STA250 Probability and Statistics

Chapter 11 Notes

Hypothesis Testing 2

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2023

Hypothesis Testing for the Mean (σ Unknown)

- Convert sample result ($\bar{x}$) to a t test statistic

Hypothesis Tests for μ

σ Known | **σ Unknown**

Consider the test

$$\boxed{\begin{aligned} H_0 &: \mu = \mu_0 \\ H_1 &: \mu > \mu_0 \end{aligned}}$$

(Assume the population is normal)

The decision rule is:

$$\text{Reject } H_0 \text{ if } t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} > t_{n-1,\alpha}$$

Hypothesis Testing for the Mean (σ Unknown)

(continued)

- For a two-tailed test:

  Consider the test

  $$\boxed{\begin{aligned} H_0 &: \mu = \mu_0 \\ H_1 &: \mu \neq \mu_0 \end{aligned}}$$

  (Assume the population is normal, and the population variance is unknown)

  The decision rule is:

Reject H_0 if $\boxed{t = \dfrac{\bar{x} - \mu_0}{\dfrac{s}{\sqrt{n}}} < -t_{n-1,\, \alpha/2}}$ or if $\boxed{t = \dfrac{\bar{x} - \mu_0}{\dfrac{s}{\sqrt{n}}} > t_{n-1,\, \alpha/2}}$

Example: Two-Tail Test

The average cost of a hotel room in New York is said to be \$168 per night. A random sample of 25 hotels resulted in $\bar{x}$ = \$172.50 and s = \$15.40. Test at the α = 0.05 level.

(Assume the population distribution is normal)

H_0: $\mu = 168$

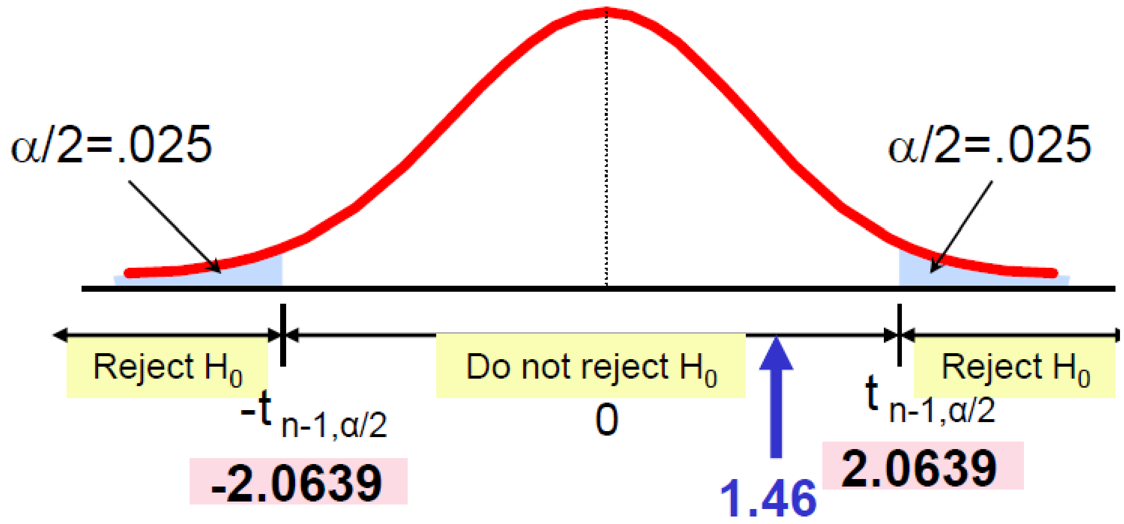
H_1: $\mu \neq 168$

Example Solution:

$H_0: \mu = 168$
$H_1: \mu \neq 168$

- $\alpha = 0.05$
- $n = 25$
- σ is unknown, so use a **t statistic**
- Critical Value:

$t_{24\,,\,.025} = \pm\, 2.0639$



$$t_{n-1} = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{172.50 - 168}{\frac{15.40}{\sqrt{25}}} = 1.46$$

Do not reject H_0: not sufficient evidence that true mean cost is different than \$168

Hypothesis Testing For Proportion

- Involves categorical variables
- Two possible outcomes
  - “Success” (a certain characteristic is present)
  - “Failure” (the characteristic is not present)
- Fraction or proportion of the population in the “success” category is denoted by P
- Assume sample size is large

Hypothesis Testing For Proportion

(continued)

