Nielsen and Chuang Solutions

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Contents

1 Chapter 1

1.1 Vector Algebra

Exercise 2.1

To prove these vectors to be linearly dependent,

$$a \begin{bmatrix} 1 \\ -1 \end{bmatrix} + b \begin{bmatrix} 1 \\ 2 \end{bmatrix} + c \begin{bmatrix} 2 \\ 1 \end{bmatrix} = 0 \qquad \text{such that: all } a, b, c \neq 0 \tag{1}$$

It can be seen that $a = 1, b = 1$ and $c = -1$ solves the aforementioned equation.

Exercise 2.2

According to the question we have,

$$A|0\rangle = |0\rangle, \quad A|1\rangle = |1\rangle \tag{2}$$

Thus for $|0\rangle, |1\rangle$ basis

$$\begin{gathered} A = \Big(A|0\rangle \;\; A|1\rangle \Big) \\ A = \Big(|0\rangle \;\; |1\rangle \Big) \\ A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \end{gathered} \tag{3}$$

Exercise 2.3

Exercise 2.4

According to the question we have,

$$A|0\rangle = |0\rangle, \quad A|1\rangle = |1\rangle \tag{4}$$

Thus for $|0\rangle, |1\rangle$ basis

$$\begin{gathered} A = \Big(A|0\rangle \;\; A|1\rangle \Big) \\ A = \Big(|0\rangle \;\; |1\rangle \Big) \\ A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \end{gathered} \tag{5}$$

1.2 Inner Products

Exercise 2.5

Exercise 2.6

$$\left(\sum_i \lambda_i |w_i\rangle, |v\rangle \right) = \left(|v\rangle, \sum_i \lambda_i |w_i\rangle \right)^* \tag{6}$$

Using equation 2.13 from the book

$$\begin{aligned} \left(|v\rangle, \sum_i \lambda_i |w_i\rangle \right)^* &= \left(\sum_i \lambda_i \Big(|v\rangle, |w_i\rangle \Big) \right)^* \\ &= \sum_i \lambda_i^* \Big(|v\rangle, |w_i\rangle \Big)^* \\ &= \sum_i \lambda_i^* \Big(|w_i\rangle, |v\rangle \Big) \end{aligned} \tag{7}$$

Exercise 2.7

For vectors to be orthogonal, the inner product of the two vectors should be 0. Thus,

$$\big(|v\rangle, |w\rangle\big) \;=\; \langle v|w\rangle \;=\; \begin{bmatrix}1 & -1\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix} \;=\; 1\times 1 + (-1)\times 1 = 0 \tag{8}$$

For normalized forms the magnitudes are,

$$\begin{aligned} ||w\rangle| = &\sqrt{\langle w|w\rangle} \;=\; \sqrt{2} \\ ||v\rangle| = &\sqrt{\langle v|v\rangle} \;=\; \sqrt{2} \end{aligned} \tag{9}$$

Thus the normalized forms are,

$$\begin{aligned} \frac{|w\rangle}{\sqrt{2}} &= \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) \\ \frac{|v\rangle}{\sqrt{2}} &= \left(\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right) \end{aligned} \tag{10}$$

Exercise 2.8

Exercise 2.9

Pauli matrices in there outer product notations are as follows,

$$\begin{aligned} \sigma_0 = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} &= \sum_{i,j=0}^{1} \langle w_j|\sigma_0|v_i\rangle |w_j\rangle\langle v_i| \\ &= 1|0\rangle\langle 0| + 0|0\rangle\langle 1| + 0|1\rangle\langle 0| + 1|1\rangle\langle 1| \\ &= |0\rangle\langle 0| + |1\rangle\langle 1| \end{aligned} \tag{11}$$

$$\begin{aligned} \sigma_1 = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix} &= 0|0\rangle\langle 0| + 1|0\rangle\langle 1| + 1|1\rangle\langle 0| + 0|1\rangle\langle 1| \\ &= |0\rangle\langle 1| + |1\rangle\langle 0| \end{aligned} \tag{12}$$

$$\begin{aligned} \sigma_2 = \begin{pmatrix}0 & -i\\ i & 0\end{pmatrix} &= 0|0\rangle\langle 0| + -i|0\rangle\langle 1| + i|1\rangle\langle 0| + 0|1\rangle\langle 1| \\ &= -i|0\rangle\langle 1| + i|1\rangle\langle 0| \end{aligned} \tag{13}$$

$$\begin{aligned} \sigma_3 = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} &= 1|0\rangle\langle 0| + 0|0\rangle\langle 1| + 0|1\rangle\langle 0| + -1|1\rangle\langle 1| \\ &= |0\rangle\langle 0| - |1\rangle\langle 1| \end{aligned} \tag{14}$$

Exercise 2.10

Exercise 2.11

PAULI X MATRIX: Using characteristic equation

$$\det(X - \lambda I) = 0 \tag{15}$$

$$\begin{vmatrix}-\lambda & 1\\ 1 & -\lambda\end{vmatrix} = \lambda^2 - 1 = 0 \qquad \therefore \lambda = \pm 1 \tag{16}$$

For $\lambda = 1$,

$$\begin{gathered} \begin{bmatrix}-1 & 1\\ 1 & -1\end{bmatrix}\begin{bmatrix}a\\ b\end{bmatrix} = \begin{bmatrix}-a+b\\ a-b\end{bmatrix} = \begin{bmatrix}0\\ 0\end{bmatrix} \\ \text{Thus, } a = b \end{gathered} \tag{17}$$

Thus the eigenvector for $\lambda = 1$,

$$|\lambda_1\rangle = \begin{bmatrix} a \\ a \end{bmatrix} \tag{18}$$

After normalizing,

$$|\lambda_1\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \tag{19}$$

For $\lambda = -1$,

$$\begin{gathered} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} a+b \\ a+b \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \\ \text{Thus, } b = -a \end{gathered} \tag{20}$$

Thus the eigenvector for λ = -1,

$$|\lambda_{-1}\rangle = \begin{bmatrix} a \\ -a \end{bmatrix} \tag{21}$$

After normalizing,

$$|\lambda_{-1}\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \tag{22}$$

Therefore,

$$\begin{aligned} X &= \lambda_1 |\lambda_1\rangle\langle\lambda_1| + \lambda_{-1}|\lambda_{-1}\rangle\langle\lambda_{-1}| \\ &= \frac{1}{2}(|0\rangle + |1\rangle)(\langle 0| + \langle 1|) - \frac{1}{2}(|0\rangle - |1\rangle)(\langle 0| - \langle 1|) \end{aligned} \tag{23}$$

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