Fuzzy Functions

Murat Osmanoglu

Kinds of Fuzzy Functions

fuzzy functions can be categorized as three groups :

- crisp function with fuzzy constraint
- crisp function that propagates the fuzziness of independent variable to dependent variable
- fuzzifying function

Crisp Function with Fuzzy Constraint

$$f : X \rightarrow Y$$

crisp function crisp sets

- let $A \subseteq X$ and $B \subseteq Y$ be fuzzy sets.

the function with fuzzy constraint $\mu_A(x) \leq \mu_B(f(x))$ on A and B

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- consider the function $f : Z^+ \rightarrow Z^+$ with the rule $f(x) = 2x$, and two fuzzy sets $A, B \subseteq Z^+$ defined as

$A = \{(1,0.3), (2,1.0)\}$ and $B = \{(2,0.6), (4,1.0)\}$

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  f can be considered as the crisp function with the fuzzy constraint $\mu_A(x) \le \mu_B(f(x))$

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  the constraint here 'a competent salesman gets higher income'

Fuzzy Extension Function

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crisp function   crisp sets

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$$\mu_{f(A)}(y) = \max_{x \text{ s.t. } f(x)=y} \mu_A(x)$$

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f: crisp function

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$$\begin{cases} \mu_A(1) = 0.6, \quad f(1) = 1 \\ \mu_A(-1) = 0.7, \quad f(-1) = 1 \end{cases}$$

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Fuzzifying Function of Crisp Value

$$f : X \rightarrow P(Y)$$

crisp set

fuzzy power set

Single Fuzzifying Function

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Let A and B two crisp sets A = {1, 2, 3}, B = {1, 2, 3, 4, 5, 6, 7}

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$f : 1 \rightarrow \{2\}$ $\alpha = 1.0$

Fuzzifying Function of Crisp Value

Fuzzy bunch of functions

- fuzzy set of crisp functions:

$$F = \{(f_1, \mu_F(f_1)), (f_2, \mu_F(f_2)), \ldots, (f_n, \mu_F(f_n))\}$$

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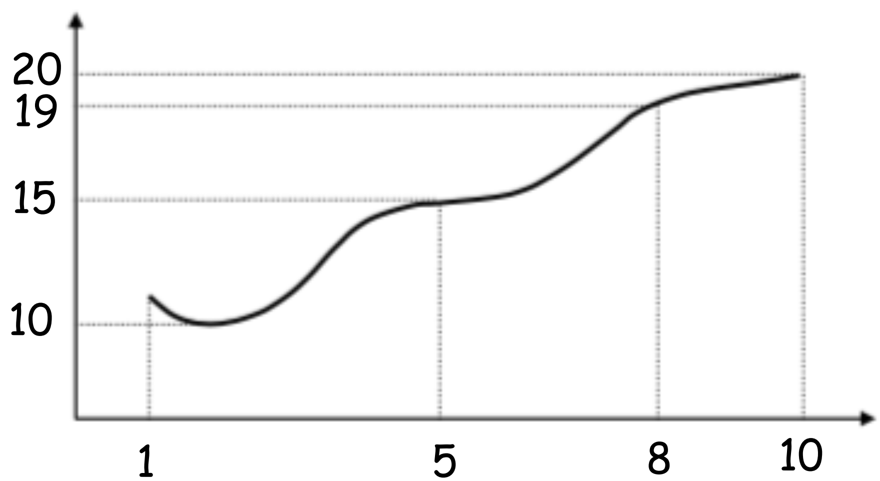
- consider the function f given with the following figure



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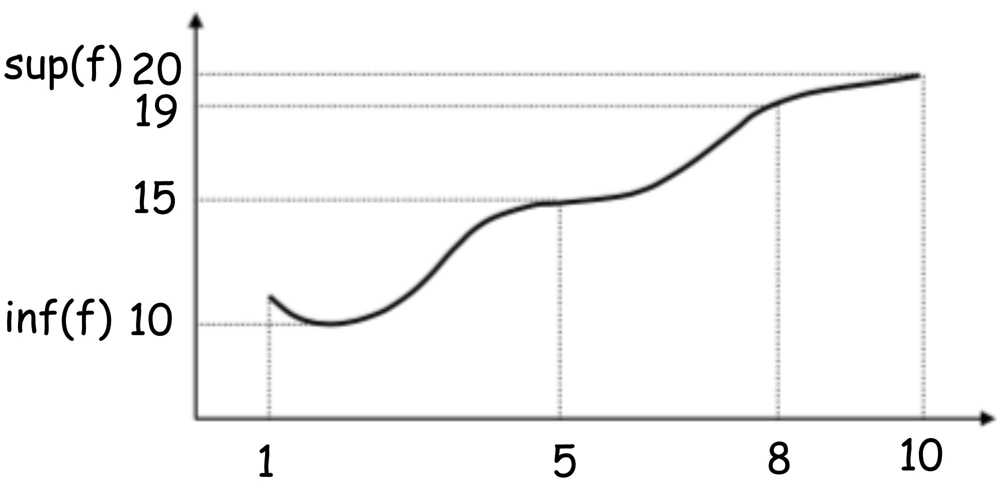