- Sample proportion in the success category is denoted by $\hat{p}$

  - $$\hat{p} = \frac{x}{n} = \frac{\text{number of successes in sample}}{\text{sample size}}$$

- When $nP(1 - P) > 9$, $\hat{p}$ can be approximated by a normal distribution with mean and standard deviation

  - $$\mu_{\hat{p}} = P \qquad \sigma_{\hat{p}} = \sqrt{\frac{P(1-P)}{n}}$$

Hypothesis Testing For Proportion

- The sampling distribution of $\hat{p}$ is approximately normal, so the test statistic is a z value:

$$z = \frac{\hat{p} - P_0}{\sqrt{\dfrac{P_0(1-P_0)}{n}}}$$

Hypothesis Tests for P

- **$nP(1 - P) > 9$**
- **$nP(1 - P) < 9$**
  - Not discussed in this chapter

Hypothesis Testing For Proportion

- The sampling distribution of $\hat{p}$ is approximately normal, so the test statistic is a z value:

$$z = \frac{\hat{p} - P_0}{\sqrt{\dfrac{P_0(1-P_0)}{n}}}$$

Hypothesis Tests for P

$nP(1 - P) > 9$

$nP(1 - P) < 9$

Not discussed in this chapter

Example:

A marketing company claims that it receives 8% responses from its mailing. To test this claim, a random sample of 500 were surveyed with 25 responses. Test at the $\alpha = .05$ significance level.

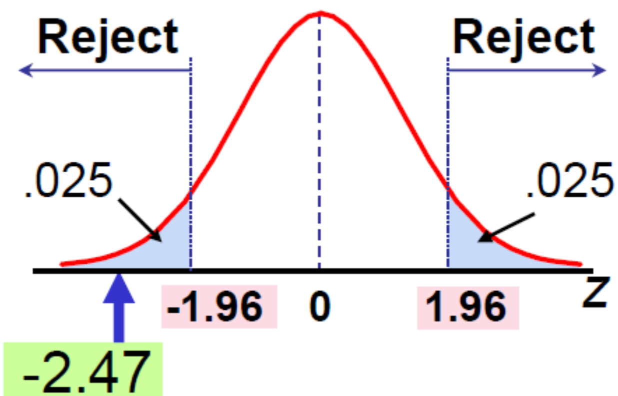
Solution:

H_0: $P = .08$
H_1: $P \neq .08$

$\alpha = .05$

$n = 500, \quad \hat{p} = .05$

Critical Values: ± 1.96



Test Statistic:

$$z = \frac{\hat{p} - P_0}{\sqrt{\frac{P_0(1-P_0)}{n}}} = \frac{.05 - .08}{\sqrt{\frac{.08(1-.08)}{500}}} = -2.47$$

Decision:

Reject H_0 at $\alpha = .05$

Conclusion:

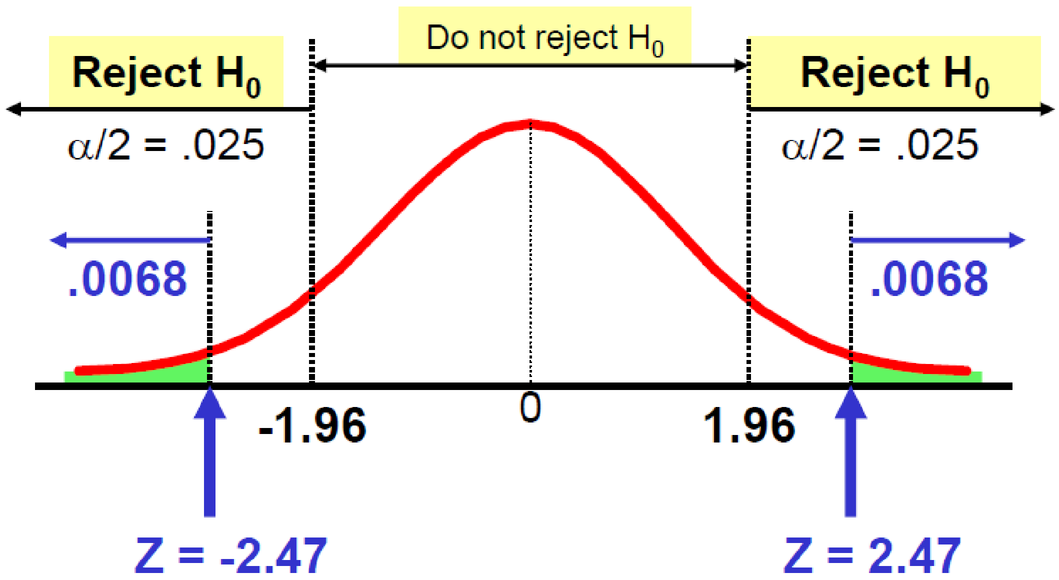
There is sufficient evidence to reject the company's claim of 8% response rate.

