Operations on Fuzzy Sets

Murat Osmanoglu

Fuzzy Operations

Standart fuzzy operations

- Complement, $\mu_C(x) = 1 - \mu_A(x)$ where $C = \neg A$

  Intersection, $\mu_C(x) = \min\{\mu_A(x), \mu_B(x)\}$ where $C = A \cap B$

  Union, $\mu_C(x) = \max\{\mu_A(x), \mu_B(x)\}$ where $C = A \cup B$

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- a desirable feature of the standard operations is their inherent prevention of the compounding of errors of operands

  if e is the error associated with $\mu_A(x)$ and $\mu_B(x)$, then maximum error associated with $\mu_C(x)$ remains e where $C = \neg A$, or $A \cap B$, or $A \cup B$

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- different functions can be used in different contexts

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- Let a fuzzy complement operation be defined as a function

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$\mu_A(x)$ specifies the degree of which x belongs to A,
$\mu_C(x)$ specifies the degree of which x does not belong to A or
x belongs to C where $C = \neg A$

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- *Axiom 1* required to get correct complements for crisp sets (since we stated that the fuzzy operations are generalizations of corresponding crisp operations)
- *Axiom 2* required to be monotonic decreasing, i.e. when μ_A increaes, the complement $c(\mu_A)$ must not increase (it may decrease or, at least remain same.

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> - a function is continuous at some x_0 in D, if f is defined at x_0, the limit of the function exist x_0, and equals to $f(x_0)$
> - the graph of a continuous function is a single unbroken curve with no holes or jumps

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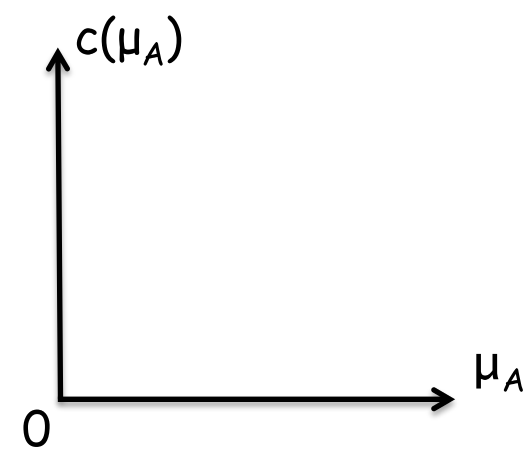

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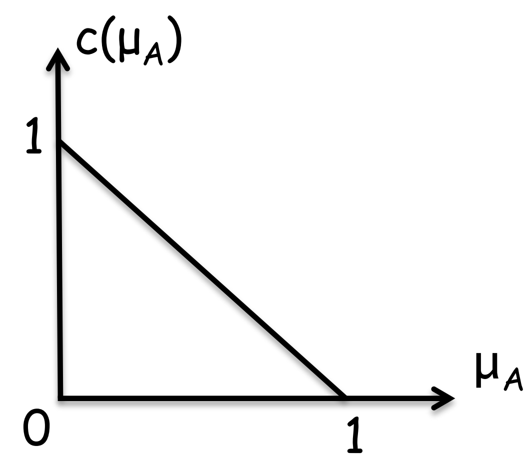

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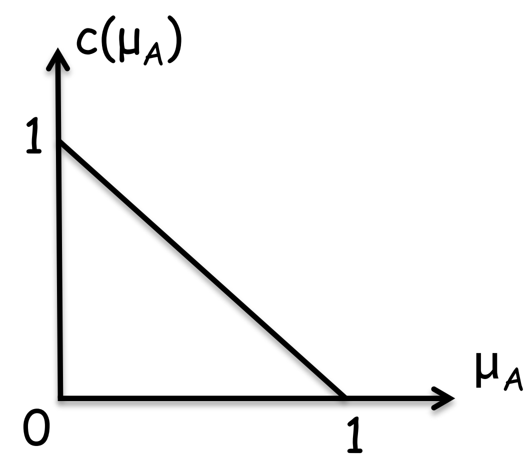

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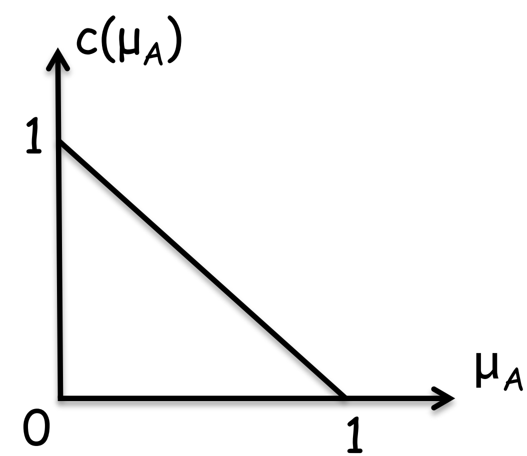

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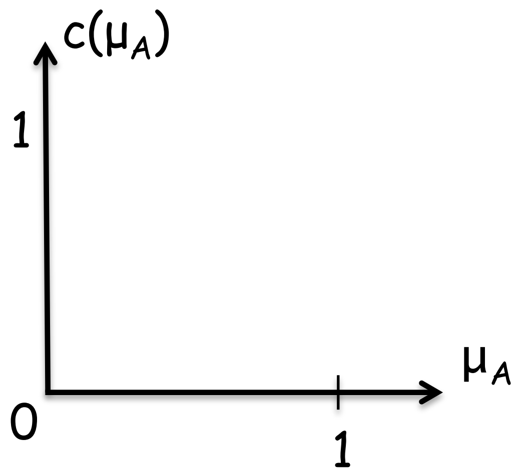

$$c(a) = \begin{cases} 1 & \text{if } a \le z \\ 0 & \text{if } a > z \end{cases}$$

