Fuzzy Sets

Murat Osmanoglu

Crisp Sets
(Classical or Nonfuzzy)

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Definition : collection of well-defined objects

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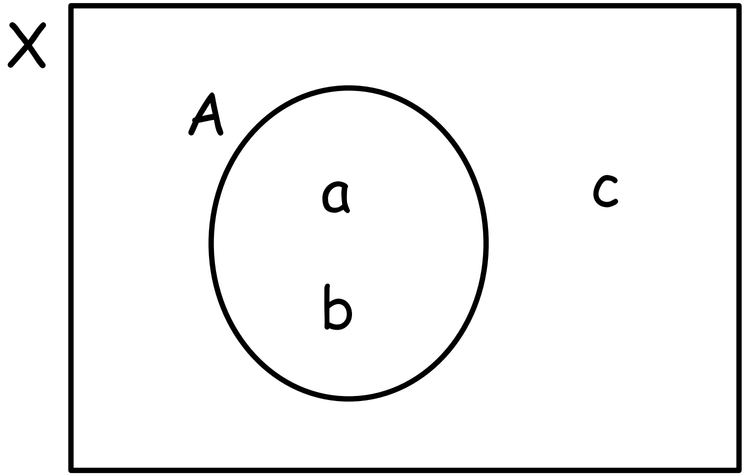

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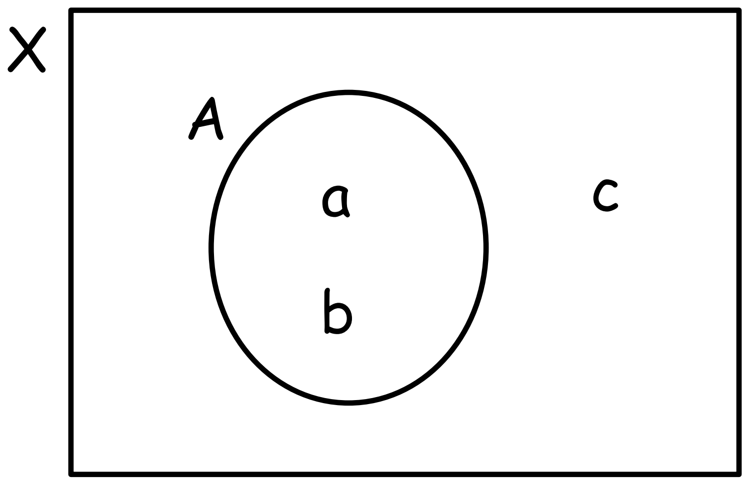

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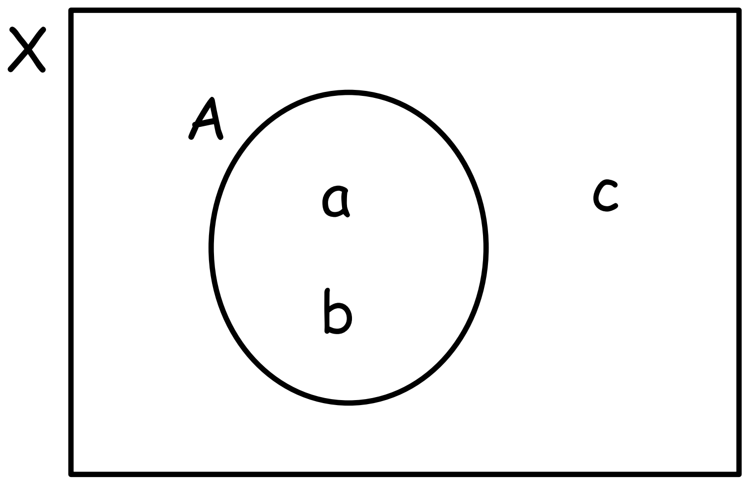

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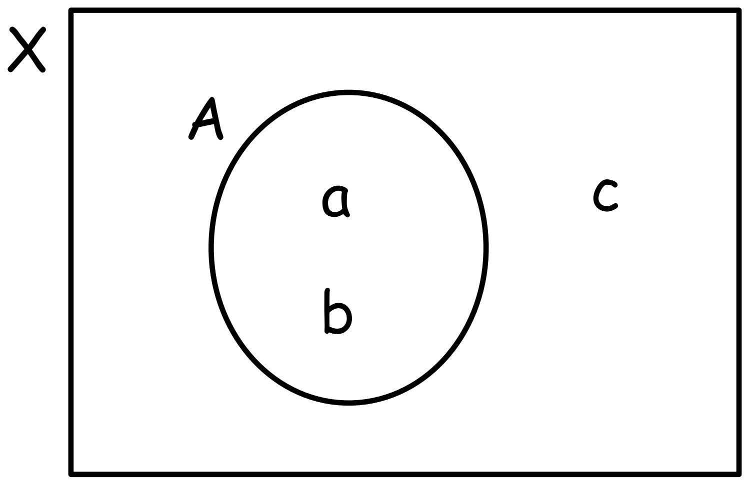

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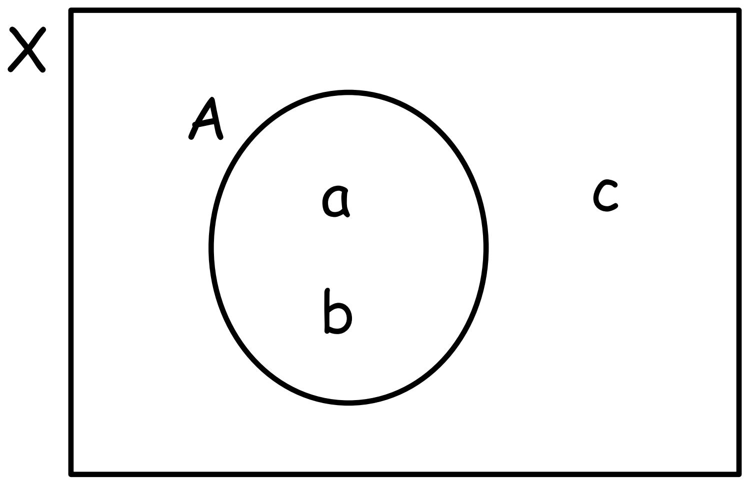

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- $\varnothing$ denotes the empty set, contains no element

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- A = $\{A_1, A_2, \ldots, A_n\}$, a family of sets, that contains sets as elements

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$$|P(A)| = 2^{|A|}$$

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$\neg(A \cup B) = \neg A \cap \neg B$, $\neg(A \cap B) = \neg A \cup \neg B$

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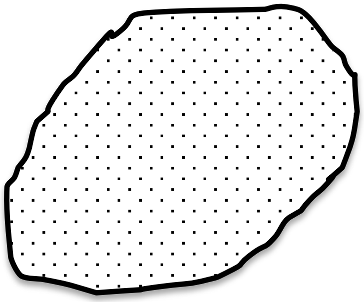

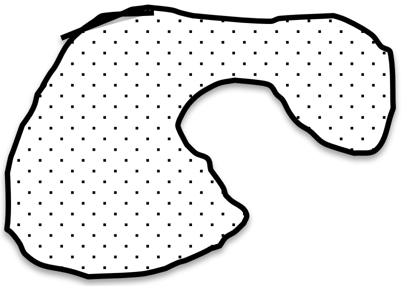

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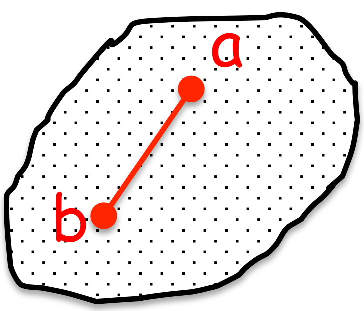

