inst 3** | 0.37 | 0.35 | 1.00 | 0.55 | 1.00 |
| **Inst 4** | 1.00 | 1.00 | 0.55 | 1.00 | 0.74 |
| **Inst 5** | 0.10 | 0.20 | 1.00 | 0.74 | 1.00 |

## Interim 2- [sim + A(y) - [sim \* A(y) ]] = 2-(a+b-(a\*b))

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **t-norm** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.00 | 1.00 | 1.63 | 1.00 | 1.90 |
| **Inst 2** | 1.00 | 1.00 | 1.65 | 1.00 | 1.80 |
| **Inst 3** | 1.63 | 1.65 | 1.00 | 1.45 | 1.00 |
| **Inst 4** | 1.00 | 1.00 | 1.45 | 1.00 | 1.26 |
| **Inst 5** | 1.90 | 1.80 | 1.00 | 1.26 | 1.00 |

## Interim (ab)/(a+b-(a\*b))

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **t-norm** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.0 | 0.54 | 0.0 | 0.19 | 0.0 |
| **Inst 2** | 0.54 | 1.0 | 0.0 | 0.29 | 0.0 |
| **Inst 3** | 0.0 | 0.0 | 1.0 | 0.0 | 0.73 |
| **Inst 4** | 0.19 | 0.29 | 0.0 | 1.0 | 0.0 |
| **Inst 5** | 0.0 | 0.0 | 0.73 | 0.0 | 1.0 |

## Interim ((1-a)^p + (1-b)^p)^(1/p) for p=0.83

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **t-norm** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 0.000000 | 0.460000 | 1.870327 | 0.810000 | 2.189314 |
| **Inst 2** | 0.460000 | 0.000000 | 1.894353 | 0.710000 | 2.072393 |
| **Inst 3** | 1.870327 | 1.894353 | 0.000000 | 1.650106 | 0.270000 |
| **Inst 4** | 0.810000 | 0.710000 | 1.650106 | 0.000000 | 1.406053 |
| **Inst 5** | 2.189314 | 2.072393 | 0.270000 | 1.406053 | 0.000000 |

## Interim min[1,((1-a)^p + (1-b)^p)^(1/p)] for p=0.83

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **t-norm** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 0.000000 | 0.460000 | 1.00 | 0.810000 | 1.00 |
| **Inst 2** | 0.460000 | 0.000000 | 1.00 | 0.710000 | 1.00 |
| **Inst 3** | 1.0 | 1.00 | 0.000000 | 1.00 | 0.270000 |
| **Inst 4** | 0.810000 | 0.710000 | 1.00 | 0.000000 | 1.00 |
| **Inst 5** | 1.00 | 1.00 | 0.270000 | 1.00 | 0.000000 |

## yager for p=0.83

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **t-norm** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 0.000000 | 0.54 | 0.00 | 0.19 | 0.00 |
| **Inst 2** | 0.54 | 0.000000 | 0.00 | 0.29 | 0.00 |
| **Inst 3** | 0.0 | 0.00 | 0.000000 | 0.00 | 0.73 |
| **Inst 4** | 0.19 | 0.31 | 0.00 | 0.000000 | 0.00 |
| **Inst 5** | 0.00 | 0.00 | 0.73 | 0.00 | 0.000000 |

## Min and product t-norm(similarity\_matrix, label\_masks)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **t-norm** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 0.0 | 0.54 | 0.0 | 0.19 | 0.0 |
| **Inst 2** | 0.54 | 0.0 | 0.0 | 0.29 | 0.0 |
| **Inst 3** | 0.0 | 0.0 | 0.0 | 0.0 | 0.73 |
| **Inst 4** | 0.19 | 0.29 | 0.0 | 0.0 | 0.0 |
| **Inst 5** | 0.0 | 0.0 | 0.73 | 0.0 | 0.0 |

**NOTE: min and product tnorm give the same results**

**Main diagonal is set to 0.0 so that sup operator ignores the information of the same instance**

## Einstein t-norm

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **t-norm** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 0.00 | 0.54 | 0.00 | 0.19 | 0.00 |
| **Inst 2** | 0.54 | 0.00 | 0.00 | 0.29 | 0.00 |
| **Inst 3** | 0.00 | 0.00 | 0.00 | 0.00 | 0.73 |
| **Inst 4** | 0.19 | 0.29 | 0.00 | 0.00 | 0.00 |
| **Inst 5** | 0.00 | 0.00 | 0.73 | 0.00 | 0.00 |

