EC813B - Recitation 12

SPRING 2022

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Question 1

(Final #3, Spring 2012) Define the golden rule and the modified golden rule capital stocks in the context of the Diamond's OLG model with production. Briefly discuss the concept of dynamic inefficiency.

Solution

Golden rule capital stock k^{GR} is the steady-state level of capital stock that maximizes steady-state consumption. Modified golden rule capital stock k^{MGR} is the steady state capital stock which is the solution of social planner's welfare maximization problem. Equilibrium steady-state capital stock k^* can be greater or smaller than k^{GR} depending on parameter values. When it is greater than k^{GR}, we say there is dynamic inefficiency because by reducing capital stock we can improve everybody's utility in Pareto sense.

Question 2

(Final #3, Spring 2018) Extending the Diamond OLG model with a bequest motive increases the likelihood that the market equilibrium is equal to what a social planner (with the same intergenerational rate of discount as the private individuals) would prefer. True or not true? Explain briefly.

Solution

It is true. We can briefly show it as following:

Social Planner's problem:

$$\max \sum_{t=0}^{\infty} \left[\frac{1}{(1+R)^t} \left[u\left(c_{t1}\right) + \beta u\left(c_{t2}\right) \right] \right] \quad \text{s.t.} \quad \text{Social Planner's Constraint}$$

Agent's problem with bequests:

$$\begin{aligned}\max V_t &= u\left(c_{t1}\right)+\beta u\left(c_{t2}\right)+\frac{1}{1+R_b}V_{t+1}\\ \text{s.t. } c_{t+1} &= w_t - s_t + b_t\\ c_{t2} &= \left(1+r_{t+1}\right)s_t - b_{t+1}\end{aligned}$$

Iterate V_t and get some function as in Social Planner's problem (given $R = R_b$). If $b > 0$ then the FOC for both problems is the same.

Question 3

(Final #1, Spring 2018) Consider an economy with overlapping generations of a constant population of an even number N of agents. All agents in the generation born time t live for two periods and have the same utility function

$$u\left(c_{t1}, c_{t2}\right) = \ln\left(c_{t1}\right) + \ln\left(c_{t2}\right)$$

where c_{t1} and c_{t2} denote consumption of an individual in the first and second periods of life. Suppose half of the population has endowment $(e_1, 0)$ and the other half has endowment $(0, e_2)$. Assume $0 < e_2 < e_1$. Each old person at $t = 1$ is endowed with M units of fiat money. No other generation is endowed with money. The stock of money is fixed over time.

(a) Find the saving function of each of the two types of young agent for $t \geq 1$.

(b) Define an equilibrium without money. Compute all such equilibria.

(c) Define an equilibrium with money. Compute all such equilibria.

(d) Argue that there is a unique stationary monetary equilibrium.

Solution

(a) Denote agents with different endowments as *Type 1* and *Type 2*. Simply we can assume that *Type 1* agents are savers and *Type 2* are borrowers. The maximization problem and the FOC lead to the saving function of *Type 1*, s_t^1,

$$\max_{s_t^1} \quad \ln\left(e_1 - s_t^1\right) + \ln\left(R_t s_t^1\right) \quad \Longrightarrow \quad s_t^1 = \frac{e_t}{2}$$

The borrowing (saving) function of *Type 2*, b_t^2 (s_t^2), is

$$\max_{b_t^2} \quad \ln\left(b_t^2\right) + \ln\left(e_2 - R_t b_t^2\right) \quad \Longrightarrow \quad b_t^2 = \frac{e_2}{2R_t} \quad \text{or} \quad s_t^2 = -\frac{e_2}{2R_t}$$

(b) *Equilibrium* is a list of $\{c_{t1}^i, c_{t2}^i\}, \{R_t\}$ s.t.

- (i) agents maximize given $\{R_t\}$
- (ii) both markets clear (loan and goods) by Walras' Law

$$\frac{N}{2}s_t^1 + \frac{N}{2}s_t^2 = 0 \quad \Longrightarrow \quad R_t = R = \frac{e_2}{e_1} < 1$$

There exists only one stationary equilibrium with interest rate lower than the inverse of the discount factor $\beta(=1)$.

(c) *Equilibrium* is a list of $\{c_{t1}^i, c_{t2}^i\}, \{R_t\}$ and a sequence of $\{q_t\}$ (value of money at t), s.t.

