Test dataset for unit testing

By Mehran Amiri

For FRsutils library (https://github.com/mehi64/FRsutils)

**Change log**

|  |  |  |
| --- | --- | --- |
| ***Date (dd.mm.YYYY)*** | ***Change*** | ***Person*** |
| 12.05.2025 | Added the document (t-norm, implicator test data) | Mehran Amiri |
| 13.05.2025 | Added similarity test data + LB for ITFRS | Mehran Amiri |
| 14.05.2025 | Added UB for ITFRS | Mehran Amiri |
| 15.05.2025 | Added data for Gaussian similarity | Mehran Amiri |
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# Implicators

## Data and outputs

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| a | b | 1-a | ab | 1-a+ab | 1-a+b | **Gaines**  **output** | **Goedel**  **output** | **KD output** | **reichenbach**  **output** | **Luk output** |
| 2.10 | 4.32 | - | - | - | - | - | - | - | - | - |
| -0.20 | -0.78 | - | - | - | - | - | - | - | - | - |
| 0.73 | 0.18 | 0.27 | 0.1314 | 0.4014 | 0.45 | 0.246575 | 0.18 | 0.27 | 0.4014 | 0.45 |
| 0.18 | 0.73 | 0.82 | 0.1314 | 0.9514 | 1.55 | 1.0 | 1.0 | 0.82 | 0.9514 | 1.00 |
| 0.88 | 0.88 | 0.12 | 0.7744 | 0.8914 | 1.00 | 1.0 | 1.0 | 0.88 | 0.8944 | 1.00 |
| 0.91 | 0.48 | 0.09 | 0.4368 | 0.5263 | 0.57 | 0.527473 | 0.48 | 0.48 | 0.5268 | 0.57 |
| 1.00 | 1.00 | 0.00 | 1.00 | 1.00 | 1.00 | 1.0 | 1.0 | 1.00 | 1.00 | 1.00 |
| 0.00 | 0.00 | 1.00 | 0.00 | 1.00 | 1.00 | 1.0 | 1.0 | 1.00 | 1.00 | 1.00 |

# Data for testing t-norms in a scalar way

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| a | b | ab | a+b | a+b-1 |  |  | **Min tnorm** | **Product tnorn** | **Lukasiewicz tnorm** |  |
| 2.10 | 4.32 | - | - | - | - | - | - | - | - |  |
| -0.20 | -0.78 | - | - | - | - | - | - | - | - |  |
| 0.73 | 0.18 | 0.1314 | 0.91 | -0.09 |  |  | 0.18 | 0.1314 | 0.00 |  |
| 0.18 | 0.73 | 0.1314 | 0.91 | -0.09 |  |  | 0.18 | 0.1314 | 0.00 |  |
| 0.88 | 0.88 | 0.7744 | 1.76 | 0.76 |  |  | 0.88 | 0.7744 | 0.76 |  |
| 0.91 | 0.48 | 0.4368 | 1.39 | 0.39 |  |  | 0.48 | 0.4368 | 0.39 |  |
| 1.00 | 1.00 | 1.00 | 2.00 | 1.00 |  |  | 1.00 | 1.00 | 1.00 |  |
| 0.00 | 0.00 | 0.00 | 0.00 | -1.00 |  |  | 0.00 | 0.00 | 0.00 |  |

# Data for testing t-norms in a map/vectorized way

Similarity map

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 1.0 | 0.2673 | 0.25456 | 0.1197 | 0.09504 |
| 0.2673 | 1.0 | 0.0658 | 0.1624 | 0.054 |
| 0.25456 | 0.0658 | 1.0 | 0.3157 | 0.53217 |
| 0.1197 | 0.1624 | 0.3157 | 1.0 | 0.53872 |
| 0.09504 | 0.054 | 0.53217 | 0.53872 | 1.0 |