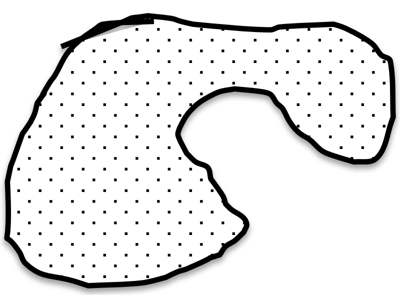

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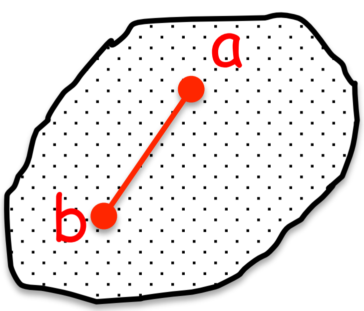

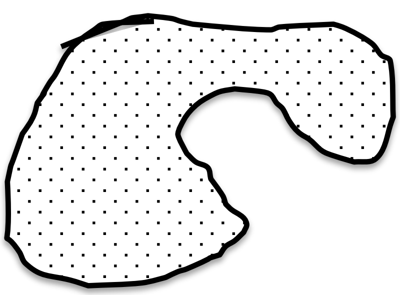

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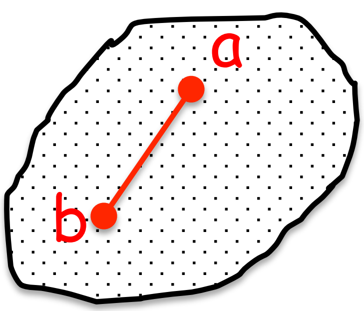

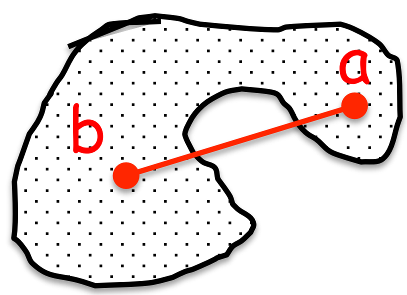

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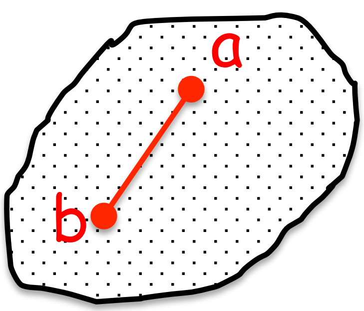

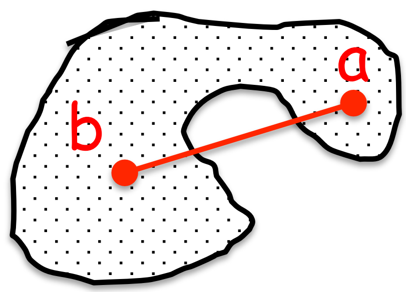

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$$A \times B = \{(a,b) \mid a \in A \text{ and } b \in B\}$$

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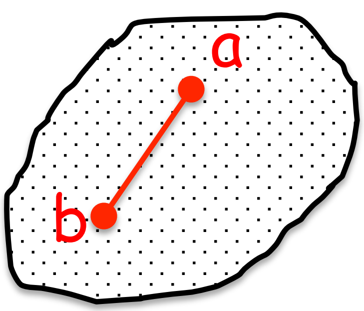

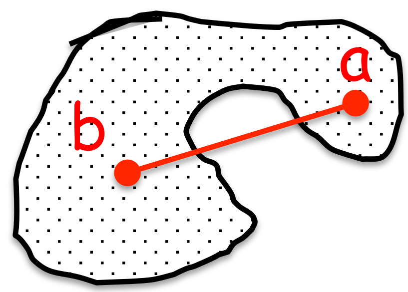

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- A is convex iff $\forall$ a, b in A, $\forall \lambda$ in [0,1], $\lambda a + (1-\lambda)b$ in A

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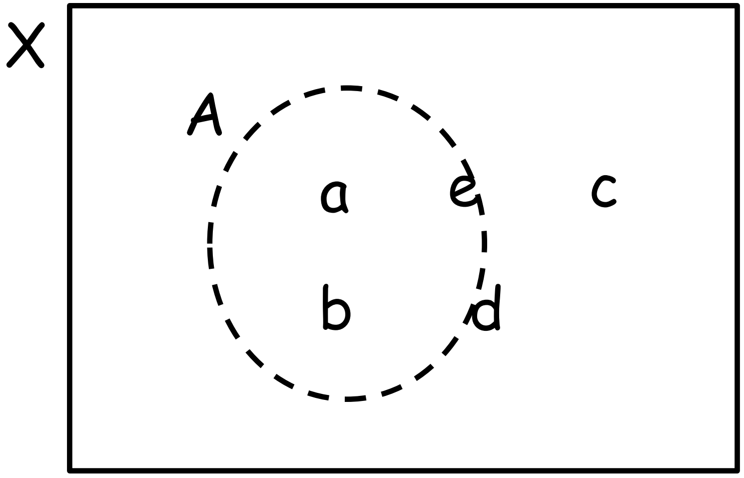

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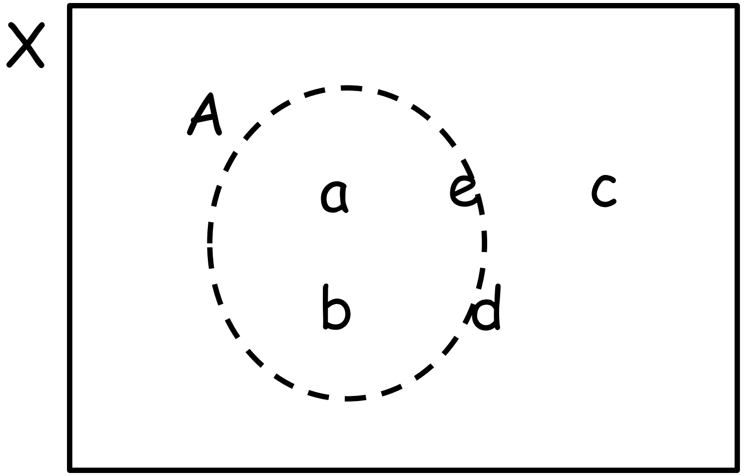

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- indicates the degree of membership

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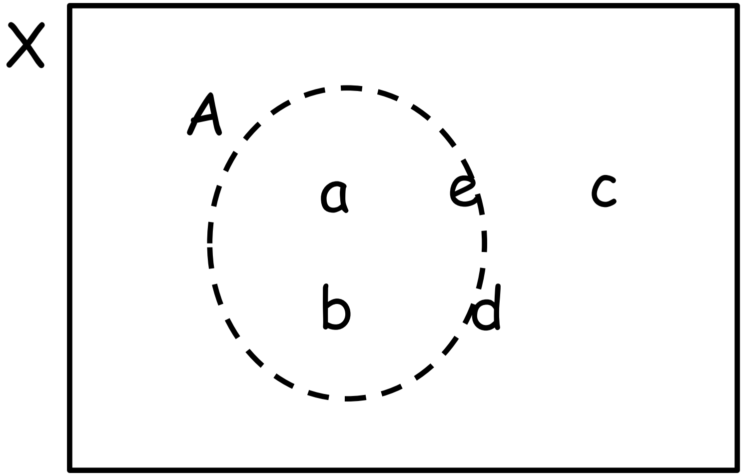