- (i) agents maximize given $\{R_t\}$ and $\{q_t\}$
  - (1) Type 1:

$$\max \quad \ln\left(c_{t1}^1\right) + \ln\left(c_{t2}^2\right) \quad \text{s.t.} \quad \begin{aligned} c_{t1}^1 &= e_1 - q_t m_t^1 - s_t^1 \\ c_{t2}^1 &= q_{t+1} m_t^1 + R_t s_t^1 \end{aligned}$$

  - (2) Type 2:

$$\max \quad \ln\left(c_{t1}^2\right) + \ln\left(c_{t2}^2\right) \quad \text{s.t.} \quad \begin{aligned} c_{t1}^2 &= -q_t m_t^2 - s_t^2 \\ c_{t2}^2 &= e_2 + q_{t+1} m_t^2 + R_t s_t^2 \end{aligned}$$

- (ii) all markets clear (three markets, but we use two markets by Walras' Law)
  - (1) $s_t^1 + s_t^2 = 0$
  - (2) $\frac{N}{2}m_t^1 + \frac{N}{2}m_t^2 = NM \quad \Longrightarrow \quad m_t^1 + m_t^2 = M$
- (iii) $\{q_t\}$ is bounded.

No arbitrage condition imposes $\frac{q_{t+1}}{q_t} = R_t$. Then, from the maximization problem, taking into account that the old generation only trades with young generation of *Type 1*, the money market clearing conditions are

$$\begin{cases} s_t^1 + q_t m_t^1 = \frac{e_1}{2} \\ s_t^2 = -\frac{e_2}{2R_t} \end{cases} \quad \Longrightarrow \quad \begin{cases} s_t^1 + q_t 2M = \frac{e_1}{2} \\ s_t^2 = -\frac{e_2}{2R_t} \end{cases}$$

The loans market clearing condition is

$$\frac{e_1}{2} - 2Mq_t = \frac{e_2}{2R_t} \quad \Longrightarrow \quad 2q_{t+1}\left(\frac{e_1}{2} - 2Mq_t\right) = e_2 q_t \qquad (*)$$

The last difference equation characterizes all equilibria. To fully characterize the equilibria, we can rewrite the last difference equation (*) as (using $q_t = \frac{1}{p_t}$)

$$2q_{t+1}\left(\frac{e_1}{2} - 2Mq_t\right) = e_2 q_t \quad \Longrightarrow \quad p_{t+1} = -\frac{4M}{e_2} + \frac{e_1}{e_2} p_t \qquad (**)$$

In a stationary equilibrium, we assume $p_t = p^*$. So we get

$$p^* = \frac{4M}{e_1 - e_2} \quad \text{or} \quad q^* = \frac{e_1 - e_2}{4M}$$

Subtracting p^* from both sides of (**), we obtain

$$p_{t+1} - p^* = -\frac{4M}{e_2} + \frac{e_1}{e_2} p_t - p^* \quad \Longrightarrow \quad p_{t+1} - p^* = \frac{e_1}{e_2}\left(p_t - p^*\right)$$

Iterating the function, one can get

$$p_{t+1} = p^* + \left(\frac{e_1}{e_2}\right)^t (p_1 - p^*)$$

Hence, we need to have $p_1 \geq p^*$ or $q_1 \leq q^*$. We showed that $\{q_t\}$ is bounded.

(d) There is a unique stationary monetary equilibrium characterized by q^* (or p^*).

Question 4

(Final #2, Spring 2018) Consider a simple "Lucas tree" pure exchange economy with one tree. The only source of consumption is the fruit that grows on the tree. This fruit is called dividends and consumed by the representative agent. The stochastic process for dividend d_t is described as follows. If d_t is not equal to d_{t-1}, then $d_{t+1} = \gamma d_t$ with probability π, and $d_{t+1} = d_t$ with probability $(1 - \pi)$. If in any pair of periods j and $j + 1, d_j = d_{j+1}$, then for all $t > j, d_t = d_j$. In words, if not stopped, the process grows at a rate γ in every period. However, once it stops growing for one period, it remains constant forever after. Let $d_0 = 1$.