Mask map

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 1.0 | 1.0 | 0.0 | 1.0 | 0.0 |
| 1.0 | 1.0 | 0.0 | 1.0 | 0.0 |
| 0.0 | 0.0 | 1.0. | 0.0 | 1.0 |
| 1.0 | 1.0 | 0.0 | 1.0 | 0.0 |
| 0.0 | 0.0 | 1.0 | 0.0 | 1.0 |

Output of product tnorm

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 1.0 | 0.2673 | 0.0 | 0.1197 | 0.0 |
| 0.2673 | 1.0 | 0.0 | 0.1624 | 0.0 |
| 0.0 | 0.0 | 1.0 | 0.0 | 0.53217 |
| 0.1197 | 0.1624 | 0.0 | 1.0 | 0.0 |
| 0.0 | 0.0 | 0.53217 | 0.0 | 1.0 |

Output of minimum tnorm

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 1.0 | 0.2673 | 0.0 | 0.1197 | 0.0 |
| 0.2673 | 1.0 | 0.0 | 0.1624 | 0.0 |
| 0.0 | 0.0 | 1.0 | 0.0 | 0.53217 |
| 0.1197 | 0.1624 | 0.0 | 1.0 | 0.0 |
| 0.0 | 0.0 | 0.53217 | 0.0 | 1.0 |

Output of Luk tnorm

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 1.0 | 0.2673 | 0.0 | 0.1197 | 0.0 |
| 0.2673 | 1.0 | 0.0 | 0.1624 | 0.0 |
| 0.0 | 0.0 | 1.0 | 0.0 | 0.53217 |
| 0.1197 | 0.1624 | 0.0 | 1.0 | 0.0 |
| 0.0 | 0.0 | 0.53217 | 0.0 | 1.0 |

# Similarities

## X matrix (each row is a data Instance)

|  |  |  |  |
| --- | --- | --- | --- |
| **Inst 1** | 0.10 | 0.32 | 0.48 |
| **Inst 2** | 0.20 | 0.78 | 0.93 |
| **Inst 3** | 0.73 | 0.18 | 0.28 |
| **Inst 4** | 0.91 | 0.48 | 0.73 |
| **Inst 5** | 1.00 | 0.28 | 0.47 |

## Linear similarity

### Element-wise calculations of |v1 - v2|, part of linear similarity

The matrix is symmetric.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **SIM** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | [0.0, 0.0, 0.0] | [0.1, 0.46, 0.45] | [0.63, 0.14, 0.20] | [0.81, 0.16, 0.25] | [0.90, 0.04, 0.01] |
| **Inst 2** | [0.1, 0.46, 0.45] | [0.0, 0.0, 0.0] | [0.53, 0.60, 0.65] | [0.71, 0.30, 0.20] | [0.80, 0.50, 0.46] |
| **Inst 3** | [0.63, 0.14, 0.20] | [0.53, 0.60, 0.65] | [0.0, 0.0, 0.0] | [0.18, 0.30, 0.45] | [0.27, 0.10, 0.19] |
| **Inst 4** | [0.81, 0.16, 0.25] | [0.71, 0.30, 0.20] | [0.18, 0.30, 0.45] | [0.0, 0.0, 0.0] | [0.09, 0.20, 0.26] |
| **Inst 5** | [0.90, 0.04, 0.01] | [0.80, 0.50, 0.46] | [0.27, 0.10, 0.19] | [0.09, 0.20, 0.26] | [0.0, 0.0, 0.0] |