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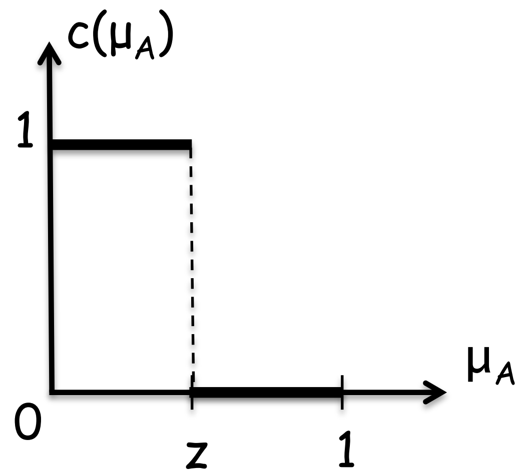

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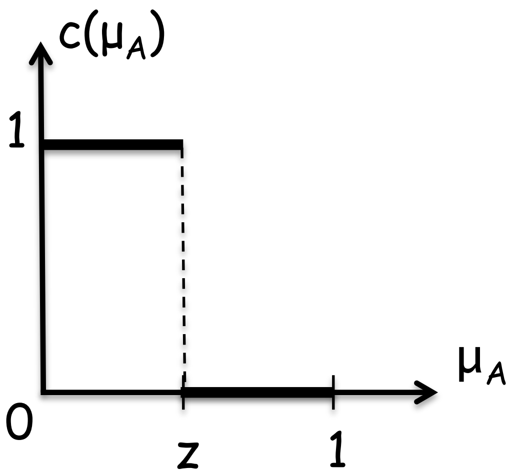

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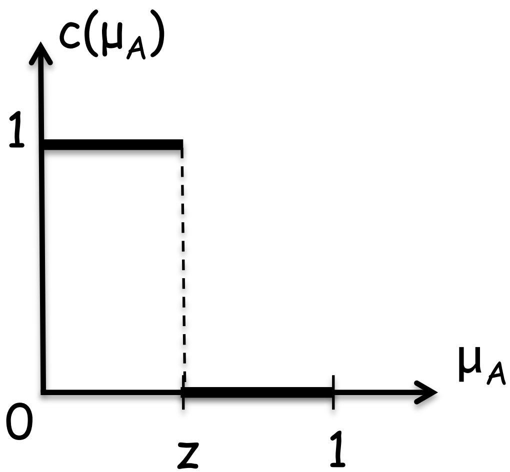

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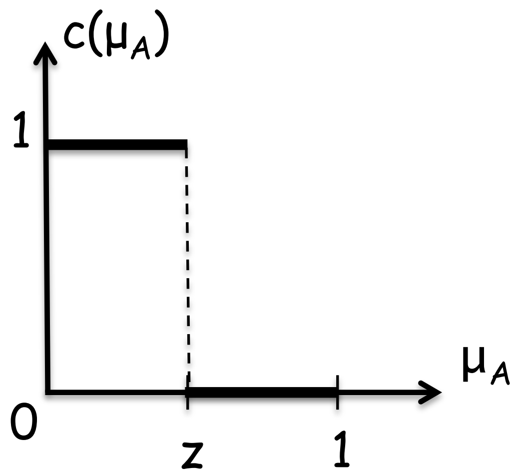

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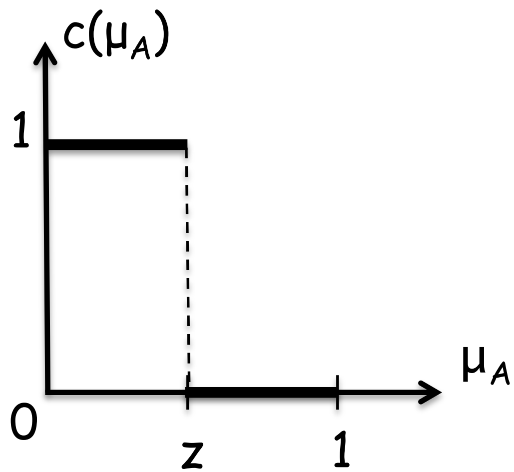

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- assume a in (0,1), then $c(c(a)) \ne a$