Preferences over stochastic processes for consumption are given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma} - 1}{1-\sigma}$$

Assume that $0 < \sigma < 1, 0 < \beta < 1, \gamma > 1$ and $\beta\gamma^{1-\sigma} < 1$.

(a) Define an equilibrium in this economy in which shares to the tree are traded.

(b) Calculate the equilibrium price of shares in this tree p_t as a function of the history of dividends.

Hint: *First, using the asset pricing equation show that the price is an expected discounted stream of future dividends. Use it to calculate the price p_t^s if the growth process is stopped. Then assume the growth process is not stopped and use the asset pricing equation to calculate p_t^g by guessing that the share price is a linear function of d_t, i.e. $p_t^g = p^g d_t$ where p^g is a constant.*

(c) Let T be the first period in which $d_{T-1} = d_T = \gamma^{T-1}$. Is $p_{T-1} > p_T$? Show conditions under which this is true.

Solution

(a) An *equilibrium* is a list of optimal decision rules and prices $\{p_t\}$ s.t.

(i) RA maximizes her discounted utility given prices, and

(ii) $c_t = d_t$ (goods market clearing implies demand for shares is equal to supply of shares.)

(b) Using the asset pricing equation, we show that the price is an expected discounted stream of future dividends. Then, if the growth process is stopped, the price p_t^s is

$$p_t^s = \sum_{s=t+1} \beta^{s-t} \frac{u'(c_s)}{u'(c_t)} d_t = \frac{\beta}{1-\beta} d_t$$

If the growth process is not stopped, the price p_t^g is

$$p_t^g = p^g d_t = \beta \left\{ \pi \frac{(\gamma d_t)^{-\sigma}}{(d_t)^{-\sigma}} \left[p^g (\gamma d_t) + \gamma d_t \right] + (1-\pi) \left[\frac{\beta}{1-\beta} d_t + d_t \right] \right\}$$

Rearranging the equation, we get

$$p^g = \frac{\beta\pi\gamma^{1-\sigma} + \frac{\beta}{1-\beta}(1-\pi)}{1-\beta\pi\gamma^{1-\sigma}} \quad \Longrightarrow \quad p_t^g = p^g d_t$$

(c) We find that

$$p_t^s = p^s d_t = \frac{\beta}{1-\beta} d_t \quad \Longrightarrow \quad p^s = \frac{\beta}{1-\beta}$$

and

$$p_t^g = p^g d_t \quad \Longrightarrow \quad p^g = \frac{\beta\pi\gamma^{1-\sigma} + \frac{\beta}{1-\beta}(1-\pi)}{1-\beta\pi\gamma^{1-\sigma}}$$

If we have $d_{T-1} = d_T = \gamma^{T-1}$, then we must have $p^g > p^s$. Hence, we need to examine

$$\frac{\beta\pi\gamma^{1-\sigma} + \frac{\beta}{1-\beta}(1-\pi)}{1-\beta\pi\gamma^{1-\sigma}} > \frac{\beta}{1-\beta}$$

Rearranging we get

$$(1-\beta)\beta\pi\gamma^{1-\sigma} + \beta(1-\pi) > \beta\left(1-\beta\pi\gamma^{1-\sigma}\right) \quad \Longrightarrow \quad \gamma^{1-\sigma} > 1$$

Thus we must have $\gamma^{1-\sigma} > 1$. As long as there is economic growth the stock price is also growing. But when the economic growth stops, the stock price drops (stock market crash) from p^g to p^s.

Question 5

(Asset Pricing with Lucas Trees) Consider a simple Lucas tree economy. It consists of (infinitely) many individuals, indexed by the real numbers between 0 and 1. There is a competitive market for equally many trees, each producing a non-storable and stochastic output d_i. In a particular period each tree produces the same amount, so there is no idiosyncratic uncertainty. By buying a tree in period t at price p_t the holder gets next period output d_{t+1} and can sell the tree at p_{t+1}. Let the representative household have preferences defined over stochastic processes of consumption given by

$$E_0\left[\sum_{t=0}^{\infty} \beta^t u\left(c_t\right)\right]$$

where $0 < \beta < 1$. Let A_t be asset holdings at period t.

(a) Set up the Bellman equation for the consumption problem.

(b) Derive the Euler equation. Now assume that $u(c_t) = \frac{c_t^{1-\alpha}-1}{1-\alpha}$, where $\alpha > 0$. Since there is no way to transfer real resources between periods it must be that $c_t = d_t$.

