Guaranteed Trade-in Price Strategy: Deconstructing Its Value for Consumers and Firms

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Guaranteed trade-in program is a relatively new ownership model that firms offer to consumers giving them the option, *but not obligating them*, to return their used durables for a pre-specified resale value when buying an upgraded model in the future; this presumably encourages them to upgrade. In this paper, we study the value of such a strategy and the source of that value from the dual perspective of firm and consumers. The basic framework of our analysis is a two-period model where a firm (retailer) sells a durable product to consumers. Consumers are non-renewable and strategic. We consider two ownership models for them: (i) just "buy" the product, and (ii) buy it with the "guaranteed trade-in" service. Buying implies consumers can sell their used products only in a volatile spot market in the second period. But the guarantee service entitles them to either exercise the option and trade-in their used products at the retailer for the guaranteed resale value or use the spot market if they get a better price there. Our analysis indicates that this service actually is a win-win situation for both the retailer and the consumers, even if it is given for free, except for some extreme cases when the consumers might be hurt. Interestingly, deconstruction of the source of this value reveals that, for the consumers, it comes from the optionality feature because of which they can use either the retailer or the spot market channel to trade-in. Although discussion about this strategy focuses on the resale value protection it provides, we demonstrate that this component of the service (without optionality) is actually harmful for the consumers. The retailer, however, extracts benefits from both the optionality and protection features. In an extension, we consider the role of the guarantor – whether it is offered by the retailer itself or by a third-party firm. Surprisingly, consumers benefit from the guarantee strategy primarily when retailers offer the service themselves; a third-party providing such service actually might make them worse off. The retailer benefits regardless of the identity of the guarantor. Most of the above insights hold true irrespective of whether the pricing mechanism used by the retailer is a static or a dynamic one. Indeed, our analysis is able to provide plausible rationale for a number of real-life aspects of this pricing strategy in terms of its product, provider and premium characteristics.

Key words: Guaranteed resale value; Strategic consumers; Consumer surplus; Pricing strategy; Trade-ins

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1. Introduction

Consider a customer buying the latest version of consumer electronics or automobiles (e.g., iPhone 6S or Tesla S). Given that regular upgrades of technology are now a key feature of such durables, the customer knows that another "latest" version is around the corner and there might be "need", actual or perceived, for buying that also. For example, there has been a new model of iPhone released every Summer/Fall from 2007 to 2015, and about 24% of the customers who bought iPhone 5S in 2013 wanted to upgrade to iPhone 6 in 2014 (this proportion is 36% for customers who bought iPhone 5 in 2012; WSJ (2014)). While this gives the customers access to the latest technology, keeping current with ever-changing gadgets can be expensive. The associated costs (e.g., actual out-of-pocket expenses for the new product, the opportunity cost of the residual value of the present one) can discourage customers from upgrading. Traditionally, one of the ways that firms have incentivized customers to continue to upgrade their durables is by offering a trade-in program. Under such a program, customers can return their used products for a credit. Although this credit can be of many forms, e.g., cash or gift card, in this paper we focus on the most common case where the credit can only be used towards the purchase of a new product. Variants of this strategy have been the norm in the automobile industry for quite some time, but is now catching on in other industries, notably consumer electronics.

A trade-in scheme certainly reduces the "upgrade cost" for customers. But, there are still significant risks from their viewpoint associated with this strategy. One of the main ones is that the future trade-in prices can vary significantly over time for a particular provider as well as among the different providers at a particular point in time. For example, the trade-in price for iPhone 5 varied between \$135 and \$205 on Gazelle.com between August and November 2014 (AppleInsider (2015)), and on a particular day the trade-in price for an iPhone 6 ranged from \$325 to \$430 and for an iPhone 5s from \$165 to \$270 across providers (MacObserver (2015))[1]. The *guaranteed* trade-in price strategy, which is the focus of this paper, is designed to alleviate this concern.

A typical guaranteed trade-in offer reads as follows.

Customers pay a service fee for the right to redeem a newly purchased durable product at a future date in exchange for a guaranteed credit amount[2].

The primary difference between the two ownership models – a traditional trade-in offer and a guaranteed one – is that in the latter case the trade-in price is locked-in (guaranteed) by the firm at the time when a customer is buying the new product. This provides the customers with full

[1] Obviously, offers from different providers differ in terms of their fine prints.

[2] The guaranteed amount differs depending on the product type and the length of the coverage.

assurance about the future resale value so that they are not sensitive to its volatility while making purchase decisions. However, the offer only acts as an option; if a customer gets a better trade-in price in the spot market other than the guarantee provider when it is time to exercise the option, s/he is free to take advantage of that. There are thus two features of the guarantee strategy that provide potential benefits for consumers: i) the *insurance/warranty* about the resale value *protects* them from downside risk, and ii) the *optionality* keeps open their upside *potential*. This strategy, depending on the industry, is also known as specified-price trade-in or resale value guarantee.

To the best of our knowledge, initially such guarantee services were provided by third-parties with the prime example being the technology start-up TechForward who partnered with retailers, e.g., Best Buy, and manufacturers, e.g., Dell, for electronics items (Parish (2012)). Very soon the idea became so popular that some of those retailers and the manufacturers started offering the service themselves with a lot of fanfare (Fortune (2012)). Indeed, this pricing strategy is now becoming a staple for retailers/manufacturers in a number of other sectors including automobile (see, e.g., Autoblog.com (2011), Tesla (2016), and Subaru (2016) for Guaranteed Trade-in Programs for Hyundai, Tesla and Subaru, respectively), heavy equipment (FASB (2000)) and musical/sports equipment (see, e.g., Amro (2016))[3]. Even Apple's new iPhone Upgrade Program is a form of guaranteed trade-in (Apple (2016)).

In this paper our goal is to understand the underlying reason behind the growing popularity of such trade-in guarantee options by investigating the source of its value for firms and customers. Specifically, we would like to understand:

- What value, if any, do consumers obtain from such an ownership strategy, especially when compared with just buying the product and utilizing non-guaranteed offers available in the spot market when it is time to trade-in? Moreover, if there is any value, which particular feature of the strategy – protection and/or potential – mainly drives the benefit for them?
- How does the provision of the guarantee service to the consumers affect the profit for the provider firm? Which of the insurance and/or optionality features of the strategy determines whether this service will also be beneficial to the firm?

In order to address the above questions, we model a value chain consisting of a firm (retailer) and a forward-looking consumer pool dealing with a durable product. They make the following decisions in two periods. In the first period, the retailer sets the price for the product and offers a guaranteed trade-in price option (which may be exercised in the beginning of period 2) for consumers who purchase the product in period 1, and possibly charges a premium for it. There is

[3] For musical/sports equipment the regular upgrades are necessary mainly due to change in physical characteristics of the user, e.g.,height or size, and not due to technological reasons.

also a volatile spot market available for transaction of used goods in period 2. In the second period, a new upgraded product is introduced, and the retailer sets its price. Simultaneously, the random spot market price is realized for the used products. Consumers who purchase in period 1 with the guarantee option have two choices in terms of the return/resale of their used products. They either exercise the option and return their used products for the guaranteed resale value to the retailer who then salvages them; or, they can let the trade-in option expire and sell their used products in the spot market. On the other hand, consumers who buy the product without the option can only use spot market to sell their used products. In addition to the return/resell decision, consumers also decide whether or not to upgrade in the second period.

We first show that under a static pricing policy the guarantee strategy results in higher retail prices compared to the no guarantee case. While this dampens the demand in the first period, assurance about the resale value boosts the number of upgrades in the second period; the total demand across the two periods actually increases. Higher prices and demand mean that offering the guarantee service is always profitable for the retailer, even if no premium is charged for it. Consumer pool as a whole also usually gains from this service since the lower upgrading cost more than compensates for the higher prices, unless the spot market is very volatile. To better understand this win-win outcome, we subsequently evaluate how much of the total value originates from the protection feature of the guarantee strategy and how much comes from optionality. This analysis establishes that just the protection component (i.e., assurance about the resale value) is indeed harmful for the consumers, especially when the spot trade-in market is quite volatile. All the benefits accruing from this feature are extracted by the retailer. However, when consumers are also offered the optionality, the upside potential of the spot market turns into a significant advantage for them. So, it is this feature that makes the guaranteed trade-in strategy valuable for consumers. Consequently, optionality increases demand ensuring that the retailer can profit even from this feature of the strategy. The positive value proposition for the retailer and consumers persists even when the retailer uses a dynamic pricing policy; the primary difference is that now the benefit for the retailer comes from higher demand and not higher prices. But, subsequently we establish that if the guarantee service is offered by a third party, and not by the retailer, the win-win outcome may not remain valid anymore. Actually, consumers are now usually hurt by the service; the retailer still gains although not as much as when offering the service itself. Indeed, our analysis possibly justifies several real-life facets of the guaranteed trade-in strategy like why it is normally offered for certain types of products, why the retailers started giving the guarantee themselves taking over from third parties and why usually the premium for the service is quite negligible.

2. Literature Review

There are several streams of literature that are relevant to our paper. Our basic modeling framework borrows from papers in Marketing and Economics fields that deal with consumers making purchasing decisions about durable products. In this stream, some of the topics addressed include whether or not to produce the old version if a new one is introduced (see, e.g., Levinthal and Purohit (1989)), firms' choice between leasing and selling strategies for durable products (see, e.g., Desai and Purohit (1998), and Desai and Purohit (1999)), how to handle the used good markets (see, e.g., Desai et al. (2004); Shulman and Coughlan (2007)) and firms' product upgrade strategies (see, e.g., Yin et al. (2010)). Consumers are often heterogenous in their valuation of the product (see, e.g., Anderson and Ginsburgh (1994)), which could increase or decrease over time due to product upgrades or deterioration of the condition due to use. Consumers use a choice model to figure out when to buy what version of the product (original, upgraded or used). Waldman (2003) provides a detailed review of this literature. Our focus on trade-in pricing strategy separates us from this literature.

Given this focus, the stream of research that studies product trade-ins is quite relevant for us. Trade-ins can be viewed as one form of sales promotion effort although the details in this case are different from other efforts like coupons, trade promotions, etc.. Consequently, it has been studied relatively extensively. Some of the examples include Levinthal and Purohit (1989), Van Ackere and Reyniers (1995), Fudenberg and Tirole (1998), Ray et al. (2005), Ferguson and Koenigsberg (2007), Zhu et al. (2008), Rao et al. (2009) and Srivastava and Chakravarti (2011). Most of these papers use a two-period framework where the pricing decision in the first period divides the market into potential replacement customers and first-time buyers in the second period. In the second period there might be multiple types of "products" available – specifically, new products of similar and upgraded models as well as used ones from the first period. So, the primary objective of using trade-in rebates here is to reduce the cannibalization of the upgraded product through price discrimination. Within this general theme, a number of unique issues are addressed such as the impact of the age distribution of the existing products in the market and customer heterogeneity with respect to quality, how the nature of information available to the firm and to the customers affects the pricing and product introduction decisions and the ramifications of how the trade-in price is presented. There is also another stream of literature focusing on the perspective of customers, rather than the firm, especially about how they make their product replacement decisions while facing trade-in offers (Heath and Fennema (1996); Okada et al. (2001)). Lastly, the reverse logistics stream in the operations literature have also examined the usefulness of trade-in rebates in acquisition of returned products and successful product recovery (for example, Heese et al. (2005), Guide Jr et al. (2003) and Agrawal et al. (2016)). There are two elements of our work that distinguish us from this stream – the volatility of the trade-in price in the spot market and the offer of guaranteed trade-in price from the retailer to counteract it.

The papers that are most relevant to us are Yin and Tang (2014) and Yin et al. (2015), which also study guaranteed trade-in prices. Our paper differs from these papers mainly from two perspectives. First, we explicitly formulate the price volatility in the used goods market in the second period as a random variable, which is one of the main factors that drives consumers' incentive to purchase the guarantee option in the first period. Yin and Tang (2014) models this price as a fixed parameter in their main model, while Yin et al. (2015) assume away the used goods market completely. Second, both Yin and Tang (2014) and Yin et al. (2015) formulate the level of product upgrade in the second period as a random variable. Our paper simplifies this aspect by assuming it to be a fixed parameter. This simplification allows us to analytically characterize the equilibrium decisions and profits which enables the subsequent detailed comparisons of various models. As a result, we are able to identify which element of the strategy – the return price guarantee or the optionality to use the spot market – is driving its value for consumers and retailers, thus providing a much richer understanding of the strategy.

3. General Model Framework

In this section, we introduce the general framework of the model that will be the basis of this paper. We divide it into two parts. The first part (§3.1) describes the salient features of our model and the sequence of events, while the second part (§3.2) develops the demand and profit functions. A list of notation is provided in Appendix A.

3.1 Model Description

Our basic model framework is a two-period one involving a value chain comprising a retailer selling a durable product (it can be a manufacturer like Tesla or a retailer like Best Buy) to end-customers along with a trade-in price guarantee service. From the product perspective, there is a new generation being released in each period with the one in period 2 (Product 2) being upgraded version of the first one (Product 1). A pool of customers arrive at the beginning of the first period who are interested in buying new Product 1 and/or new Product 2. Customers are heterogenous in terms of their valuations of these products. Without loss of generality, we normalize the size of the customer pool to be 1 for the rest of the paper.

Our model starts with the retailer setting the price for Product 1 in period 1 (say, p_1) as well as the details of the *trade-in price guarantee* service. Specifically, along with p_1, the retailer also announces that any period-1 customer who wishes to trade-in her/his Product 1 for Product 2 at the beginning of period 2 will be assured a trade-in price of w as long as she pays a fixed *premium* αw upfront for this service, where $\alpha \in [0,1)$ is an exogenous parameter. In reality, α can be quite low, even zero. However, sometimes the conditions of the guarantee are such that, while there are no explicit fees, there is an indirect charge[4], which effectively increases the value of α. For simplicity and tractability, we model it as a given parameter. Although the product is durable, each generation provides utility to the customers only for one period and after that the particular generation is not valuable to those who bought it. Any customer who buys Product 1 gets a value of v_1 for the product in period 1, where $v_1 \sim U[0,1]$. Period 1 customers have three choices available to them: i) buy the product without the guarantee option, ii) buy it with the guarantee option, and iii) do not buy.

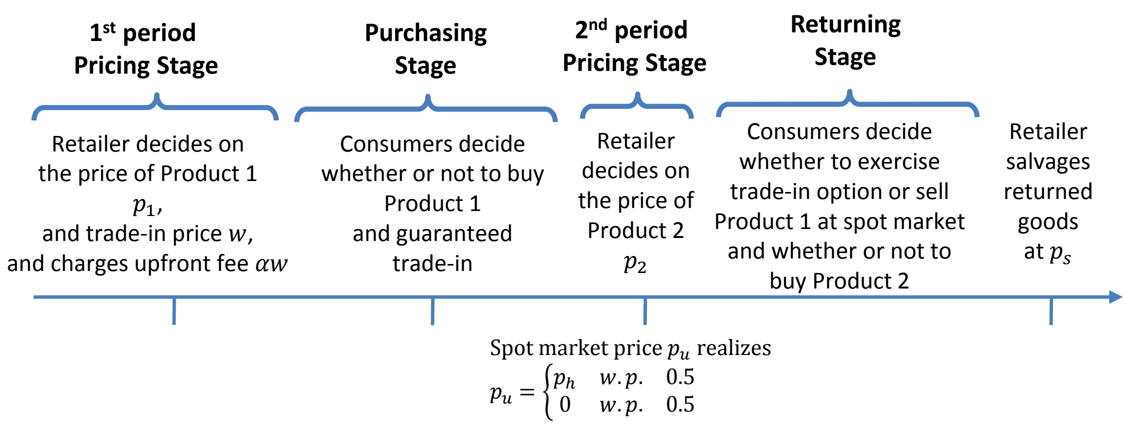
At the end of period 1, the used products do not have any utility for the customers who bought it, but they might still be valuable to others. So, these customers need to decide where to sell such used products. In our setting, there are two channels available to them for such transactions: i) trade-in for a new generation Product 2 and get a discount w, or ii) sell it in the spot trade-in market (e.g., through Gazelle.com or Amazon). The first channel is available only to customers who bought the option to trade-in by paying αw; but, they are not obligated to use that channel. All customers can access the spot market for used products. As shown in §1, such spot markets usually are characterized by price volatility that is affected by a number of industry level market forces such as the total supply/demand, the number of different generations of the product available and the prices of new products. We capture this phenomenon by assuming that the used goods price p_u exhibits exogenous randomness and can be high (say, p_h) or low (say, 0) with equal probability. Clearly, the difference between high and low prices (in this case, p_h) measures the *degree of volatility* in this market. To avoid trivial returns of used products to the retailers, we restrict $w \le p_h$. However, the retailer can set $w = p_h$ to ensure full returns (i.e., no customer would use the spot market regardless of its price).

Once the randomness in the spot market realizes, i.e., it is known whether the price is p_h or 0, at the end of period 1, customers who paid the premium for the guarantee service need to decide whether to trade-in for Product 2 or sell their used products in the spot market; ones who did not buy the service can only use the spot market. If the trade-in option is utilized, the used products

[4] For example, in the case of Tesla's Guaranteed Trade-in program (also known as Resale Value Guarantee), the customers who wish to enter into this program need to finance their purchases through one of the Tesla's official vehicle financing programs, which usually come with extra cost-bearing restrictions on minimum length of loan, rate of financing, etc.

are returned to the retailer who receives a value of p_s per unit by salvaging them. This *salvage value* represents the residual value that the retailer can extract from remanufacturing/reselling of the returned products or some of their parts[5]. Simultaneously with the realization of p_u, the retailer also announces the price p_2 for Product 2 in period 2. Customers buying Product 2 in period 2 derive a value of $v_2 \equiv \theta v_1$, where $\theta \geq 1$ represents the relative upgrade of Product 2 compared to Product 1. We assume away the production cost of both generations of the product throughout the paper. Figure 1 captures the sequence of events in our setting.

Figure 1 **Sequence of Events**



3.2 Demand and Profit Functions

A distinguishing feature of this paper is modeling of the customer choice process from which the demand functions are derived. Note that the set of customers in our model is forward-looking. So, they will make certain strategic choices at time 0 keeping in mind all the options they have over both periods. As indicated before, in period 1, customers have the following three options: i) buy Product 1 with the guaranteed trade-in price service, ii) buy Product 1 without the guarantee, and iii) do not buy Product 1. The options available to the same set of customers in period 2 will depend on what they choose in period 1. For example, if they decide to buy the guarantee service, only then they may trade-in Product 1 for Product 2 and receive a discount of w in period 2. Figure 2 depicts the decision tree for the customers over the two periods and the utilities that they will receive in each period depending on which "branch" they choose.

Based on Figure 2, there are two important issues to point out here. (1) While deciding on

[5] In some sense, the salvage value p_s can also be interpreted as the expected wholesale price charged in B2B markets. Note that in rational expectations model, the (expected) salvage value would be related to the mean spot market price, i.e., $p_s = E[p_u] = \frac{p_h}{2}$. However, in our model, we allow p_s to be less (or greater) than $E[p_u]$ (as long as $p_s \leq p_h$) in order to capture (illiquid) risk premium. Along these lines, the difference between p_s and $E[p_u]$ represents the risk premium between volume-driven relatively stable B2B and volatile, customer-driven spot markets.

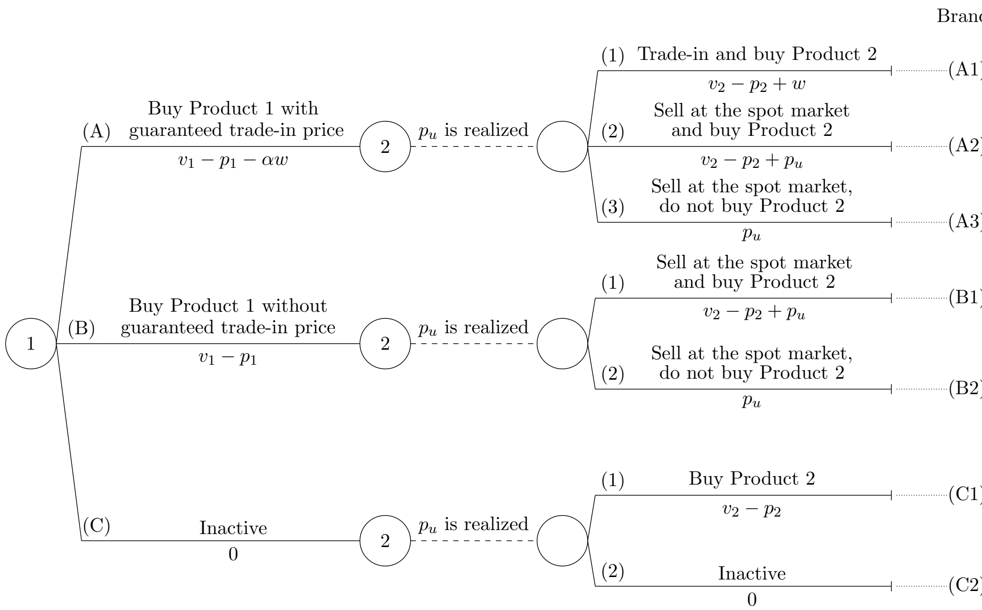


Figure 2 Customers' Choices with Guaranteed Trade-in

which of the three branches to choose in period 1, customers consider the expected utility over two periods with respect to p_u (expectation is required since p_u is random). (2) Customer choice for period 2 will take place only after the price volatility in the spot market has realized (so the second period choice is contingent on spot market realization). Subsequently, we can then express customers' expected total utility over two periods as follows.

