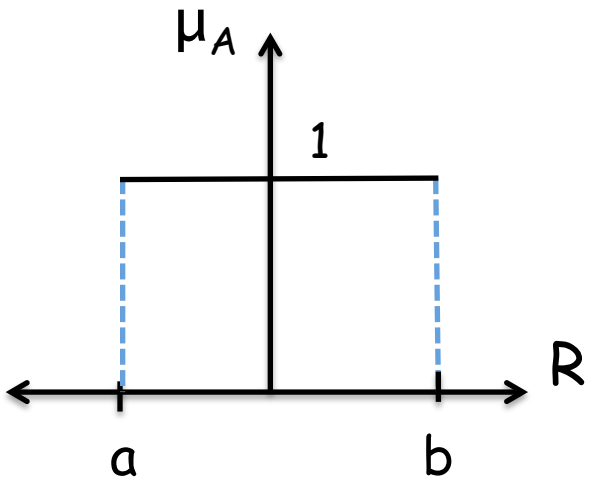
Fuzzy Number

Murat Osmanoglu

Fuzzy Number

- consider an interval $A = [a,b]$ defined on R

Fuzzy Number



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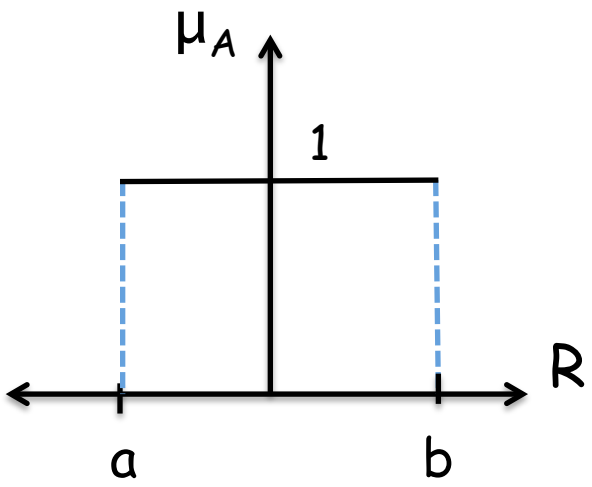

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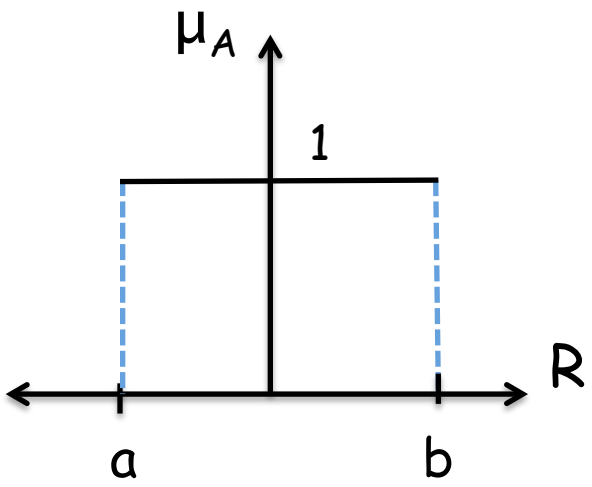

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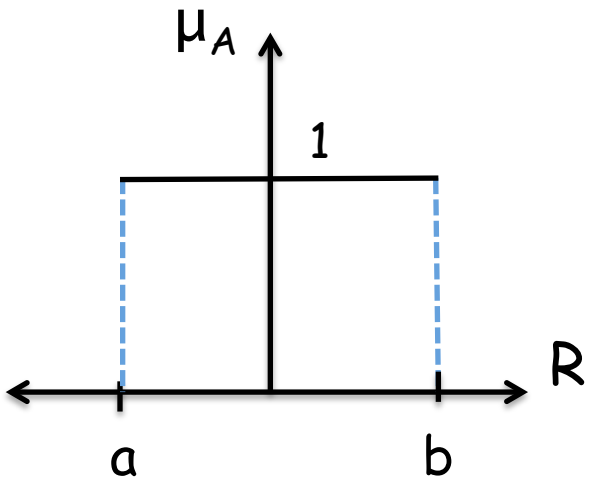

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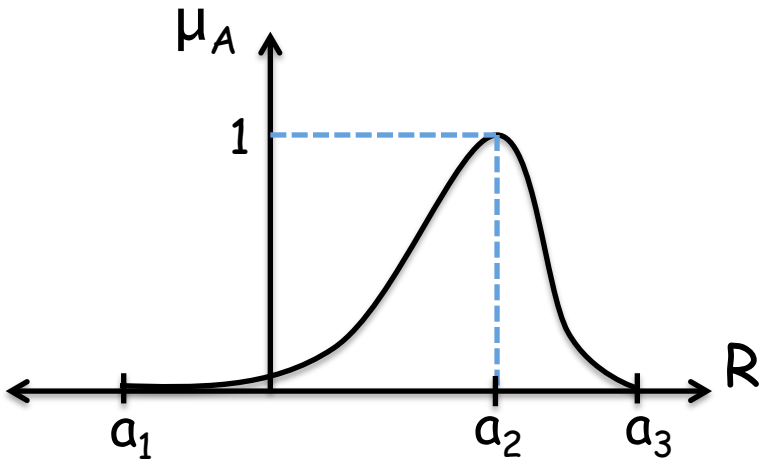