(c) Use the assumptions above and the Euler equation to derive an expression for the price of the Lucas asset. Now assume that dividends, d_t, follow

$$\ln d_{t+1} = \ln d_t + \varepsilon_{t+1}$$

where $\varepsilon_t \sim N(0,1)$. Guess that the asset price is linear in the current dividend so that we can write $p_t = pd_t$ with p, the price-dividend ratio, being constant for all t. Use this guess together with the Euler equation to get an explicit solution for p in terms of expectations of d_{t+1}/d_t.

(d) Use the result that if $\varepsilon \sim N(\mu, \sigma^2)$ then $E[\exp(t\varepsilon)] = \exp\left(\mu t + \frac{\sigma^2 t^2}{2}\right)$ to express p as a function of the parameters. What is the price-dividend ratio when α equals 1 and how does it change when $\alpha \gtrless 1$? *(HINT: For any constant $t, x_t = \exp(t \ln x)$.)*

Solution

(a) The Bellman equation for the consumption problem is

$$V(A_t, d_t) = \max\left\{u(d_t A_t + p_t A_t - p_t A_{t+1}) + \beta EV\left(A_{t+1}, d_{t+1}\right)\right\}$$

(b) Using the FOC and the envelope theorem, the Euler equation is

$$\begin{aligned}
&- u'(c_t)\, p_t + \beta E V_1\left(A_{t+1}, d_{t+1}\right) = 0 \\
&V_1\left(A_t, d_t\right) = u'(c_t)\left(d_t + p_t\right) \\
\implies\quad & u'(c_t) p_t = \beta E\left[u'\left(c_{t+1}\right)\left(d_{t+1} + p_{t+1}\right)\right]
\end{aligned}$$

Assuming $u(c_t) = \frac{c_t^{1-\alpha}-1}{1-\alpha}$ and $c_t = d_t$, we have $u'(c_t) = c_t^{-\alpha} = d_t^{-\alpha}$. Plugging these into the Euler equation, we get the asset pricing equation as

$$d_t^{-\alpha} p_t = \beta E\left[d_{t+1}^{-\alpha}\left(d_{t+1} + p_{t+1}\right)\right] \quad \implies \quad p_t = \beta E\left[\left(\frac{d_t}{d_{t+1}}\right)^{\alpha}\left(d_{t+1} + p_{t+1}\right)\right]$$

(c) We assume that dividends, d_t, follow $\ln d_{t+1} = \ln d_t + \varepsilon_{t+1}$ where $\varepsilon_t \sim N(0,1)$. Guess that $p_t = pd_t$ and plug it into the Euler equation. Hence we get

$$pd_t = \beta E\left[\left(\frac{d_t}{d_{t+1}}\right)^{\alpha}(d_{t+1} + pd_{t+1})\right]$$
$$p = \beta(1+p)E\left[\left(\frac{d_{t+1}}{d_t}\right)^{1-\alpha}\right]$$
$$p = \frac{\beta E\left[\left(\frac{d_{t+1}}{d_t}\right)^{1-\alpha}\right]}{1-\beta E\left[\left(\frac{d_{t+1}}{d_t}\right)^{1-\alpha}\right]}$$

(d) We find $\varepsilon_{t+1} = \ln\left(\frac{d_{t+1}}{d_t}\right)$ using $\ln d_{t+1} = \ln d_t + \varepsilon_{t+1}$. Combining these, we can get

$$\frac{d_{t+1}}{d_t} = \exp(\varepsilon_{t+1}) \quad \Longrightarrow \quad \left(\frac{d_{t+1}}{d_t}\right)^{1-\alpha} = \exp\left((1-\alpha)\varepsilon_{t+1}\right)$$

Using the result that if $\varepsilon \sim N(\mu, \sigma^2)$ then $E[\exp(t\varepsilon)] = \exp\left(\mu t + \frac{\sigma^2 t^2}{2}\right)$ for $\varepsilon_t \sim N(0,1)$, we obtain

$$E\left[\left(\frac{d_{t+1}}{d_t}\right)^{1-\alpha}\right] = \exp\left(\frac{(1-\alpha)^2}{2}\right)$$