- Branch (A): $u_{\mathrm{A}} = (v_1 - p_1 - \alpha w) + \mathrm{E}_{p_u} \max(v_2 - p_2 + w, v_2 - p_2 + p_u, p_u)$;
- Branch (B): $u_{\mathrm{B}} = (v_1 - p_1) + \mathrm{E}_{p_u} \max(v_2 - p_2 + p_u, p_u)$;
- Branch (C): $u_{\mathrm{C}} = 0 + \max(v_2 - p_2, 0)$.

Note that if $\alpha = 0$, i.e., the guarantee premium is zero, the branch of buying Product 1 without guarantee will be dominated by the branch of buying Product 1 with guarantee. This is because the latter gives all customers access to both the channels for used goods – trade-in at w or use the spot market at no cost – once the spot market price randomness realizes. Comparing the expected utilities over the three branches, the customers will make their (utility-maximizing) choices in period 1 and then, after p_u realizes, they will compare the utilities for the corresponding branches in period 2 to make their choices about period 2. This will give rise to the demand functions D_{Rt} and D_{Gt}, $t = 1, 2$, for the two periods as presented in Table 1. D_{R1} and D_{R2} represent the demands for the two generations of the product from the retailer, while D_{G1} and D_{G2} show how many customers would buy the guarantee service in period 1 and how many of them would decide to exercise the option in period 2, respectively. Note that we simply assume that the price in the used goods market exhibits exogenous volatility (see, e.g., Yin and Tang (2010)), rather than explicitly deriving it. For this reason and also for model simplicity, we assume that, if customers decide not to buy Product 1 in the first period (i.e., branch (C)), then they can either buy Product 2 or decide not to buy in the second period. We do not consider the option for them to buy a used product in this paper. Since Product 2 is an upgrade of Product 1, it is natural to assume that the price of Product 2 will be equal or higher than the price of Product 1, i.e., $p_2 \geq p_1$; and since the maximum valuation that customers can derive from Product 2 is θ, we also assume that $p_2 \leq \theta$. Given above we can derive the following demand expressions. The proofs for all the theoretical results are presented in Appendix B.

PROPOSITION 1. *When customers are offered with the guaranteed trade-in service, the demands for the product and the guarantee service in the two periods are as shown in Table 1.*

Table 1 Demands for Products and Guarantee Service

	(i) If $p_2 \leq p_2^b$	(ii) If $p_2^b < p_2 \leq p_2^a$	(iii) If $p_2 \geq p_2^a$
Period 1 product demand D_{R1}	$1-p_1+\frac{p_h+w(1-2\alpha)}{2}$	$\frac{2+\theta-2p_1+p_h-p_2+w(1-2\alpha)}{2+\theta}$	$1-p_1+\frac{p_h}{2}$
Period 2 product demand D_{R2}	$1-\frac{p_2}{\theta}$	$\frac{\theta(4+2\theta-2p_1+p_h+w(1-2\alpha))-2p_2(1+\theta)}{2\theta(2+\theta)}$	$\frac{2(\theta-p_2)+w(1-2\alpha)}{2\theta}$
Period 1 guarantee demand D_{G1}	$1-p_1+\frac{p_h+w(1-2\alpha)}{2}$	$\frac{2+\theta-2p_1+p_h-p_2+w(1-2\alpha)}{2+\theta}$	$\frac{\theta-p_2+w(1-2\alpha)}{\theta}$
Period 2 option exercises D_{G2}	$\frac{1}{2}\left(1-p_1+\frac{p_h+w(1-2\alpha)}{2}\right)$	$\frac{2+\theta-2p_1+p_h-p_2+w(1-2\alpha)}{2(2+\theta)}$	$\frac{\theta-p_2+w(1-2\alpha)}{2\theta}$

Note: $p_2^a = \frac{\theta(2p_1-p_h)}{2} + w(1-2\alpha)$ and $p_2^b = \frac{\theta(2p_1-p_h)}{2} - \frac{\theta w(1-2\alpha)}{2}$.

The main difference between the three demand cases are as follows. In case (iii), when the new generation Product 2's price is sufficiently high (i.e., $p_2 \geq p_2^a$), all branches (A), (B) and (C) in Figure 2 are likely to be chosen by end-customers. That is, a customer may choose to buy Product 1 with guarantee, or buy Product 1 without such a service, or not buy Product 1 at all, respectively. This happens because, if p_2 is high, then only high valuation customers are considering buying Product 2 and so they would like to buy the trade-in price guarantee in order to be able to upgrade without incurring too much cost; but others, i.e., those with low valuations, are either not thinking to do so and hence do not require the insurance or are content with the spot market to finance their upgrades when p_u turns out to be high. But, when the second period p_2 is relatively low, i.e., $p_2 \leq p_2^a$, even low valuation customers are planning to buy Product 2. So, anyone who buys Product 1 buys the guarantee service in order to lower her upgrading cost and so branch (B) does not come into the picture anymore. What differentiates case (i) from (ii) is only the indifference point between branches (A) and (C). Also, p_2^a increases in trade-in price w, i.e., the guarantee option becomes more valuable for the customers in reducing their upgrading costs as w increases.

Note that, depending on the branches, not only are the utilities derived different for the customers, but the revenues collected by the retailer are also different. For example, in branch (A1) the retailer receives $(p_1 + \alpha w)$ and p_2, respectively, from the customers in the two periods, receives p_s from salvaging the used products and pays w in period 2 to those who exercise the option. In branch (A2), the retailer does not receive p_s and does not have to pay w. Lastly, in branch (A3), the retailer only receives $(p_1 + \alpha w)$ in period 1. Taking all possible such revenues/costs, we get the following profit functions for the retailer over the two periods.

$$\Pi_{R1} = p_1 D_{R1} + \alpha w D_{G1} \text{ and } \Pi_{R2} = p_2 D_{R2} + (p_s - w) D_{G2}. \tag{1}$$

Note that the expectation with respect to the spot market price p_u has already been considered in calculating the demand functions and so is not included in the profit functions.

4. Value of Guaranteed Trade-in Price Service

In order to determine the source of the value (if any) of the trade-in price guarantee service for consumers and the retailer, we analyze and compare the following two models in this section.

No Guarantee (Pure Sell) Model (Model NG): In this case, the retailer sells the product without the guarantee service. So, consumers can only buy the product in anticipation of a volatile spot trade-in market and use that market to sell at the end of period 1 once the randomness realizes. Effectively, this means only branches B and C of Figure 2 are available for the consumers.

Guaranteed Trade-in Model (Model G): This is the model discussed in §3.1 above and shown in Figure 2 where the guarantee service gives consumers an option to use either the spot market or the retailer for their used products depending on the realization of the price in the spot market.

For tractability purpose, in §§4 and 5 we focus on a "static" pricing scenario whereby the retailer commits to equal prices for both product generations at the beginning of period 1, i.e., $p_1 = p_2$. However, we also study in §5.1 how our insights are affected by "dynamic" pricing when p_2 is set only in period 2 and is not necessarily equal to p_1 (i.e., the framework discussed in §3).

4.1 Analysis of the No Guarantee Model

In this case, the retailer is selling two generations of the product over two periods without guaranteed trade-in price. So, the demand functions can be derived by only considering branches (B) and (C) in Figure 2. Following a similar, but simpler, process as in Proposition 1, we have the following demand functions for the retailer for each period:

$$D_{R1} = 1 - p_1 + \frac{p_h}{2}; \text{ and } D_{R2} = 1 - \frac{p_2}{\theta}.$$

D_{G1} and D_{G2} are irrelevant here since there is no guarantee. The retailer's total profit function over the two periods is given by:

$$\Pi_R = p_1 D_{R1} + p_2 D_{R2}.$$

While the retailer's value is given by its profit, that for the consumers will be given by the consumer surplus (CS) obtained by the entire pool as follows:

$$CS = \underbrace{\int_{\bar{p}}^{1} u_B dv}_{\text{CS on Branch (B)}} + \underbrace{\int_{0}^{\bar{p}} u_C dv}_{\text{CS on Branch (C)}}$$

where u_B, and u_C are provided in §3.2, and $\bar{p}$ represents the valuation of marginal customer who is indifferent between choosing branches (B) and (C), respectively.

An equilibrium analysis of the retailer's problem leads to the following result:

Proposition 2. *(***No Guarantee Model***) The optimal values of the relevant decisions, the retailer's profit and the consumer surplus for the no guarantee model under static pricing are:*

- *Retail prices in two periods:* $p_1 = p_2 = \frac{\theta(4+p_h)}{4(1+\theta)}$;
- *Retailer's demands in two periods:* $D_{R1} = \frac{4+p_h(2+\theta)}{4(1+\theta)}$ *and* $D_{R2} = \frac{4\theta - p_h}{4(1+\theta)}$;
- *Retailer's total profit:* $\Pi_R = \frac{\theta(4+p_h)^2}{16(1+\theta)}$; *and*
- *Consumer surplus:* $CS = \frac{16\theta^2 - \theta(16+8p_h-p_h^2)+4(2+p_h)^2}{32(1+\theta)}$.

4.2 Analysis of Guaranteed Trade-in Model

In this case, the retailer offers the trade-in price guarantee service. So, all the branches in Figure 2 are available to the consumers and demands are given by Proposition 1. The retailer's profit function is given by:

$$\Pi_R = p_1 D_{R1} + \alpha w D_{G1} + p_2 D_{R2} + (p_s - w) D_{G2}. \tag{2}$$

The consumer surplus CS is given by:

$$CS = \underbrace{\int_{\bar{\bar{p}}}^{1} u_A dv}_{\text{CS on Branch (A)}} + \underbrace{\int_{\bar{p}}^{\bar{\bar{p}}} u_B dv}_{\text{CS on Branch (B)}} + \underbrace{\int_{0}^{\bar{p}} u_C dv}_{\text{CS on Branch (C)}}$$

where u_A, u_B, and u_C are provided in §3.2, and $\bar{\bar{p}}$ and $\bar{p}$ represent the valuation of marginal customers who are indifferent between choosing branches (A) and (B) and between branches (B) and (C), respectively.

Due to complexity in the demand functions of this case, we first analyze the retailer's problem assuming that the product upgrade in the second period is negligible, i.e., $\theta = 1$. This represents reality for products like musical/sports equipment. For a general $\theta > 1$, as is the case for automobile/electronics products, we resort to a numerical study. The analytical result for $\theta = 1$ is described in the result below. The numerical outcomes for a general θ will be omitted here but they will be used to compare with the no guarantee model in the next subsection.

PROPOSITION 3. (**Guaranteed Trade-in Price Model**) *The optimal values of the relevant decisions, the retailer's profit and consumers' surplus for the guaranteed trade-in model under static pricing and negligible product upgrade, i.e., $\theta = 1$, are characterized in Table 2.*

Table 2 Optimal Outcomes for Guaranteed Trade-in Price Model under Static Pricing and $\theta = 1$

	(i) If $p_s \le \frac{2p_h}{3}$	(ii) If $p_s \ge \frac{2p_h}{3}$
Trade-in price w	$\frac{p_h+3p_s}{6(1-2\alpha)}$	$\frac{p_h}{2(1-2\alpha)}$
Retail price $p_1(=p_2)$	$\frac{3+p_h}{6}$	$\frac{4+2p_h-p_s}{8}$
Period 1 product demand D_{R1}	$\frac{3+2p_h}{6}$	$\frac{4+2p_h+p_s}{8}$
Period 2 product demand D_{R2}	$\frac{6-p_h+3p_s}{12}$	$\frac{4+p_s}{8}$
Period 1 guarantee demand D_{G1}	$\frac{1+p_s}{2}$	$\frac{4+2p_h+p_s}{8}$
Period 2 option exercises D_{G2}	$\frac{1+p_s}{4}$	$\frac{4+2p_h+p_s}{16}$
R's total profit Π_R	$\frac{p_h^2+6p_h+3(p_s^2+2p_s+4)}{24}$	$\frac{4p_h(2+p_s)+(4+p_s)^2}{32}$
Consumer surplus CS	$\frac{p_h^2+2p_h+4+2p_s+p_s^2}{16}$	$\frac{4p_h^2+2p_h(4+p_s)+(4+p_s)^2}{64}$

4.3 Value of the Guaranteed Trade-in Service

In this section we compare the optimal outcomes for the two models – no guarantee and trade-in guarantee – to understand whether the guarantee strategy is indeed valuable for the retailer and/or consumers as its growing popularity would suggest. In this context, we first start with the case when Product 2 is not really an upgrade compared to Product 1, i.e., $\theta = 1$. The comparison for this case is shown in Table 3, which directly leads to Proposition 4.

Table 3 Guaranteed Trade-in vs. No Guarantee Models (G – NG) under Static Pricing and $\theta = 1$

	(i) If $p_s \le \frac{2p_h}{3}$	(ii) If $p_s \ge \frac{2p_h}{3}$
Retail price $p_1(=p_2)$	$\frac{p_h}{24} > 0$	$\frac{p_h-p_s}{8} > 0$
Period 1 product demand D_{R1}	$-\frac{p_h}{24} < 0$	$\frac{p_s-p_h}{8} < 0$
Period 2 product demand D_{R2}	$\frac{p_h+6p_s}{24} > 0$	$\frac{p_h+p_s}{8} > 0$
Total product demand $D_{R1}+D_{R2}$	$\frac{p_s}{4} > 0$	$\frac{p_s}{4} > 0$
R's total profit Π_R	$\frac{p_h^2+24p_s+12p_s^2}{96} > 0$	$\frac{-p_h(p_h-4p_s)+p_s(8+p_s)}{32} > 0$
Consumer surplus CS	$\frac{-p_h^2+4p_s(2+p_s)}{64} \begin{cases} \le 0, \text{ if } p_s \le p_s' \\ > 0, \text{ otherwise} \end{cases}$	$\frac{-(p_h-p_s)^2+2p_s(4+p_s)}{64} > 0$

Note: $p_s' = \sqrt{1+\frac{p_h^2}{4}} - 1$.

PROPOSITION 4. *(***Value of Guarantee Service***) In the scenario that the retailer follows static pricing and the degree of upgrade for the durable product is negligible, i.e.,* $\theta = 1$*, provision of the trade-in price guarantee results in:*

(1) a higher profit for the retailer;

(2) a higher consumer surplus unless p_h *is relatively very high; and*

(3) higher retail price and total demand over two periods, but lower demand in the first period.

So, the retailer indeed gains value from the guaranteed trade-in price strategy even though it is a service geared to benefit the customers. In fact, the strategy is profitable for the retailer even when it is provided for free, i.e., the retailer does not charge any premium for the service ($\alpha = 0$). A main contributor to the positive effect on the retailer is that this service encourages more customers to upgrade their products in the second period, as compared to the case without such a service. Recall that without the guarantee, customers would be willing to pay a premium to upgrade their products only when the spot market price turns out to be high. However, with the guarantee, customers might be willing to upgrade even when the spot market price is low, especially if they are assured a good trade-in value. Compared to the pure selling model without the guarantee, this protection option decreases the cost of upgrading, which in turn increases the number of customers who purchase new products in period 2 as indicated by the positive increase in D_{R2}. The retailer increases the selling price in order to extract a portion of surplus gained by the period 2 customers; this slightly decreases the sales in the first period because the same price is charged for period 1 customers. Overall, the retailer always benefits from the guarantee due to its demand-enhancing effect (an increase in demand for upgraded product and hence the total demand) together with its revenue-enhancing effect (an increase in the retail price). Moreover, this benefit increases in both the salvage value, p_s, and the volatility of the spot market p_h. Note that an increase in p_s enables the retailer to offer more protection (measured by the trade-in price w in Table 2), which, in turn, leads to a higher demand-enhancing effect. Although a higher p_h increases the volatility, for customers who have bought the guarantee it effectively means an increased upside potential of their trade-in value (since they are protected from the downside risk). This enables the retailer to charge higher prices yielding a higher revenue-enhancing effect of the guarantee.

As for the customers, the guarantee strategy affects them in two contrasting ways: (i) on one hand, it leads to a reduction in the upgrading cost, hence, creates surplus in period 2 for them; (ii) on the other hand, it increases the retail price, which dampens their utility and hence demand in period 1. The former surplus-enhancing effect increases in salvage price p_s, whereas the latter surplus-destroying effect increases in p_h. Also, note that the effect of former is stronger than that of the latter. Hence, unless p_h is very high compared to p_s (i.e., the risk premium is high), which is a relatively rare sub-region in a realistic parametric space, customers generally benefit from the guarantee. This is expected because of the protection of the resale value of their used products as well as their potential for gaining from the spot market. The above discussion on the surplus-enhancing or destroying effect also indicates that the value of the guarantee to the consumers would increase in salvage value p_s and decrease in the volatility of the spot market p_h.

Finally, we observe from Table 2 in Proposition 3 that neither Π_R nor CS in the guarantee model depends on the premium parameter α. It simply plays a transactional role (affecting only w) between the retailer and consumers, and does not impact variables such as sales and prices. Consequently, the value of the guarantee to the retailer and the consumers is not dependent on it.

The analytical comparison of the models in Proposition 4 is based on $\theta = 1$. For a general level of product upgrade $\theta > 1$, the numerical comparison of the two models is presented in the Table C.1. in the Appendix C. Based on the table we can conclude that the guarantee service continues to be a win-win outcome for the retailer and consumers even for $\theta > 1$ due mainly to its demand-enhancing effect (as measured by the increase in the total demand). Moreover, we also note that, in general, as the degree of product upgrade θ increases, the retailer comparatively benefits more from the guarantee service and the consumers comparatively less. A possible rationale for the above pattern is as follows. As θ increases, Product 2 becomes vertically more differentiated and customers start preferring the second generation more over the first generation. Hence, without the guarantee service, customers tend to delay their purchases, waiting to buy the new generation in the second period. Guaranteed trade-ins, on the other hand, incentivizes them not to wait, and buy the first generation, i.e., the loss in period 1 demand as θ increases is not as high with the guarantee service compared to the no guarantee case. Effectively, this service counteracts the strategic customer behavior and the retailer is able to extract this benefit. On the other hand, guaranteed trade-ins hurt customers as θ goes up because it inter-temporally segments them more in that case.

5. Deconstruction of the Value of the Guarantee Service

While the previous section established that the guarantee strategy is always valuable for the retailer and will mostly benefit the customers, our goal in this paper is to go one step further and investigate the source of this value. Recall that there are two features of the guarantee strategy that separate it from a pure selling one: i) the *insurance/warranty* about the resale value protection, and ii) the *optionality* to use the spot market if the price there turns out to be higher than the guaranteed value. In this section, we deconstruct the overall value obtained by the retailer and consumers to identify the value added individually by insurance and optionality features of the guarantee strategy. For simplicity, we again focus on the case when $\theta = 1$.

In order to do so, we first develop and analyze an "intermediate" model that conceptually lies between the no guarantee (NG) and the guaranteed trade-in (G) ones. We term it as the *obligatory* guarantee model (Model OG). In this model, the retailer guarantees a trade-in price of w to the consumers who buy the guarantee service by paying a premium αw at the beginning of period 1, but this service now specifies that the consumers are *obligated* to return their used products to the retailer, i.e., they do not have the option to use the spot market even if the price in that market turns out to be higher than the guaranteed w. This model, clearly, serves the protection role but not the potential role, when compared to the no guarantee model.

In the context of Figure 2, the consumers now consider branches (A1), (B) and (C). The retailer's total profit can be written as:

$$\Pi_R = p_1(D_{R1} + D_{R2}) + \alpha w D_{G1} + (p_s - w) D_{G2}$$

and consumer surplus function can be written as

$$CS = \underbrace{\int_{\bar{\bar{p}}}^{1} u_A dv}_{\text{CS on Branch (A) without return option}} + \underbrace{\int_{\bar{p}}^{\bar{\bar{p}}} u_B dv}_{\text{CS on Branch (B)}} + \underbrace{\int_{0}^{\bar{p}} u_C dv}_{\text{CS on Branch (C)}}$$

where u_A is the utility of branch (A) without return option (i.e., $u_A = (v - p - \alpha w) + \mathrm{E}_{p_u} \max(v - p + p_u, p_u)$), u_B and u_C are provided in §3.2, and $\bar{\bar{p}}$ and $\bar{p}$ represent the valuation of marginal customers who are indifferent between choosing branches (A) without return option and (B) and between branches (B) and (C), respectively. In order to be analytically consistent with NG and G models we focus on static pricing and $\theta = 1$. Analyzing the retailer's problem in the OG model, we derive the following.