## luk

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **t-norm** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 0.00 | 0.54 | 0.00 | 0.19 | 0.00 |
| **Inst 2** | 0.54 | 0.00 | 0.00 | 0.29 | 0.00 |
| **Inst 3** | 0.00 | 0.00 | 0.00 | 0.00 | 0.73 |
| **Inst 4** | 0.19 | 0.29 | 0.00 | 0.00 | 0.00 |
| **Inst 5** | 0.00 | 0.00 | 0.73 | 0.00 | 0.00 |

## Drastic

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **t-norm** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 0.00 | 0.54 | 0.00 | 0.19 | 0.00 |
| **Inst 2** | 0.54 | 0.00 | 0.00 | 0.29 | 0.00 |
| **Inst 3** | 0.00 | 0.00 | 0.00 | 0.00 | 0.73 |
| **Inst 4** | 0.19 | 0.29 | 0.00 | 0.00 | 0.00 |
| **Inst 5** | 0.00 | 0.00 | 0.73 | 0.00 | 0.00 |

## Nilpotent

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **t-norm** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 0.00 | 0.54 | 0.00 | 0.19 | 0.00 |
| **Inst 2** | 0.54 | 0.00 | 0.00 | 0.29 | 0.00 |
| **Inst 3** | 0.00 | 0.00 | 0.00 | 0.00 | 0.73 |
| **Inst 4** | 0.19 | 0.29 | 0.00 | 0.00 | 0.00 |
| **Inst 5** | 0.00 | 0.00 | 0.73 | 0.00 | 0.00 |

## Hamacher

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **t-norm** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.0 | 0.54 | 0.0 | 0.19 | 0.0 |
| **Inst 2** | 0.54 | 1.0 | 0.0 | 0.29 | 0.0 |
| **Inst 3** | 0.0 | 0.0 | 1.0 | 0.0 | 0.73 |
| **Inst 4** | 0.19 | 0.29 | 0.0 | 1.0 | 0.0 |
| **Inst 5** | 0.0 | 0.0 | 0.73 | 0.0 | 1.0 |

## Upper approximations

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Min t-norm** | 0.54 | 0.54 | 0.73 | 0.29 | 0.73 |
| **Prod t-norm** | 0.54 | 0.54 | 0.73 | 0.29 | 0.73 |
| **Einstein** | 0.54 | 0.54 | 0.73 | 0.29 | 0.73 |
| **luk** | 0.54 | 0.54 | 0.73 | 0.29 | 0.73 |
| **Drastic** | 0.54 | 0.54 | 0.73 | 0.29 | 0.73 |
| **Nilpotent** | 0.54 | 0.54 | 0.73 | 0.29 | 0.73 |
| **Hamacher** | 0.54 | 0.54 | 0.73 | 0.29 | 0.73 |
| **yager** | 0.54 | 0.54 | 0.73 | 0.29 | 0.73 |

# OWAFRS lower approximation

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| lower\_approximation\_weights | 0.1 | 0.2 | 0.3 | 0.4 |

## KD Implicator results with making main diagonal 0.0

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 0.0 | 1.0 | 0.63 | 1.0 | 0.9 |
| **Inst 2** | 1.0 | 0.0 | 0.65 | 1.0 | 0.8 |
| **Inst 3** | 0.63 | 0.65 | 0.0 | 0.45 | 1.0 |
| **Inst 4** | 1.0 | 1.0 | 0.45 | 0.0 | 0.26 |
| **Inst 5** | 0.9 | 0.8 | 1.0 | 0.26 | 0.0 |

## Descending sorted KD Implicator results after making main diagonal 0.0

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.0 | 1.0 | 0.9 | 0.63 | 0.0 |
| **Inst 2** | 1.0 | 1.0 | 0.8 | 0.65 | 0.0 |
| **Inst 3** | 1.0 | 0.65 | 0.63 | 0.45 | 0.0 |
| **Inst 4** | 1.0 | 1.0 | 0.45 | 0.26 | 0.0 |
| **Inst 5** | 1.0 | 0.9 | 0.8 | 0.26 | 0.0 |

## OWAinf calculations for KD Implicator

|  |
| --- |
| 0.1+0.2+0.27+0.252=0.822 |
| 0.1+0.2+0.24+0.26=0.8 |
| 0.1+0.13+0.189+0.18=0.599 |
| 0.1+0.2+0.135+0.104=0.539 |
| 0.1+0.18+0.24+0.104=0.624 |

|  |
| --- |
| 0.1 |
| 0.2 |
| 0.3 |
| 0.4 |

x

=

|  |  |  |  |
| --- | --- | --- | --- |
| 1.0 | 1.0 | 0.9 | 0.63 |
| 1.0 | 1.0 | 0.8 | 0.65 |
| 1.0 | 0.65 | 0.63 | 0.45 |
| 1.0 | 1.0 | 0.45 | 0.26 |
| 1.0 | 0.9 | 0.8 | 0.26 |