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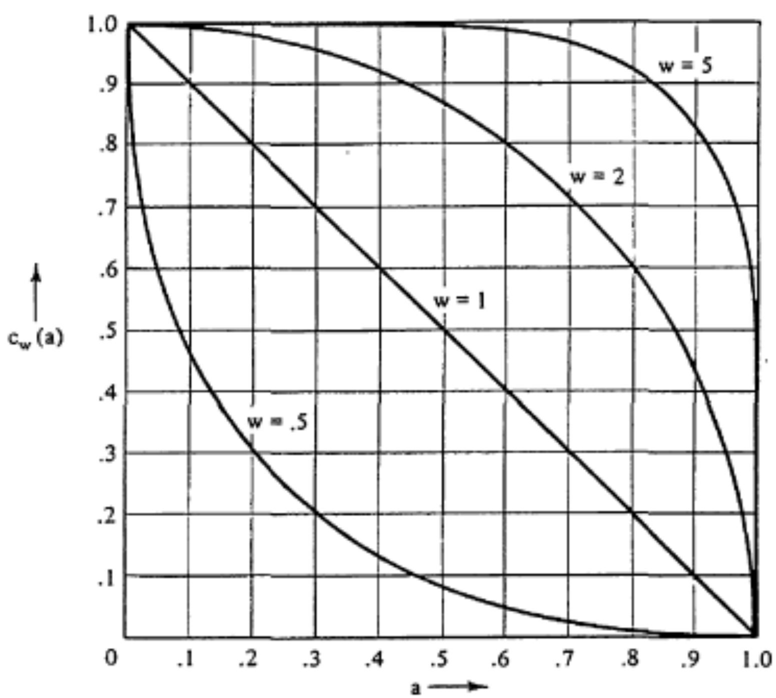
Yager Class

$c_w(a) = (1 - a^w)^{1/w}$

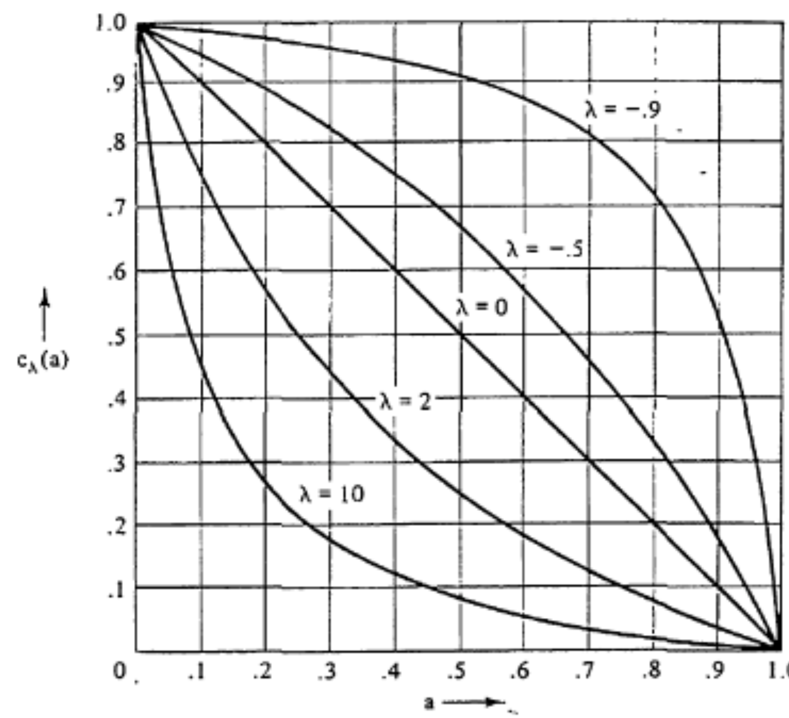
where w in $(0,\infty)$



Sugeno Class

$c(a) = (1 - a) / (1 + c.a)$

where λ in $(-1,\infty)$



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- Let a fuzzy intersection operation be defined as a function

$$i : [0,1] \times [0,1] \rightarrow [0,1]$$

$$(\mu_A(x), \mu_A(x)) \rightarrow i(\mu_A(x), \mu_B(x))$$

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- **Axiom 3** : if $b \leq d$, then $i(a,b) \leq i(a,d)$ (monotonicity)
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- monotonicity and commutativity express that a decrease in the degree of membership in sets A and B cannot produce an increase in the degree of membership of intersection

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– assume $b \le d$; if $a < b$, then $i(a,b) = a \le i(a,d) = a$
if $b < a$, then $i(a,b) = b \le i(a,d)$

– $i(i(a,b),c) = i(a,i(b,c))$

Fuzzy Operations

Fuzzy Intersection (t-norm)

- Let a fuzzy intersection operation be defined as a function

$$i : [0,1] \times [0,1] \rightarrow [0,1]$$

$$(\mu_A(x), \mu_A(x)) \rightarrow i(\mu_A(x), \mu_B(x))$$

- **Axiom 5 :** i is a continuous function (continuity)
- **Axiom 6 :** $i(a,a) = a$ (idempotency)
- Standart Intersection ; $i(a,b) = \min\{a,b\}$ where $a=\mu_A(x)$ and $b=\mu_B(x)$

– $i(0,a) = \min\{0,a\} = 0$ and $i(1,1) = \min\{1,1\} = 1$

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– i is continuous function

– $i(a,a) = a$

Fuzzy Operations

Fuzzy Intersection (t-norm)

- Algebraic Product $(A \bullet B)$; $i(a,b) = a.b$

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Fuzzy Intersection (t-norm)

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Fuzzy Operations

Fuzzy Intersection (t-norm)

- Algebraic Product (A•B); i(a,b) = a.b
  - i(0,a) = 0 and i(1,1) = 1
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- Algebraic Product ($A \bullet B$); $i(a,b) = a.b$
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  - $i(i(a,b),c) = i(a,i(b,c)) \rightarrow a.b.c = a.b.c$

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  - $i(i(a,b),c) = i(a,i(b,c)) \rightarrow a.b.c = a.b.c$
  - $i(a,a) = a^2 \neq a$