PROPOSITION 5. *(***Obligatory Guarantee Model***) The optimal values of the relevant decisions, the retailer's profit and consumer surplus for the obligatory guarantee model under static pricing and a negligible degree of product upgrade, i.e., $\theta = 1$, are characterized in Table 4.*

The primary reason for the obligatory guarantee model is that when we compare it to the no guarantee one, we obtain the value of the insurance role of the guarantee strategy about the protection of the resale value. On the other hand, comparison of this model with the trade-in guarantee one establishes the value of the optionality role in the guarantee offer. We discuss them in the next two subsections.

Table 4 Optimal Outcomes in the Obligatory Guarantee Model under Static Pricing and $\theta = 1$

	(i) If $p_s \le \tilde{p}_s$	(ii) If $\tilde{p}_s < p_s \le p_h(1-2\alpha)$	(ii) If $p_s \ge p_h(1-2\alpha)$
Trade-in price w	$w \sim 0$	$\frac{p_s+p_h}{2(1-\alpha)}$	p_h
Retail price $p_1(=p_2)$	$\frac{4+p_h}{8}$	$\frac{2+p_h}{4}$	$\frac{2-p_s+2p_h(1-\alpha)}{4}$
R's period 1 demand D_{R1}	$\frac{4+3p_h}{8}$	$\frac{2+p_h}{4}$	$\frac{p_s+2(1+\alpha p_h)}{4}$
R's period 2 demand D_{R2}	$\frac{4-p_h}{8}$	$\frac{2(1+p_s)-p_h}{4}$	$\frac{p_s+2(1-\alpha p_h)}{4}$
Guarantee demand D_{G1} = Option Exercises D_{G2}	N.A.	$\frac{2(1+p_s)-p_h}{4}$	$\frac{p_s+2(1-\alpha p_h)}{4}$
R's total profit Π_R	$\frac{(4+p_h)^2}{32}$	$\frac{(2+p_s)^2+(p_h-p_s)^2}{8}$	$\frac{(2+p_s)^2-4\alpha p_h(p_s-p_h(1-\alpha))}{8}$
Consumer surplus CS	$\frac{5p_h^2+8p_h+16}{64}$	$\frac{(2+p_s)^2+(p_h-p_s)^2}{16}$	$\frac{(2+p_s)^2+(2\alpha p_h)^2}{16}$

Note: $\tilde{p}_s = (2(p_h-2)+\sqrt{2(8-p_h^2)})/4$.

5.1 Value of Protection Only due to Resale Value Insurance

In this subsection, to determine the value of the resale price protection role of the guarantee service for the retailer and consumers, we compare the optimal outcomes for the obligatory guarantee and no guarantee models. This entails comparing Propositions 2 and 5, which is provided below.

Proposition 6. (**Value of Resale Price Protection Only**) *When the retailer employs static pricing and the level of upgrade is negligible, i.e., $\theta = 1$, offering the obligatory guarantee (for resale price protection only) results in:*

(1) a higher profit for the retailer;

(2) a lower consumer surplus, unless p_h is sufficiently low; and

(3) a higher retail price and second period demand, but a lower first period demand for the retailer; and a higher total demand, if p_h is sufficiently low.

Recall that the protection provided by the retailer in the guaranteed trade-in strategy translates into more upgrades in period 2 and higher prices, which lead to higher retail profits. The same holds true also for OG model, where the protection is offered in an obligatory fashion. As for the consumers, obligatory protection affects their upgrading costs in a slightly different manner than when the protection is coupled with optionality. In the latter, the consumers always choose the best option to finance their upgrading decision, i.e., exercising the guarantee if the spot trade-in market price turns out to be low or selling in the spot market otherwise. However, in the former, the consumers are stuck with the first option, and receive a trade-in value irrespective of the spot market realization. Compared to the no guarantee model, this still creates a positive surplus for the consumers as long as the protection is sufficiently high, which is the case when the spot market volatility is sufficiently low, i.e., when p_h is low. Indeed, both the retailer and the consumers are better off with (obligatory) protection as long as p_h is below a threshold value. But when p_h goes above the threshold, consumers prefer no (obligatory) protection as they then lose the opportunity to generate potentially a big surplus by selling their used goods in the spot market (when p_u turns out to be p_h), while incurring higher costs in the form of higher prices. This dampens the demand for the obligatory guarantee and hence reduces the value of protection from the retailer's perspective. But, since the retailer can still charge a higher price for giving the guarantee, it continues to extract positive value from the OG model (compared to the no guarantee one). Indeed, the sensitivity analysis indicates that the lower the spot market volatility, the more benefit the retailer and consumers can obtain from the obligatory guarantee service. Also, both the retailer and consumers prefer a higher salvage value.

However, in terms of the parameter α (i.e., the fixed premium for obtaining the guarantee service), the retailer and consumers have opposite preferences. Specifically, it is interesting to observe that higher premiums improve the value of the obligatory guarantee to the consumers, while reducing its value to the retailer. As α increases, in the OG model, the level of protection (i.e., w) increases and gradually approaches p_h. It is known that offering resale value protection increases the retail price. However, this up push on the retail price is dampened when α increases. Consequently, consumers prefer the guarantee more if there is an increase in α, due to both an increase in the trade-in value and a lower increase in the retail price. Due to the same reasons, the retailer actually prefers the guarantee service less as α increases.

Finally, if we compare the value of obligatory guarantee (in Proposition 6) to that of the guarantee coupled with optionality (in Proposition 4), we observe that the latter is more likely to benefit the consumers. That is, the threshold value of p_h below which the consumers are better off under option-included guarantee service is higher than that under obligatory guarantee service. This difference is caused by the optionality feature, the value of which is discussed in the next subsection.

5.2 Value of Potential due to Optionality

In this subsection, we try to understand the value of the potential due to optionality feature in the trade-in guarantee service whereby consumers can return their used products in the spot market if it turns out to be higher than the guaranteed trade-in value from the retailer. This entails comparing the results of Propositions 3 (for the guaranteed trade-in price G model) and 5 (for the obligatory guarantee OG model).

PROPOSITION 7. (**Value of Optionality**) *When the retailer employs static pricing and the level of upgrade is negligible, i.e., $\theta = 1$, offering optionality in the guarantee service that provides consumers the option of using the spot market in addition to trading-in with the retailer results in:*

(1) a higher profit for the retailer;

(2) a higher consumer surplus; and

(2) a lower retail price and trade-in value, and higher demands in both periods.

Note that the only difference between guaranteed trade-in and obligatory guarantee services is the presence of optionality in the former policy. As shown in the above proposition, this very feature is the crux of the service that really creates a win-win outcome for both retailer and consumers. The positive impact on the consumers comes from the fact that the optional protection enables them to optimize their upgrading decision and choose whatever is best from their perspective depending on the spot market realization. The consumers finance their upgrades by either exercising guaranteed trade-in option if p_u turns out to be low or selling their used goods in the spot market otherwise. Overall, this decreases the cost of upgrading for the consumers and encourages them to buy more across the two periods. Interestingly, optionality also decreases the retail price and the trade-in value. Compared to obligatory protection, the retailer can reduce the amount of protection (i.e., trade-in price w) in guaranteed trade-in because there is no need to insure the consumers in the case of high spot market state. This reduces the cost of protection for the retailer, which frees some cash to provide discount on the selling price. This in turn increases the sales in the first-period under model G compared to model OG. To summarize, demands in both periods 1 and 2 would increase under optional protection model, which results in a positive return for the retailer. As suggested by this discussion, as the spot market volatility p_h increases, it also opens up the possibility for more consumers to take advantage of the upside potential (since they are protected from downside risk). This also benefits the retailer because more consumers would prefer option-included guarantee service under such circumstances. Indeed, our sensitivity analysis states that an increase in p_h would lead to a higher value of the optionality feature for both the retailer and consumers, whereas an increase in the salvage value p_s would reduce its value for both. In terms of the premium to be paid for obtaining the guarantee service α, we observe that higher premiums reduce the value of the (additional) optionality to the consumers but improve its value to the retailer. This is in contrast with the impact of α on the value of the obligatory guarantee discussed earlier in the previous subsection.

6. Model Generalizations

The analysis until now about the value of the trade-in price guarantee strategy is based on a number of assumptions. Two of them we consider to be major are: i) The retailer commits to equal prices for both periods at the beginning of period 1, and ii) the guarantee service is offered by the retailer itself. In this section, we relax them, one at a time, to study whether the value-added property of the guarantee strategy is general enough and remains valid even under those circumstances.

6.1 Dynamic Pricing

What if the retailer cannot commit to prices, but decides on them separately in each period, as described in Figure 1 of §3. That is, the retailer decides on the price of Product 1, p_1, at the beginning of period 1 and of Product 2, p_2, at the beginning of period 2. Note that p_2 is decided simultaneously with the realization of the spot market price and so is not contingent on the realization. The demand and profit functions for the dynamic pricing variant of the trade-in guarantee model are already provided in §3 (see Table 1 and Equation (1)). Similarly, we can develop the demand and profit function for the no guarantee model assuming the trade-in value is equal to zero. Analysis of the no guarantee model results in the following.

Proposition 8. (**No Guarantee Model under Dynamic Pricing**) *The optimal values of the relevant decisions, the retailer's profit and the consumer surplus for the no guarantee model under dynamic pricing are*[6] *in Table 5.*

Table 5 Optimal Outcomes in the No Guarantee Model under Static and Dynamic Pricing

	Static Pricing	Dynamic Pricing (i) If $\theta \le 1+\frac{p_h}{2}$	Dynamic Pricing (ii) If $\theta \ge 1+\frac{p_h}{2}$
R's period 1 price p_1	$\frac{\theta(4+p_h)}{4(1+\theta)}$	$\frac{\theta(4+p_h)}{4(1+\theta)}$	$\frac{2+p_h}{4}$
R's period 2 price p_2	$\frac{\theta(4+p_h)}{4(1+\theta)}$	$\frac{\theta(4+p_h)}{4(1+\theta)}$	$\frac{\theta}{2}$
R's period 1 demand D_{R1}	$\frac{4+p_h(2+\theta)}{4(1+\theta)}$	$\frac{4+p_h(2+\theta)}{4(1+\theta)}$	$\frac{2+p_h}{4}$
R's period 2 demand D_{R2}	$\frac{4\theta-p_h}{4(1+\theta)}$	$\frac{4\theta-p_h}{4(1+\theta)}$	$\frac{1}{2}$
R's total profit Π_R	$\frac{\theta(4+p_h)^2}{16(1+\theta)}$	$\frac{\theta(4+p_h)^2}{16(1+\theta)}$	$\frac{4\theta+(2+p_h)^2}{16}$
Consumer surplus CS	$\frac{16\theta^2-\theta H_0+4(2+p_h)^2}{32(1+\theta)}$	$\frac{16\theta^2-\theta H_0+4(2+p_h)^2}{32(1+\theta)}$	$\frac{4\theta+(2+p_h)^2}{32}$

Note: $H_0 = 16+8p_h-p_h^2$.

Comparison of the expressions in Table 5 indicates that static and dynamic pricing would result in identical optimal outcomes if the product upgrade θ is not too high, i.e., $\theta \le 1+\frac{p_h}{2}$. However, when the second generation is a much higher upgrade than the first generation, dynamic pricing would lead to lower retail prices but higher demand in period 1 and higher retail prices but lower demand in period 2. This suggests that in the case of a major product upgrade, and only then, dynamic pricing enables the retailer to segment the customer pool inter-temporally, which in turn increases his profit compared to static pricing. However, this occurs at the expense of the reduced level of surplus for the consumers.

[6] The results for the static pricing case presented in Proposition 2 are included here for comparison purpose.

The analysis of the dynamic pricing case for the guaranteed trade-in model is analytically more difficult, especially for a general θ. Hence, we numerically study the guarantee model and compare it with the no guarantee model under dynamic pricing. Based on the numerical experiments, we can deduce whether dynamic pricing qualitatively changes the value of the guarantee service, relative to static pricing (and, if so, how?). To do so, we present the numerical differences between the models with and without guarantee under static and dynamic pricing policies in Table C.2. of the Appendix C. By comparing left (under static pricing) and right (under dynamic pricing) panels of Table C.2., we observe that the value of the guarantee strategy to the retailer and consumers is quite consistent regardless of how retail prices are set. That is, the guarantee strategy is, in general, valuable for both the retailer and the consumers. This is mainly driven by lower upgrading cost that leads to a higher demand in the second period. However, dynamic pricing could significantly impact the retailer's pricing strategy before and after guarantee is offered. Recall from §4.3 that under static pricing (the retailer commits to the same price for both periods), the retailer was able to charge a higher retail price due to the customers' higher willingness to upgrade in the second period after the guarantee service is offered. The new feature of dynamic pricing is that the retailer can now set different prices for products sold in the two periods. In this case, offering guarantee under dynamic pricing does not actually lead to higher prices as in the case of static pricing. Hence, the retailer can then induce more customers to upgrade in period 2 by offering them a better resale value. Indeed, although the guarantee service results in win-win outcome under both pricing policies, providing customers with a better resale value without necessarily increasing the retail prices means the value of guarantee service is even higher for the consumer (and lower for the retailer) under dynamic pricing, compared to the static one.

6.2 Third-party Guarantee Provider

If the trade-in price guarantee is not provided by the retailer but by a third party guarantor, will such a guarantee be still valuable for the retailer and consumers? Recall from §1 that one of the first popular instances of such guarantee service was indeed provided by a third party (Techforward). In order to be consistent with our analysis of the model in §4, we consider a static pricing scenario whereby the retailer first sets the prices for both generations and the guarantor follows by setting the guaranteed return price of w for a premium of αw, both at the beginning of period 1. The rest of the events unfolds like the guaranteed trade-in model of §4 except that the customers who want to exercise the guarantee option need to return their used products to the third-party guarantor who then salvages them for p_s per unit. So, the demand notation D_{G1} and D_{G2} (representing the number of customers who purchase the guarantee service in the first period and how many of them would exercise the option in the second period, respectively) will appear in the guarantor's profit functions. Using backward induction, we are able to fully characterize the equilibrium decisions, profits and consumer surplus under static pricing. For brevity, we only present the profits and the consumer surplus in the result below.

PROPOSITION 9. *(**Guarantee Model with a Third-party Guarantor**) The equilibrium values of profits and the consumer surplus in the model with the guarantee service offered by a third party and under static pricing are summarized in Table 6:*

Table 6 Equilibrium Outcomes in the Guarantee Model with a Third Party Guarantor under Static Pricing

	(i) If $p_h \le \min(\bar{p}_h, 1)$	(ii) If $p_h \ge \min(\bar{p}_h, 1)$ & $p_s \le \min(\bar{p}_s, p_h)$	(iii) If $p_h \ge \min(\bar{p}_h, 1)$ & $p_s \ge \min(\bar{p}_s, p_h)$
R's total profit Π_R	$\frac{\theta(4+p_h)^2}{16(1+\theta)}$	$\frac{\theta(4+p_h)^2}{16(1+\theta)}$	$\frac{(p_s+\theta(7+2p_h))^2}{16\theta(3+4\theta)}$
G's total profit Π_G	$\frac{p_s(4+p_h(2+\theta))}{8(1+\theta)}$	$\frac{p_s(4\theta-p_h)}{8(1+\theta)}$	$\frac{(\theta(8\theta-2p_h-1)+p_s(5+8\theta))^2}{32\theta(3+4\theta)^2}$
Consumer surplus CS	$\frac{16\theta^2-\theta H_0+4(2+p_h)^2}{32(1+\theta)}$	$\frac{16\theta^2-\theta H_0+4(2+p_h)^2}{32(1+\theta)}$	$\frac{320\theta^4+\theta^3 H_1+\theta^2 H_2+\theta H_3+29p_s^2}{64\theta(3+4\theta)^2}$

Note: $\bar{p}_h = \frac{4(\theta-1)}{3+\theta}, \bar{p}_s = \theta\left((4+p_h)\sqrt{\frac{3+4\theta}{1+\theta}} - (7+2p_h)\right), H_1 = 32p_h^2 - 128p_h + 128p_s - 72, H_2 = 116p_h^2 + 260p_h - 64p_hp_s + 64p_s^2 - 16p_s + 101, H_3 = 72p_h^2 + 288p_h - 52p_hp_s + 88p_s^2 - 98p_s + 288$, and $H_0 = 16 + 8p_h - p_h^2$.

In the following comparison, we will focus on the impact of guarantee and its provider on the retailer and consumers. First of all, we have the following result:

PROPOSITION 10. *Assuming static pricing and a negligible product upgrade ($\theta = 1$), we can rank the no guarantee model and the two guarantee models with different service providers as follows:*

– *From the retailer's profit perspective*

$$\text{Guarantee by retailer} \ge \text{Guarantee by third-party} \ge \text{No guarantee}$$

– *From consumers surplus perspective, the ranking is the following, unless p_h is significantly higher than p_s:*

$$\text{Guarantee by retailer} \ge \text{No guarantee} \ge \text{Guarantee by third-party;}$$

otherwise, guarantee by retailer hurts the consumers the most while the ranking between the other two models stays the same.

Note that the profit for the third party is positive only when she is offering the guarantee service. Indeed, our further numerical experiments indicate that the above insight in Proposition 10 remains valid for a general degree of product upgrade θ under either static or dynamic pricing. Recall that we have already seen the validation of this when we compared the no guarantee model and the retailer-offered guarantee model for a general θ under static pricing in the end of §4 and under dynamic pricing in §6.1. To verify Proposition 10 for a general θ under either static or dynamic pricing, we only need to compare the third-party offered guarantee model with the no guarantee and the retailer-offered guarantee model. Again, our numerical study demonstrates that both the static and dynamic pricing scenarios generate a consistent outcome.[7]

An immediate implication is that when the guarantee is offered by a third-party rather than the retailer, its value decreases for the consumers; actually, they prefer the no-guarantee scenario compared to a guarantee from a third party because the latter leads to lower protection and higher prices, both of which hurts the consumers' surplus when compared to the no-guarantee scenario. The value of the guarantee decreases even for the retailer because of lower demand; however, the value is still positive when compared to the no-guarantee scenario. So, the consumers always prefer no-guarantee over a guarantee service provided by a third party and consider a guarantee service provided by the retailer to be the best scenario, unless the spot market is highly volatile. The retailer wants a guarantee service to be provided regardless, although it prefers the provider to be himself. However, our analysis also indicates that the total demand is highest when the guarantee is provided by the retailer. This might have some adverse environmental consequences. From that perspective, a third-party guarantee seems to be the best option.

7. Managerial Insights and Concluding Remarks

Innovations in products and services have opened entirely new markets that did not exist a decade ago. However, they have also shortened the product life cycles and proliferated the number of offerings, together leaving the customers with some hard choices to make in terms of their purchasing/upgrading decisions. Companies are offering new services and ownership models for the customers to protect them against this frenzy. A popular one among these is the trade-in price guarantee (or resale value guarantee). This pricing strategy provides the consumers at the time of purchase the option of returning their used products at guaranteed trade-in prices when they decide to upgrade (within a specific period), rather than being subject to the whims of a volatile spot trade-in market. In this paper, we analyze (i) what role such price guarantees play in determining the pricing and purchasing decisions of firms and consumers, respectively, (ii) how they impact consumer surplus, and firm profits, and finally (ii) in what fashion the different features of the guarantee service contribute to its impact on consumer surplus and firm profits.

In order to address these research questions, we develop a model in which a firm (retailer) sells two generations of a durable product in two periods to a set of forward-looking consumers. There is also a volatile spot market for transacting used goods. In the first period, the retailer sets the

[7] For space consideration, the numerical values are omitted here but are available upon request from the authors.

retail price of the first generation product, offers guaranteed trade-in for this product and collects an upfront fee in exchange for this service from the consumers. Consumers decide whether or not to buy the product from the retailer, and if yes, whether or not to also opt for the guarantee service that provides them with the insurance against any downside risk for the resale value of their used products. In the second period, the retailer sells the second generation (possibly upgraded) product, and spot market resale value for the first generation product is realized for the consumers. Depending on their first-period choices, consumers in the second period have the following choices: (i) whether to exercise the guarantee option or to use the spot market to sell their used products, and (ii) whether or not to buy the new upgraded product.

In order to answer the first question and shed light on the value of the guarantee, we analyze the above model with and without guaranteed trade-in service. As expected, the guarantee lowers the upgrading cost for the consumers by establishing an assured channel for them to trade-in their used goods. Especially, this comes in handy for the consumers when the spot market price turns out to be low. The provision of guarantee increases the retail prices in both periods, which dampens the demand for the first generation. But, this service also helps more consumers to upgrade, boosting the demand for the new generation in the second period. Indeed, the demand-enhancing effect in the second period trumps the negative effect on the first-period's demand. The demand- and revenue-enhancing effects of the guarantee service lead to sure winnings for the retailer. With regards to consumers, the overall value of guaranteed trade-in is generally positive, except for scenarios with high spot market volatility and low salvage value.