### Element-wise similarity of Instances (linear similarity)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **SIM** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | [1.0, 1.0,1.0] | [0.9, 0.54, 0.55] | [0.37, 0.86, 0.8 ] | [0.19, 0.84, 0.75] | [0.1 , 0.96, 0.99] |
| **Inst 2** | [0.90 , 0.54, 0.55] | [1.0, 1.0,1.0] | [0.47, 0.4 , 0.35] | [0.29, 0.7 , 0.8 ] | [0.2 , 0.5 , 0.54] |
| **Inst 3** | [0.37, 0.86, 0.80 ] | [0.47, 0.4 , 0.35] | [1.0, 1.0,1.0] | [0.82, 0.7 , 0.55] | [0.73, 0.9 , 0.81] |
| **Inst 4** | [0.19, 0.84, 0.75] | [0.29, 0.70 , 0.8 ] | [0.82, 0.7 , 0.55] | [1.0, 1.0,1.0] | [0.91, 0.8 , 0.74] |
| **Inst 5** | [0.10 , 0.96, 0.99] | [0.20 , 0.5 , 0.54] | [0.73, 0.9 , 0.81] | [0.91, 0.8 , 0.74] | [1.0, 1.0,1.0] |

### Final similarity\_matrix\_with\_linear\_similarity\_minimum\_tnorm

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **SIMILARITIES** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.0 | 0.54 | 0.37 | 0.19 | 0.1 |
| **Inst 2** | 0.54 | 1.0 | 0.35 | 0.29 | 0.2 |
| **Inst 3** | 0.37 | 0.35 | 1.0 | 0.55 | 0.73 |
| **Inst 4** | 0.19 | 0.29 | 0.55 | 1.0 | 0.74 |
| **Inst 5** | 0.10 | 0.20 | 0.73 | 0.74 | 1.0 |

### Final similarity\_matrix\_with\_linear\_similarity\_product\_tnorm

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **SIMILARITIES** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.00000 | 0.26730 | 0.25456 | 0.11970 | 0.09504 |
| **Inst 2** | 0.26730 | 1.00000 | 0.06580 | 0.16240 | 0.05400 |
| **Inst 3** | 0.25456 | 0.06580 | 1.00000 | 0.31570 | 0.53217 |
| **Inst 4** | 0.11970 | 0.16240 | 0.31570 | 1.00000 | 0.53872 |
| **Inst 5** | 0.09504 | 0.05400 | 0.53217 | 0.53872 | 1.00000 |

### Final similarity\_matrix\_with\_linear\_similarity\_luk\_tnorm

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **SIMILARITIES** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.0000 | 0.0000 | 0.0300 | 0.0000 | 0.0500 |
| **Inst 2** | 0.0000 | 1.0000 | 0.0000 | 0.0000 | 0.0000 |
| **Inst 3** | 0.0300 | 0.0000 | 1.0000 | 0.0700 | 0.4400 |
| **Inst 4** | 0.0000 | 0.0000 | 0.0700 | 1.0000 | 0.4500 |
| **Inst 5** | 0.0500 | 0.0000 | 0.4400 | 0.4500 | 1.0000 |

## Gaussian Similarity

### Element-wise calculations of (v1 - v2)2, part of the Gaussian similarity

The matrix is symmetric.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **SIM** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | [0.0, 0.0, 0.0] | [0.0100, 0.2116, 0.2025] | [0.3969, 0.0196, 0.0400] | [0.6561, 0.0256, 0.0625] | [0.8100, 0.0016, 0.0001] |
| **Inst 2** | [0.0100, 0.2116, 0.2025] | [0.0, 0.0, 0.0] | [0.2809, 0.3600, 0.4225] | [0.5041, 0.0900, 0.0400] | [0.6400, 0.2500, 0.2116] |
| **Inst 3** | [0.3969, 0.0196, 0.0400] | [0.2809, 0.3600, 0.4225] | [0.0, 0.0, 0.0] | [0.0324, 0.0900, 0.2025] | [0.0729, 0.0100, 0.0361] |
| **Inst 4** | [0.6561, 0.0256, 0.0625] | [0.5041, 0.0900, 0.0400] | [0.0324, 0.0900, 0.2025] | [0.0, 0.0, 0.0] | [0.0081, 0.0400, 0.0676] |
| **Inst 5** | [0.81, 0.0016, 0.0001] | [0.6400, 0.2500, 0.2116] | [0.0729, 0.0100, 0.0361] | [0.0081, 0.0400, 0.0676] | [0.0, 0.0, 0.0] |