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- Bounded Product ($A \odot B$); $i(a,b) = \max\{0, a+b-1\}$
- Yager Class ; $i_w(a,b) = 1 - \min\{1, ((1 - a)^w+(1 - b)^w)^{1/w}\}$

  where w in $(0,\infty)$

Fuzzy Operations

Fuzzy Union (s-norm)

- Let a fuzzy union operation be defined as a function

$$u : [0,1] \times [0,1] \rightarrow [0,1]$$

$$(\mu_A(x), \mu_A(x)) \rightarrow u(\mu_A(x), \mu_B(x))$$

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- **Axiom 1** : $u(1,1) = 1, u(1,0) = 1, u(0,1) = 1, u(0,0) = 0$ (boundary condition)
- **Axiom 2** : $u(a,b) = u(b,a)$ (comutativity)
- **Axiom 3** : if $b \leq d$, then $u(a,b) \leq u(a,d)$ (monotonicity)
- **Axiom 4** : $u(u(a,b),c) = u(a,u(b,c))$ (associativity)

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- these axioms ensure that the fuzzy union becomes the classical union when A and B are crisp
- monotonicity and commutativity express that a decrease in the degree of membership in sets A and B cannot produce an increase in the degree of membership of union

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- ***Axiom 6*** **:** u(a,a) = a (idempotency)

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Fuzzy Operations

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Fuzzy Operations

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– u(a,b) = u(b,a)

– assume $b \le d$; if $d < a$, then $u(a,b) = a \le u(a,d) = a$

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Fuzzy Union (s-norm)

- Probabilistic Sum ; $u(a,b) = a + b - a.b$

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  - assume $b \le d$, then $u(a,b) = a + b(1 - a) \le u(a,d) = a + d(1 - a)$

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  - $u(a,a) = 2a - a^2 \neq a$

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  - $u(a,a) = 2a - a^2 \neq a$
- Bounded Sum ($A \oplus B$); $u(a,b) = \min\{1, a+b\}$

Fuzzy Operations

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- Bounded Sum ($A \oplus B$); $u(a,b) = \min\{1, a+b\}$
- Yager Class ; $u_w(a,b) = \min\{1, (a^w + b^w)^{1/w}\}$

  where w in $(0,\infty)$

Fuzzy Operations

Other Operations on Fuzzy Sets

- Disjunctive Sum ; $C = (A \cap \neg B) \cup (\neg A \cap B)$

  $\mu_C(x) = \max\{\min[\mu_A(x), 1 - \mu_B(x)], \min[\mu_B(x), 1 - \mu_A(x)]\}$

Fuzzy Operations

Other Operations on Fuzzy Sets

- Disjunctive Sum ; $C = (A \cap \neg B) \cup (\neg A \cap B)$

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- Disjoint Sum ; $C = A \Delta B$

  $\mu_C(x) = |\mu_A(x) - \mu_B(x)|$

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- Disjoint Sum ; $C = A \Delta B$

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- Set Difference ; $C = A - B = A \cap \neg B$

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Other Operations on Fuzzy Sets

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- Distance in Fuzzy Set ;

  Minkowski distance, $d_w(A,B) = (\Sigma_{x \text{ in } X} |\mu_A(x) - \mu_B(x)|^w)^{1/w}$

Fuzzy Operations

- $\mu_A(x) = 1/(x^2+1)$ and $\mu_B(x) = x^2/(x^2+1)$

  $C = \neg A$, $D = \neg B$, $E = A \cap B$, $F = A \cup B$, $\mu_C, \mu_D, \mu_E, \mu_F = ?$

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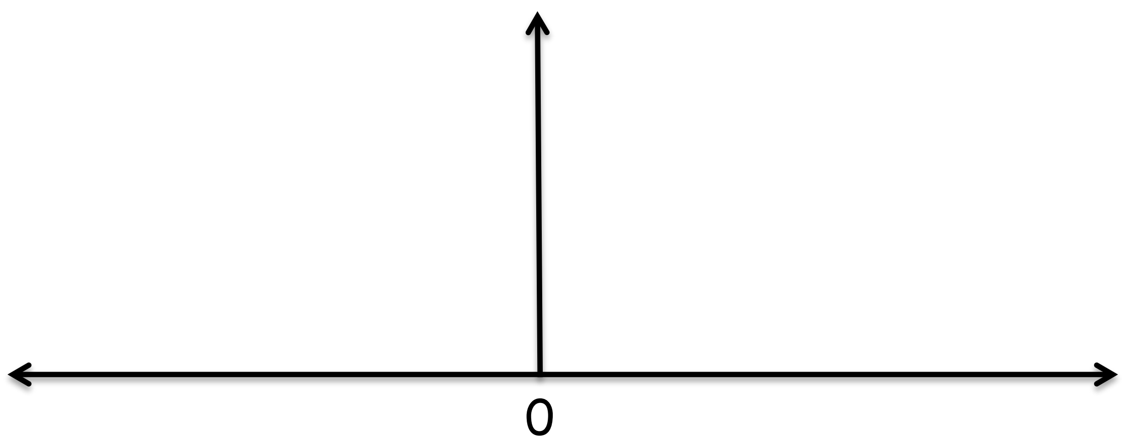