We know that $p_t = pd_t$. So the price-dividend ratio is $p = \frac{p_t}{d_t}$. Now, we can plug this result into the equation for p. The price-dividend ratio, when $\alpha \gtrless 1$, is

$$p = \frac{p_t}{d_t} = \frac{\beta E\left[\left(\frac{d_{t+1}}{d_t}\right)^{1-\alpha}\right]}{1-\beta E\left[\left(\frac{d_{t+1}}{d_t}\right)^{1-\alpha}\right]} = \frac{\beta\exp\left(\frac{(1-\alpha)^2}{2}\right)}{1-\beta\exp\left(\frac{(1-\alpha)^2}{2}\right)}$$

Thus, when $\alpha = 1$, the price-dividend ratio becomes $p = \frac{p_t}{d_t} = \frac{\beta}{1-\beta}$.

Question 6

(Asset Pricing with Lucas Trees) Suppose that a representative agent has preferences

$$E_0\left[\sum_{t=0}^{\infty}\beta^t u(c_t)\right]$$

over the single non-storable consumption good, where $u(c) = \frac{c^{1-\sigma}-1}{1-\sigma}$ for $\sigma > 0$. The endowment of the good is governed by a Markov process with transition function $F(x', x)$. Let p_t denote the price of a claim to the endowment process (a tree) at time t; a_t holdings of tree (i.e., stock, claims to the yield of the tree, fruit or dividends); x_t dividends paid out by the tree (fruit). The representative agent maximizes the objective function subject to the following constraints: $c_t + p_t a_{t+1} = (p_t + x_t)$ at $c_t \geq 0, a_t \geq 0, x_t \geq 0, p_t \geq 0, \beta \in (0, 1)$ with conditional expectations (for dividends) formed using $F(x', x)$. Assume that $p_t = p(x_t)$ (pricing function); agents have rational expectations.

(a) Set up the Bellman equation and derive the Euler equation with a market in claims to the endowment process (trees).

(b) When the consumer has logarithmic utility, $u(c) = \ln c$, what is the equilibrium price-dividend ratio of a claim to the entire consumption stream? How does it depend on the distribution of consumption growth? Explain intuitively why an increase in expected future dividends does not affect asset prices.

Solution

(a) The Bellman equation is

$$V(a_t, x_t) = \max_{a_{t+1}} \{u(p_t a_t + x_t a_t - p_t a_{t+1}) + \beta E V(a_{t+1}, x_{t+1})\}$$

Using the FOC and the envelope theorem, the Euler equation is

$$\begin{aligned} & -u'(c_t) p_t + \beta E V_1(a_{t+1}, x_{t+1}) = 0 \\ & V_1(a_t, x_t) = u'(c_t)(p_t + x_t) \\ \Longrightarrow \quad & u'(c_t) p_t = \beta E[u'(c_{t+1})(p_{t+1} + x_{t+1})] \end{aligned}$$

and, then the asset pricing equation is

$$p_t = \beta E\left[\frac{u'(c_{t+1})}{u'(c_t)}(p_{t+1} + x_{t+1})\right]$$

Iterating the asset pricing equation for both p_t and x_t, we get

$$p_t = E_t\left[\sum_{s=1}^{T} \beta^s \frac{u'(c_{t+s})}{u'(c_t)} x_{t+s} + \beta^T \frac{u'(c_{t+T})}{u'(c_t)} p_{t+T}\right]$$

Now, we take the limit as $T \to \infty$. We get

$$\begin{aligned}
p_t &= \lim_{T\to\infty} E_t \left[\sum_{s=1}^{T} \beta^s \frac{u'(c_{t+s})}{u'(c_t)} x_{t+s} + \underbrace{\beta^T \frac{u'(c_{t+T})}{u'(c_t)} p_{t+T}}_{=0} \right] \\
&= E_t \left[\sum_{s=1}^{\infty} \beta^s \frac{u'(c_{t+s})}{u'(c_t)} x_{t+s} \right] \\
&= E_t \left[\sum_{s=1}^{\infty} \beta^s \frac{u'(x_{t+s})}{u'(x_t)} x_{t+s} \right]
\end{aligned}$$

since $c_t = x_t$ in equilibrium (market clearing condition).