In order to understand the above effects, we decompose the overall value of the guaranteed trade-in strategy to its two contributing orthogonal factors: (i) value due to (resale value) protection and (ii) value due to optionality. To factorize the value, we develop an intermediate model that lacks the feature of optionality but has the protection component, and benchmark it against models with and without guaranteed trade-ins. Our analysis reveals that values associated with protection and optionality features change in opposite directions. Namely, the former increases in salvage value and decreases in volatility of the spot market, whereas the latter decreases in salvage value and increases in volatility of the spot market. Based on this and other results, we develop several findings. First, the retailer obtains value from both protection and optionality features of guaranteed trade-in service and its profits increase in both salvage value and volatility of the spot market. Specifically, any increase in salvage value benefits by amplifying the value of the protection component, whereas any increase in volatility of the spot market benefits by increasing the value of the optionality component. However, this is not the case for the consumers. The resale value protection part of the guarantee service by itself is actually harmful for them. However, the optionality feature adds so much value for them that overall they also benefit from the guaranteed trade-in service. We also analyze how the premium paid for the guarantee service affects the value decomposition, and find out that the more expensive (resp., cheaper) it is, the more protective (resp., optional) value it creates for the consumers. This suggests that consumers value protection (resp., optionality) more as they pay high (resp., low) premium price.

Finally, we develop two extensions to analyze the implications of dynamic pricing and third-party provision of guaranteed trade-ins. The first generalization shows that the guarantee strategy is still valuable for both the retailer and consumers, although the latter now benefits even more compared to that under static pricing. Interestingly, when the guaranteed trade-in policy is offered by the third party, the retailer still generates positive return with respect to no-guarantee. However, consumers now can be worse off due to the fact that a guarantee offered by a third party can both decrease the guaranteed resale value offered and increase the retail price, both of which decrease the consumers' surplus.

The above-mentioned results provide a number of important managerial insights about the real-life instances of trade-in price guarantee offers. First, recall that for the firm, the value of the guarantee is always positive and increases in salvage value and volatility of the spot market. This finding corroborates with the observation that the guarantee option has emerged first for the products that can be salvaged quickly and for which spot market volatilities are the highest. Note that the former condition is satisfied by the products whose secondary markets are quite liquid and hence have relatively high expected salvage values for the provider firm. Latter condition holds for products which has short life cycles and hence exposes the consumers to high spot market risk. At the intersection lies the high-tech products (e.g., smart-phones) and products in the automotive sector, where the entire industry introduces new models in relatively high volumes and in regular intervals[8]. This keeps a constant flow of the products into secondary market that reduces the access cost for the firms. Indeed, there are instances where companies have discontinued guarantee for models that have become almost commodity items, but have kept them for high-end products, i.e., for products with higher spot market volatility (e.g., see Hyundai's strategy in MotorTrend (2012)).

Second, the differentiating feature of guaranteed trade-in over other ownership models (such as traditional leasing model) is the provision of optionality. Our results show that the value of optionality is highest when the premium for this service is kept at its lowest value. This suggests

[8] Products like musical/sports equipment also satisfy the two conditions, although not due to technological reasons like automobile/electronics.

that guaranteed trade-in would be offered to the customers with as minimum premium as possible in order to differentiate the unique feature of this ownership model from the others. Indeed, guaranteed trade-in with zero (or neglible) premiums is exactly the format in which it is offered in many of the markets discussed in §1 (e.g., automotive, musical instruments, etc.).

Our comparative analysis between no guarantee, third party guarantee, and guarantee offered by the retailer suggests an optimal market structure for the provision of this service. Namely, it is most effective from both perspectives of consumer and provider when it is offered by the firm that sells the product. This finding also explains for the trend that has been recently observed in the industry. Namely, even though some of the examples of guaranteed trade-in are historically first offered by third-party guarantors (e.g., TechForward), they are quickly replaced by the guaranteed trade-in policies offered by the original sellers (e.g., Best Buy). Finally, it is important to point out that the benefits brought forth by the guaranteed trade-ins for the consumers and associated increase in demand for new goods might have some adverse consequences for the environment, and this needs to be kept in mind while touting the value of this service.

Our model can be extended in different ways. First, we assume that consumers who have bought the product in the first period will have no residual value of their used goods in the second period. Even though this assumption fits well with products such as musical instruments or strollers, one may relax this assumption by setting the low state of the spot market price to be a positive value. Recall that in the current paper, the random spot market price has a two-point distribution (high and low) with the value of the low state being zero. Moreover, this randomness is exogenous. One can also make the randomness to be endogenous that depends on the decisions in the first period. This extension would be quite involved since it will add another layer of interaction between the two periods. Third, we can make the degree of upgrade θ itself to be random, which would in turn increase consumers' valuation risk. Even though considering both product upgrade and spot market uncertainties complicates the analysis considerably, we predict that this would make the trade-in option more valuable for the consumers and hence more valuable for the retailers. Finally, our model can be extended to allow consumers to buy used products in the second period. This change would extend the tree of the consumers' choices in Figure 2 from seven branches/choices to ten which will significantly increase the complexity of the consumers' choice model and hence the demand derivation, profit function and the equilibrium analysis and comparison. We hope that our paper would act as a stepping stone for these extensions so that we can better understand the issue of trade-ins and associated product upgrade friendly pricing strategies.

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Appendices: Guaranteed Trade-in Price Strategy: Deconstructing Its Value for Consumers and Firms

Appendix A: Parameters and Decision Variables

Tale A.1: Parameters and Decision Variables

Symbol	Definition
t	Period; $t=1,2$
N	Potential market size for each period; $N=1$
v_t	Consumer valuation for new goods in period t; $v_1 \sim U[0,1]$, and $v_2=\theta v_1$
θ	Upgrade of Product 2 compared to Product 1; $\theta \geq 1$
p_t	Selling price of new goods in period t; $p_1 \in (0,1]$, and $p_2 \in [p_1, \theta]$
p_u	Random variable for spot market price; $p_u \in \{0, p_h\}$
p_h	Upside potential for spot market price; $p_h \geq 0$
p_s	Salvage value of per unit used good; $p_s \in (0, p_h]$
w	Trade-in price; $w \in (0, p_h]$
α	αw refers to upfront fee; $\alpha \in [0,1)$
D_{Rt}	Retailer's demand in period t
D_{G1}	Demand of the guarantee in period 1
D_{G2}	Demand of option exercises in period 2
Π_{Rt}	Retailer's profit in period t
Π_{G1}	Profit generated from selling the guarantee in period 1
Π_{G2}	Payment made to consumers who exercise their option in period 2
Π_R	Retailer's total profit
Π_G	Guarantor's total profit if the guarantee is offered by a third-party
CS	Consumer surplus

Appendix B: Proofs

Proof of Proposition 1. N customers arrive in the beginning of the first period. Their valuation of Product 1 (measured by v_1) is uniformly distributed between 0 and 1, i.e., $v_1 \sim U[0,1]$. Their valuation of Product 2 is $v_2 = \theta v_1$, where $\theta \geq 1$ measures the level of product upgrade in the second period. Without loss of generality, we normalize $N = 1$. We follow closely the decision tree presented in Figure 2 of the paper to derive the demand functions in two periods. First of all, a consumer entering in the first period has the following three options:

Option 1 – Branch (A): buy Product 1 at p_1 and the guaranteed trade-in option at an upfront fee αw;
Option 2 – Branch (B): buy Product 1 at p_1 without the trade-in price guarantee; and
Option 3 – Branch (C): do not buy Product 1 (or stay inactive in period 1).

To determine which branch to choose, customers need to evaluate the total expected utility across both periods, which is given below and also in Table 1 of the paper.

Table B.1 Customers' Total Expected Utilities across both Periods

Branch	Expected utilities
A	$u_{\mathrm{A}} = (v_1 - p_1 - \alpha w) + \mathrm{E}_{p_u} \max(v_2 - p_2 + w, v_2 - p_2 + p_u, p_u)$
B	$u_{\mathrm{B}} = (v_1 - p_1) + \mathrm{E}_{p_u} \max(v_2 - p_2 + p_u, p_u)$
C	$u_{\mathrm{C}} = 0 + \max(v_2 - p_2, 0)$

Their utility generated in the first period is straightforward. That is, $v_1 - p_1 - \alpha w$, $v_1 - p_1$ or 0 with respect to branch (A), (B) or (C), respectively. However, customers' utility in the second period is a bit complex since it is contingent on their choice made in the first period AND also the realization of the used goods price in the spot market (which could be either high at p_h or low at $p_l = 0$).

If the spot market price is high, i.e., $p_u = p_h$, with probability of $\frac{1}{2}$, due to the assumption that $w \leq p_h$, the trade-in option (if available) is dominated by the spot market. Hence, the decision tree presented in Figure 2 of the paper can be simplified as in Figure B.1.

According to Figure B.1, irrespective of the choice made in the first period, consumers' decision in the second period is between buying versus not buying Product 2. It is easy to conclude that consumers would buy if $v_2 = \theta v_1 \geq p_2$ (or equivalently, $v_1 \geq p_2/\theta$) and not buy otherwise.

If the spot market price turns out to be low (at zero) with probability of $\frac{1}{2}$, due to the assumption that $w \geq 0$, the spot market option is dominated by the trade-in option (if available). Hence, the decision tree presented in Figure 2 of the paper can be simplified as in Figure B.2.

According to Figure B.2, the second period choice is contingent on the first period decision. If the decision in the first period is to buy Product 1 with guarantee, then the customer will trade-in and buy Product 2 if $v_2 - p_2 + w \geq 0$ (or equivalently, $v_1 \geq \frac{p_2 - w}{\theta}$), or otherwise, the customer would sell at spot market and not buy. The indifference point of Branch B or C remains the same as that in the case when $p_u = p_h$.

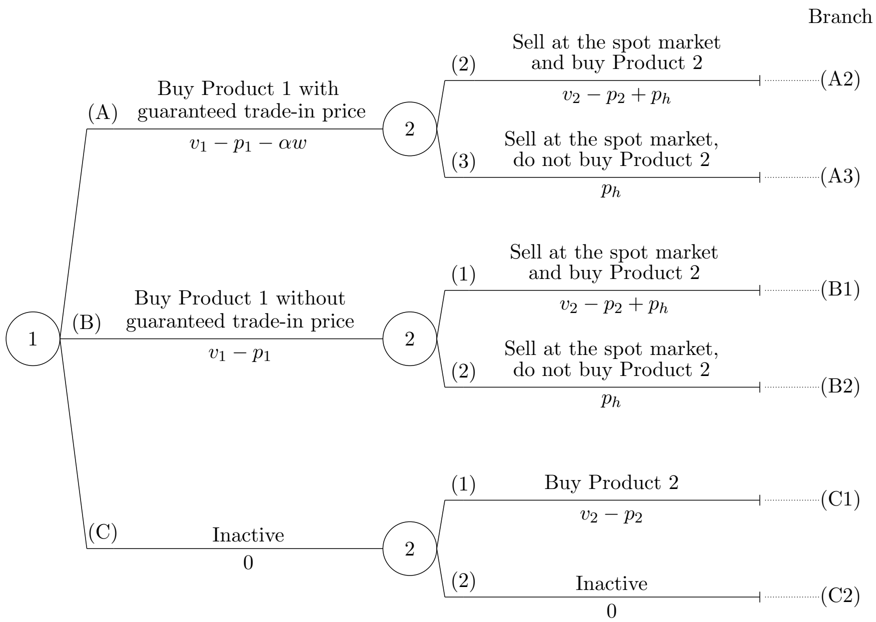

Figure B.1 **Customers' Choices when** $p_u = p_h$

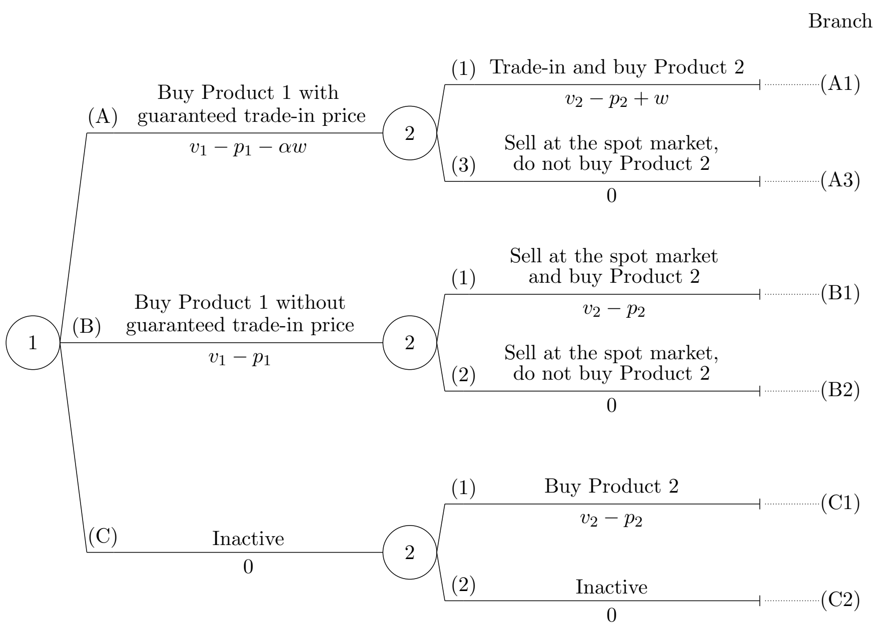

Figure B.2 **Customers' Choices when** $p_u = 0$

With the above analysis under scenarios with high and low spot market price, we can fully characterize the total utility for each branch and work out the indifference point between any two branches as follows:

- Branch (A) dominates if $v_1 \geq v_1^{AB} = \frac{p_2 - w(1-2\alpha)}{\theta}$, and branch (B) dominates otherwise.
- Branch (A) dominates if $v_1 \geq v_1^{AC}$, and branch (C) dominates otherwise, where:

$$v_1^{AC} = \begin{cases} v_1^{AC1} = \frac{2p_1 - p_h + 2\alpha w}{2}, & \text{if } \; v_1 \leq \frac{p_2 - w}{\theta}; \\ v_1^{AC2} = \frac{2p_1 - p_h + p_2 - w(1-2\alpha)}{2+\theta}, & \text{if } \; \frac{p_2 - w}{\theta} \leq v_1 \leq \frac{p_2}{\theta}; \\ v_1^{AC3} = \frac{2p_1 - p_h - w(1-2\alpha)}{2}, & \text{otherwise.} \end{cases}$$

- Branch (B) dominates if $v_1 \geq v_1^{BC} = p_1 - \frac{p_h}{2}$, and branch (C) dominates otherwise.

In order to compare all of the three branches together, we need to compare the three indifference points all together as well. With some algebra, we can show that the ranking of the three indifference points will meet one and only one of the following three possible cases depending on the prices and other model parameters:

- Case (i): If $p_2 \leq p_2^b = \frac{\theta(2p_1 - p_h)}{2} - \frac{\theta w(1-2\alpha)}{2}$, then we have $v_1^{AB} \leq v_1^{AC3} \leq v_1^{BC}$.
- Case (ii): If $p_2^b \leq p_2 \leq p_2^a = \frac{\theta(2p_1 - p_h)}{2} + w(1-2\alpha)$, then we have $v_1^{AB} \leq v_1^{AC2} \leq v_1^{BC}$.
- Case (iii): If $p_2 \geq p_2^a$, then we have $v_1^{BC} \leq v_1^{AC1} \leq v_1^{AB}$.

Note that for cases (i) and (ii), it is evident that branch (B) is always dominated so the choice is in between branches (A) and (C). However, for case (iii), indeed, all of the three branches are possible, which leads to the most complicated characterization of the demand functions. So, as long as we explain the process of deriving the demand functions in case (iii) well, the process for cases (i) and (ii) are similar and simpler with different indifference points. So, let us start with case (iii) first.

In case (iii), there are effectively two indifference points for three possible branches (or choices), i.e., v_1^{BC} and v_1^{AB}. More specifically, customers with high valuation, i.e., $v_1 \geq v_1^{AB}$, will choose branch (A) (or buy Product 1 with guarantee); customers with low valuation, i.e., $v_1 \leq v_1^{BC}$, will choose branch (C) (or be inactive); customers with medium valuation will choose branch (B) (or buy Product 1 without guarantee). As a result, the retailer's demand in period 1, D_{R1}, is:

$$\begin{aligned} D_{R1} = N \cdot \big(&\Pr\{\text{Buy Product 1 with guarantee}\} + \Pr\{\text{Buy Product 1 without guarantee}\}\big) \\ &= 1 \cdot (\Pr\{v_1 \geq v_1^{AB}\} + \Pr\{v_1^{BC} \leq v_1 \leq v_1^{AB}\}) = 1 - v_1^{BC} \\ &= 1 - p_1 + \frac{p_h}{2} \end{aligned}$$

The number of customers who purchase the guarantee, D_{G1}, is:

$$\begin{aligned} D_{G1} &= N \cdot \Pr\{\text{Buy Product 1 with guarantee}\} \\ &= 1 \cdot \Pr\{v_1 \geq v_1^{AB}\} = 1 - v_1^{AB} \\ &= \frac{\theta - p_2 + w(1-2\alpha)}{\theta} \end{aligned}$$

Retailer's demand in period 2, D_{R2}, yields:

$$\begin{aligned} D_{R2} = & N \cdot \sum_{u=\{h,l\}} \Pr\{p_u\} \cdot \Big(\Pr\{\text{Buy Product 1 with guarantee, and buy Product 2}\} \\ & \qquad + \Pr\{\text{Buy Product 1 without guarantee, and buy Product 2}\} + \Pr\{\text{Inactive, and buy Product 2}\}\Big) \\ = & N \cdot \sum_{u=\{h,l\}} \Pr\{p_u\} \cdot \Big(\Pr\{\text{Buy Product 1 with guarantee}\} \cdot \Pr\{\text{Buy Product 2} \mid \text{Buy Product 1 with guarantee}\} \\ & \qquad + \Pr\{\text{Buy Product 1 without guarantee}\} \cdot \Pr\{\text{Buy Product 2} \mid \text{Buy Product 1 without guarantee}\} \\ & \qquad + \Pr\{\text{Inactive}\} \cdot \Pr\{\text{Buy Product 2} \mid \text{Inactive}\}\Big) \\ = & 1 \cdot \Big[\Pr\{p_u = p_h\} \Big(\Pr\{\text{Buy Product 1 with guarantee}\} \cdot \Pr\{\text{Sell in spot market and buy Product 2} \mid \text{Buy Product 1 with guarantee}\}) \\ & \qquad + \Pr\{\text{Buy Product 1 without guarantee}\} \cdot \Pr\{\text{Sell in spot market and buy Product 2} \mid \text{Buy Product 1 without guarantee}\} \\ & \qquad + \Pr\{\text{Inactive}\} \cdot \Pr\{\text{Buy Product 2} \mid \text{Inactive}\}\Big) \\ & + \Pr\{p_u = 0\} \Big(\Pr\{\text{Buy Product 1 with guarantee}\} \cdot \Pr\{\text{Trade-in and buy Product 2} \mid \text{Buy Product 1 with guarantee}\}) \\ & \qquad + \Pr\{\text{Buy Product 1 without guarantee}\} \cdot \Pr\{\text{Sell in spot market and buy Product 2} \mid \text{Buy Product 1 without guarantee}\} \\ & \qquad + \Pr\{\text{Inactive}\} \cdot \Pr\{\text{Buy Product 2} \mid \text{Inactive}\}\Big)\Big] \\ = & \underbrace{\frac{1}{2} \Big(\Pr\{v_1 \geq v_1^{AB}\} \cdot \Pr\{v_1 \geq \tfrac{p_2}{\theta} \mid v_1 \geq v_1^{AB}\} + \Pr\{v_1^{BC} \leq v_1 \leq v_1^{AB}\} \cdot \Pr\{v_1 \geq \tfrac{p_2}{\theta} \mid v_1^{BC} \leq v_1 \leq v_1^{AB}\} + \Pr\{v_1 \leq v_1^{BC}\} \cdot \Pr\{v_1 \geq \tfrac{p_2}{\theta} \mid v_1 \leq v_1^{BC}\}\Big)}_{p_u = p_h \ w.p.\ 1/2} \end{aligned}$$

$$+\underbrace{\frac{1}{2}\big(\Pr\{v_1\geq v_1^{AB}\}\cdot\Pr\{v_1\geq \tfrac{p_2-w}{\theta}\mid v_1\geq v_1^{AB}\}+\Pr\{v_1^{BC}\leq v_1\leq v_1^{AB}\}\cdot\Pr\{v_1\geq \tfrac{p_2}{\theta}\mid v_1^{BC}\leq v_1\leq v_1^{AB}\}+\Pr\{v_1\leq v_1^{BC}\}\cdot\Pr\{v_1\geq \tfrac{p_2}{\theta}\mid v_1\leq v_1^{BC}\}\big)}_{p_u=p_h\ w.p.\ 1/2}$$

$$=\tfrac{1}{2}\Big((1-v_1^{AB})\Big(\tfrac{1-\frac{p_2}{\theta}}{1-v_1^{AB}}\Big)+(v_1^{AB}-v_1^{BC})\cdot 0+(v_1^{BC}-0)\cdot 0\Big)+\tfrac{1}{2}\big((1-v_1^{AB})\cdot 1+(v_1^{AB}-v_1^{BC})\cdot 0+(v_1^{BC}-0)\cdot 0\big)$$
$$=\frac{1}{2}\Big((1-\frac{p_2}{\theta})+(1-v_1^{AB})\Big)=\frac{2(\theta-p_2)+w(1-2\alpha)}{2\theta}$$