p-value Solution:

(continued)

Calculate the p-value and compare to α

(For a two sided test the p-value is always two sided)



p-value = .0136:

$$P(Z \le -2.47) + P(Z \ge 2.47)$$

$$= 2(.0068) = 0.0136$$

Reject H_0 since p-value = .0136 < α = .05

Type II Error

Assume the population is normal and the population variance is known. Consider the test

$$H_0: \mu = \mu_0$$
$$H_1: \mu > \mu_0$$

The decision rule is:

$$\text{Reject } H_0 \text{ if } z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} > z_\alpha$$

or

$$\text{Reject } H_0 \text{ if } \bar{x} = \bar{x}_c > \mu_0 + Z_\alpha \sigma / \sqrt{n}$$

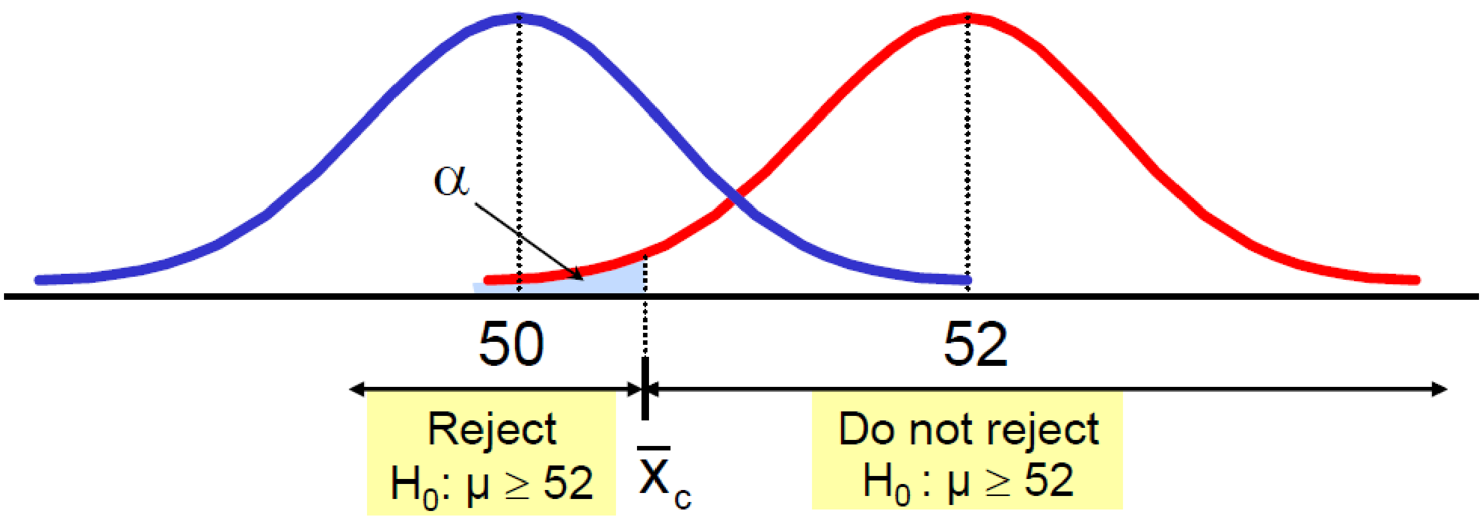
If the null hypothesis is false and the true mean is μ*, then the probability of type II error is

$$\beta = P(\bar{x} < \bar{x}_c \mid \mu = \mu^*) = P\left(z < \frac{\bar{x}_c - \mu^*}{\sigma / \sqrt{n}} \right)$$

Type II Error Example:

- Type II error is the probability of failing to reject a false H_0

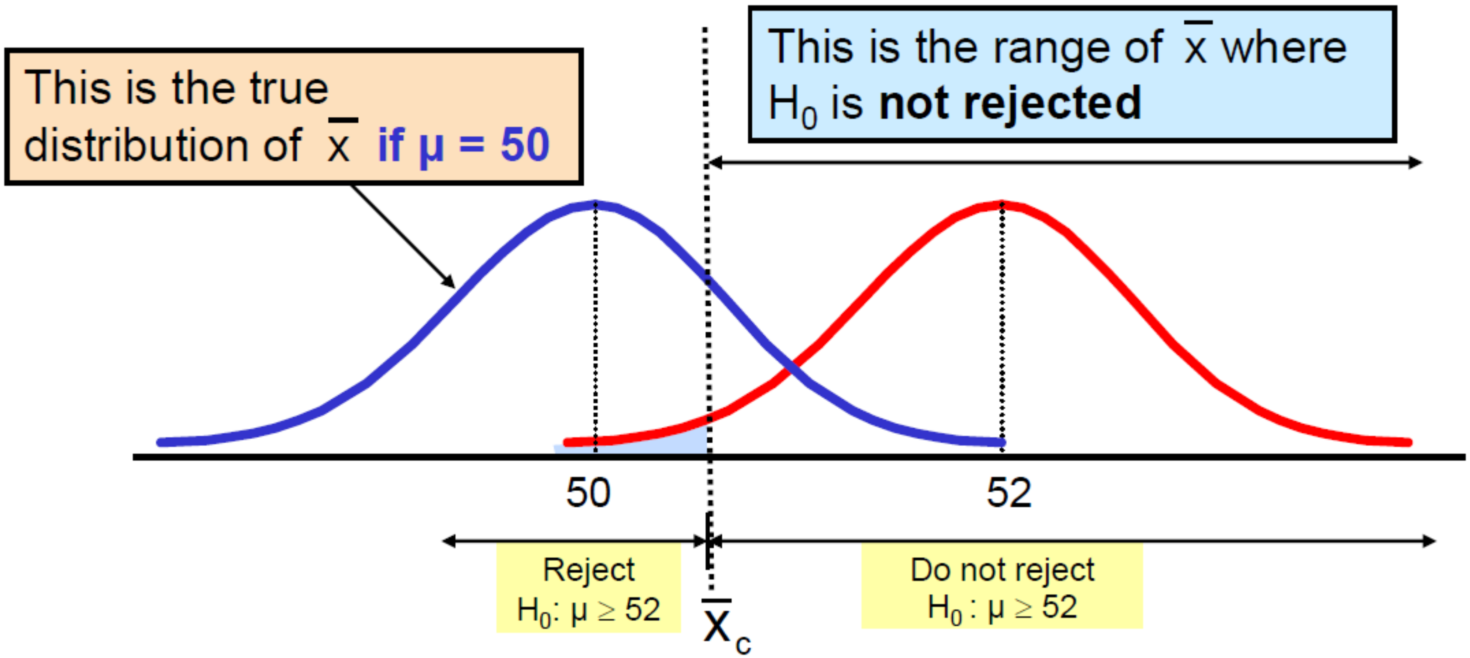
Suppose we fail to reject **H_0: $\mu \geq 52$** when in fact the true mean is **$\mu^* = 50$**



Type II Error Example:

(continued)