(b) We assume that $u(c) = \ln c$ and define the price-dividend ratio as $\frac{p_t}{x_t}$. Plugging these into the asset pricing equation, the price-dividend ratio becomes

$$\begin{aligned}
p_t &= E_t \left[\sum_{s=1}^{\infty} \beta^s \frac{x_t}{x_{t+s}} x_{t+s} \right] \\
\frac{p_t}{x_t} &= \frac{1}{x_t} E_t \left[\sum_{s=1}^{\infty} \beta^s \frac{x_t}{x_{t+s}} x_{t+s} \right] \\
\frac{p_t}{x_t} &= E_t \left[\sum_{s=1}^{\infty} \beta^s \frac{x_t}{x_{t+s}} \frac{x_{t+s}}{x_t} \right] = \sum_{s=1}^{\infty} \beta^s \\
\frac{p_t}{x_t} &= \frac{\beta}{1-\beta}
\end{aligned}$$

Using the price-divident ratio we get the assert pricing equation $p_t = \frac{\beta}{1-\beta} x_t$ for the special case in which we have logarithmic utility. Hence, an increase in expected future dividends does not affect asset prices as it only depends on the current dividend value.

Question 7

(Prelim #2, Spring 2013) Consider the standard Lucas asset pricing model with n trees that produce stochastic output $\{y_{it}\}_{i=1}^{n}$, described by the following cdf, $F(y', y) = \Pr(y_{t+1} \le y' \mid y_t = y)$. There are n perfectly divisible assets, one asset per tree. Each asset entitles the agent to the output of the corresponding tree. The representative agent maximizes

$$E\sum_{t=0}^{\infty}\beta^{t}u\left(c_{t}\right)$$

by choosing future asset holding subject to a constraint, where consumption is limited by dividend income and portfolio adjustment income (same model as in class).

(a) Write down the Bellman equation and derive the asset pricing equations.

(b) Define equilibrium and show that in the linear utility case the price of an asset is determined by the present discounted value of expected future dividends.

Solution

(a) The Bellman equation can be written as

$$V(A_t, y_t) = \max\left\{u(y_tA_t + p_tA_t - p_tA_{t+1}) + \beta EV\left(A_{t+1}, y_{t+1}\right)\right\}$$

Using the FOC and the envelope theorem, the asset pricing equation is

$$\begin{aligned} & -u'\left(c_t\right)p_t + \beta EV_1\left(A_{t+1}, y_{t+1}\right) = 0 \\ & V_1\left(A_t, y_t\right) = u'\left(c_t\right)\left(y_t + p_t\right) \\ \Longrightarrow \quad & u'(c_t)p_t = \beta E\left[u'\left(c_{t+1}\right)\left(y_{t+1} + p_{t+1}\right)\right] \end{aligned}$$

and, then the asset pricing equation is

$$p_t = \beta E\left[\frac{u'\left(c_{t+1}\right)}{u'(c_t)}\left(y_{t+1} + p_{t+1}\right)\right]$$

Notice that this asset pricing equation is for the ith tree. Aggregating for n trees, we can rewrite the Euler equation as following

$$u'\left(\sum_{i=1}^{n} y_{it}\right)p_{it} = \beta E\left[u'\left(\sum_{i=1}^{n} y_{i,t+1}\right)\left(y_{i,t+1} + p_{i,t+1}\right)\right]$$

and, the asset pricing equation becomes

$$p_t = \beta E \left[\frac{u'\left(\sum_{i=1}^{n} y_{i,t+1}\right)}{u'(\sum_{i=1}^{n} y_{it})} \left(y_{t+1} + p_{t+1}\right) \right]$$

(b) An *equilibrium* is a list of optimal decision rules and prices $\{p_{it}\}$ s.t.

   (i) RA maximizes her discounted utility given prices, and

   (ii) $c_t = \sum_{i=1}^{n} y_{it}$ (goods market clearing implies demand for shares is equal to supply of shares.)

Now, we need to show that in the linear utility case the price of an asset is determined by the present discounted value of expected future dividends. We can write the asset pricing equation as

$$p_{it} = E\left[\beta\left(y_{i,t+1} + p_{i,t+1}\right)\right]$$

since we have linear utility. Iterating the asset pricing equation, we get

$$p_{it} = E\left[\sum_{s=1}^{\infty} \beta^s y_{i,t+s}\right]$$

Hence, we show that the price of an asset is determined by the present discounted value of expected future dividends.