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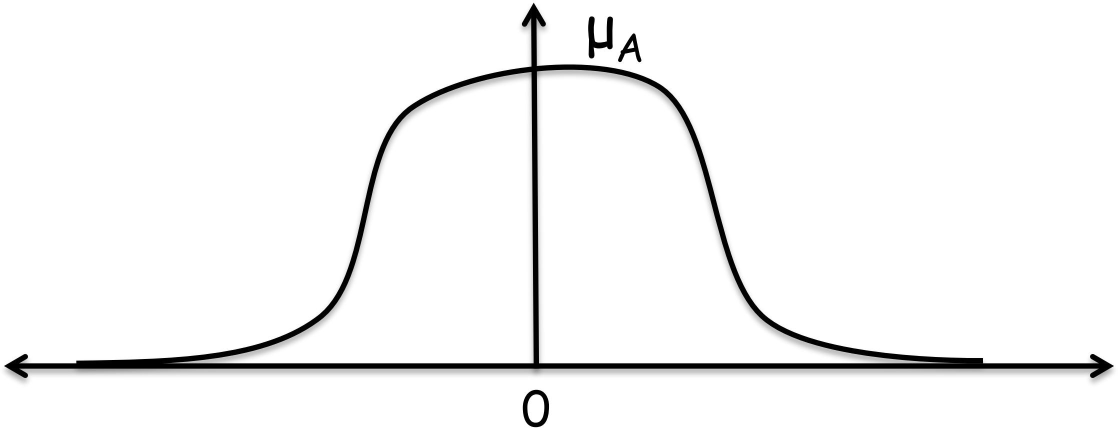

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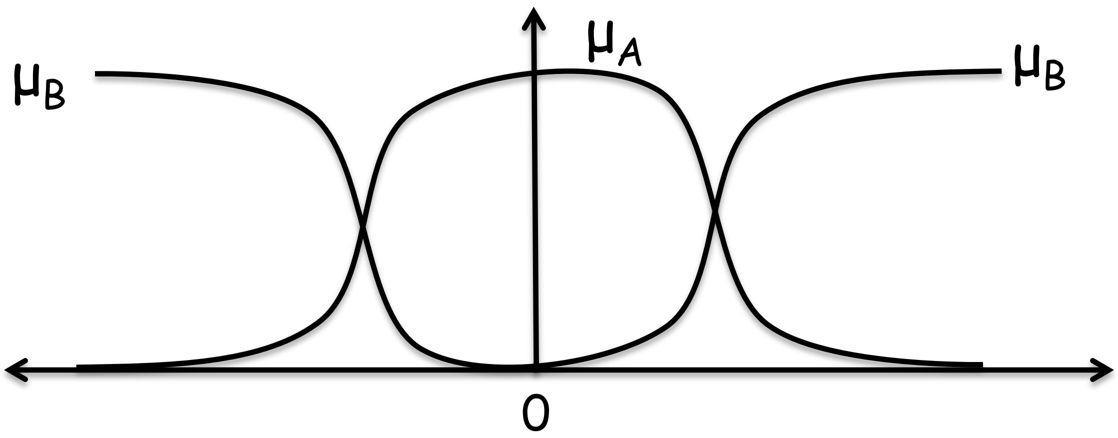

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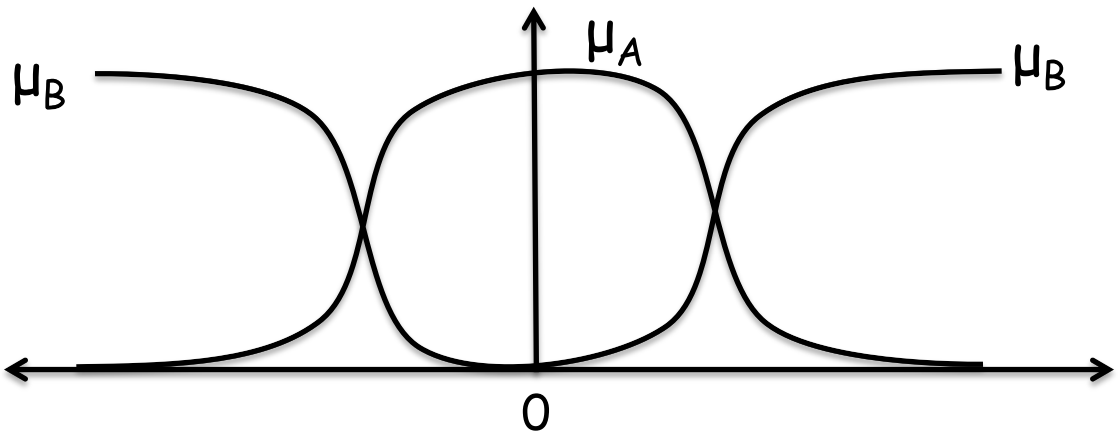