The number of customers who exercise the guarantee option, D_{G2}, is:

$$D_{G2}=N\cdot\sum_{u=\{h,l\}}\Pr\{p_u\}\cdot\Pr\{\text{Buy Product 1 with guarantee, and trade-in and buy Product 2}\}$$
$$=N\cdot\sum_{u=\{h,l\}}\Pr\{p_u\}\cdot\big(\Pr\{\text{Buy Product 1 with guarantee}\}\cdot\Pr\{\text{Trade-in and buy Product 2}\mid\text{Buy Product 1 with guarantee}\}\big)$$
$$=\underbrace{\frac{1}{2}\left(\Pr\{v_1\geq v_1^{AB}\}\cdot\Pr\{v_1\geq\frac{p_2-w}{\theta}\mid v_1\geq v_1^{AB}\}\right)}_{p_u=0\ w.p.\ 1/2}$$
$$=\frac{1}{2}(1-v_1^{AB})=\frac{\theta-p_2+w(1-2\alpha)}{2\theta}$$

Demand functions in case (i) where $p_2\leq p_2^b$ can be derived similarly. The difference is that there is only one indifference point v_1^{AC3} above which customers will pick branch (A), otherwise they will pick branch (C). Hence, the demands for the retailer and the number of customers who purchase the guarantee, D_{R1} and D_{G1}, can be identically shown as:

$$\begin{aligned}D_{R1}=D_{G1}&=N\cdot\Pr\{\text{Buy Product 1 with guarantee}\}\\&=1\cdot\Pr\{v_1\geq v_1^{AC3}\}=1-v_1^{AC3}\\&=1-p_1+\frac{p_h+w(1-2\alpha)}{2}\end{aligned}$$

Retailer's demand in period 2, D_{R2}, gives:

$$D_{R2}=N\cdot\sum_{u=\{h,l\}}\Pr\{p_u\}\cdot\big(\Pr\{\text{Buy Product 1 with guarantee, and buy Product 2}\}+\Pr\{\text{Inactive, and buy Product 2}\}\big)$$
$$\begin{aligned}=N\cdot\sum_{u=\{h,l\}}\Pr\{p_u\}\cdot\big(&\Pr\{\text{Buy Product 1 with guarantee}\}\cdot\Pr\{\text{Buy Product 2}\mid\text{Buy Product 1 with guarantee}\}\\&+\Pr\{\text{Inactive}\}\cdot\Pr\{\text{Buy Product 2}\mid\text{Inactive}\}\big)\end{aligned}$$
$$\begin{aligned}=1\cdot\Big[\Pr\{p_u=p_h\}\big(&\Pr\{\text{Buy Product 1 with guarantee}\}\cdot\Pr\{\text{Sell in spot market and buy Product 2}\mid\text{Buy Product 1 with guarantee}\})\\&+\Pr\{\text{Inactive}\}\cdot\Pr\{\text{Buy Product 2}\mid\text{Inactive}\}\big)\\+\Pr\{p_u=0\}\big(&\Pr\{\text{Buy Product 1 with guarantee}\}\cdot\Pr\{\text{Trade-in and buy Product 2}\mid\text{Buy Product 1 with guarantee}\})\\&+\Pr\{\text{Inactive}\}\cdot\Pr\{\text{Buy Product 2}\mid\text{Inactive}\}\big)\Big]\end{aligned}$$
$$=\underbrace{\frac{1}{2}\left(\Pr\{v_1\geq v_1^{AC3}\}\cdot\Pr\{v_1\geq\frac{p_2}{\theta}\mid v_1\geq v_1^{AC3}\}+\Pr\{v_1\leq v_1^{AC3}\}\cdot\Pr\{v_1\geq\frac{p_2}{\theta}\mid v_1\leq v_1^{AC3}\}\right)}_{p_u=p_h\ w.p.\ 1/2}$$
$$+\underbrace{\frac{1}{2}\left(\Pr\{v_1\geq v_1^{AC3}\}\cdot\Pr\{v_1\geq\frac{p_2-w}{\theta}\mid v_1\geq v_1^{AC3}\}+\Pr\{v_1\leq v_1^{AC3}\}\cdot\Pr\{v_1\geq\frac{p_2}{\theta}\mid v_1\leq v_1^{AC3}\}\right)}_{p_u=0\ w.p.\ 1/2}$$
$$=\frac{1}{2}\left((1-v_1^{AC3})\cdot 1+(v_1^{AC3}-0)\cdot\left(\frac{v_1^{AC3}-\frac{p_2}{\theta}}{v_1^{AC3}-0}\right)\right)+\frac{1}{2}\left((1-v_1^{AC3})\cdot 1+(v_1^{AC3}-0)\cdot\left(\frac{v_1^{AC3}-\frac{p_2}{\theta}}{v_1^{AC3}-0}\right)\right)$$
$$=1-\frac{p_2}{\theta}$$

The number of customers who decide to exercise the guarantee option, D_{G2}, is:

$$D_{G2}=N\cdot\sum_{u=\{h,l\}}\Pr\{p_u\}\cdot\Pr\{\text{Buy Product 1 with guarantee, and trade-in and buy Product 2}\}$$
$$=N\cdot\sum_{u=\{h,l\}}\Pr\{p_u\}\cdot\big(\Pr\{\text{Buy Product 1 with guarantee}\}\cdot\Pr\{\text{Trade-in and buy Product 2}\mid\text{Buy Product 1 with guarantee}\}\big)$$

$$= \underbrace{\frac{1}{2}\left(\Pr\{v_1 \geq v_1^{AC3}\}\cdot\Pr\{v_1 \geq \frac{p_2-w}{\theta} \mid v_1 \geq v_1^{AC3}\}\right)}_{p_u=0\ w.p.\ 1/2}$$
$$= \frac{1}{2}(1-v_1^{AC3}) = \frac{1}{2}\left(1-p_1+\frac{p_h+w(1-2\alpha)}{2}\right)$$

Similarly, there is only one indifference point in case (ii), i.e., v_1^{AC2}, above which customers will choose branch (A) and otherwise they will choose branch (C). Accordingly, the demand functions for the retailer and the number of customers who purchase the guarantee, D_{R1} and D_{G1}, can be shown as:

$$\begin{aligned} D_{R1} = D_{G1} &= N\cdot\Pr\{\text{Buy Product 1 with guarantee}\} \\ &= 1\cdot\Pr\{v_1 \geq v_1^{AC2}\} = 1-v_1^{AC2} \\ &= \frac{2+\theta-2p_1+p_h-p_2+w(1-2\alpha)}{2+\theta} \end{aligned}$$

Retailer's demand in period 2, D_{R2}, leads to:

$$\begin{aligned} D_{R2} =& N\cdot\sum_{u=\{h,l\}}\Pr\{p_u\}\cdot\Big(\Pr\{\text{Buy Product 1 with guarantee, and buy Product 2}\}+\Pr\{\text{Inactive, and buy Product 2}\}\Big) \\ =& N\cdot\sum_{u=\{h,l\}}\Pr\{p_u\}\cdot\Big(\Pr\{\text{Buy Product 1 with guarantee}\}\cdot\Pr\{\text{Buy Product 2} \mid \text{Buy Product 1 with guarantee}\} \\ &+\Pr\{\text{Inactive}\}\cdot\Pr\{\text{Buy Product 2} \mid \text{Inactive}\}\Big) \\ =& 1\cdot\Big[\Pr\{p_u=p_h\}\Big(\Pr\{\text{Buy Product 1 with guarantee}\}\cdot\Pr\{\text{Sell in spot market and buy Product 2} \mid \text{Buy Product 1 with guarantee}\}) \\ &+\Pr\{\text{Inactive}\}\cdot\Pr\{\text{Buy Product 2} \mid \text{Inactive}\}\Big) \\ &+\Pr\{p_u=0\}\Big(\Pr\{\text{Buy Product 1 with guarantee}\}\cdot\Pr\{\text{Trade-in and buy Product 2} \mid \text{Buy Product 1 with guarantee}\}) \\ &+\Pr\{\text{Inactive}\}\cdot\Pr\{\text{Buy Product 2} \mid \text{Inactive}\}\Big)\Big] \\ =& \underbrace{\frac{1}{2}\left(\Pr\{v_1 \geq v_1^{AC2}\}\cdot\Pr\{v_1 \geq \frac{p_2}{\theta} \mid v_1 \geq v_1^{AC2}\}+\Pr\{v_1 \leq v_1^{AC2}\}\cdot\Pr\{v_1 \geq \frac{p_2}{\theta} \mid v_1 \leq v_1^{AC2}\}\right)}_{p_u=p_h\ w.p.\ 1/2} \\ &+\underbrace{\frac{1}{2}\left(\Pr\{v_1 \geq v_1^{AC2}\}\cdot\Pr\{v_1 \geq \frac{p_2-w}{\theta} \mid v_1 \geq v_1^{AC2}\}+\Pr\{v_1 \leq v_1^{AC2}\}\cdot\Pr\{v_1 \geq \frac{p_2}{\theta} \mid v_1 \leq v_1^{AC2}\}\right)}_{p_u=0\ w.p.\ 1/2} \\ =& \frac{1}{2}\left((1-v_1^{AC2})\cdot\left(\frac{1-\frac{p_2}{\theta}}{1-v_1^{AC2}}\right)+(v_1^{AC2}-0)\cdot 0\right)+\frac{1}{2}\left((1-v_1^{AC2})\cdot 1+(v_1^{AC2}-0)\cdot 0\right) \\ =& \frac{1}{2}\left(2-v_1^{AC2}-\frac{p_2}{\theta}\right) = \frac{\theta(4+2\theta-2p_1+p_h+w(1-2\alpha))-2p_2(1+\theta)}{2\theta(2+\theta)} \end{aligned}$$

The number of customers who exercise the guarantee option, D_{G2}, is:

$$\begin{aligned} D_{G2} =& N\cdot\sum_{u=\{h,l\}}\Pr\{p_u\}\cdot(\Pr\{\text{Buy Product 1 with guarantee, and trade-in and buy Product 2}\}) \\ =& N\cdot\sum_{u=\{h,l\}}\Pr\{p_u\}\cdot\Big(\Pr\{\text{Buy Product 1 with guarantee}\}\cdot\Pr\{\text{Trade-in and buy Product 2} \mid \text{Buy Product 1 with guarantee}\}\Big) \\ =& \underbrace{\frac{1}{2}\left(\Pr\{v_1 \geq v_1^{AC2}\}\cdot\Pr\{v_1 \geq \frac{p_2-w}{\theta} \mid v_1 \geq v_1^{AC2}\}\right)}_{p_u=0\ w.p.\ 1/2} \\ =& \frac{1}{2}(1-v_1^{AC2}) = \frac{2+\theta-2p_1+p_h-p_2+w(1-2\alpha)}{2+\theta} \end{aligned}$$

Proof of Proposition 2. This proof is to analyze the no guarantee model under static pricing. When static pricing policy is used, the retailer sets p_1 and $p_2(=p_1)$ simultaneously in period 1, and its profit-maximizing problem can be formulated as:

$$\max_{0<p_1\leq 1} \Pi_R = p_1\left((1-p_1+\frac{p_h}{2})+(1-\frac{p_1}{\theta})\right).$$

It is easy to verify Π_R is strictly concave in p_1. Using the first-order conditions we get the optimal retail price for products of each period, $p_t^* = \frac{\theta(4+p_h)}{4(\theta+1)}$ $(t=1,2)$, which is less than 1 by fact that $\theta \leq 4$. Substituting p_1^* and p_2^* into the retailer's profit function yields the optimal profit $\Pi_R^* = \frac{\theta(4+p_h)^2}{16(\theta+1)}$. The corresponding optimal demand for Product 1 and 2 is $D_{R1}^* = \frac{4+p_h(2+\theta)}{4(\theta+1)}$ and $D_{R2}^* = \frac{4\theta-p_h}{4(\theta+1)}$, respectively. Next, we calculate consumer surplus. By the derivations process of demand functions, a consumer is indifferent in buying Product 2 and not buying at $v_2^{BN} = \frac{p_2}{\theta}$ in period 2, s/he buys Product 2 if her/his valuation of Product 2 v_2 is greater than v_2^{BN}, otherwise not buy; a customer is indifferent between buying Product 1 and keeping inactive at $v_1^{BC} = p_1 - \frac{p_h}{2}$ in period 1. Consumers whose valuation of Product 1 is above v_1^{BC} choose to buy Product 1, otherwise not buy.

When $v_1^{BC} \leq v_2^{BN}$: (i) $0 \leq v_1 \leq v_1^{BC}$. Consumers do not buy in both periods and hence gain a utility of zero; (ii) $v_1^{BC} \leq v_1 \leq v_2^{BN}$. Consumers buy Product 1 but not buy Product 2, hence gain a utility $(v_1 - p_1) + \mathrm{E}_{p_u}(p_u)$; and (iii) $v_2^{BN} \leq v_1 \leq 1$. Consumers buy in both periods, the corresponding utility is $(v_1 - p_1) + \mathrm{E}_{p_u}(v_2 - p_2 + p_u)$. Therefore, consumer surplus can be written as:

$$CS = \int_0^{v_1^{BC}} 0 \cdot f(v_1)\mathrm{d}v_1 + \int_{v_1^{BC}}^{v_2^{BN}} (v_1 - p_1 + \frac{p_h}{2}) f(v_1)\mathrm{d}v_1 + \int_{v_2^{BN}}^{1} ((v_1 - p_1) + (\theta v_1 - p_1 + \frac{p_h}{2})) f(v_1)\mathrm{d}v_1.$$

When $v_1^{BC} \geq v_2^{BN}$: (i) $0 \leq v_1 \leq v_2^{BN}$. Consumers will not buy in both periods, and gain a utility of zero; (ii) $v_2^{BN} \leq v_1 \leq v_1^{BC}$. Consumers buy Product 2 but do not buy Product 1, gaining a utility $v_2 - p_1$; and (iii) $v_1^{BC} \leq v_1 \leq 1$. Consumers choose to buy in each period, the corresponding utility is $(v_1 - p_1) + \mathrm{E}_{p_u}(v_2 - p_2 + p_u)$. Thus, consumer surplus is formulated as:

$$CS = \int_0^{v_2^{BN}} 0 \cdot f(v_1)\mathrm{d}v_1 + \int_{v_2^{BN}}^{v_1^{BC}} (\theta v_1 - p_1) f(v_1)\mathrm{d}v_1 + \int_{v_1^{BC}}^{1} ((v_1 - p_1) + (\theta v_1 - p_1 + \frac{p_h}{2})) f(v_1)\mathrm{d}v_1.$$

Substituting the optimal retail prices $p_t^* = \frac{\theta(4+p_h)}{4(\theta+1)}$ $(t=1,2)$ into the above two consumer surplus functions leads to the equilibrium consumer surplus, which can be identical shown as $CS^* = \frac{16\theta^2 - \theta(16+8p_h-p_h^2)+4(2+p_h)^2}{32(1+\theta)}$. This complete the proof of Proposition 2. □

Proof of Proposition 3. This proof is to analyze the guaranteed trade-in price model under static pricing. In this model, the retailer sets retail prices of Product 1 and 2 and subsequently decides on the trade-in price, both in the first period. We use backward induction to solve for the equilibrium outcomes. For description convenience, we will use some notations appeared in the proof of other propositions but with different values. Note that demand functions in Table 1 is also applicable to this model except that Case (i) is gone because $w^b = \frac{2p_1(\theta-1)-\theta p_h}{\theta(1-2\alpha)}$ is less than zero when $\theta = 1$. Substitute $\theta = 1$ into the demands in Table 1 yields the demand functions for the guaranteed trade-in price model, as shown in Table B.2.

The retailer's total expected profit is given by:

$$\Pi_R = \Pi_{R1} + \Pi_{R2}, \tag{B-1}$$

where $\Pi_{R1} = p_1 D_{R1} - C(\theta) + \alpha w D_{G1}$ and $\Pi_{R2} = p_1 D_{R2} + (p_s - w) D_{G2}$. It is easy to verify that Π_R is continuous at the cut-off value $\frac{p_h}{2(1-2\alpha)}$. Substituting the demand functions in Table B.2 into equation (B-1), using the first-order conditions we get the unconstraint optimal w, as follows: For case (ii), $w_2^* = \frac{3(2p_1-1)+p_s-p_h}{2(1-2\alpha)}$;

Table B.2 Demand Functions: Trade-in Price Guarantee from the retailer under Static Pricing, $\theta = 1$

	(ii) If $w \geq \frac{p_h}{2(1-2\alpha)}$	(iii) otherwise
R's demand in period 1 D_{R1}	$1-p_1+\frac{p_h+w(1-2\alpha)}{3}$	$1-p_1+\frac{p_h}{2}$
R's demand in period 2 D_{R2}	$1-p_1+\frac{p_h+w(1-2\alpha)}{6}$	$1-p_1+\frac{w(1-2\alpha)}{2}$
G's demand in period 1 D_{G1}	$1-p_1+\frac{p_h+w(1-2\alpha)}{3}$	$1-p_1+w(1-2\alpha)$
G's demand in period 2 D_{G2}	$\frac{1}{2}\left(1-p_1+\frac{p_h+w(1-2\alpha)}{3}\right)$	$\frac{1}{2}\left(1-p_1+w(1-2\alpha)\right)$

for case (iii), $w_3^* = \frac{2p_1-1+p_s}{2(1-2\alpha)}$. A comparison of the unconstraint optimal w and its lower/upper bound values i.e., 0 and p_h, yields the following results. Under case (ii), if $p_1 \leq p_1^{2a}$, $w^* = \frac{p_h}{2(1-2\alpha)}$, where $p_1^{2a} = \frac{3-p_s+2p_h}{6}$; if $p_1^{2a} \leq p_1 \leq p_1^{2b}$, $w^* = w_2^*$, where $p_1^{2b} = \frac{3-p_s+p_h(3-4\alpha)}{6}$; otherwise $w^* = p_h$. Under case (iii), if $p_1 \leq p_1^{3a}$, $w^* \sim 0$, where $p_1^{3a} = \frac{1-p_s}{2}$; if $p_1^{3a} \leq p_1 \leq p_1^{3b}$, $w^* = w_3^*$, where $p_1^{3b} = \frac{1-p_s+p_h}{6}$; otherwise $w^* = \frac{p_h}{2(1-2\alpha)}$. Next, we compare the above cut-offs of p_1 and its lower/upper bound values to determine the optimal w. The analysis can be divided into the following three cases:

(i) $p_s \leq p_s^b$, which gives $0 < p_1^{3a} \leq p_1^{2a} \leq p_1^{2b} \leq p_1^{3b} \leq 1$, where $p_s^b = 2\alpha p_h$. Based on the optimal w analysis, if $p_1 \leq p_1^{3a}$, the optimal w in case (ii) and (iii) is $\frac{p_h}{2(1-2\alpha)}$ and 0, respectively. Moreover, one can verify that the retailer's total expected profits at $\frac{p_h}{2(1-2\alpha)}$ in case (ii) and (iii) are identical, showing that Π_R is continuous at $\frac{p_h}{2(1-2\alpha)}$. Hence, $w^* = \frac{p_h}{2(1-2\alpha)}$ is dominated by $w^* \sim 0$. For any $p_1 \leq p_1^{3a}$, we get case (iii) and the optimal w is approaching to zero. Following this logic, one can easily verify that if $p_1^{3a} \leq p_1 \leq p_1^{2a}$, $w^* = w_3^*$; if $p_1 \geq p_1^{3b}$, $w^* = p_h$. Things will be a bit different when $p_1^{2a} \leq p_1 \leq p_1^{3b}$ because one cannot directly compare the retailer's total expected profits in those two cases. If $p_1^{2a} \leq p_1 \leq p_1^{2b}$, we know that the optimal w in the case (ii) and (iii) is w_2^* and w_3^*, respectively. A comparison of the retailer's total expected profit gives the threshold value $p_1^a = \frac{\sqrt{3}}{12}(p_h(1+\sqrt{3}) + 2(\sqrt{3}-p_s))$ at which $\Pi_R(w = w_2^*) \equiv \Pi_R(w = w_3^*)$. Specifically, if $p_1^{2a} \leq p_1 \leq \min(p_1^{2b}, p_1^a)$, the optimal w yields w_3^*, otherwise it is w_2^*. Compare p_1^a with p_1^{2a} and p_1^{2b} leads to: (1) if $p_s \leq p_s^c$, then $p_1^{2a} \leq p_1^{2b} \leq p_1^a$, where $p_s^c = \frac{p_h(1+\sqrt{3})(8\alpha+\sqrt{3}-3)}{4}$. Hence, for any $p_1^{2a} \leq p_1 \leq p_1^{2b}$, the optimal w is w_3^*. (2) If $p_s^c \leq p_1 \leq p_s^b$, then $p_1^{2a} \leq p_1^a \leq p_1^{2b}$. So, if $p_1^{2a} \leq p_1^a$, the optimal w gives w_3^*, otherwise it is w_2^*. On the other hand, when $p_1^{2b} \leq p_1 \leq p_1^{3b}$, the optimal w in the case (ii) and (iii) is p_h and w_3^*, respectively. We derive another threshold value $p_1^b = \frac{\sqrt{3}}{6}(\sqrt{3}(1-p_s) - 2\sqrt{p_h(1-4\alpha)(p_h(1-2\alpha)-p_s)})$ at which $\Pi_R(w = p_h) \equiv \Pi_R(w = w_3^*)$. Particularly, if $p_1^{2b} \leq p_1 \leq \min(p_1^{3b}, p_1^b)$, the optimal w yields w_3^*, otherwise it is p_h. By the similar logic as the analysis when $p_1^{2a} \leq p_1 \leq p_1^{2b}$, we have: (1) If $p_s \leq p_s^c$, we have $p_1^{2b} \leq p_1^b \leq p_1^{3b}$. Therefore, if $p_1^{2b} \leq p_1 \leq p_1^b$, the optimal w is w_3^*, otherwise it gives p_h. (2) If $p_s^c \leq p_1 \leq p_s^b$, $p_1^b \leq p_1^{2b} \leq p_1^{3b}$ and the optimal w is p_h. Based on the above analysis, for any $p_s \leq p_s^b$, the optimal w is can be summarized as follows. When $p_s \leq p_s^c$, $w^* \sim 0$ if $p_1 \leq p_1^{3a}$, $w^* = w_3^*$ if $p_1^{3a} \leq p_1 \leq p_1^b$, and $w^* = p_h$ otherwise. When $p_s^c \leq p_s \leq p_s^b$, $w^* \sim 0$ if $p_1 \leq p_1^{3a}$, $w^* = w_3^*$ if $p_1^{3a} \leq p_1 \leq p_1^a$, $w^* = w_2^*$ if $p_1^a \leq p_1 \leq p_1^{2b}$, and $w^* = p_h$ otherwise.