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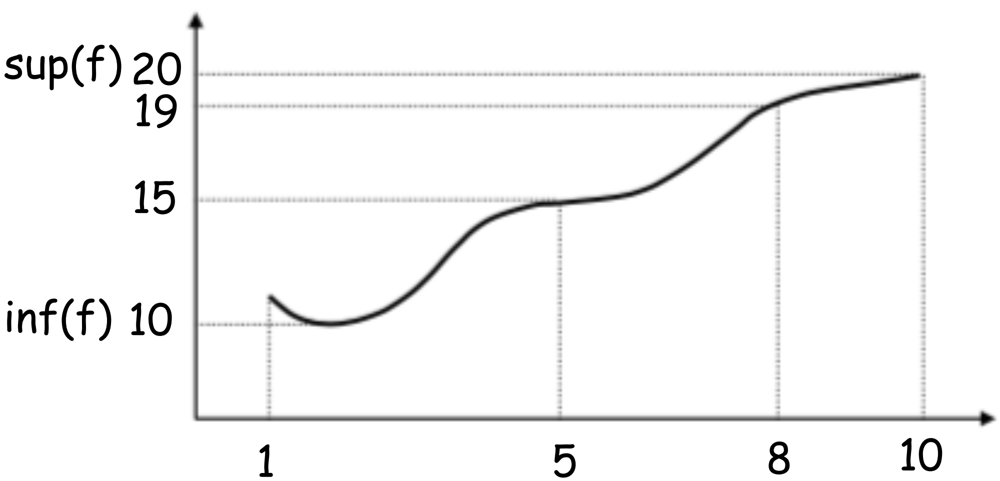

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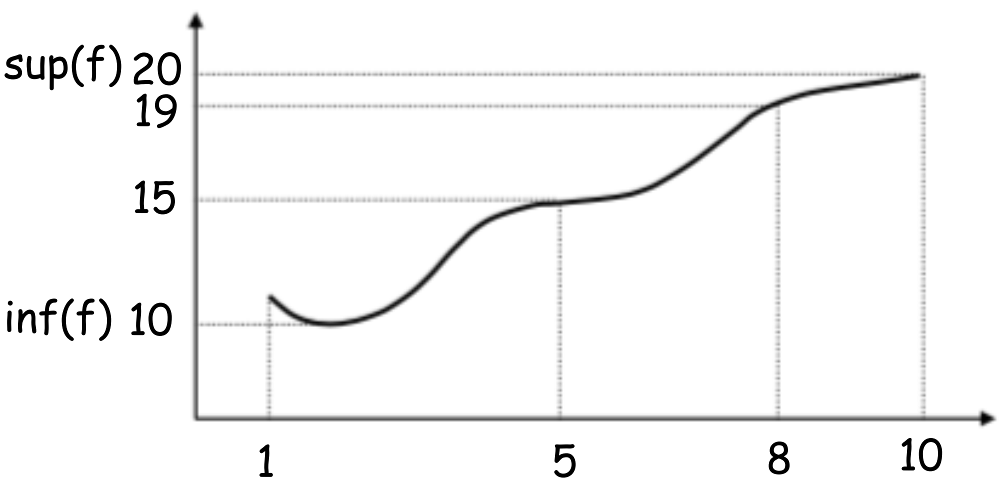