$\mu_C(x) = 1 - \mu_A(x)$

$\mu_C(x) = 1 - 1/(x^2+1)$

$\mu_C(x) = x^2/(x^2+1) = \mu_B(x)$

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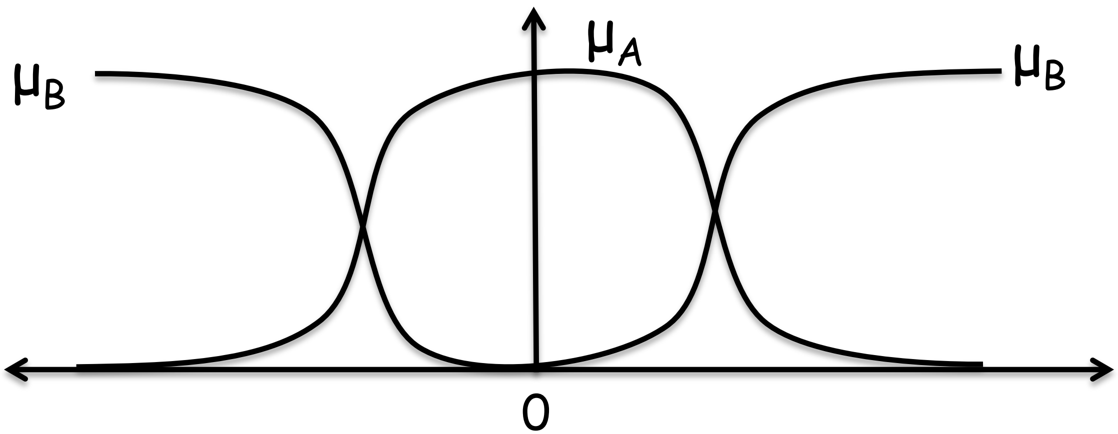

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$\mu_D(x) = 1 - \mu_B(x)$

$\mu_D(x) = 1 - x^2/(x^2+1)$

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Fuzzy Operations

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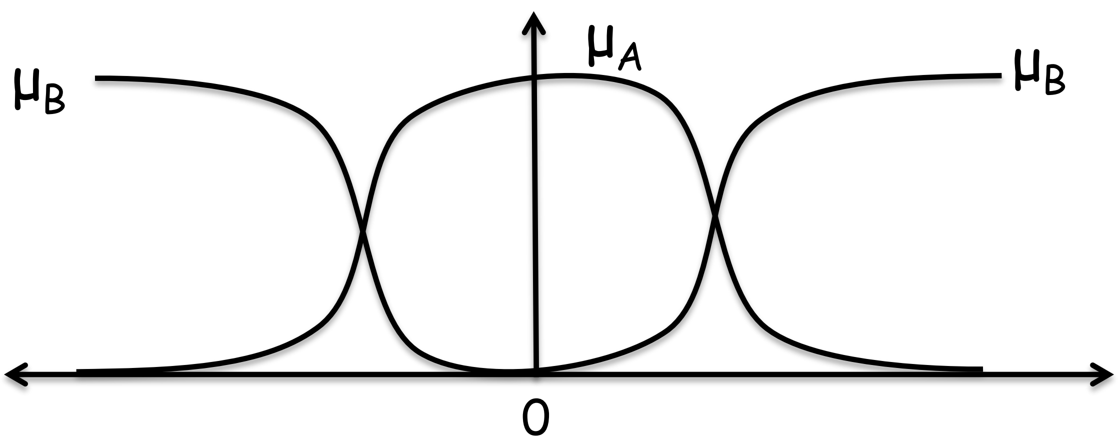

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$\mu_D(x) = 1 - \mu_B(x)$

$\mu_D(x) = 1 - x^2/(x^2+1)$

$\mu_C(x) = 1/(x^2+1) = \mu_A(x)$

$\mu_E(x) = \min\{\mu_A(x), \mu_B(x)\}$

$\mu_F(x) = \max\{\mu_A(x), \mu_B(x)\}$

Fuzzy Operations

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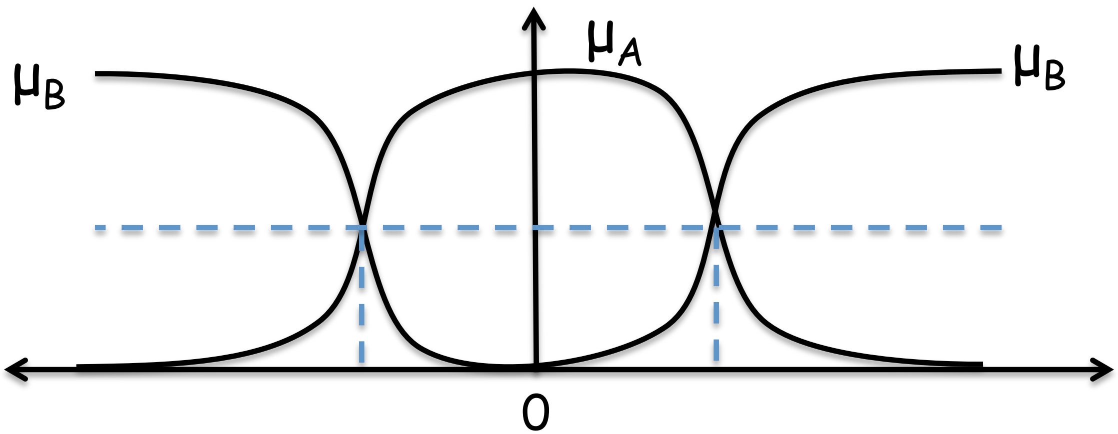