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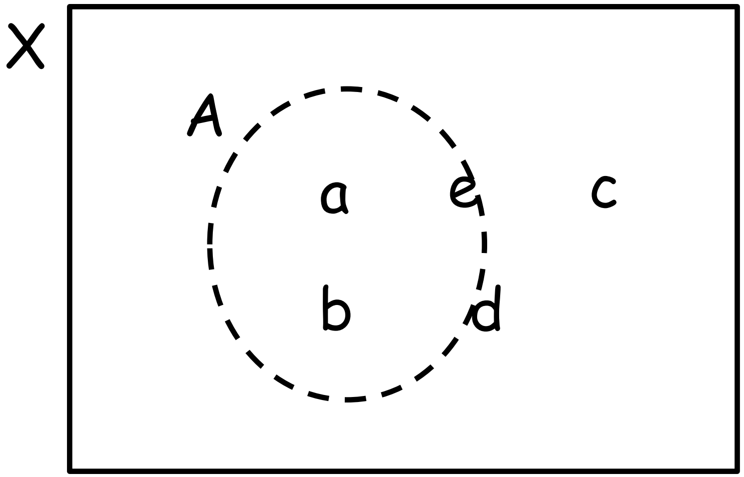

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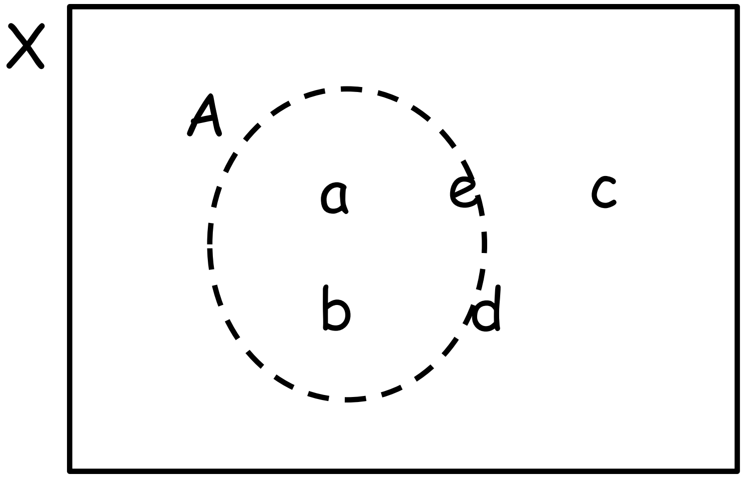

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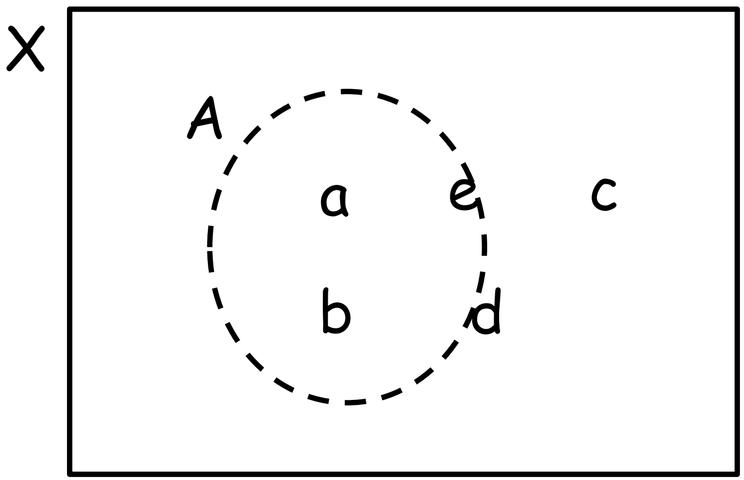

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- $A = 1/a + 1/b + 0.3/d + 0.7/e$

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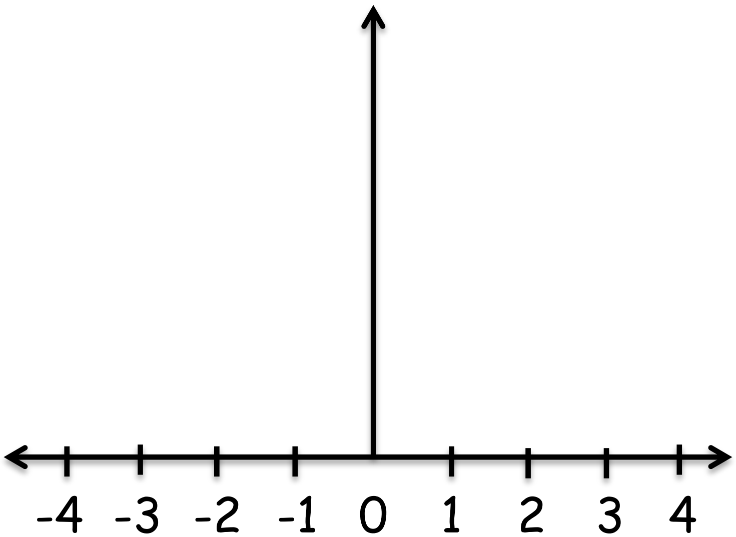

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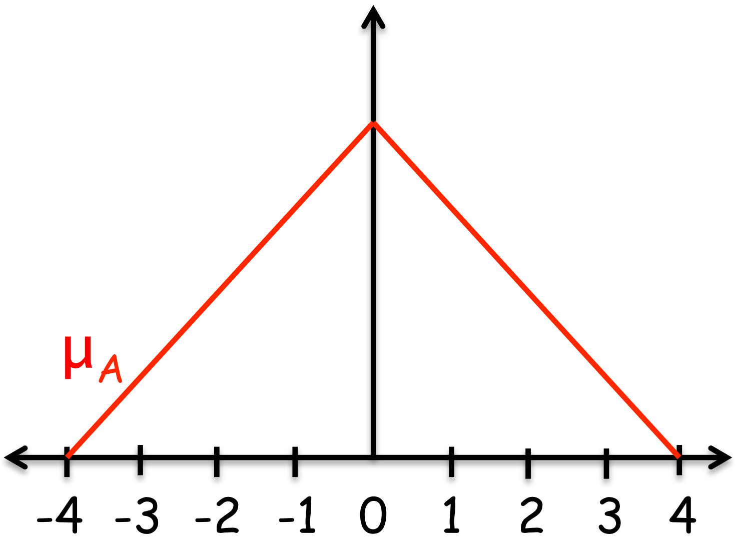

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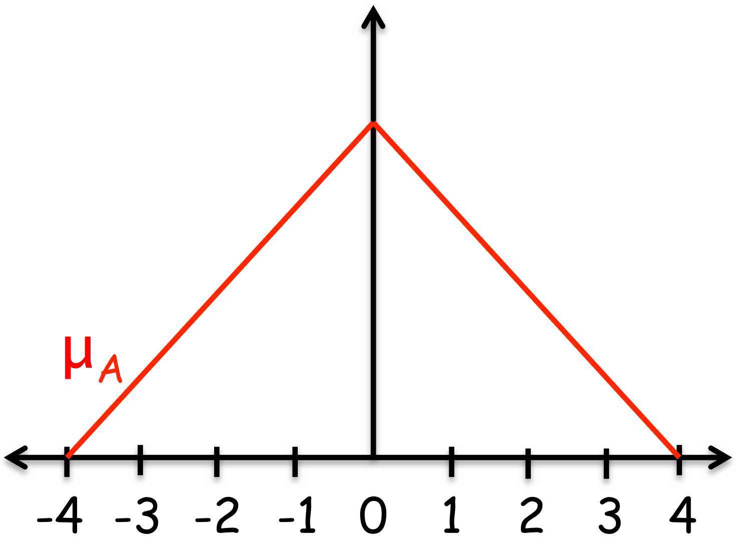