The results of Reichenbach, luk and KD are the same

## Goedel Implicator results with making main diagonal 0.0

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 0.0 | 1.0 | 0.0 | 1.0 | 0.0 |
| **Inst 2** | 1.0 | 0.0 | 0.0 | 1.0 | 0.0 |
| **Inst 3** | 0.0 | 0.0 | 0.0 | 0.0 | 1.0 |
| **Inst 4** | 1.0 | 1.0 | 0.0 | 0.0 | 0.0 |
| **Inst 5** | 0.0 | 0.0 | 1.0 | 0.0 | 0.0 |

## Descending sorted Goedel Implicator results after making main diagonal 0.0

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.0 | 1.0 | 0.0 | 0.0 | 0.0 |
| **Inst 2** | 1.0 | 1.0 | 0.0 | 0.0 | 0.0 |
| **Inst 3** | 1.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| **Inst 4** | 1.0 | 1.0 | 0.0 | 0.0 | 0.0 |
| **Inst 5** | 1.0 | 0.0 | 0.0 | 0.0 | 0.0 |

## OWAinf calculations for Goedel Implicator

|  |
| --- |
| 0.1+0.2=0.3 |
| 0.1+0.2=0.3 |
| 0.1=0.1 |
| 0.1+0.2=0.3 |
| 0.1=0.1 |

|  |
| --- |
| 0.1 |
| 0.2 |
| 0.3 |
| 0.4 |

x

=

|  |  |  |  |
| --- | --- | --- | --- |
| 1.0 | 1.0 | 0.0 | 0.0 |
| 1.0 | 1.0 | 0.0 | 0.0 |
| 1.0 | 0.0 | 0.0 | 0.0 |
| 1.0 | 1.0 | 0.0 | 0.0 |
| 1.0 | 0.0 | 0.0 | 0.0 |

The results of goedel and gaines implicators are the same.

# OWAFRS upper approximation

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| upper\_approximation\_weights | 0.4 | 0.3 | 0.2 | 0.1 |

# Min and product t-norm(similarity\_matrix, label\_masks)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **t-norm** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 0.0 | 0.54 | 0.0 | 0.19 | 0.0 |
| **Inst 2** | 0.54 | 0.0 | 0.0 | 0.29 | 0.0 |
| **Inst 3** | 0.0 | 0.0 | 0.0 | 0.0 | 0.73 |
| **Inst 4** | 0.19 | 0.29 | 0.0 | 0.0 | 0.0 |
| **Inst 5** | 0.0 | 0.0 | 0.73 | 0.0 | 0.0 |

## Descending sorted min results after making main diagonal 0.0

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **t-norm** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 0.54 | 0.19 | 0.0 | 0.0 | 0.0 |
| **Inst 2** | 0.54 | 0.29 | 0.0 | 0.0 | 0.0 |
| **Inst 3** | 0.73 | 0.0 | 0.0 | 0.0 | 0.0 |
| **Inst 4** | 0.29 | 0.19 | 0.0 | 0.0 | 0.0 |
| **Inst 5** | 0.73 | 0.0 | 0.0 | 0.0 | 0.0 |

## OWAsup calculations for min tnorm

|  |
| --- |
| 0.273 |
| 0.303 |
| 0.292 |
| 0.173 |
| 0.292 |

|  |
| --- |
| 0.4 |
| 0.3 |
| 0.2 |
| 0.1 |

x

=

|  |  |  |  |
| --- | --- | --- | --- |
| 0.54 | 0.19 | 0.0 | 0.0 |
| 0.54 | 0.29 | 0.0 | 0.0 |
| 0.73 | 0.0 | 0.0 | 0.0 |
| 0.29 | 0.19 | 0.0 | 0.0 |
| 0.73 | 0.0 | 0.0 | 0.0 |

The results of minimum and product tnorm are the same.