### Gaussian similarity elementwise (sigma = 0.67)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **SIM** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | [1.0000, 1.0000, 1.0000] | [0.9889, 0.7900, 0.7981] | [0.6427, 0.9784, 0.9564] | [0.4815, 0.9719, 0.9328] | [0.4057, 0.9982, 0.9999] |
| **Inst 2** | [0.9889, 0.7900, 0.7981] | [1.0000, 1.0000, 1.0000] | [0.7313, 0.6697, 0.6246] | [0.5704, 0.9046, 0.9564] | [0.4902, 0.7569, 0.7900] |
| **Inst 3** | [0.6427, 0.9784, 0.9564] | [0.7313, 0.6697, 0.6246] | [1.0000, 1.0000, 1.0000] | [0.9646, 0.9046, 0.7981] | [0.9220, 0.9889, 0.9606] |
| **Inst 4** | [0.4815, 0.9719, 0.9328] | [0.5704, 0.9046, 0.9564] | [0.9646, 0.9046, 0.7981] | [1.0000, 1.0000, 1.0000] | [0.9910, 0.9564, 0.9275] |
| **Inst 5** | [0.4057, 0.9982, 0.9999] | [0.4902, 0.7569, 0.7900] | [0.9220, 0.9889, 0.9606] | [0.9910, 0.9564, 0.9275] | [1.0000, 1.0000, 1.0000] |

### Gaussian similarity with product tnorm

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **SIM** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.0000 | 0.6235 | 0.6014 | 0.4365 | 0.4049 |
| **Inst 2** | 0.6235 | 1.0 | 0.3059 | 0.4935 | 0.2932 |
| **Inst 3** | 0.6014 | 0.3059 | 1.0 | 0.6964 | 0.8759 |
| **Inst 4** | 0.4365 | 0.4935 | 0.6964 | 1.0 | 0.8791 |
| **Inst 5** | 0.4049 | 0.2932 | 0.8759 | 0.8791 | 1.0 |

### Gaussian similarity with minimum tnorm

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **SIM** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.0 | 0.7900 | 0.6427 | 0.4815 | 0.4057 |
| **Inst 2** | 0.7900 | 1.0 | 0.6246 | 0.5704 | 0.4902 |
| **Inst 3** | 0.6427 | 0.6246 | 1.0 | 0.7981 | 0.9220 |
| **Inst 4** | 0.4815 | 0.5704 | 0.7981 | 1.0 | 0.9275 |
| **Inst 5** | 0.4057 | 0.4902 | 0.9220 | 0.9275 | 1.0 |

### Gaussian similarity with luk tnorm

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **SIM** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | [1.0000 | 0.5770 | 0.5775 | 0.3862 | 0.4038] |
| **Inst 2** | [0.5770 | 1.0000 | 0.0256 | 0.4314 | 0.0371] |
| **Inst 3** | [0.5775 | 0.0256 | 1.0000 | 0.6673 | 0.8715] |
| **Inst 4** | [0.3862 | 0.4314 | 0.6673 | 1.0000 | 0.8749] |
| **Inst 5** | [0.4038 | 0.0371 | 0.8715 | 0.8749 | 1.0000] |

# Lower and Upper approximations (ITFRS)

## y (labels)

|  |  |
| --- | --- |
| **Inst 1** | 1.0 |
| **Inst 2** | 1.0 |
| **Inst 3** | 0.0 |
| **Inst 4** | 1.0 |
| **Inst 5** | 0.0 |

## label\_masks (b)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Label masks** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.0 | 1.0 | 0.0 | 1.0 | 0.0 |
| **Inst 2** | 1.0 | 1.0 | 0.0 | 1.0 | 0.0 |
| **Inst 3** | 0.0 | 0.0 | 1.0 | 0.0 | 1.0 |
| **Inst 4** | 1.0 | 1.0 | 0.0 | 1.0 | 0.0 |
| **Inst 5** | 0.0 | 0.0 | 1.0 | 0.0 | 1.0 |