- A = 'real numbers close to 0'

$$\mu_A(x) = \begin{cases} x/4 + 1 & \text{if } x \text{ in } [-4,0] \\ 1 - x/4 & \text{if } x \text{ in } [0,4] \end{cases}$$

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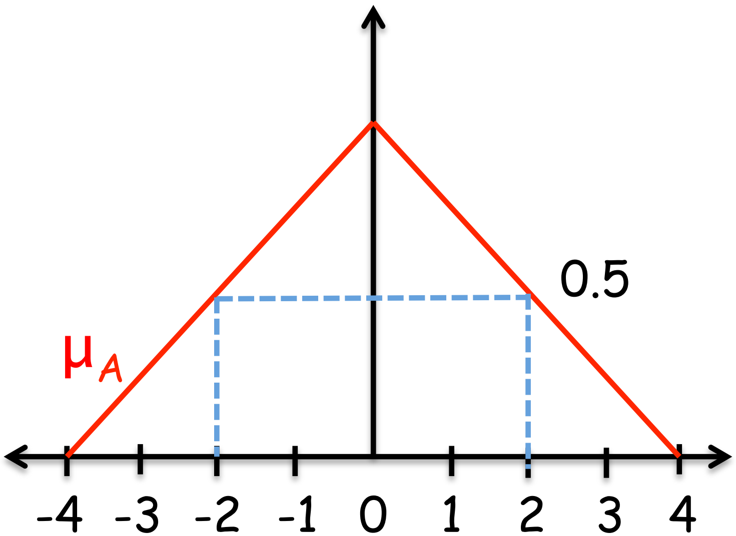

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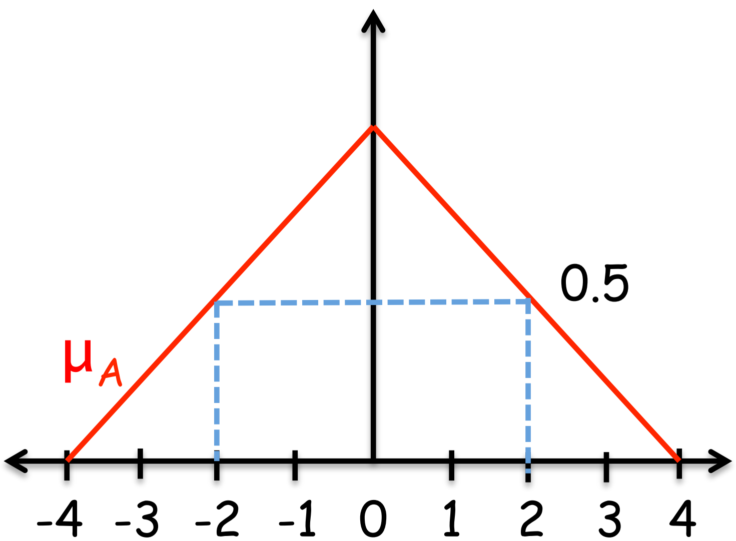

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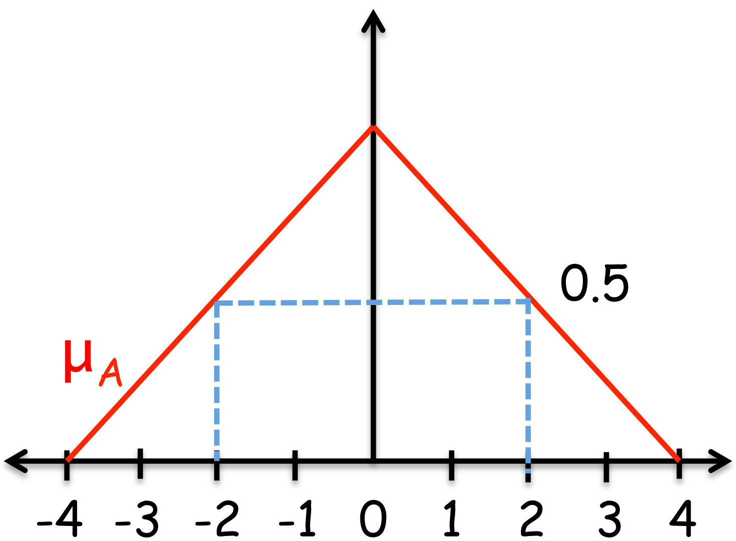

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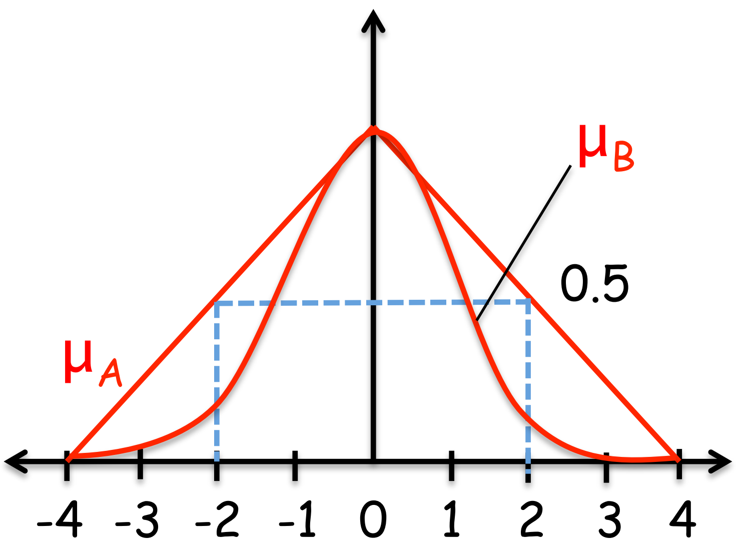

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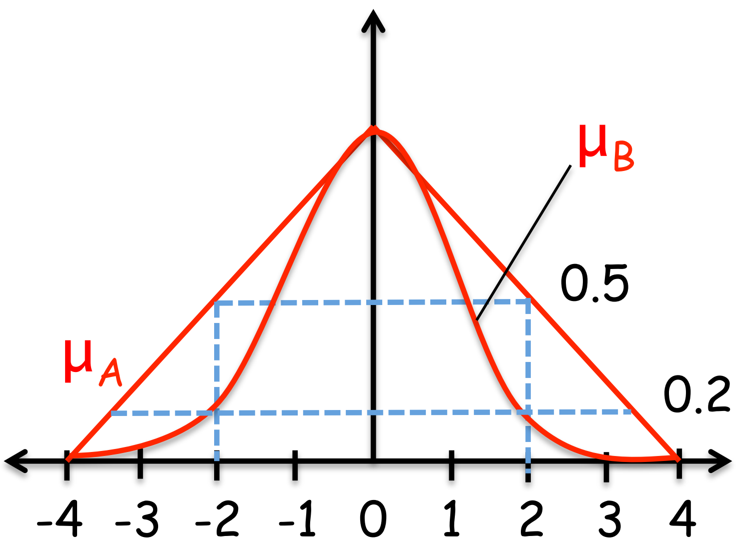