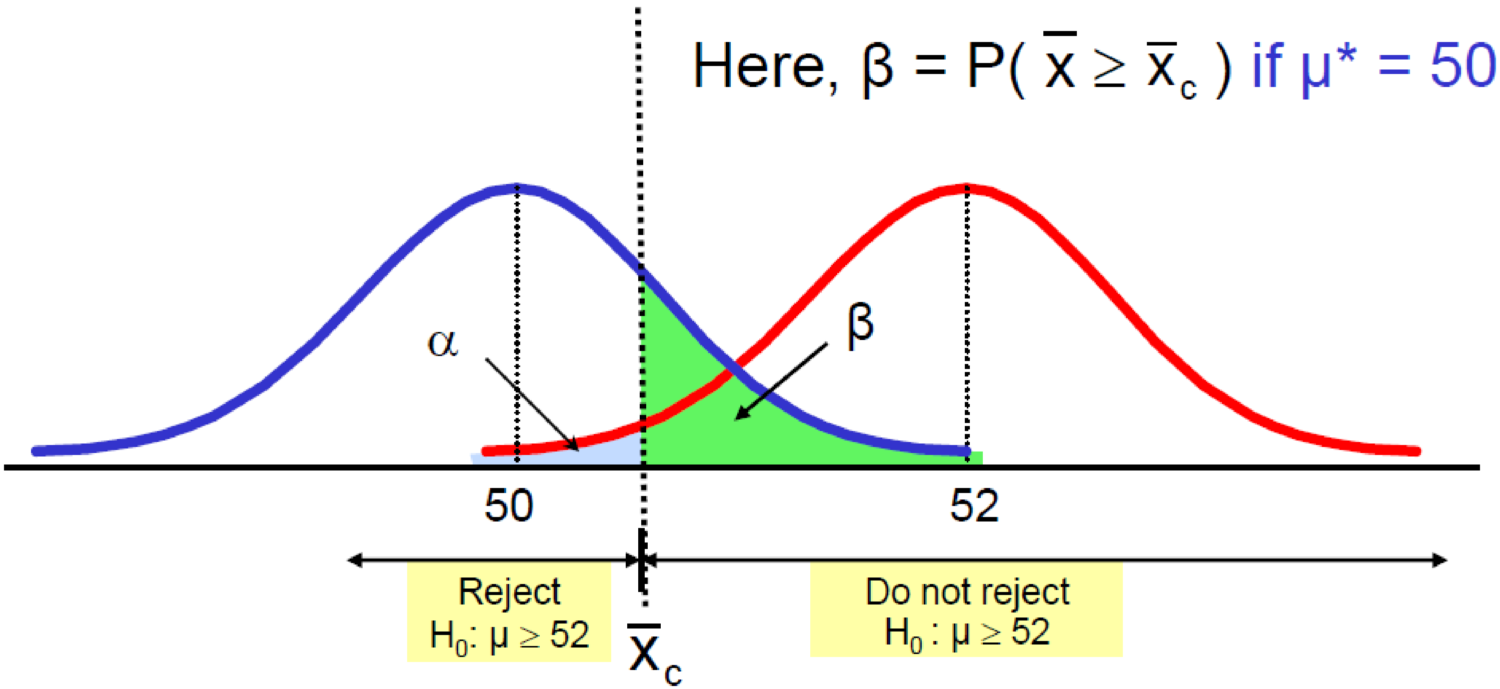
- Suppose we do not reject H_0: $\mu \geq 52$ when in fact the true mean is $\mu^* = 50$



Type II Error Example:

(continued)

- Suppose we do not reject H_0: $\mu \geq 52$ when in fact the true mean is $\mu^* = 50$