(ii) $p_s^b \leq p_s \leq p_s^a$, which gives $0 < p_1^{3a} \leq p_1^{2a} \leq p_1^{3b} \leq p_1^{2b} \leq 1$, where $p_s^a = \frac{p_h}{2}$. By the similar logic as the analysis when $p_s \leq p_s^b$. One can easily verify that if $p_1 \leq p_1^{3a}$, $w^* \sim 0$; if $p_1^{3a} \leq p_1 \leq p_1^a$, $w^* = w_3^*$; if $p_1^a \leq p_1 \leq p_1^{2b}$, $w^* = w_2^*$; otherwise $w^* = p_h$.

(iii) $p_s \geq p_s^a$, which gives $0 < p_1^{3a} \leq p_1^{3b} \leq p_1^{2a} \leq p_1^{2b} \leq 1$. Using the similar logic, we have: if $p_1 \leq p_1^{3a}$, $w^* \sim 0$; if $p_1^{3a} \leq p_1 \leq p_1^{3b}$, $w^* = w_3^*$; if $p_1^{3b} \leq p_1 \leq p_1^{2a}$, $w^* = \frac{p_h}{2(1-2\alpha)}$; if $p_1^{2a} \leq p_1 \leq p_1^{2b}$, $w^* = w_2^*$; otherwise $w^* = p_h$.

Take the above three cases into consideration, it is easy to see that the optimal w in the case with $p_s^c \leq p_s \leq p_s^b$ has the same structure as that in the case with $p_s^b \leq p_s \leq p_s^a$. Hence, those two cases can be combined together and the cut-off value p_s^b is gone. Compare p_s^a and p_s^c with the lower/upper bound values of p_s (i.e., 0 and p_h), we know that, if $\alpha \leq 0.1585$, $p_s^c \leq 0 \leq p_s^a < p_h$, otherwise $0 \leq p_s^c \leq p_s^a < p_h$. Given a specific value of p_s, one can easily find out the optimal w according to the above analysis.

Next, we turn to optimize p_1. When $w \leq \frac{p_h}{2(1-2\alpha)}$, we have case (iii) in which the optimal trade-in price could be 0, w_3^*, or $\frac{p_h}{2(1-2\alpha)}$. Substitute the optimal w into the retailer's total expected profit in equation (B-1) yields: $\Pi_R(0) = \frac{1}{2}(p_1(4(1-p_1)+p_h-p_s)+p_s) - C(\theta)$, $\Pi_R(w_3^*) = \frac{1}{8}(4p_1(3(1-p_1)+p_h)+(1+p_s)^2) - C(\theta)$ and $\Pi_R(\frac{p_h}{2(1-2\alpha)}) = \frac{1}{8}(-16p_1^2 - (2+p_h)(p_h-2p_s) + 4p_1(4+2p_h-p_s)) - C(\theta)$. One can easily verify that each of the profit functions is strictly concave in p_1. Using the first-order conditions gives the unconstraint optimal p_1, which is $p_1^c = \frac{4+p_h-p_s}{8}, p_1^d = \frac{3+p_h}{6}$, and $p_1^e = \frac{4+2p_h-p_s}{8}$, respectively. Following the similar logic, one can examine that the retailer's total expected profit is also strictly concave in p_1 under case (ii) where the optimal w could be $\frac{p_h}{2(1-2\alpha)}, w_2^*$, or p_h. The corresponding unconstraint optimal p_1 gives $p_1^e = \frac{4+2p_h-p_s}{8}, p_1^f = \frac{1}{2}$, and $p_1^g = \frac{4-p_s+p_h(3-4\alpha)}{8}$, respectively. We compare each of the above optimal p_1 with its lower/upper bound to find out the optimal p_1, since it will depend on other variables. For instance, if $\alpha \leq 0.1585$, then $p_s^c \leq 0 \leq p_s^a < p_h$. For any $p_s \leq p_s^a$, the order $0 \leq p_1^{3a} \leq p_1^a \leq p_1^{3b} \leq 1$ holds. If $p_1 \leq p_1^{3a}$, the optimal w is approaching to zero, and we must compare p_1^c with 0 and p_1^{3a} to get the optimal p_1. By following this logic, given the specific values of α and p_s, we are able to show the optimal w and p_1, as follows. (1) If $\alpha \leq 0.1585$, $p_s^c \leq 0 \leq p_s^a \leq \frac{2p_h}{3}$. We get $w^* = w_3^*$ and $p_1^* = p_1^d$ when $p_s \in [0, p_s^a) \cup [p_s^a, \frac{2p_h}{3}]$, otherwise $w^* = \frac{p_h}{2(1-2\alpha)}$ and $p_1^* = p_1^e$. (2) If $\alpha \geq 0.1585$, $0 \leq p_s^c \leq p_s^a \leq \frac{2p_h}{3}$. We have $w^* = w_3^*$ and $p_1^* = p_1^d$ when $p_s \in [0, p_s^c) \cup [p_s^c, p_s^a) \cup [p_s^a, \frac{2p_h}{3}]$, otherwise $w^* = \frac{p_h}{2(1-2\alpha)}$ and $p_1^* = p_1^e$. Clearly, the structure of the retailer's optimal pricing decisions does not affect by α. A combination of the above two cases leads to the following results: if $p_s \geq \frac{2p_h}{3}$, $p_1^* = p_1^e$ and $w^* = \frac{p_h}{2(1-2\alpha)}$, otherwise $p_1^* = p_1^d$ and $w^* = w_3^*$. Substitute p_1^d into w_3^* immediately gives $w^* = \frac{p_h+3p_s}{6(1-2\alpha)}$. Substituting the optimal prices into the demand functions in Table B.2 and the retailer's total expected profit in equation (B-1), some algebra leads to the optimal decisions and profits.

Consumer surplus can be calculated as follows. If $p_s \leq \frac{2p_h}{3}$, we have case (iii) in which all the branches (A), (B), (C) are likely to be chosen, depending on two indifference points v_1^{BC} and v_1^{AB}. Moreover, in equilibrium, it is easy to verify that $v_1^{BC} \leq v_1^{AB} \leq p_1 \leq 1$ and $p_1 - w \leq v_1^{AB}$. According to consumers' total expected utilities across both periods in Table 1, consumers' surplus CS can be written as follows:

$$CS = \int_{v_1^{BC}}^{v_1^{AB}} (v_1 - p_1 + \frac{p_h}{2})\mathrm{d}v_1 + \frac{1}{2}\int_{v_1^{AB}}^{p_1} (3(v_1-p_1)+p_h+w(1-2\alpha))\mathrm{d}v_1 + \frac{1}{2}\int_{p_1}^{1} (4(v_1-p_1)+p_h+w(1-2\alpha))\mathrm{d}v_1 \tag{B-2}$$

Substitute $w^* = \frac{p_h+3p_s}{6(1-2\alpha)}$ and $p_1^* = \frac{3+p_h}{6}$ into equation (B-2), some algebra gives the optimal consumer surplus $CS^* = \frac{p_h^2+2p_h+4+2p_s+p_s^2}{16}$. On the other hand, if $p_s \geq \frac{2p_h}{3}$, consumer can choose either branch (A) or (C), depending on the indifference point v_1^{AC2}. In addition, one can easily verify $p_1 - w \leq v_1^{AC2} \leq p_1$. Consumers surplus is formulated as:

$$CS = \frac{1}{2}\int_{v_1^{AC2}}^{p_1} (3(v_1-p_1)+p_h+w(1-2\alpha))\mathrm{d}v_1 + \frac{1}{2}\int_{p_1}^{1} (4(v_1-p_1)+p_h+w(1-2\alpha))\mathrm{d}v_1 \tag{B-3}$$

Substitute $w^* = \frac{p_h}{2(1-2\alpha)}$ and $p_1^* = \frac{4+2p_h-p_s}{8}$ into equation (B-3) and some algebraic calculation yields the optimal consumer surplus $CS^* = \frac{4p_h^2+2p_h(4+p_s)+(4+p_s)^2}{64}$. This completes the proof of proposition 4. □

Proof of Proposition 4. This proposition can be obtained through comparing the equilibrium outcomes in proposition 3 and proposition 2. We omit the details. □

Proof of Proposition 5. This proof is to analyze the obligatory guarantee model (i.e., resale value protection only) under static pricing. In this model, customers who bought Product 1 and the guaranteed trade-in price in period 1 cannot sell the used goods anywhere but to the retailer in period 2. However, customers who bought Product 1 while did not join the guarantee still could sell the used goods at the spot market. Hence, there are three options for customers to choose in period 1:

Option 1 – Branch (A): buy Product 1 at p_1 and the guaranteed trade-in option at an upfront fee αw;

Option 2 – Branch (B): buy Product 1 at p_1 without joining the guarantee; and

Option 3 – Branch (C): do not buy Product 1.

In the first period, a customer gains a utility $v_1 - p_1 - \alpha w$, $v_1 - p_1$ and 0 when s/he picks branch (A), (B) and (C), respectively. In the second period, customers who bought Product 1 with the guarantee only have one option to transact the used goods, i.e., the option of trading in and buying Product 2, and the corresponding utility yields $v_1 - p_1 + w$. Customers who bought Product 1 without the guarantee have two options in period 2, i.e., sell the used goods at the spot market and then determine whether or not to buy Product 2, and the utility gives $v_1 - p_1 + p_u$ and p_u, respectively. Customers who did not buy Product 1 have two options in the second period, either buy Product 2 at p_1 or do not buy, and the utility is $v_1 - p_1$ and 0, respectively. Therefore, the total utility over two periods a customer could gain is $u_A = 2(v_1 - p_1) + (1-\alpha)w$, $u_B = v_1 - p_1 + \max(v_1 - p_1 + p_u, p_u)$, and $u_C = \max(v_1 - p_1, 0)$ when s/he pick branch (A), (B) and (C), respectively.

The difference of u_A and u_B yields $u_A - u_B = w(1-\alpha) - p_h/2$ if $v_1 \geq p_1$, otherwise $u_A - u_B = v_1 - p_1 + w(1-\alpha) - p_h/2$. One can examine that for any $w > \frac{p_h}{2(1-\alpha)}$, a customer chooses branch (A) if her/his valuation of Product 1 is greater than $v_1^{AB} = p_1 - w(1-\alpha) + p_h/2$, otherwise s/he picks branch (B), meaning that v_1^{AB} is customers' indifference point between those two branches. However, for a low trade-in price (i.e., $w \leq \frac{p_h}{2(1-\alpha)}$), it is easy to verify that branch (A) becomes unattractive and the model will be identical with the model without guarantee. A comparison of u_A, u_B and u_C leads to two indifference points $v_1^{AC} = p_1 - w(1-\alpha)/2$ and $v_1^{BC} = p_1 - p_h/2$. When customers' valuation of Product 1 is above v_1^{AC}, they pick branch (A), otherwise they pick branch (C); when customers valuation of Product 1 is above v_1^{BC}, they pick branch (B), otherwise they pick branch (C). In addition, it is easy to see that $v_1^{BC} \leq v_1^{AC} \leq v_1^{AB} \leq p_1$, which implies there are effectively two indifference points, v_1^{BC} and v_1^{AB}, for the three possible branches (or choices). More specifically, customers with high valuation, i.e., $v_1 \geq v_1^{AB}$, will choose branch (A); customers with low valuation, i.e., $v_1 \leq v_1^{BC}$, will choose branch (C); customers with medium valuation will choose branch (B). As a result, the retailer's demand in period 1 is,

$$D_{R1} = N \cdot \big(\Pr\{\text{Buy Product 1 with guarantee}\} + \Pr\{\text{Buy Product 1 without guarantee}\}\big)$$

$$
\begin{aligned}
&= 1 \cdot (\Pr\{v_1 \geq v_1^{AB}\} + \Pr\{v_1^{BC} \leq v_1 \leq v_1^{AB}\}) = 1 - v_1^{BC} \\
&= 1 - p_1 + \frac{p_h}{2}
\end{aligned}
$$

Note that customers who bought Product 1 and the guarantee will definitely choose to trade-in and buy Product 2, meaning that the amount of customers joined the guarantee in period 1 equals that of customers execute the guaranteed trade-in option. However, in both branches (B) and (C), a customer buys Product 2 iff her/his valuation of Product 2 is larger than p_1, which is bigger than the indifference points v_1^{BC} and v_1^{AC}. Thus, the numbers of customers who purchase the guarantee, as well as the retailer's second period demand, can be identically given by:

$$
\begin{aligned}
D_{R2} = D_{G1} = D_{G2} &= N \cdot \Pr\{\text{Buy Product 1 with guarantee}\} \\
&= 1 - v_1^{AB} \\
&= 1 - p_1 + w(1-\alpha) - \frac{p_h}{2}
\end{aligned}
$$

The retailer's total profit and consumer utility can be written as $\Pi_R = p_1(D_{R1} + D_{R2}) + \alpha w D_{G1} + (p_s - w)D_{G2}$ and $CS = \int_0^{v_1^{BC}} u_C dv_1 + \int_{v_1^{BC}}^{v_1^{AB}} u_B dv_1 + \int_{v_1^{AB}}^{1} u_A dv_1$, respectively. One can examine that when customers' valuation of Product 1 is less than p_1, then $u_C = 0$, $u_B = v_1 - p_1 + p_h/2$ and $u_A = 2(v_1 - p_1) + w(1-\alpha)$. Substitute the above demand functions into Π_R, by using the first-order conditions and some algebra leads to the equilibrium outcomes for the obligatory guarantee model. □

Proof of Proposition 6. Value added individually by the insurance only. See Table B.3.

Table B.3 Obligatory Guarantee vs. No Guarantee Models under Static Pricing and $\theta = 1$

	(i) If $p_s \leq \tilde{p}_s$	(ii) If $\tilde{p}_s \leq p_s \leq p_h(1-2\alpha)$	(ii) If $p_s \geq p_h(1-2\alpha)$
$p_1(=p_2)$	0	$\frac{p_h}{8} \geq 0$	$-\frac{2p_s - p_h(3-4\alpha)}{8} \geq 0$
D_{R1}	0	$-\frac{p_h}{8} \leq 0$	$-\frac{p_h(3-4\alpha)-2p_s}{8} \leq 0$
D_{R2}	0	$-\frac{p_h - 4p_s}{8} \geq 0$	$\frac{2p_s + p_h(1-4\alpha)}{8} \geq 0$
$D_{R1} + D_{R2}$	0	$-\frac{p_h - 2p_s}{4} \begin{cases} \leq 0, \text{ if } p_s \leq \frac{p_h}{2} \\ \geq 0, \text{ otherwise} \end{cases}$	$-\frac{p_h - 2p_s}{4} \geq 0$
Π_R	0	$-\frac{8(p_h(1+p_s) - p_s(1+p_s)) - 3p_h^2}{32} \geq 0$	$-\frac{p_h(8(1+2\alpha p_s) + p_h(1-16\alpha(1-\alpha))) - 4p_s(4+p_s)}{32} \geq 0$
CS	0	$-\frac{p_h^2 + 8(p_h(1+p_s) - p_s(2+p_s))}{64} \begin{cases} \leq 0, \text{ if } p_s \leq \min(p_{s1}, p_{s2}) \\ \geq 0, \text{ o/w} \end{cases}$	$-\frac{p_h(8 + p_h(5-16\alpha^2)) - 4p_s(4+p_s)}{64} \begin{cases} \leq 0, \text{ if } p_s \leq \min(p_{s1}, p_{s2}) \\ \geq 0, \text{ o/w} \end{cases}$

Notes: $\tilde{p}_s = \frac{2(p_h-2)+\sqrt{2(8-p_h^2)}}{4}$, $p_{s1} = \frac{2(p_h-2)+\sqrt{2(8+3p_h^2)}}{4}$, and $p_{s2} = \frac{\sqrt{(5-16\alpha^2)p_h^2 + 8p_h + 16}}{2} - 2$.

Proof of Proposition 7. Value of optionality. See Tables B.4 and B.5.