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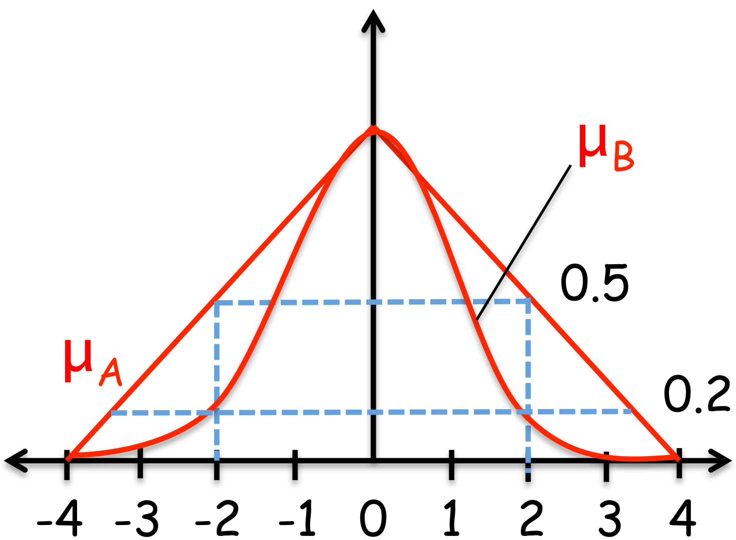

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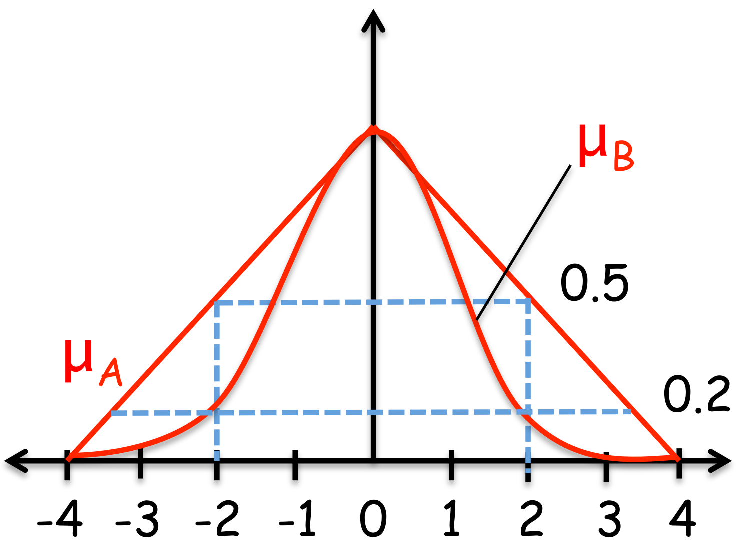

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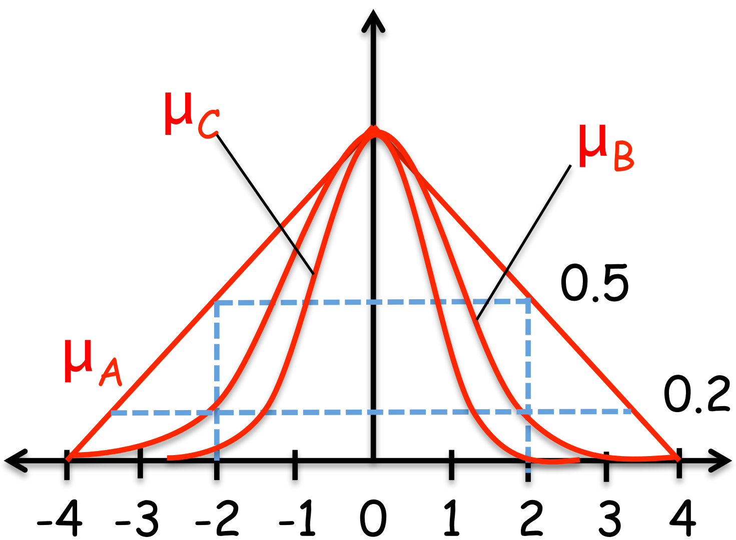
Fuzzy Sets



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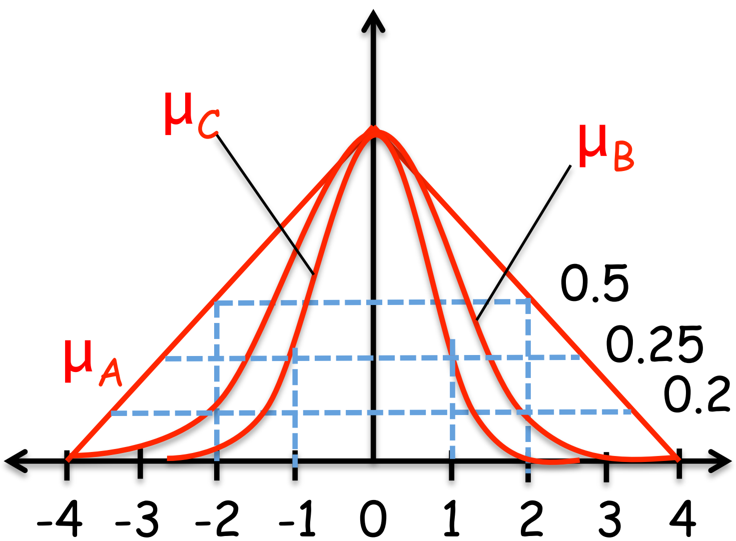

- *A* = 'real numbers close to 0'

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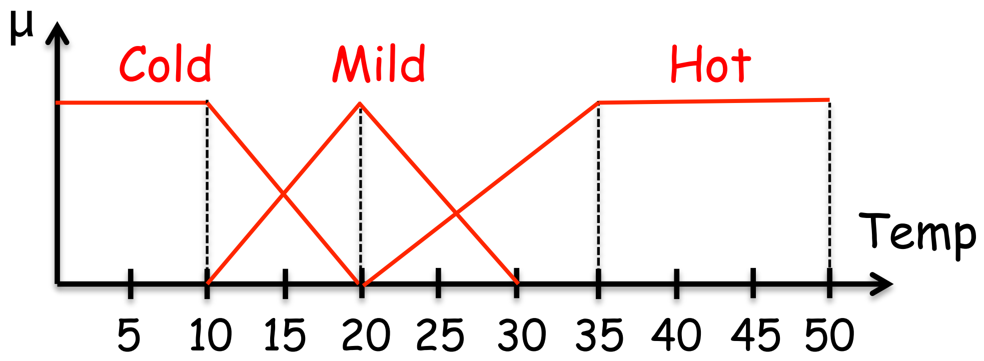
Fuzzy Sets

- X = {5, 10, 15, 20, 25, 30, 35, 40, 45, 50}



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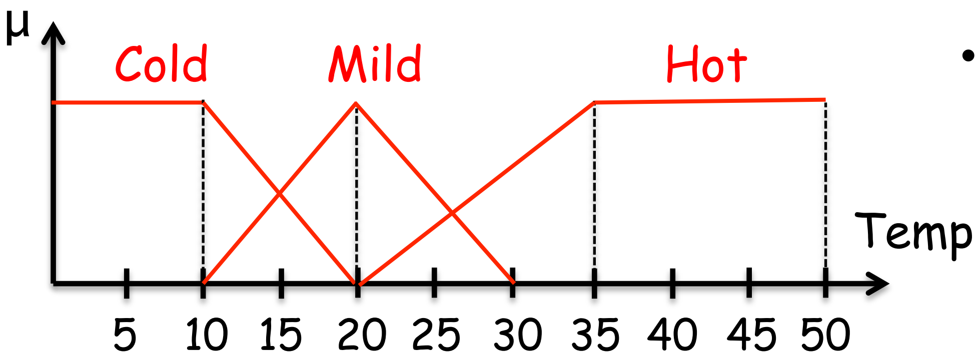