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$\mu_D(x) = 1 - \mu_B(x)$

$\mu_D(x) = 1 - x^2/(x^2+1)$

$\mu_C(x) = 1/(x^2+1) = \mu_A(x)$

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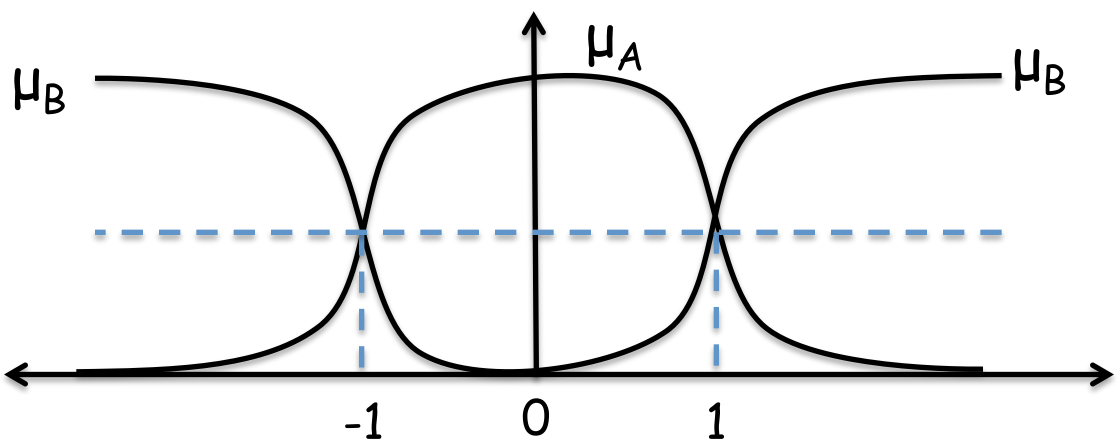
$1/(x^2+1) = x^2/(x^2+1)$

Fuzzy Operations

- $\mu_A(x) = 1/(x^2+1)$ and $\mu_B(x) = x^2/(x^2+1)$

  $C = \neg A$, $D = \neg B$, $E = A \cap B$, $F = A \cup B$, $\mu_C, \mu_D, \mu_E, \mu_F$ = ?



$\mu_C(x) = 1 - \mu_A(x)$
$\mu_C(x) = 1 - 1/(x^2+1)$
$\mu_C(x) = x^2/(x^2+1) = \mu_B(x)$

$\mu_D(x) = 1 - \mu_B(x)$
$\mu_D(x) = 1 - x^2/(x^2+1)$
$\mu_C(x) = 1/(x^2+1) = \mu_A(x)$

$\mu_E(x) = \min\{\mu_A(x),\mu_B(x)\}$

$\mu_F(x) = \max\{\mu_A(x),\mu_B(x)\}$

$$1/(x^2+1) = x^2/(x^2+1)$$
$$x = \pm 1$$

Fuzzy Operations

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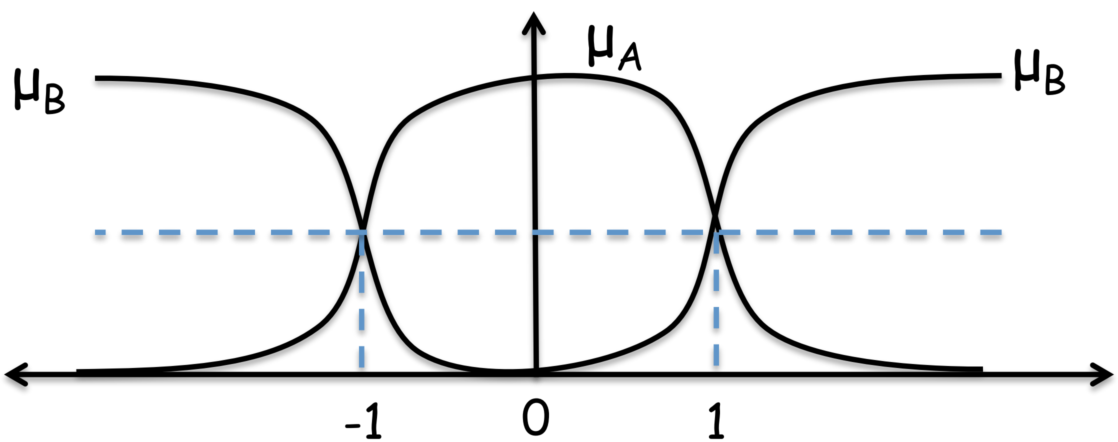

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Fuzzy Operations

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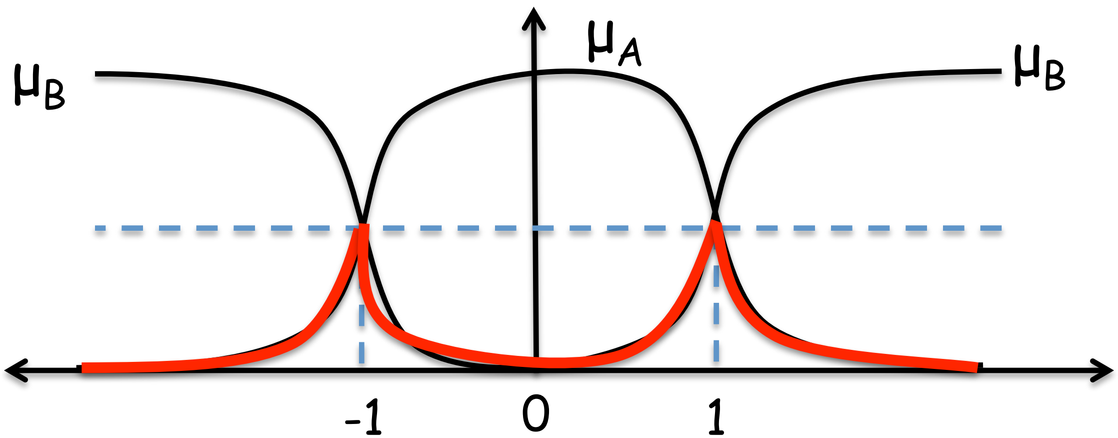