Proof of Proposition 8. This proof is to analyze the no guarantee model under dynamic pricing. In this model, the retailer sets p_1 in period 1, and subsequently p_2 in period 2. We use backward induction to obtain

Table B.4 Guaranteed Trade-in vs. Obligatory Guarantee Models under Static Pricing and $\theta=1$ (for low guarantee premium $\alpha\le\frac{1}{6}$)

	(i) If $\tilde{p}_s\le p_s\le\frac{2p_h}{3}$	(ii) If $\frac{2p_h}{3}\le p_s\le p_h(1-2\alpha)$	(ii) If $p_s\ge p_h(1-2\alpha)$
w	$\frac{3\alpha p_s-p_h(2-5\alpha)}{6(2\alpha^2-3\alpha+1)}<0$	$\frac{\alpha p_h-p_s(1-2\alpha)}{2(2\alpha^2-3\alpha+1)}<0$	$\frac{p_h(4\alpha-1)}{2(1-2\alpha)}<0$
$p_1(=p_2)$	$-\frac{p_h}{12}<0$	$-\frac{p_s}{8}<0$	$\frac{p_s-2p_h(1-2\alpha)}{8}<0$
D_{R1}	$\frac{p_h}{12}>0$	$\frac{p_s}{8}>0$	$\frac{2p_h(1-2\alpha)-p_s}{8}>0$
D_{R2}	$\frac{2p_h-3p_s}{12}>0$	$\frac{2p_h-3p_s}{8}\le 0$	$\frac{4\alpha p_h-p_s}{8}\le 0$
$D_{R1}+D_{R2}$	$\frac{p_h-p_s}{4}>0$	$\frac{p_h-p_s}{4}>0$	$\frac{p_h-p_s}{4}>0$
D_{G1}	$\frac{p_h}{4}>0$	$\frac{4p_h-3p_s}{8}>0$	$\frac{2p_h(1+2\alpha)-p_s}{8}>0$
D_{G2}	$\frac{p_h-p_s-1}{4}<0$	$\frac{6p_h-7p_s-4}{16}<0$	$\frac{2p_h(1+4\alpha)-3p_s-4}{16}<0$
Π_R	$\frac{p_h^2+3(p_h-p_s)(2+p_s-p_h)}{24}>0$	$\frac{4p_h(2+3p_s-p_h)-p_s(8+7p_s)}{32}>0$	$\frac{4p_h(2+p_s(1+4\alpha))-16p_h^2\alpha(1-\alpha)-p_s(8+3p_s)}{32}>0$
CS	$\frac{2(p_h-p_s)+p_s(2p_h-p_s)}{16}>0$	$\frac{8(p_h-p_s)+p_s(10p_h-7p_s)}{64}>0$	$\frac{2p_h(4+p_s)+4p_h^2(1-4\alpha^2)-p_s(8+3p_s)}{64}>0$

Table B.5 Guaranteed Trade-in vs. Obligatory Guarantee Models under Static Pricing and $\theta=1$ (for high guarantee premium$\alpha\ge\frac{1}{6}$)

	(i) If $\tilde{p}_s\le p_s\le p_h(1-2\alpha)$	(ii) If $p_h(1-2\alpha)\le p_s\le\frac{2p_h}{3}$	(ii) If $p_s\ge\frac{2p_h}{3}$
w	$\frac{3\alpha p_s-p_h(2-5\alpha)}{6(2\alpha^2-3\alpha+1)}<0$	$\frac{p_h+3p_s}{6(1-2\alpha)}-p_h<0$	$\frac{p_h(4\alpha-1)}{2(1-2\alpha)}<0$
$p_1(=p_2)$	$-\frac{p_h}{12}<0$	$\frac{3p_s-2p_h(2-3\alpha)}{12}<0$	$\frac{p_s-2p_h(1-2\alpha)}{8}<0$
D_{R1}	$\frac{p_h}{12}>0$	$\frac{2p_h(2-3\alpha)-3p_s}{12}>0$	$\frac{2p_h(1-2\alpha)-p_s}{8}>0$
D_{R2}	$\frac{2p_h-3p_s}{12}>0$	$\frac{p_h(6\alpha-1)}{12}>0$	$\frac{4\alpha p_h-p_s}{8}\begin{cases}\ge 0, \text{ if } p_s\le 4\alpha p_h\\ <0, \text{ otherwise}\end{cases}$
$D_{R1}+D_{R2}$	$\frac{p_h-p_s}{4}>0$	$\frac{p_h-p_s}{4}>0$	$\frac{p_h-p_s}{4}>0$
D_{G1}	$\frac{p_h}{4}>0$	$\frac{p_s+2\alpha p_h}{4}>0$	$\frac{2p_h(1+2\alpha)-p_s}{8}>0$
D_{G2}	$\frac{p_h-p_s-1}{4}<0$	$\frac{2\alpha p_h-1}{4}<0$	$\frac{2p_h(1+4\alpha)-3p_s-4}{16}<0$
Π_R	$\frac{p_h^2+3(p_h-p_s)(2+p_s-p_h)}{24}>0$	$\frac{6(p_h-p_s(1-2\alpha p_h))+p_h^2(1-12\alpha(1-\alpha))}{24}>0$	$\frac{4p_h(2+p_s(1+4\alpha))-16p_h^2\alpha(1-\alpha)-p_s(8+3p_s)}{32}>0$
CS	$\frac{2(p_h-p_s)+p_s(2p_h-p_s)}{16}>0$	$\frac{p_h(2+p_h-4\alpha^2p_h)-2p_s}{16}>0$	$\frac{2p_h(4+p_s)+4p_h^2(1-4\alpha^2)-p_s(8+3p_s)}{64}>0$

the optimal decisions and profit. In period 2, the retailer optimize her profit over two periods by choosing p_2, i.e.,

$$\max_{p_1\le p_2\le\theta}\ \Pi_R=p_1(1-p_1+\frac{p_h}{2})+p_2(1-\frac{p_2}{\theta}).$$

Optimize the above mathematical programming, we get the optimal price of Product 2, $p_2^*=p_1$ if $p_1\ge\frac{\theta}{2}$, otherwise $p_2^*=\frac{\theta}{2}$. Note that when the price of Product 1 is sufficiently large, i.e., $p_1\ge\frac{\theta}{2}$, the retailer will adopt static pricing policy and the optimal prices are $p_t^*=\frac{\theta(4+p_h)}{4(\theta+1)}$ $(t=1,2)$. In other words, static pricing is the retailer's best choice when $p_t^*=\frac{\theta(4+p_h)}{4(\theta+1)}\ge\frac{\theta}{2}\Rightarrow\theta\le 1+\frac{p_h}{2}$, i.e., the upgrade level of Product 2 is low. On the other hand, when the upgrade level of Product 2 is high ($\theta\ge 1+\frac{p_h}{2}$), the optimal price of

Product 2 is $p_2^* = \frac{\theta}{2}$. Thus, the retailer's period 1 problem can be written as: $\max_{p_1} \Pi_R = p_1(1-p_1+\frac{p_h}{2})+\frac{\theta}{4}$, such that $p_1 \le \min(1,\frac{\theta}{2})$. Solving for p_1 gives the optimal price of Product 1, $p_1^* = \frac{2+p_h}{4}$, and the retailer's optimal total profit $\Pi_R^* = \frac{4\theta+(2+p_h)^2}{16}$. The corresponding optimal demand in period 1 and 2 is, $D_{R1} = \frac{2+p_h}{4}$ and $D_{R2} = \frac{1}{2}$, respectively. In what follows, we will calculate consumer surplus. According to the value of θ, the analysis can be divided into the following two scenarios: (i) When $\theta \le 1+\frac{p_h}{2}$, the optimal prices $p_1^* = p_2^* = \frac{\theta(4+p_h)}{4(\theta+1)}$, the retailer adopts static pricing policy and the optimal consumer surplus is directly given by $CS^* = \frac{16\theta^2-\theta(16+8p_h-p_h^2)+4(2+p_h)^2}{32(\theta+1)}$. (ii) When $\theta \ge 1+\frac{p_h}{2}$, the optimal prices $p_1^* = \frac{2+p_h}{4}$ and $p_2^* = \frac{\theta}{2}$. Hence, $v_2^{BN} = \frac{p_2^*}{\theta} \ge v_1^{BC} = p_1^* - \frac{p_h}{2}$ and consumer surplus is:

$$CS = \int_0^{v_1^{BC}} 0\cdot f(v_1)\mathrm{d}v_1 + \int_{v_1^{BC}}^{v_2^{BN}} (v_1 - p_1 + \frac{p_h}{2}) f(v_1)\mathrm{d}v_1 + \int_{v_2^{BN}}^{1} ((v_1-p_1)+(\theta v_1 - p_1 + \frac{p_h}{2}))f(v_1)\mathrm{d}v_1.$$

Substituting the optimal prices into the above equation and some algebraic calculations leads to the optimal consumer surplus $CS^* = \frac{4\theta+(2+p_h)^2}{32}$. This complete the proof of Proposition 8. □

Proof of Proposition 9. This proof is to analyze the guarantee model with a third-party guarantor under static pricing. In this model, the retailer determines retail prices $p_1 = p_2$ and subsequently, the guarantor decides on the trade-in price w. Thus, demand functions in Table 1 will depend on w instead of p_2. After some algebra, demand functions for the the guarantee model with a third-party guarantor under static pricing scenario can be shown in Table B.6.

Table B.6 Demand Functions: Trade-in Price Guarantee from a Third Party Guarantor under Static Pricing

	(i) If $p_1 \ge \min(p_1^a, 1)$ & $w \le \min(w^b, p_h)$	(ii) Otherwise	(iii) If $p_1 \le \min(p_1^a, 1)$ & $w \le \min(w^a, p_h)$
R's period 1 demand D_{R1}	$1-p_1+\frac{p_h+w(1-2\alpha)}{2}$	$\frac{2+\theta-3p_1+p_h+w(1-2\alpha)}{2+\theta}$	$1-p_1+\frac{p_h}{2}$
R's period 2 demand D_{R2}	$1-\frac{p_1}{\theta}$	$\frac{\theta(4+2\theta-2p_1+p_h+w(1-2\alpha))-2p_1(1+\theta)}{2\theta(2+\theta)}$	$\frac{2(\theta-p_1)+w(1-2\alpha)}{2\theta}$
G's period 1 demand D_{G1}	$1-p_1+\frac{p_h+w(1-2\alpha)}{2}$	$\frac{2+\theta-3p_1+p_h+w(1-2\alpha)}{2+\theta}$	$\frac{\theta-p_1+w(1-2\alpha)}{\theta}$
G's period 2 demand D_{G2}	$\frac{1}{2}\Big(1-p_1+\frac{p_h+w(1-2\alpha)}{2}\Big)$	$\frac{2+\theta-3p_1+p_h+w(1-2\alpha)}{2(2+\theta)}$	$\frac{\theta-p_1+w(1-2\alpha)}{2\theta}$

Notes: $p_1^a = \frac{\theta p_h}{2(\theta-1)}$, $w^a = \frac{\theta p_h - 2p_1(\theta-1)}{2(1-2\alpha)}$, and $w^b = \frac{2p_1(\theta-1)-\theta p_h}{\theta(1-2\alpha)}$.

We use backward induction to calculate the optimal decision and profit for each party. The guarantor's problem is to set a trade-in price w to maximize her total expected profit, i.e.:

$$\max_{w}\ \Pi_G = \alpha w D_{G1} + (p_s - w)D_{G2} \tag{B-4}$$

According to the guarantor's demand functions in Table B.6, we need to consider the following three cases: Case (i): Equation (B-4) can be re-written as $\max_w \Pi_G = \frac{1}{2}(p_s - w(1-2\alpha))\Big(1-p_1+\frac{p_h+w(1-2\alpha)}{2}\Big)$. Using the first-order conditions leads to the unconstraint optimal trade-in price $\tilde{w}_1 = \frac{2(p_1-1)+(p_s-p_h)}{2(1-2\alpha)}$, which is less than zero due to the assumptions $p_s \le p_h$ and $p_1 \le 1$. For any $w \le \min(w^b, p_h)$, Π_G is decreasing in w and thus the optimal w is approaching to zero. Case (ii): Equation (B-4) is formulated as $\max_w \Pi_G = (p_s - w(1-2\alpha))\Big(\frac{2+\theta-3p_1+p_h+w(1-2\alpha)}{2(2+\theta)}\Big)$. One can easily verify that the unconstraint optimal trade-in price

$\tilde{w}_2 = \frac{(3p_1-\theta-2)+(p_s-p_h)}{2(1-2\alpha)}$ is less than zero. Note that if $p_1 \le p_1^a$, $w^b \le 0 \le w^a$, otherwise $w^a \le 0 \le w^b$. For any $w \ge \min(w^a, p_h)$ or $w \ge \min(w^b, p_h)$, Π_G decreases in w. Thus, if $p_1 \le p_1^a$, $w_2^* = \min(w^a, p_h)$, otherwise $w_2^* = \min(w^b, p_h)$. Case (iii): Equation (B-4) can be shown as $\max_w \ \Pi_G = (p_s - w(1-2\alpha))\left(\frac{\theta-p_1+w(1-2\alpha)}{2\theta}\right)$. The unconstrained optimal w gives $\tilde{w}_3 = \frac{p_1+p_s-\theta}{2(1-2\alpha)}$. The optimal trade-in price can be zero, $\tilde{w}_3$ or $\min(w^a, p_h)$, depends on p_1 value, as will be shown later.

It is easy to verify that Π_G as a function of w^b in Case (i) equals that in Case (ii), implying that Π_G is continuous at w^b. From the perspective of the guarantor's total expected profit, for any $p_1 \ge \min(p_1^a, 1)$, the optimal w will be approaching to zero. Hence, we only focus on the scenario where $p_1 \le \min(p_1^a, 1)$, which gives $w^b \le 0 \le w^a$. A comparison of w^a and p_h (the upper bound of w) yields the following two scenarios: 1) If $p_1 \le p_1^b$, then $w^a \ge p_h$ and only Case (iii) exists, where $p_1^b = \frac{p_h(\theta-2(1-2\alpha))}{2(\theta-1)}$. The optimal w is as follows:

$$w^* = \begin{cases} 0, & \text{if } \ p_1 \le p_1^{3a}; \\ \tilde{w}_3, & \text{if } \ p_1^{3a} \le p_1 \le p_1^{3c}; \\ p_h, & \text{otherwise.} \end{cases}$$

where $p_1^{3a} = \theta - p_s$ and $p_1^{3c} = \theta - p_s + 2p_h(1-2\alpha)$. 2) If $p_1^b \le p_1 \le p_1^a$, then $w^a \le p_h$. For any $w \le w^a$, Case (iii) happens, otherwise we have Case (ii). Note that for any $p_1 \le p_1^a$, the optimal w in Case (ii) is $\min(w^a, p_h) \equiv w^a$, which, from the perspective of the guarantor's total expected profit, will be dominated by the optimal w in Case (iii), as follows:

$$w^* = \begin{cases} 0, & \text{if } \ p_1 \le p_1^{3a}; \\ \tilde{w}_3, & \text{if } \ p_1^{3a} \le p_1 \le p_1^{3b}; \\ w^a, & \text{otherwise.} \end{cases}$$

where $p_1^{3b} = \frac{\theta(1+p_h)-p_s}{2\theta-1}$.

Based on the above analysis, we will prove that for any $p_1 \le p_1^a$, the optimal w can be $0, \tilde{w}_3$ or w^a, depending on the cut-offs of p_1. It is helpful to let $\Gamma = 2(\theta-1)(\theta-p_s) + p_h(3\theta - 2 - 4\alpha(2\theta-1))$. Clearly, Γ decreases in α and p_s. For any $\alpha \le 0.25$ and $p_s \le p_h$, the minimum of Γ gives $(\theta-1)(2\theta-p_h)$, which is nonnegative due to the assumptions $\theta \ge 1$ and $p_h \le 1$. Note that $p_1^{3b} - p_1^b = \frac{\Gamma}{2(\theta-1)(2\theta-1)}$ and $p_1^{3c} - p_1^{3b} = \frac{\Gamma}{(2\theta-1)}$, meaning that $p_1^b \le p_1^{3b} \le p_1^{3c}$. Thus, for any $p_1 \le p_1^b$, the optimal w never reaches to p_h. In summary, if $p_1 \le \min(p_1^{3a}, p_1^a)$, w^* is approaching to zero; if $\min(p_1^{3a}, p_1^a) \le p_1 \le p_1^{3b}$, $w^* = \tilde{w}_3$; otherwise $w^* = w^a$. Compare the above cut-offs of p_1 and its lower/upper bounds, after some algebra, we can summarize the optimal w as follows:

- $\theta \le 2$, we get $0 \le p_h^d \le p_h^a \le 1$, where $p_h^a = 2(1-\frac{1}{\theta})$ and $p_h^d = \frac{2\theta(\theta-1)}{3\theta-2}$. In this case: 1) if $p_h \le p_h^d$, for any $p_s \le p_h$ and $p_1 \le 1$, $w^* \sim 0$. 2) If $p_h^d \le p_h \le p_h^a$ and $p_s \le p_s^a$, where $p_s^a = \frac{\theta(2(\theta-1)-p_h)}{2(\theta-1)}$, then for any $p_1 \le 1$, $w^* \sim 0$; if $p_h^d \le p_h \le p_h^a$ and $p_s \ge p_s^a$, $w^* \sim 0$ when $p_1 \le p_1^{3a}$ or $p_1 \ge p_1^a$; $w^* = \tilde{w}_3$ when $p_1^{3a} \le p_1 \le p_1^{3b}$; and $w^* = w^a$ when $p_1^{3b} \le p_1 \le p_1^a$. 3) If $p_h \ge p_h^a$ and $p_s \le p_s^b$, where $p_s^b = \theta - 1$. For any $p_1 \le 1$, $w^* \sim 0$; if $p_h \ge p_h^a$ and $p_s^b \le p_s \le p_s^c$, where $p_s^c = 1 - \theta(1-p_h)$, then $w^* \sim 0$ when $p_1 \le p_1^{3a}$, and $w^* = \tilde{w}_3$ when $p_1 \ge p_1^{3a}$; if $p_h \ge p_h^a$ and $p_s \ge p_s^c$, $w^* \sim 0$ when $p_1 \le p_1^{3a}$, $w^* = \tilde{w}_3$ when $p_1^{3a} \le p_1 \le p_1^{3b}$, and $w^* = w^a$ when $p_1 \ge p_1^{3b}$.
- $\theta \ge 2$, for any $p_h \le 1$, $p_s \le p_h$, and $p_1 \le 1$, the optimal w is approaching to zero.

Given the optimal w contingent on p_1, next we will compute $p_1 (= p_2)$ that maximizes the retailer's total expected profit. When $w^* \sim 0$, the retailer's total expected profit gives $\Pi_R = \frac{p_1(\theta(4+p_h)-2p_1(\theta+1))}{2\theta} - C(\theta)$. It is easy to verify Π_R is strictly concave in p_1. Using the first-order conditions we get the unconstraint optimal $\check{p}_1 = \frac{\theta(4+p_h)}{4(1+\theta)}$. Similarly, when $w^* = \tilde{w}_3$ and $w^* = w^a$, the unconstraint optimal p_1 is $\tilde{p}_1 = \frac{\theta(7+2p_h)+p_s}{2(3+4\theta)}$ and

$\hat{p}_1 = \frac{\theta(8+3p_h)}{4(1+3\theta)}$, respectively. By following the similar logic as the optimal w analysis, one can compare those unconstraint optimal values with the lower/upper bound of p_1 to find out the final optimal p_1. Omit the detailed processes, we obtain the following conditions characterize the retailer's optimal pricing decision:

- $\theta \le 2$, which gives $0 \le \bar{p}_h \le p_h^d \le p_h^a \le 1$, where $\bar{p}_h = \frac{4(\theta-1)}{\theta+3}$. In this case: 1) $p_h \le \bar{p}_h$. For any $p_s \le p_h$, we have $0 \le p_1^a \le \check{p}_1 \le 1$. Hence, $p_1 \le p_1^a$ leads to Case (iii) in which the optimal p_1 is p_1^a, otherwise Case (i) happens and the optimal p_1 gives $\check{p}_1$. Moreover, one can verify that Π_R as a function of p_1 in Case (i) equals that in Case (iii), meaning that Π_R is continuous at p_1^a and hence p_1^a is dominated by $\check{p}_1$. Therefore, for any $p_h \le \bar{p}_h$, only Case (i) exists and the optimal $p_1^* = \check{p}_1$. 2) $\bar{p}_h \le p_h \le p_h^d$. For any $p_s \le p_h$, $0 \le \check{p}_1 \le p_1^a \le 1$. Under this setting, $p_1 \le p_1^a$ gives Case (iii) in which the optimal $p_1^* = \check{p}_1$, otherwise we have Case (i) where $p_1^* = p_1^a$. Clearly, in terms of the retailer's total expected profit, Case (i) will be dominated by Case (iii) and the final optimal p_1 yields $\check{p}_1$. 3) $p_h^d \le p_h \le p_h^a$. By following the similar logic as the analysis when $p_h \le p_h^d$, one can verify that for any $p_s \le p_s^a$, we obtain Case (iii) where the optimal $p_1^* = \check{p}_1$. When $p_s \ge p_s^a$ and $p_1 \le p_1^a$, we have Case (iii) in which $p_1^* = \check{p}_1$ if $p_1 \le p_1^{3a}$; $p_1^* = \tilde{p}_1$ if $p_1^{3a} \le p_1 \le p_1^{3b}$; and $p_1^* = p_1^{3b}$ otherwise. On the other hand, when $p_s \ge p_s^a$ and $p_1 \ge p_1^a$, we get Case (i) where the optimal $p_1^* = p_1^a$. Note that the retailer's total expected profit function is continuous between p_1^{3a} and 1. Therefore, p_1^a is dominated by p_1^{3b}, which is again dominated by $\tilde{p}_1$, meaning that for any $p_1 \ge p_1^{3a}$, the optimal p_1 gives $\tilde{p}_1$. As a result, for any $p_1 \le 1$, the optimal p_1 can be either $\check{p}_1$ or $\tilde{p}_1$, depending on the indifference point of the retailer's total expected profit. In particular, if $p_s \le \bar{p}_s$, $p_1^* = \check{p}_1$, otherwise $p_1^* = \tilde{p}_1$, where $\bar{p}_s = \theta((4+p_h)\sqrt{(3+4\theta)/(1+\theta)} - (7+2p_h))$. 4) $p_h \ge p_h^a$, which leads to Case (iii). Using the similar logic as the analysis when $p_h^d \le p_h \le p_h^a$, one can verify that if $p_s \le p_s^b$, the optimal p_1 gives $\check{p}_1$. If $p_s \ge p_s^b$ and $p_1 \le p_1^{3a}$, $p_1^* = \check{p}_1$, otherwise $p_1^* = \tilde{p}_1$. Therefore, the optimal p_1 also depends on the cut-off value of p_s, i.e., if $p_s \le \bar{p}_s$, then $p_1^* = \check{p}_1$, otherwise $p_1^* = \tilde{p}_1$.
- $\theta \ge 2$, it is easy to see that the optimal w is approaching to zero, the optimal p_1 can be either $\check{p}_1$ or 1. Specifically, if $p_h \le \frac{4}{\theta}$, $p_1^* = \check{p}_1$, otherwise $p_1^* \to 1$, which means the optimal retail price approaches to the maximum valuation of Product 1. In what follows, we are only interested in the setting where $\frac{4}{\theta} \le 1$, i.e., $\theta \le 4$. A comparison of $\check{p}_1$, p_1^a, and the lower/upper bound of p_1, we can see that if $2 \le \theta \le \frac{7}{3}$, then $0 < \bar{p}_h \le 1$. In this case, $0 < p_h \le \bar{p}_h$ leads to $0 < p_1^a \le \check{p}_1 \le 1$ and hence, Case (i) happens and the optimal p_1 gives $\check{p}_1$. Otherwise, $0 < \check{p}_1 \le p_1^a \le 1$, which gives Case (i) where the optimal p_1 yields $\check{p}_1$. By the similar logic as the above analysis, one can verify that for any $\frac{7}{3} \le \theta \le 4$, $\bar{p}_h \ge 1$ and Case (i) happens, the optimal $p_1^* = \check{p}_1$.