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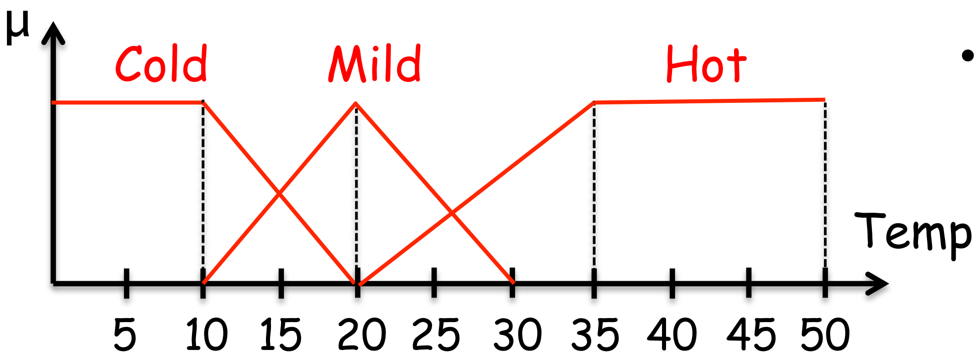

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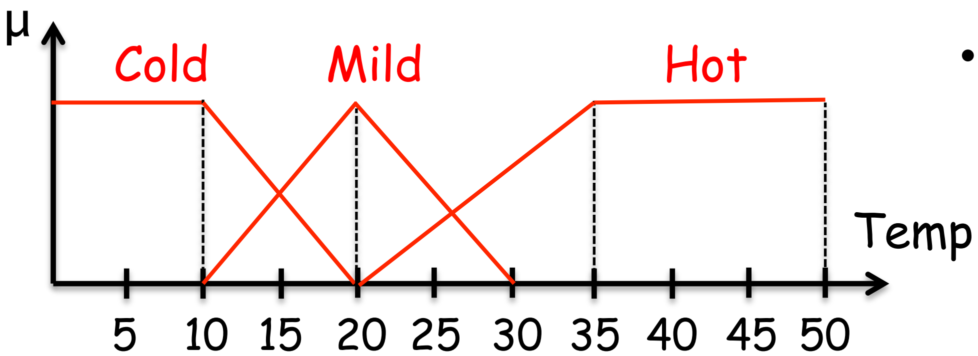

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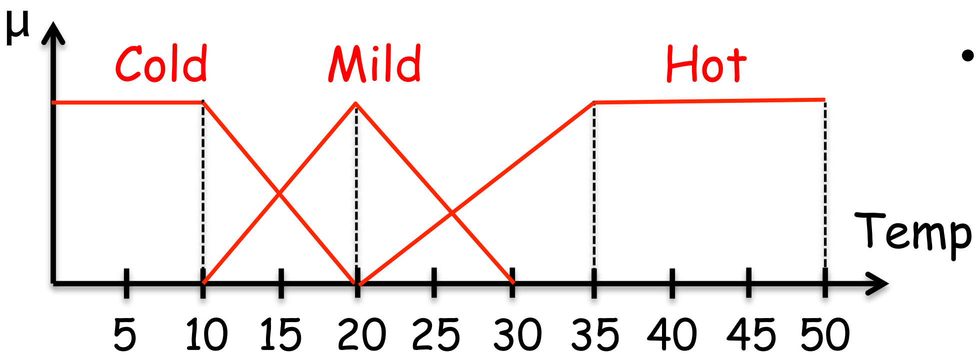

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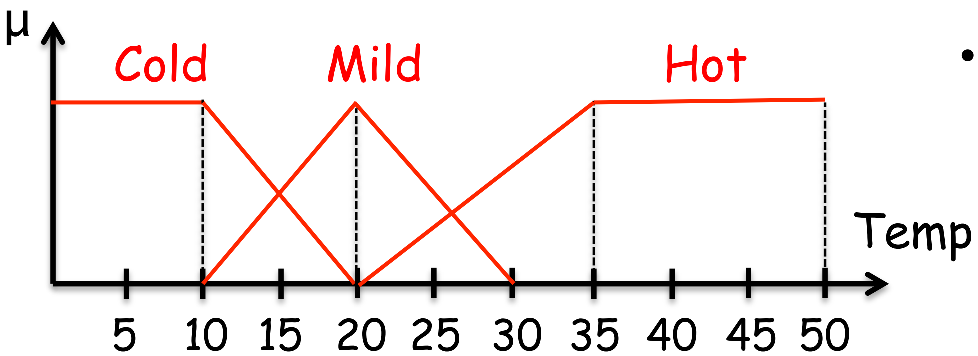

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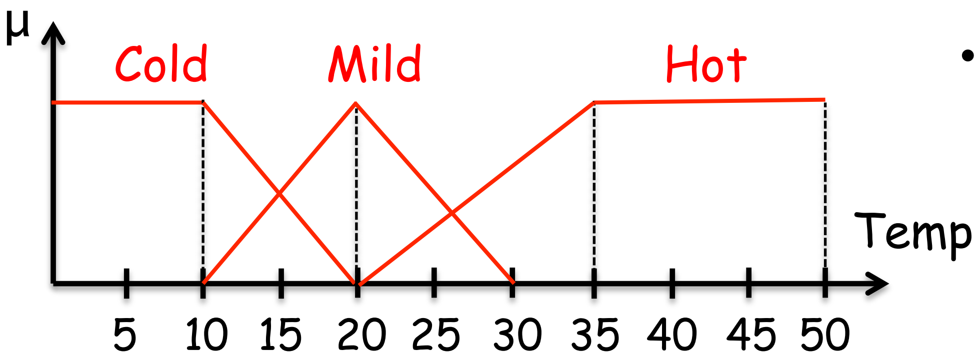

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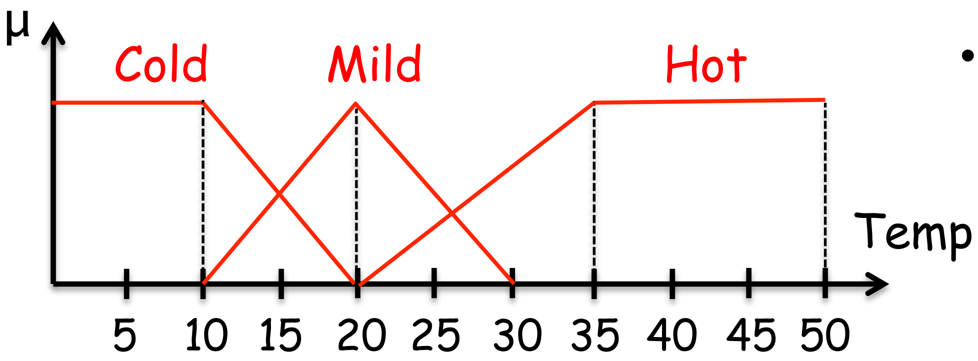

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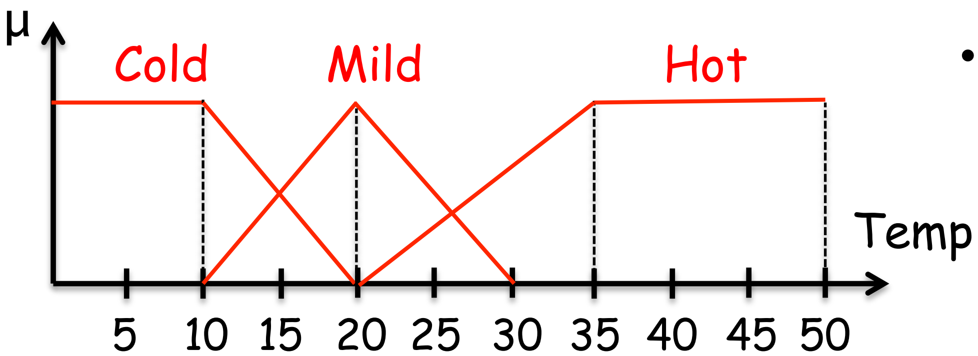

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$Cold_{0.5}$

$Cold_1 =$

> - The maximum membership value is called the height of the fuzzy set.