## similarity\_matrix (a)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **SIMILARITIES** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.00 | 0.54 | 0.37 | 0.19 | 0.10 |
| **Inst 2** | 0.54 | 1.00 | 0.35 | 0.29 | 0.20 |
| **Inst 3** | 0.37 | 0.35 | 1.00 | 0.55 | 0.73 |
| **Inst 4** | 0.19 | 0.29 | 0.55 | 1.00 | 0.74 |
| **Inst 5** | 0.10 | 0.20 | 0.73 | 0.74 | 1.00 |

## Interim 1 - sim

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 0.00 | 0.46 | 0.63 | 0.81 | 0.90 |
| **Inst 2** | 0.46 | 0.00 | 0.65 | 0.71 | 0.80 |
| **Inst 3** | 0.63 | 0.65 | 0.00 | 0.45 | 0.27 |
| **Inst 4** | 0.81 | 0.71 | 0.45 | 0.00 | 0.26 |
| **Inst 5** | 0.90 | 0.80 | 0.27 | 0.26 | 0.00 |

## Interim Sim \* A(y)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.0 | 0.54 | 0.0 | 0.19 | 0.0 |
| **Inst 2** | 0.54 | 1.0 | 0.0 | 0.29 | 0.0 |
| **Inst 3** | 0.0 | 0.0 | 1.0 | 0.0 | 0.73 |
| **Inst 4** | 0.19 | 0.29 | 0.0 | 1.0 | 0.0 |
| **Inst 5** | 0.0 | 0.0 | 0.73 | 0.0 | 1.0 |

## Interim 1 - sim + A(y)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.0 | 1.46 | 0.63 | 1.81 | 0.9 |
| **Inst 2** | 1.46 | 1.0 | 0.65 | 1.71 | 0.8 |
| **Inst 3** | 0.63 | 0.65 | 1.0 | 0.45 | 1.27 |
| **Inst 4** | 1.81 | 1.71 | 0.45 | 1.0 | 0.26 |
| **Inst 5** | 0.9 | 0.8 | 1.27 | 0.26 | 1.0 |

## Luk Implicator results

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.0 | 1.0 | 0.63 | 1.0 | 0.9 |
| **Inst 2** | 1.0 | 1.0 | 0.65 | 1.0 | 0.8 |
| **Inst 3** | 0.63 | 0.65 | 1.0 | 0.45 | 1.0 |
| **Inst 4** | 1.0 | 1.0 | 0.45 | 1.0 | 0.26 |
| **Inst 5** | 0.9 | 0.8 | 1.0 | 0.26 | 1.0 |

## KD Implicator results

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.0 | 1.0 | 0.63 | 1.0 | 0.9 |
| **Inst 2** | 1.0 | 1.0 | 0.65 | 1.0 | 0.8 |
| **Inst 3** | 0.63 | 0.65 | 1.0 | 0.45 | 1.0 |
| **Inst 4** | 1.0 | 1.0 | 0.45 | 1.0 | 0.26 |
| **Inst 5** | 0.9 | 0.8 | 1.0 | 0.26 | 1.0 |

## Reichenbach Implicator results

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.0 | 1.0 | 0.63 | 1.0 | 0.9 |
| **Inst 2** | 1.0 | 1.0 | 0.65 | 1.0 | 0.8 |
| **Inst 3** | 0.63 | 0.65 | 1.0 | 0.45 | 1.0 |
| **Inst 4** | 1.0 | 1.0 | 0.45 | 1.0 | 0.26 |
| **Inst 5** | 0.9 | 0.8 | 1.0 | 0.26 | 1.0 |
|  |  |  |  |  |  |