In summary, for any $\theta \le 4$: if $p_h \le \min(\bar{p}_h, 1)$, for any $p_s \le p_h$, Case (i) holds. The optimal prices are $w^* \sim 0$ and $p_1^* = \check{p}_1$. Otherwise, $0 < \bar{p}_s \le p_h$ and either Case (ii) or Case (iii) will happen. The optimal p_1 is contingent on p_s value. Specifically, if $p_s \le \min(\bar{p}_s, p_h)$, the optimal prices are $w^* \sim 0$ and $p_1^* = \check{p}_1$, otherwise $w^* = \tilde{w}_3$ and $p_1^* = \tilde{p}_1$. Substitute $\tilde{p}_1$ into $\tilde{w}_3$ and some algebra gives $w^* = \frac{7p_s+\theta(1+2p_h+8p_s-8\theta)}{4(1-2\alpha)(3+4\theta)}$. Substitute the equilibrium prices into the demand functions in Table B.6, the guarantor's total expected profit in equation (B-4), and the retailer's total expected profit, some algebraic calculation directly yields the optimal demands and profits, we omit the details.

Next, we show the derivation process of consumer surplus. Under Case (i), consumers can pick branch (A) or (C), depends on the indifference point v_1^{AC3}. Based on customers' total expected utilities across both periods in Table 1, a customer gains a utility $u_A = (v_1 - p_1 - \alpha w) + \mathrm{E}_{p_u} \max(v_2 - p_2 + w, v_2 - p_2 + p_u, p_u)$ and

$u_C = \max(v_2 - p_2, 0)$ when s/he picks branch (A) and (C), respectively. Moreover, given the optimal prices $w^* \sim 0$ and $p_1^* = \frac{\theta(4+p_h)}{4(1+\theta)}$, it is easy to see that $v_1^{AC3} \geq \frac{p_1}{\theta}$. Hence, consumer surplus is shown as:

$$CS = \int_{\frac{p_1}{\theta}}^{v_1^{AC3}} (\theta v_1 - p_1)\mathrm{d}v_1 + \frac{1}{2}\int_{v_1^{AC3}}^{1} (2((1+\theta)v_1 - 2p_1) + p_h + w(1-2\alpha))\mathrm{d}v_1 \tag{B-5}$$

Substituting the equilibrium prices into equation (B-5), some algebraic calculation yields the optimal consumer surplus $CS = \frac{16\theta^2 - \theta(16+8p_h-p_h^2)+4(2+p_h)^2}{32(1+\theta)}$. Under Case (ii) and Case (iii), all branches are likely to be chosen, depending on the indifference points v_1^{BC} and v_1^{AB}. Specifically, customer whose valuation of Product 1 $v_1 \leq v_1^{BC}$ picks branch (C); if $v_1^{BC} \leq v_1 \leq v_1^{AB}$, s/he selects branch (B); otherwise s/he chooses branch (A). Thus, consumer surplus is given by $CS = \int_0^{v_1^{BC}} u_C \mathrm{d}v_1 + \int_{v_1^{BC}}^{v_1^{AB}} u_B \mathrm{d}v_1 + \int_{v_1^{AB}}^{1} u_A \mathrm{d}v_1$, where u_A, u_B, and u_C are defined in Table 1. In addition, it is easy to verify that $\frac{p_1-w}{\theta} \leq v_1^{AB} \leq \frac{p_1}{\theta}$. Consumer surplus is obtained by:

$$\begin{aligned} CS = &\int_{v_1^{BC}}^{v_1^{AB}} (v_1 - p_1 + \frac{p_h}{2})\mathrm{d}v_1 + \frac{1}{2}\int_{v_1^{AB}}^{\frac{p_1}{\theta}} ((2+\theta)v_1 - 3p_1 + p_h + w(1-2\alpha))\mathrm{d}v_1 \\ &+ \frac{1}{2}\int_{v_1^{AB}}^{1} (2((1+\theta)v_1 - 2p_1) + p_h + w(1-2\alpha))\mathrm{d}v_1 \end{aligned} \tag{B-6}$$

Note that the equilibrium prices in Case (iii) depend on the cut-off, $\bar{p}_s$. If $p_s \leq \min(\bar{p}_s, p_h)$, the optimal prices are $w^* \sim 0$ and $p_1^* = \frac{\theta(4+p_h)}{4(1+\theta)}$, otherwise $w^* = \frac{7p_s+\theta(1+2p_h+8p_s-8\theta)}{4(1-2\alpha)(3+4\theta)}$ and $p_1^* = \frac{\theta(7+2p_h)+p_s}{2(3+4\theta)}$. Substitute the equilibrium prices into equation (B-6) leads to optimal consumer surplus. We omit the detailed process. □

Proof of Proposition 10. We make comparisons of the equilibrium outcomes in the guarantee model with a third-party guarantor (Model "G3") with the no guarantee model (Model "NG") and the guaranteed trade-in model (Model "G") to rank the retailer's optimal profit and consumer surplus.

First, we compare the equilibrium outcomes in model "G3" with those in model "NG". Under the static pricing scenario and $\theta = 1$, note that the threshold $\bar{p}_h \equiv 0$ and case (i) of the Table 6 is gone. Hence, if $p_s \leq \min(\bar{p}_s, p_h)$, the optimal w is approaching to zero and those two models are identical, meaning that the guaranteed trade-in price offered by a third-party is ineffective. However, when p_s is sufficiently high, i.e., $p_s \geq \min(\bar{p}_s, p_h)$, the guaranteed trade-in price becomes effective and its impact on the retailer's optimal profit and consumer surplus are as follows: (i) The impact on the retailer's optimal profit. Note that in the analysis of the optimal retail price, the retailer is indifferent between setting $\check{p}_1$ and $\tilde{p}_1$ at $\bar{p}_s$. If $p_s \geq \min(\bar{p}_s, p_h)$, the optimal p_1 gives $\tilde{p}_1$, which means $\Pi_R(\tilde{p}_1) \geq \Pi_R(\check{p}_1)$, otherwise $\Pi_R(\tilde{p}_1) < \Pi_R(\check{p}_1)$. Hence, for any $p_s \geq \min(\bar{p}_s, p_h)$, we get $\Pi_R^{G3} \geq \Pi_R^{NG}$. (ii) The difference on consumer surplus in models "G3" and "NG" yields:

$$\Delta CS = \frac{p_s^2(64\theta^3+152\theta^2+117\theta+29) - 2p_s\theta(2p_h(16\theta^2+29\theta+13) - 64\theta^3 - 56\theta^2 + 57\theta + 49) - \theta^2(2p_h^2(11+14\theta) - p_h(128\theta^2+4\theta-76) + 192\theta^3 + 8\theta^2 + 3\theta + 91)}{64\theta(\theta+1)(4\theta+3)^2}$$

In what follows, we prove ΔCS is less than zero. It is easy to see ΔCS is convex in p_s. Solving for $\Delta CS = 0$ yields two solutions $p_s^a = \frac{\theta(A+B)}{(\theta+1)(64\theta^2+88\theta+29)}$ and $p_s^b = \frac{-\theta(2p_h^2(14\theta+11)+4p_h(32\theta^2+\theta-19)-(192\theta^3+8\theta^2+3\theta+91))}{A+B}$, where $A = (\theta+1)(49+8\theta-64\theta^2+2p_h(16\theta+13))$ and $B = (4\theta+3)\sqrt{2(\theta+1)(8(\theta(8\theta-5)(8\theta+3)+35)+p_h^2(88\theta+73)-24p_h(16\theta^2-11))}$. We prove that $p_s^a > p_h$ and $p_s^b < 0$. One can verify that $A+B$ is positive. Moreover, the numerator of p_s^b is strictly concave in p_h and its discriminant gives $-8(\theta+1)(40\theta+31)(\theta(4\theta+3))^2 < 0$. Hence, $p_s^b < 0$. On the other hand, $p_s^a - p_h = \frac{\theta B - C}{(\theta+1)(64\theta^2+88\theta+29)}$, where $C = 64\theta^4 + 8\theta^3(7+4p_h) + \theta^2(94p_h - 57) + 7\theta(13p_h - 7) + 29p_h$, which is increasing in p_h and the maximal value of C gives $\max(C) = (\theta+1)(\theta(8\theta(8\theta+3)+13)+29)$. Note that $(\theta B)^2 - (\max(C))^2 = (\theta+1)(2p_h^2(\theta(3+4\theta))^2(73+88\theta) - 48p_h(\theta(3+4\theta))^2(16\theta^2-11) + 16(\theta(3+4\theta))^2(35+\theta(8\theta-5)(8\theta+3)) - (\theta+1)(29+\theta(13+8\theta(3+8\theta)))^2)$. Clearly, $(\theta B)^2 - (\max(C))^2$ is convex in θ and its discriminant is $\Omega = 8(1+\theta)^3(\theta(3+4\theta))^2(61393+\theta(129050+\theta(126017+8\theta(9697-8\theta(7679+8\theta(1949+8\theta(97-24\theta)))))))$. For any $\theta \le 4$, numerical result shows that Ω is strictly less than 0, meaning that $(\theta B)^2 - (\max(C))^2 > 0$, which leads to $p_s^a > p_h$. In summary, ΔCS is convex in p_s, and two solutions to $\Delta CS = 0$ follow $p_s^b < 0$ and $p_s^a > p_h$. Hence, $\Delta CS < 0$. To sum up, for any $0 < p_s \le p_h$, the ranks of the retailer's optimal profit and consumer surplus follows: $\Pi_R^{G3} \ge \Pi_R^{NG}$ and $CS^{G3} \le CS^{NG}$.

Second, we compare the equilibrium outcomes in model "G3" with those in model "G". According to Propositions 3 and 6, the analysis can be divided into the following three scenarios: (i) $0 < p_h \le \frac{5\sqrt{14}-14}{11}$, which leads to $\frac{2p_h}{3} \le p_h \le \bar{p}_s$. In this case, if $0 < p_s \le \frac{2p_h}{3}$, the difference on the retailer's optimal profit gives: $\Pi_R^G - \Pi_R^{G3} = \frac{p_h^2+12p_s(2+p_s)}{96}$, which is obviously greater than zero. The difference on consumer surplus is: $CS^G - CS^{G3} = \frac{4p_s(2+p_s)-p_h^2}{64}$, one can examine that $p_s \le p_s'$ gives $CS^G \le CS^{G3}$, otherwise $CS^G > CS^{G3}$, where $p_s' = \sqrt{1+\frac{p_h^2}{4}} - 1$ and for any $0 < p_h < 1$, we have $p_s' < \min(\bar{p}_s, \frac{2p_h}{3})$. On the other hand, if $p_s \ge \frac{2p_h}{3}$, $\Pi_R^G - \Pi_R^{G3} = \frac{p_s^2+8p_s+4p_sp_h-p_h^2}{32}$, which is convex in p_s. The two solutions to $\Pi_R^G - \Pi_R^{G3} = 0$ yields $-\sqrt{5p_h^2+16p_h+16} - 2p_h - 4$ and $\sqrt{5p_h^2+16p_h+16} - 2p_h - 4$, it is easy to verify that both of them are strictly less than $\frac{2p_h}{3}$, meaning that for any $p_s \ge \frac{2p_h}{3}$, $\Pi_R^G - \Pi_R^{G3} > 0$. In addition, $CS^G - CS^{G3} = \frac{p_s^2+8p_s+2p_sp_h-p_h^2}{64}$, which is also convex in p_s and the two solutions to $CS^G - CS^{G3} = 0$, $-\sqrt{2(8+4p_h+p_h^2)} - p_h - 4$ and $\sqrt{2(8+4p_h+p_h^2)} - p_h - 4$, are all less than $\frac{2p_h}{3}$. Hence, for any $p_s \ge \frac{2p_h}{3}$, $CS^G > CS^{G3}$. (ii) $\frac{5\sqrt{14}-14}{11} \le p_h \le \frac{3(11\sqrt{14}-28)}{65}$, which gives $\frac{2p_h}{3} \le \bar{p}_s \le p_h$. By the similar logic as when $0 < p_h \le \frac{5\sqrt{14}-14}{11}$, one can examine that for any $0 \le p_h \le \bar{p}_s$, $\Pi_R^G > \Pi_R^{G3}$, and $CS^G \le CS^{G3}$ iff $p_s \ge p_s'$. On the other hand, if p_s is sufficiently high, i.e., $p_s \ge \bar{p}_s$, we get $\Pi_R^G - \Pi_R^{G3} = \frac{2(7-4p_h^2)+p_s(28+5p_s+20p_h)}{224}$, which is positive. In terms of consumer surplus, $CS^G - CS^{G3} = \frac{-6p_s(22p_s-63)-24p_h^2+2p_h(107p_s-14)+147}{3136}$, which is concave in p_s. Two solutions, $\frac{189+107p_h-7\sqrt{169p_h^2+750p_h+1125}}{132}$ and $\frac{189+107p_h+7\sqrt{169p_h^2+750p_h+1125}}{132}$, lead to $CS^G - CS^{G3} = 0$, with the former less than zero and the latter greater than 1. Thus, $CS^G > CS^{G3}$. (iii) $\frac{3(11\sqrt{14}-28)}{65} \le p_h \le 1$, which yields $\bar{p}_s \le \frac{2p_h}{3} \le p_h$. By the similar logic as when $\frac{5\sqrt{14}-14}{11} \le p_h \le \frac{3(11\sqrt{14}-28)}{65}$, one can verify that if p_s is low (i.e., $p_s \le \bar{p}_s$), we get $\Pi_R^G > \Pi_R^{G3}$, and $CS^G \ge CS^{G3}$ iff $p_s \ge p_s'$. If p_s is high (i.e., $p_s \ge \frac{2p_h}{3}$), then $\Pi_R^G > \Pi_R^{G3}$ and $CS^G \ge CS^{G3}$. When p_s is in an intermediate level, i.e., $\bar{p}_s \le p_s \le \frac{2p_h}{3}$, then $\Pi_R^G - \Pi_R^{G3} = \frac{21(1+p_s)^2+2(p_h-3p_s)^2}{336}$, which is also greater than zero. Additionally, $CS^G - CS^{G3} = \frac{15p_s^2+21(18p_s+7)+4p_h(29p_s-6p_h-7)}{3136}$, which is convex in p_s, and one can easily examine that the two solutions to $CS^G - CS^{G3} = 0$, $\frac{-189-58p_h-14\sqrt{19}(3+p_h)}{15}$ and $\frac{-189-58p_h+14\sqrt{19}(3+p_h)}{15}$, are all less than zero, meaning that $CS^G > CS^{G3}$. In summary, for any $0 < p_s \le p_h$, the ranks of the retailer's optimal profit and consumer surplus follows $\Pi_R^G > \Pi_R^{G3}$, and $CS^G \ge CS^{G3}$ iff $p_s \ge p_s'$.

Finally, according to the comparisons between models "G" and "NG" in Table 3, as well as the above analysis, we can hence make comprehensive rankings of the retailer's optimal profit and consumer surplus, as follows: if $0 < p_s \le p_s'$, $\Pi_R^G > \Pi_R^{G3} \ge \Pi_R^{NG}$ and $CS^G \le CS^{G3} \le CS^{NG}$, otherwise $\Pi_R^G > \Pi_R^{G3} \ge \Pi_R^{NG}$ and $CS^G \ge CS^{NG} \ge CS^{G3}$. This completes the proof of proposition 10. □

Appendix C: Numerical Studies

Table C.1 The Guarantee vs. No Guarantee Models under Static Pricing and Guarantee Premium $\alpha = 0.0$

$p_h = 0.20$

	$p_s = 0.05$					$p_s = 0.15$				
θ	1.1	1.2	1.3	1.4	1.5	1.1	1.2	1.3	1.4	1.5
p_1	0.00000	0.02513	0.05025	0.05528	0.05528	0.00000	0.02513	0.03518	0.04020	0.04020
p_2	0.00000	0.02513	0.05025	0.05528	0.05528	0.00000	0.02513	0.03518	0.04020	0.04020
D_{R1}	0.00000	0.01181	0.02944	0.04100	0.04472	0.00423	0.02751	0.04314	0.05430	0.05980
D_{R2}	0.01051	-0.00230	-0.02303	-0.03682	-0.03685	0.02722	0.00555	-0.01038	-0.02479	-0.02680
$\sum_{t=1}^{2} D_{Rt}$	0.01051	0.00951	0.00641	0.00418	0.00787	0.03145	0.03306	0.03276	0.02951	0.03300
Π_R	0.01279	0.01358	0.01669	0.02118	0.02485	0.03995	0.04092	0.04379	0.04792	0.05110
CS	0.00592	0.00506	-0.00042	-0.00722	-0.00901	0.01819	0.01880	0.01572	0.00885	0.00718

$p_h = 0.40$

	$p_s = 0.05$					$p_s = 0.15$				
θ	1.1	1.2	1.3	1.4	1.5	1.1	1.2	1.3	1.4	1.5
p_1	0.00503	0.00000	-0.00503	0.05528	0.09045	0.00503	0.00000	-0.01005	0.05528	0.09045
p_2	0.00503	0.00000	-0.00503	0.05528	0.09045	0.00503	0.00000	-0.01005	0.05528	0.09045
D_{R1}	-0.00503	0.00000	0.00503	0.00958	0.02268	-0.00503	0.00000	0.01005	0.02436	0.03704
D_{R2}	0.02193	0.00921	0.00387	-0.00684	-0.02852	0.04477	0.03015	0.02242	0.00055	-0.02134
$\sum_{t=1}^{2} D_{Rt}$	0.01690	0.00921	0.00889	0.00274	-0.00584	0.03974	0.03015	0.03247	0.02491	0.01570
Π_R	0.01320	0.01276	0.01309	0.01411	0.01874	0.04047	0.03985	0.04006	0.04279	0.04731
CS	0.00899	0.00565	0.00554	0.00098	-0.01010	0.02270	0.01924	0.02149	0.01540	0.00427

Table C.2 The Guarantee vs. No Guarantee Models under Static and Dynamic Pricing Policies ($\alpha = 0.0$)

$p_h = 0.20$ and $\theta = 1.25$

	Static Pricing					Dynamic Pricing				
p_s	0.050	0.075	0.100	0.125	0.150	0.050	0.075	0.100	0.125	0.150
p_1	0.04020	0.04020	0.04020	0.04020	0.03518	0.00000	0.00000	0.00000	0.03015	0.01508
p_2	0.04020	0.04020	0.04020	0.04020	0.03518	0.00000	0.00000	0.00000	-0.00020	-0.00040
D_{R1}	0.02122	0.02524	0.02895	0.03081	0.03545	0.00000	0.00000	0.00000	-0.00222	0.00155
D_{R2}	-0.01376	-0.01175	-0.00990	-0.00897	-0.00464	0.01005	0.01528	0.02010	0.02525	0.02722
$\sum_{t=1}^{2} D_{Rt}$	0.00746	0.01349	0.01906	0.02184	0.03081	0.01005	0.01528	0.02010	0.02303	0.02877
Π_R	0.01490	0.02166	0.02846	0.03530	0.04219	0.01275	0.01931	0.02600	0.03282	0.03972
CS	0.00235	0.00588	0.00916	0.01081	0.01622	0.00640	0.00983	0.01306	0.01534	0.01878

$p_h = 0.40$ and $\theta = 1.50$

	Static Pricing					Dynamic Pricing				
p_s	0.050	0.075	0.100	0.125	0.150	0.050	0.075	0.100	0.125	0.150
p_1	0.09045	0.09045	0.09045	0.09045	0.09045	0.00000	0.00000	0.00000	0.00000	0.00000
p_2	0.09045	0.09045	0.09045	0.09045	0.09045	0.00000	0.00000	0.00000	0.00000	0.00000
D_{R1}	0.02268	0.02613	0.02958	0.03302	0.03704	0.00000	0.00000	0.00000	0.00000	0.00000
D_{R2}	-0.02852	-0.02680	-0.02508	-0.02335	-0.02134	0.00871	0.01340	0.01742	0.02144	0.02546
$\sum_{t=1}^{2} D_{Rt}$	-0.00584	-0.00067	0.00450	0.00967	0.01570	0.00871	0.01340	0.01742	0.02144	0.02546
Π_R	0.01874	0.02581	0.03293	0.04010	0.04731	0.01271	0.01922	0.02583	0.03255	0.03938
CS	-0.01010	-0.00668	-0.00325	0.00021	0.00427	0.00663	0.01029	0.01348	0.01672	0.02001