YILDIZ TECHNICAL UNIVERSITY

Department of Computer Engineering

Algorithm Analysis
Homework-1

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1)

a) $c_1 \cdot g(n) \leq (n^2+1)^{10}$ ve $c_2 \cdot g(n) \geqslant (n^2+1)^{10}$

$g(n) = n^{20} \Rightarrow c_1 \cdot n^{20} \leq (n^2+1)^{10}$

$n_0 = 1 \Rightarrow n_0 \leq n \Rightarrow c_1 = 1,$

$c_2 \cdot n^{20} \geqslant (n^2+1)^{10} \Rightarrow c_2 \cdot (n^2)^{10} \geqslant (n^2+1)^{10}$

$\sqrt[10]{c_2} \cdot n^2 \geqslant n^2+1 \Rightarrow n_0 = 1$

$n_0 \leq n$ için $\sqrt[10]{c_2} = 2 \Rightarrow c_2 = 2^{10}$

$(n^2+1)^{10} \in \Theta(n^{20})$

b)

$c_1 \cdot g(n) \leq \sqrt{10n^2+7n+3} \leq c_2 \cdot g(n)$

$c_1 \cdot g(n) \leq \sqrt{10n^2+7n+3} \Rightarrow g(n) = \sqrt{n^2}$

$n_0 = 1$ ve $n_0 \leq n$ için $c_1 = \sqrt{10}$

$c_2 \cdot g(n) \geqslant \sqrt{10n^2+7n+3}$ $n_0 = 1$ iken

$n_0 \leq n$ için $c_2 = \sqrt{20}$

$\sqrt{10n^2+7n+3} \in \Theta(n)$

2) $\sum_{i=0}^{n-1} \sum_{j=0}^{i-1} (i+j)$ $\sum_{j=0}^{i-1}(i) + \sum_{j=0}^{i-1}(j) = i^2 + \frac{(i-1) \cdot i}{2}$

$\sum_{i=0}^{n-1}\left(\frac{3i^2}{2} - \frac{i}{2}\right) = \frac{3}{2}\sum_{i=0}^{n-1}(i^2) - \frac{1}{2}\sum_{i=0}^{n-1}(i)$

$\frac{3}{2} \cdot \left(\frac{(n-1) \cdot n \cdot (2n-1)}{6}\right) - \frac{1}{2} \cdot \left(\frac{(n-1) \cdot n}{2}\right)$

$\frac{2n^3 - 3n^2 + n}{4} - \frac{(n^2-n)}{4} = \frac{2n^3 - 4n^2 + 2n}{4} = \frac{n^3 - 2n^2 + n}{2}$

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3) a) Mathematical Notation of the code => $\sum_{k=i}^{n} \sum_{j=i+1}^{n-1} \sum_{i=0}^{n-2} 1$

$$\sum_{i=0}^{n-2} 1 = n-2-0+1 = n-1$$

$$\sum_{j=i+1}^{n-1} (n-1) = \sum_{j=i+1}^{n-1} (n) - \sum_{j=i+1}^{n-1} 1 = (n-1-(i+1)+1).n - (n-1-(i+1)+1).1$$

$$= (n-i-1).n - (n-i-1)$$

$$= n^2 - i.n + n - n + i + 1$$

$$= n^2 - i.n + i + 1$$

$$\sum_{k=i}^{n} (n^2) - n.\sum_{k=i}^{n} i + \sum_{k=i}^{n} i + \sum_{k=i}^{n} 1 = (n-i+1).n^2 - \left(\frac{n.(n+1)}{2} - \frac{(i-1).i}{2}\right).(n+1) + (n-i)$$

$$= n^3 - i.n^2 + n^2 - \left(\frac{n^2+n-i^2+i}{2}\right).(n+1) + (n-i)$$

The fastest growing term n^3 olduğundan

Big-Oh $= O(n^3)$ //

b) A[j][i]'nin 0 olduğu değerler için en içteki for döngüsü hiç işlem yapmayacağı için while döngüsüne çevrilmeli.

```
for i<-0 to n-2 do
    for j<-i+1 to n-1 do
        k<-i;
        while(k<n and A[j][i] !=0)
            A[j][k] =A[j][k] - A[i][k]* A[j][i]/A[i][i];
```

4)

$$\left.\begin{aligned} T(n) &= T(n/3) + 1 \\ T(n/3) &= T(n/9) + 1 \\ T(n/9) &= T(n/27) + 1 \end{aligned}\right) \Rightarrow T(n) = T(n/27) + 3$$

$$T(n) = T\left(\frac{n}{3^k}\right) + k$$

$$n = 3^k \Rightarrow k = \log_3 n$$

$$T(n) = \underbrace{T(1)}_{0} + \log_3 n \Rightarrow T(n) = \log_3 n$$