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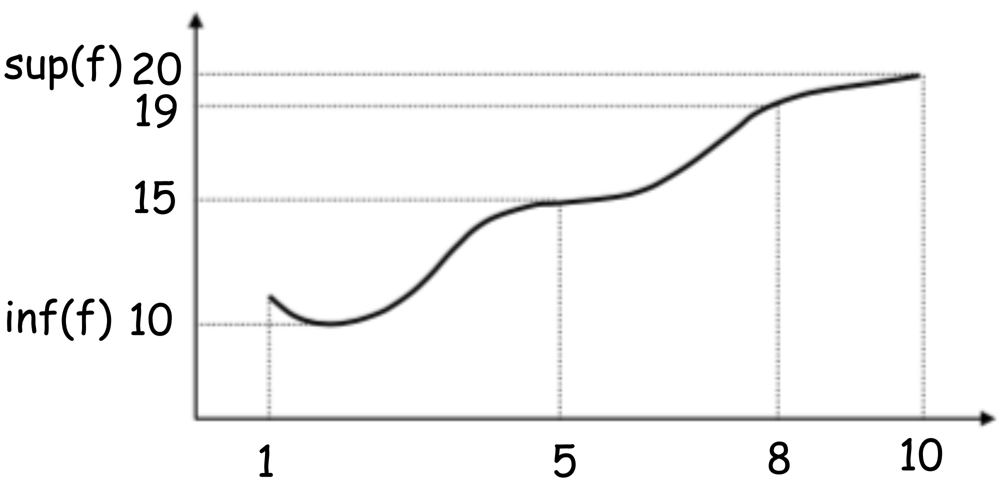

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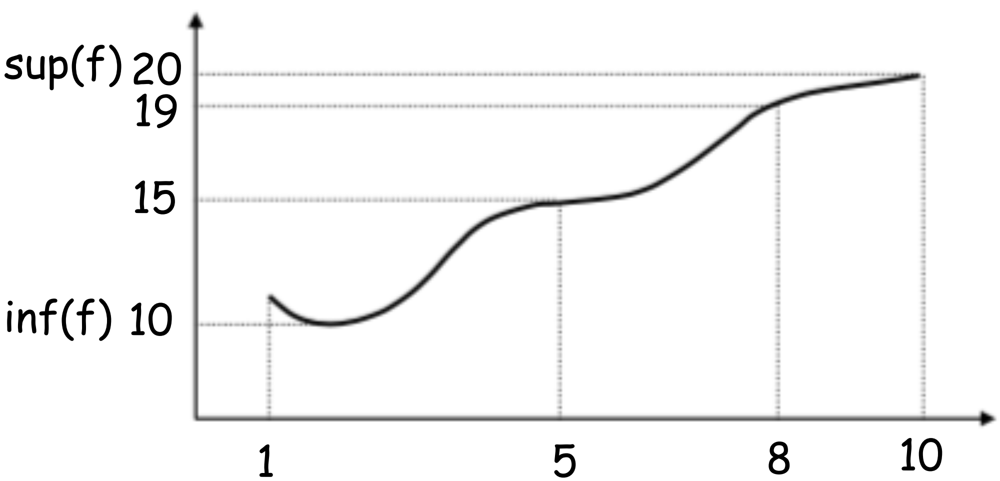

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$$\mu_M(\pi/3) = \frac{\cos(\pi/3) + 1}{2} = 3/4$$

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Integration of fuzzifying function in crisp interval

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- consider the fuzzy bunch of function $F = \{(f_1, 0.6), (f_2, 0.9), (f_3, 0.5)\}$

  where $f_1(x) = 3x$, $f_2(x) = x^2$, $f_3(x) = x - 1$

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$$\tilde{I}(1,2) = \left\{\left(\frac{9}{2}, 0.6\right), \left(\frac{7}{3}, 0.9\right), (\frac{1}{2}, 0.5)\right\}$$

Integration of Fuzzy Function

Integration of crispfunction in fuzzy interval

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Differentiation of Fuzzy Function

Differentiation of fuzzifying function on crisp point

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Differentation of fuzzifying function on crisp point

- consider the fuzzy bunch of function $F = \{(f_1, 0.6), (f_2, 0.9), (f_3, 0.5)\}$

  where $f_1(x) = 4x$, $f_2(x) = x^2$, $f_3(x) = x - 1$

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- calculate the differentiation of F on $x_0 = 2$

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$$f_1'(x) = 4,\ f_2'(x) = 2x,\ f_3'(x) = 1$$

$$F'(2) = \{(4, 0.6), (4, 0.9), (1, 0.5)\}$$

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- consider the function $f(x) = 3x^3$

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- consider the function $f(x) = 3x^3$
- calculate differentiation of f at A

  where $A = \{(-2, 0.5), (0, 1.0), (2, 0.7)\}$

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- consider the function $f(x) = 3x^3$
- calculate differentiation of f at *A*

where $A = \{(-2, 0.5), (0, 1.0), (2, 0.7)\}$

$$f'(x) = 9x^2$$

Differentiation of Fuzzy Function

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- consider the function $f(x) = 3x^3$
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$$f'(x) = 9x^2$$

$$f'(A) = \{(36, 0.5), (0, 1.0), (36, 0.7)\}$$

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