$Mild_{0.5} = \{15, 20, 25\}$ $\quad$ $Hot_1 = \{35,40,45,50\}$

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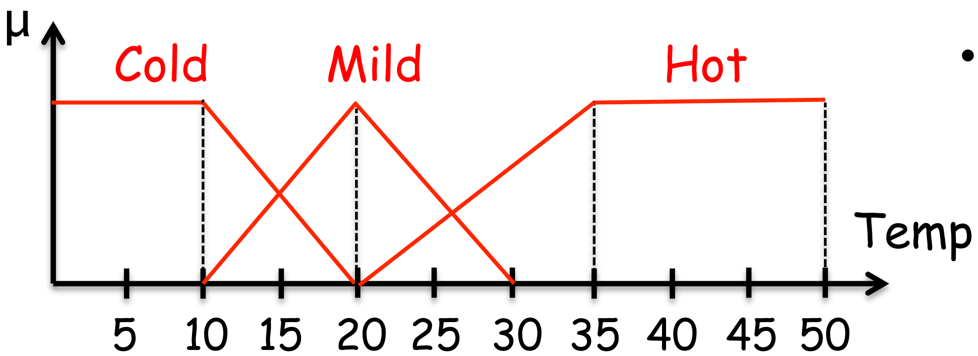

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- The maximum membership value is called the height of the fuzzy set.
- If the height of a fuzzy set is 1, then it is called normalized.

- If $u > v$, then $A_u \subseteq A_v$

Fuzzy Sets

- A is a subset of B, $A \subseteq B$, if $\mu_A(x) \leq \mu_B(x)$ for all x in X

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- Let A be a fuzzy set. $\forall$ a, b in A, if $\mu_A(\lambda a + (1-\lambda)b) \geq \min\{\mu_A(a), \mu_A(b)\}$ where λ in [0,1], then A is a convex fuzzy set

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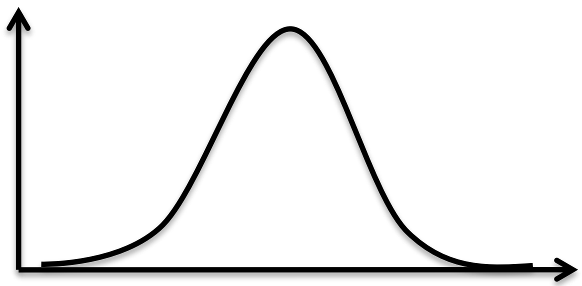

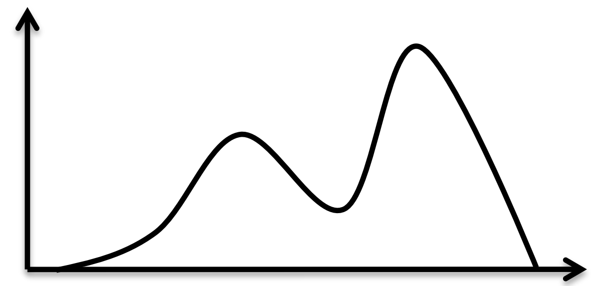

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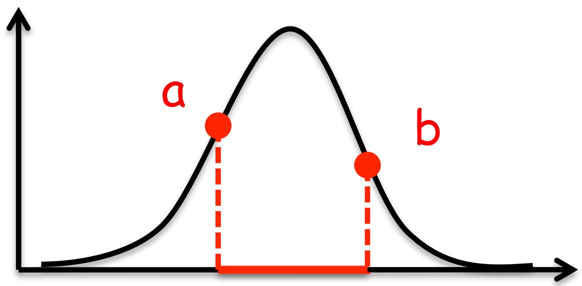

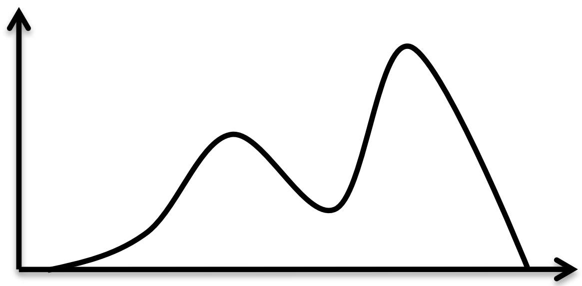

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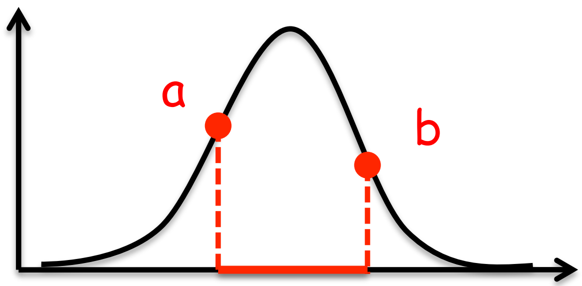

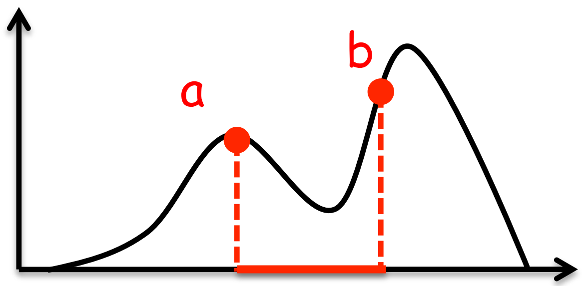

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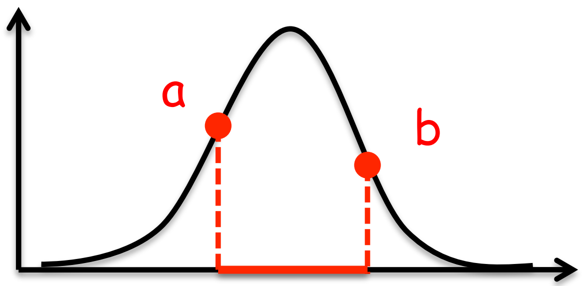

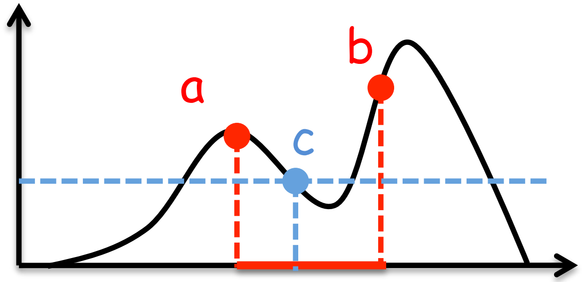

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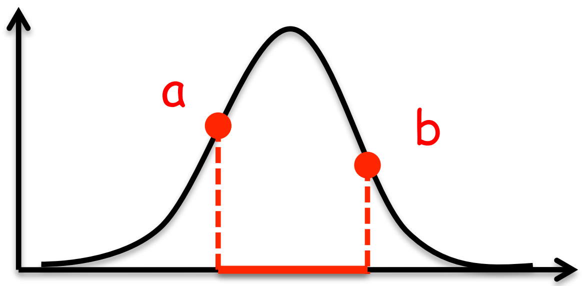

