Homework 2

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2) $${}^{c}t_{w} = \begin{bmatrix} -5 \\ 0.5 \\ 4 \end{bmatrix}$$

$$w_{t_c} = -({}^{c}R_{w})^{T} \cdot {}^{c}t_{w} = -\begin{pmatrix} 0.707 & -0.707 & 0 \\ 0.707 & 0.707 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} -5 \\ 0.5 \\ 4 \end{pmatrix}$$

$$\Rightarrow \boxed{w_{t_c} = \begin{pmatrix} 6.363 \\ 3.1815 \\ -4 \end{pmatrix}}$$

4) $$R_1 = \begin{pmatrix} c45 & -s45 & 0 \\ s45 & c45 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0.707 & 0.707 & 0 \\ -0.707 & 0.707 & 0 \\ 0 & 0 & 1 \end{pmatrix} = {}^{A}R_{B}$$

$${}^{A}t_{B} = [1, 1, 0]^{T}$$

$${}^{A}T_{B} = \begin{pmatrix} 0.707 & 0.707 & 0 & 1 \\ -0.707 & 0.707 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$ {}^{B}T_{A} = \begin{bmatrix} ({}^{A}R_{B})^{T} & -({}^{A}R_{B})^{T} \cdot {}^{A}t_{B} \end{bmatrix} $$

$$ ({}^{A}R_{B})^{T} = \begin{pmatrix} 0.707 & -0.707 & 0 \\ 0.707 & 0.707 & 0 \\ 0 & 0 & 1 \end{pmatrix} $$

$$ {}^{B}t_{A} = -\begin{pmatrix} 0.707 & -0.707 & 0 \\ 0.707 & 0.707 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -1.414 \\ 0 \end{pmatrix} $$

$$ {}^{B}T_{A} = \begin{pmatrix} 0.707 & -0.707 & 0 & 0 \\ 0.707 & 0.707 & 0 & -1.414 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

$$ {}^{B}p = {}^{B}T_{A} \cdot p^{A} = \begin{pmatrix} 0.707 & -0.707 & 0 & 0 \\ 0.707 & 0.707 & 0 & -1.414 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 0 \\ 4 \\ 1 \end{pmatrix} $$

$$ \Rightarrow \boxed{ {}^{B}p = \begin{pmatrix} -0.707 \\ -2.121 \\ 4 \\ 1 \end{pmatrix} \Rightarrow \begin{pmatrix} -0.707 \\ -2.121 \\ 4 \end{pmatrix} } $$

5) $R_1 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{pmatrix} = Rot(\hat{j}, 90°)$

$R_2 = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} = Rot(\hat{z}, 90°)$

$${}^{A}R_B = R_2 \cdot R_1 = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{pmatrix}$$

$${}^{A}R_B = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{pmatrix} \quad ; \quad {}^{A}t_B = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

$${}^{A}T_B = \begin{pmatrix} 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

a) $${}^{A}\hat{y}_B = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$$

b) ${}^{B}\vec{y}_{A} = {}^{A}R_{B}^{T} \cdot {}^{A}\vec{y}_{B} = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$

$$ {}^{B}\vec{y}_{A} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} $$

c) ${}^{B}T_{A} = \left[\left({}^{A}R_{B}\right)^{T} \quad - \left({}^{A}R_{B}\right)^{T} \cdot {}^{A}t_{B} \right]$

$$ \left({}^{A}R_{B}\right)^{T} = \begin{pmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} $$

$$ -\left({}^{A}R_{B}\right)^{T} \cdot {}^{A}t_{B} = -\begin{pmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} $$

$$ {}^{B}T_{A} = \begin{pmatrix} 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

$$ {}^{B}P = {}^{B}_{A}T \cdot {}^{A}P = \begin{pmatrix} 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 8 \\ 1 \end{pmatrix} $$

$$ \Rightarrow \boxed{ {}^{B}P = \begin{pmatrix} -8 \\ 0 \\ -1 \\ 1 \end{pmatrix} \Rightarrow \begin{pmatrix} -8 \\ 0 \\ -1 \end{pmatrix} } $$

b) a) $ {}^{A}\vec{x}_B = \begin{pmatrix} -\sin 45 \\ 0 \\ \cos 45 \end{pmatrix} $; $ {}^{A}\vec{y}_B = \begin{pmatrix} \cos 45 \\ 0 \\ \sin 45 \end{pmatrix} $; $ {}^{A}\vec{z}_B = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} $

$$ \Rightarrow \boxed{ {}^{A}R_B = \begin{pmatrix} -0.707 & 0.707 & 0 \\ 0 & 0 & 1 \\ 0.707 & 0.707 & 0 \end{pmatrix} } $$

b) $\boxed{ {}^{A}t_B = \begin{pmatrix} 11 \\ 0 \\ 6 \end{pmatrix} }$