## Goedel Implicator results

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.0 | 1.0 | 0.0 | 1.0 | 0.0 |
| **Inst 2** | 1.0 | 1.0 | 0.0 | 1.0 | 0.0 |
| **Inst 3** | 0.0 | 0.0 | 1.0 | 0.0 | 1.0 |
| **Inst 4** | 1.0 | 1.0 | 0.0 | 1.0 | 0.0 |
| **Inst 5** | 0.0 | 0.0 | 1.0 | 0.0 | 1.0 |

## Gaines Implicator results

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 1.0 | 1.0 | 0.0 | 1.0 | 0.0 |
| **Inst 2** | 1.0 | 1.0 | 0.0 | 1.0 | 0.0 |
| **Inst 3** | 0.0 | 0.0 | 1.0 | 0.0 | 1.0 |
| **Inst 4** | 1.0 | 1.0 | 0.0 | 1.0 | 0.0 |
| **Inst 5** | 0.0 | 0.0 | 1.0 | 0.0 | 1.0 |

NOTES:

1. KD and Reichenbach results are the same for this example.

Since for the calculations of lower approximation, we calculate *Inf* which is basically a minimum, to exclude the same instance from calculations we don’t need anything because the diagonal is set to 1.0 which is ignored by min operator. To be sure all is correct, inside code, we set main diagonal to 1.0

Since for the calculations of upper approximation, we calculate sup which is basically a maximum, to exclude the same instance from calculations we need to set the main diagonal to 0.0 which is ignored by max operator. Otherwise all upper approxamations will be 1.0.

## Lower approximation with all Implicators

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Richenbach** | 0.63 | 0.65 | 0.45 | 0.26 | 0.26 |
| **KD** | 0.63 | 0.65 | 0.45 | 0.26 | 0.26 |
| **Luk** | 0.63 | 0.65 | 0.45 | 0.26 | 0.26 |
| **Goedel** | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| **Gaines** | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |

## Min and product t-norm(similarity\_matrix, label\_masks)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **t-norm** | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Inst 1** | 0.0 | 0.54 | 0.0 | 0.19 | 0.0 |
| **Inst 2** | 0.54 | 0.0 | 0.0 | 0.29 | 0.0 |
| **Inst 3** | 0.0 | 0.0 | 0.0 | 0.0 | 0.73 |
| **Inst 4** | 0.19 | 0.29 | 0.0 | 0.0 | 0.0 |
| **Inst 5** | 0.0 | 0.0 | 0.73 | 0.0 | 0.0 |

**NOTE: min and product tnorm give the same results**

**Main diagonal is set to 0.0 so that sup operator ignores the information of the same instance**

## Upper approximations

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **Inst 1** | **Inst 2** | **Inst 3** | **Inst 4** | **Inst 5** |
| **Min t-norm** | 0.54 | 0.54 | 0.73 | 0.29 | 0.73 |
| **Prod t-norm** | 0.54 | 0.54 | 0.73 | 0.29 | 0.73 |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |

# OWA\_weights data for test

 owa\_infimum\_weights\_linear\_len\_5 = np.array([0.06666667, 0.13333333, 0.2, 0.26666667, 0.33333333])

        owa\_infimum\_weights\_linear\_len\_10 = np.array([0.01818182, 0.03636364, 0.05454545, 0.07272727, 0.09090909, 0.10909091, 0.12727273, 0.14545455, 0.16363636, 0.18181818])

        owa\_suprimum\_weights\_linear\_len\_8 = np.array([0.22222222, 0.19444444, 0.16666667, 0.13888889, 0.11111111, 0.08333333, 0.05555556, 0.02777778])

        owa\_supriimum\_weights\_linear\_len\_13 = np.array([0.14285714, 0.13186813, 0.12087912, 0.10989011, 0.0989011,  0.08791209, 0.07692308, 0.06593407, 0.05494505, 0.04395604, 0.03296703, 0.02197802, 0.01098901])