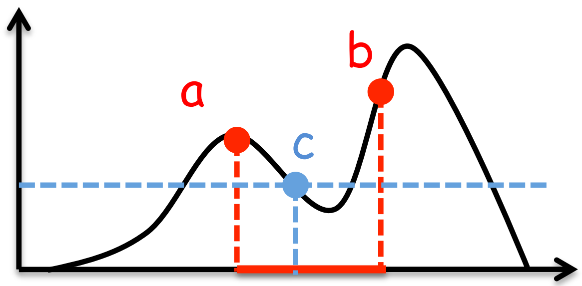

Magnitude of Fuzzy Set

- Scalar Cardinality, $|A| = \Sigma\, \mu_A(x)$,

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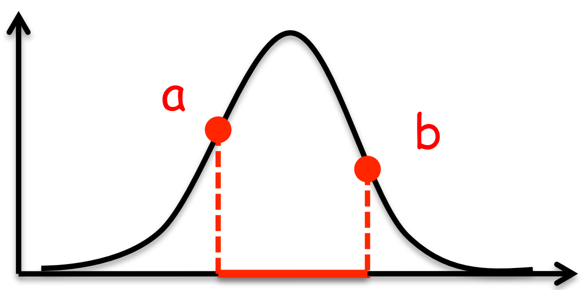

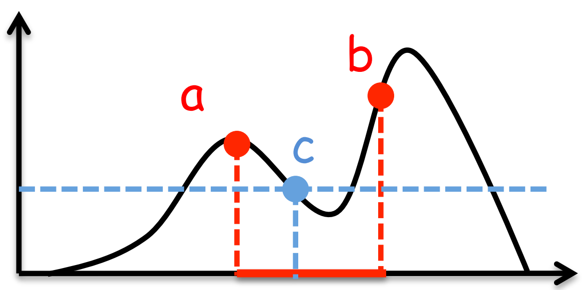

Magnitude of Fuzzy Set

- Scalar Cardinality, $|A| = \Sigma\, \mu_A(x)$,

  $|Hot| = 0.33 + 0.66 + 1 + 1 + 1 + 1 = 5$

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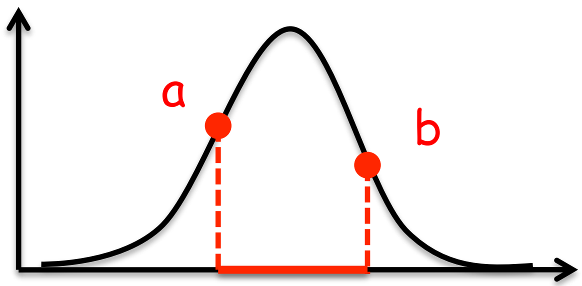

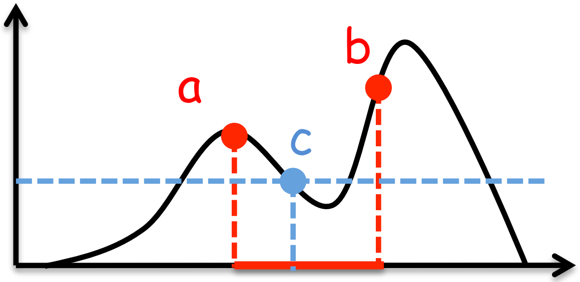

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- Relative Cardinality, $||A|| = |A| / |X|$

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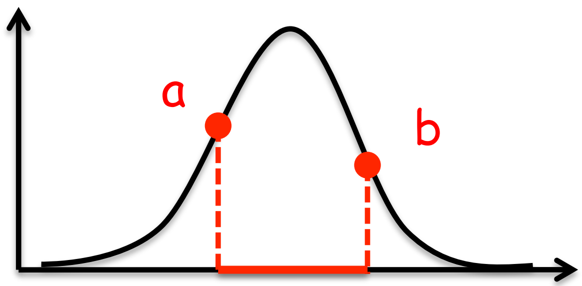

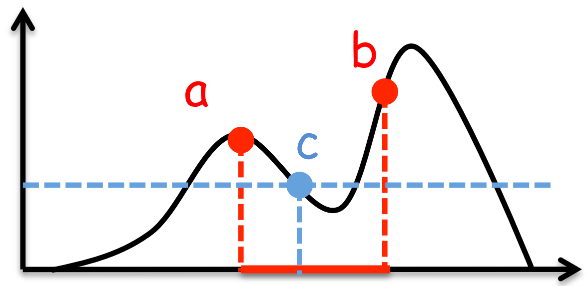

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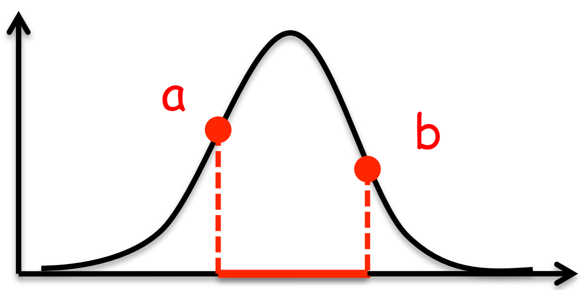
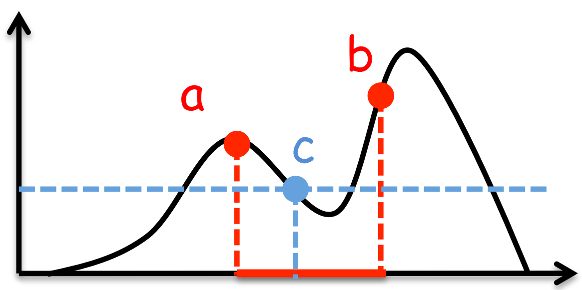
Convex Fuzzy Set

- Let A be a fuzzy set. ∀ a, b in A, if $\mu_A(\lambda a + (1-\lambda)b) \geq \min\{\mu_A(a), \mu_A(b)\}$ where λ in [0,1], then A is a convex fuzzy set





Magnitude of Fuzzy Set

- Scalar Cardinality, $|A| = \Sigma\, \mu_A(x)$,

  $|Hot| = 0.33 + 0.66 + 1 + 1 + 1 + 1 = 5$

- Relative Cardinality, $||A|| = |A| / |X|$

  $||Hot|| = 5/10 = 0.5$

- Fuzzy Cardinality, $|A|_F = \{(|A_u|,u)\}$,

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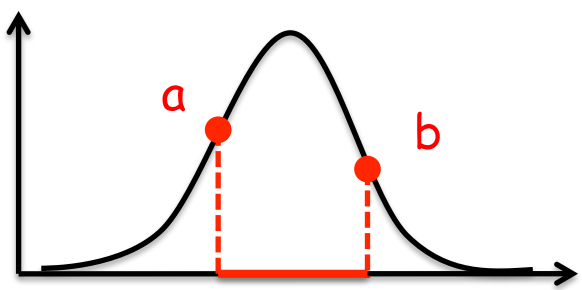

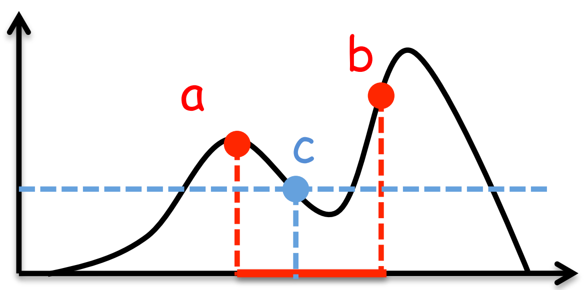

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  $|Hot|_F = \{(6,0.33), (5,0.66), (4,1)\}$