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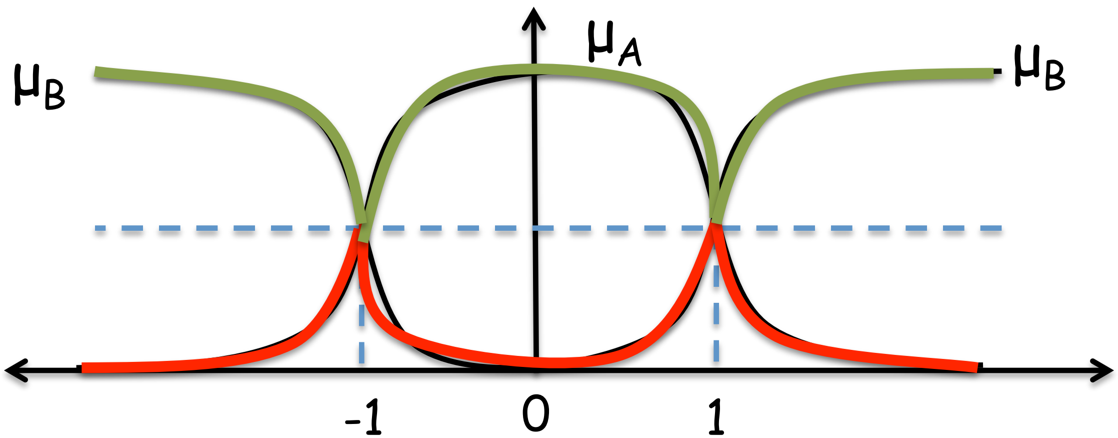

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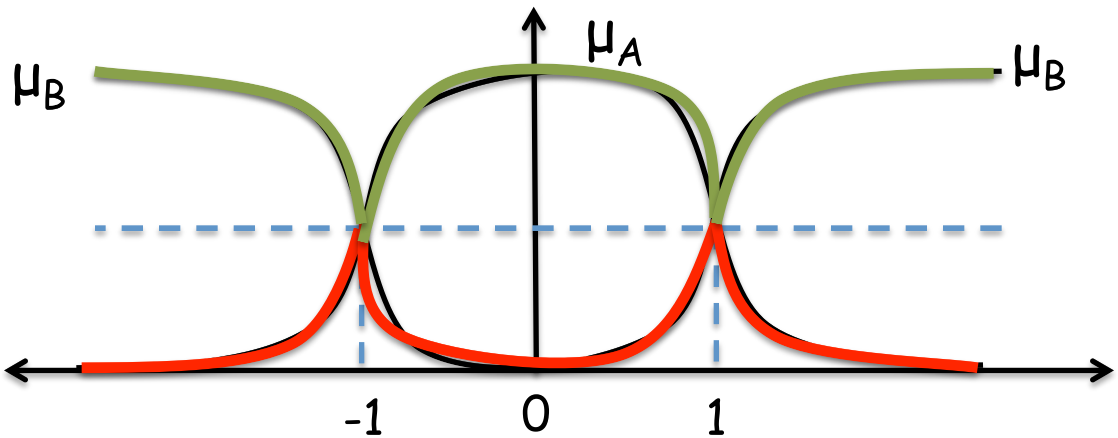

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$\mu_F(x) = \max\{\mu_A(x), \mu_B(x)\}$

$$\mu_E(x) = \begin{cases} \mu_A(x) & \text{if } x < -1 \\ \mu_B(x) & \text{if } -1 \leq x \leq 1 \\ \mu_A(x) & \text{if } x > 1 \end{cases}$$

Fuzzy Operations

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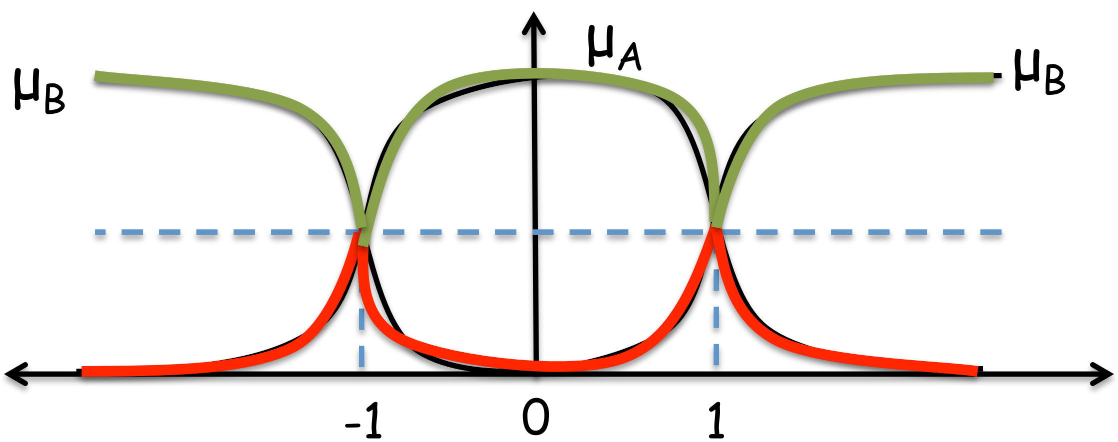

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