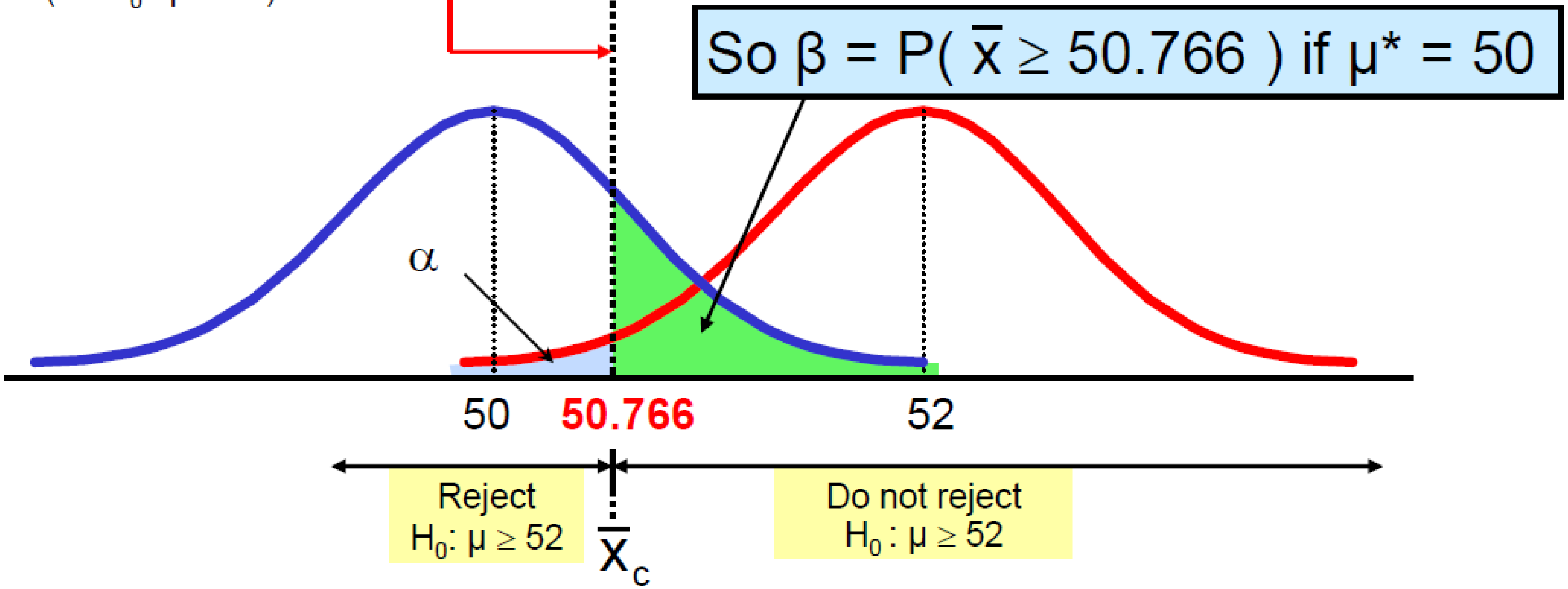


Calculating β

- Suppose n = 64 , σ = 6 , and α = .05

$$\bar{x}_c = \mu_0 - z_\alpha \frac{\sigma}{\sqrt{n}} = 52 - 1.645 \frac{6}{\sqrt{64}} = \boxed{50.766}$$

(for $H_0 : \mu \geq 52$)

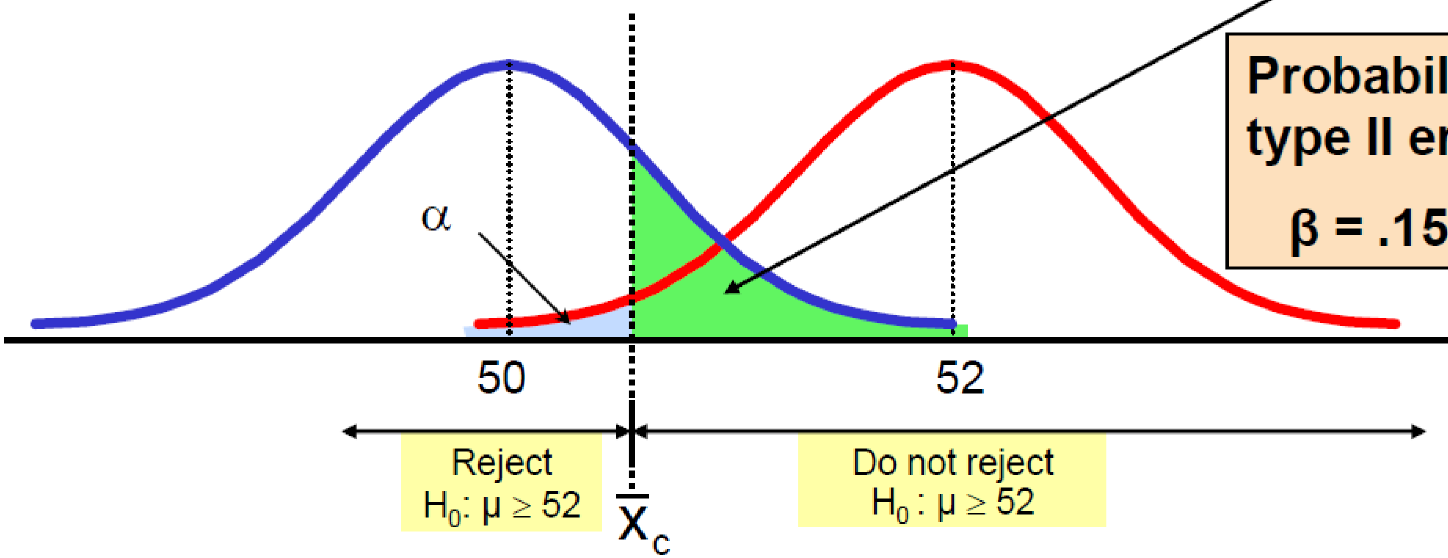


Calculating β

(continued)

- Suppose n = 64 , σ = 6 , and α = .05

$$P(\bar{x} \geq 50.766 \mid \mu^* = 50) = P\left(z \geq \frac{50.766-50}{6/\sqrt{64}} \right) = P(z \geq 1.02) = .5 - .3461 = \boxed{.1539}$$



Probability of type II error:

β = .1539

Calculating power of the Test Example

If the true mean is $\mu^* = 50$,

- The probability of Type II Error = $\beta = 0.1539$
- The power of the test = $1 - \beta = 1 - 0.1539 = 0.8461$

Next Lesson

- **Hypothesis Testing-3**

See you☺