# Lower and Upper approximations (VQRS)

## y (labels)

|  |  |
| --- | --- |
| **Inst 1** | 1.0 |
| **Inst 2** | 1.0 |
| **Inst 3** | 0.0 |
| **Inst 4** | 1.0 |
| **Inst 5** | 0.0 |

## label\_masks

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Label masks** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.0 | 1.0 | 0.0 | 1.0 | 0.0 |
| **Inst 2** | 1.0 | 1.0 | 0.0 | 1.0 | 0.0 |
| **Inst 3** | 0.0 | 0.0 | 1.0 | 0.0 | 1.0 |
| **Inst 4** | 1.0 | 1.0 | 0.0 | 1.0 | 0.0 |
| **Inst 5** | 0.0 | 0.0 | 1.0 | 0.0 | 1.0 |

## similarity\_matrix after setting 0.0 into main diagonal elements

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **SIMILARITIES** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 0.00 | 0.54 | 0.37 | 0.19 | 0.10 |
| **Inst 2** | 0.54 | 0.00 | 0.35 | 0.29 | 0.20 |
| **Inst 3** | 0.37 | 0.35 | 0.00 | 0.55 | 0.73 |
| **Inst 4** | 0.19 | 0.29 | 0.55 | 0.00 | 0.74 |
| **Inst 5** | 0.10 | 0.20 | 0.73 | 0.74 | 0.00 |

## VQRS nominator step1 (min[R(x,y),A(x)])

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 0.00 | 0.54 | 0.0 | 0.19 | 0. 0 |
| **Inst 2** | 0.54 | 0.00 | 0.0 | 0.29 | 0. 0 |
| **Inst 3** | 0.0 | 0.0 | 0.00 | 0.0 | 0.73 |
| **Inst 4** | 0.19 | 0.29 | 0.0 | 0.00 | 0.0 |
| **Inst 5** | 0.00 | 0.00 | 0.73 | 0.0 | 0.00 |

## Nominator {sum(min[R(x,y),A(x)])}

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 0.73 | 0.83 | 0.73 | 0.48 | 0.73 |

## Denominator (sum [R(x,y)])

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 1.2 | 1.38 | 2.0 | 1.77 | 1.77 |

## Division results (nominator/denominator)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 0.608333 | 0.60144928 | 0.365 | 0.27118644 | 0.412429379 |

## Upper approximation (alpha =0.2, beta = 1.0) with quad fuzzy quantifier

For 0.608333:

2 \* (0.608333 - 0.2)2 / (1 - 0.2)2 = 2\*(0.166735838889 / 0.64) = 2 \* 0.2605247482640625 =0.521049496528125

-------------------------------------------------------------------------------------------------------

For 0.60144928:

2 \* (0.60144928- 0.2)2 / (1 - 0.2)2 = 2\*(0.1611615244125184/ 0.64) = 2 \* 0.25181488189456 =0.50362976378912

-------------------------------------------------------------------------------------------------------

For 0.365:

2 \* (0.365- 0.2)2 / (1 - 0.2)2 = 2\*(0.027225/ 0.64) = 2 \* 0.027225=0.085078125

-------------------------------------------------------------------------------------------------------

For 0.27118644:

2 \* (0.27118644- 0.2)2 / (1 - 0.2)2 = 2\*(0.0050675092398736/ 0.64) = 2 \* 0.0079179831873025 =0.015835966374605

-------------------------------------------------------------------------------------------------------

For 0.412429379:

2 \* (0.412429379- 0.2)2 / (1 - 0.2)2 = 2\*(0.045126241062325641/ 0.64) = 2 \* 0.0705097516598838140625 =0.141019503319767628125

-------------------------------------------------------------------------------------------------------

Final results for upper approximation:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 0.521049496528125 | 0.50362976378912 | 0.085078125 | 0.015835966374605 | 0.141019503319767628125 |

## Lower approximation (alpha =0.1, beta = 0.6) with quad fuzzy quantifier

For 0.608333:

=1.0

-------------------------------------------------------------------------------------------------------

For 0.60144928:

=1.0

-------------------------------------------------------------------------------------------------------

For 0.365:

1 - 2 \* (0.365- 0.6)2 / (0.6 - 0.1)2 = 1 - 2\*(0.055225/ 0.25) = 1 - 2 \* 0.2209 =1 - 0.4418 =0.5582

-------------------------------------------------------------------------------------------------------

For 0.27118644:

2 \* (0.27118644- 0.1)2 / (0.6 - 0.1)2 = 2\*(0.0293047972398736/ 0.25) = 2 \* 0.1172191889594944=0.2344383779189888

-------------------------------------------------------------------------------------------------------

For 0.412429379:

1 - 2 \* (0.412429379- 0.6)2 / (0.6 - 0.1)2 =1 - 2\*(0.035182737862325641/ 0.25) = 1 - 2 \* 0.140730951449302564 =0.718538097101394872

Final results for lower approximation:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 1.0 | 1.0 | 0.5582 | 0.2344383779189888 | 0.718538097101394872 |