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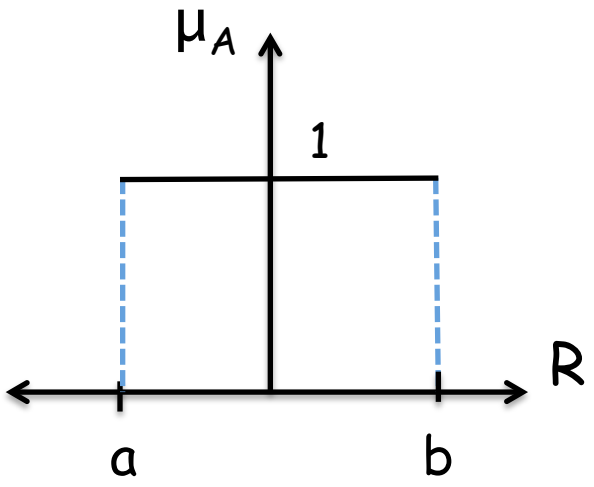

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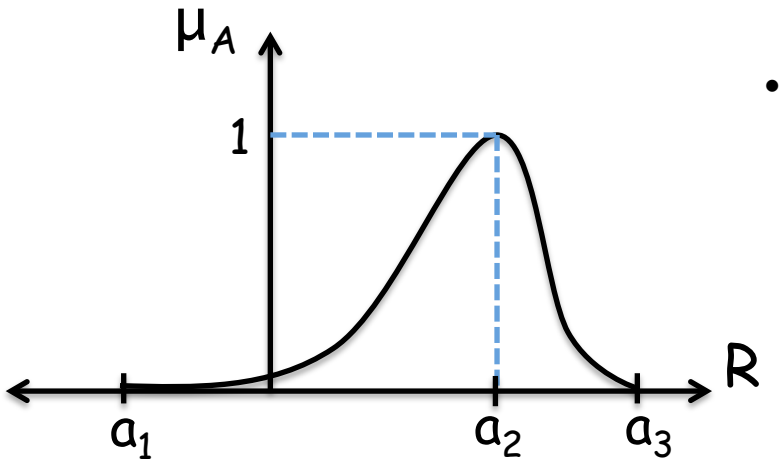

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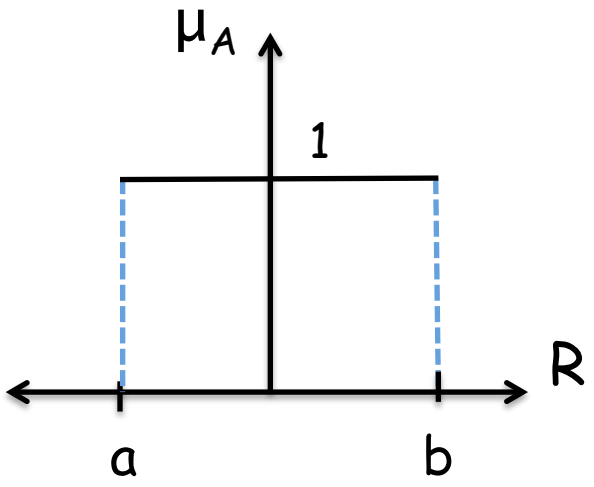

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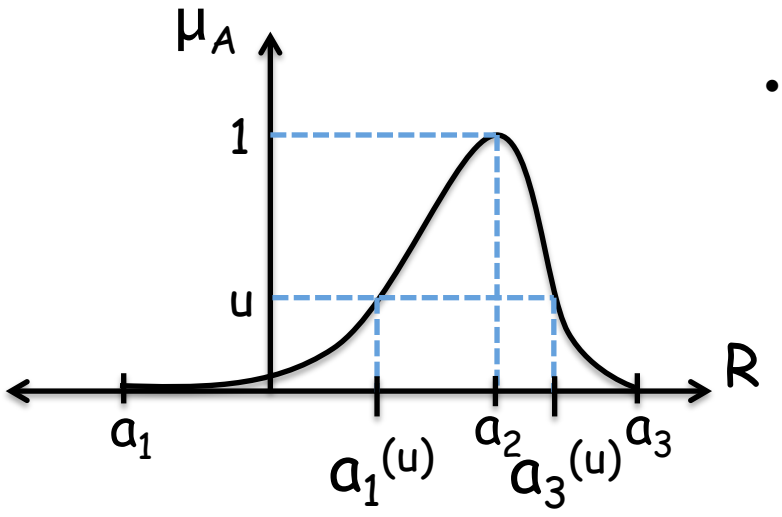

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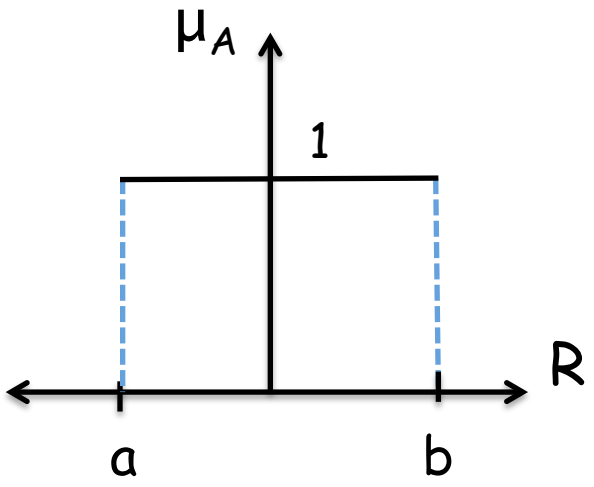

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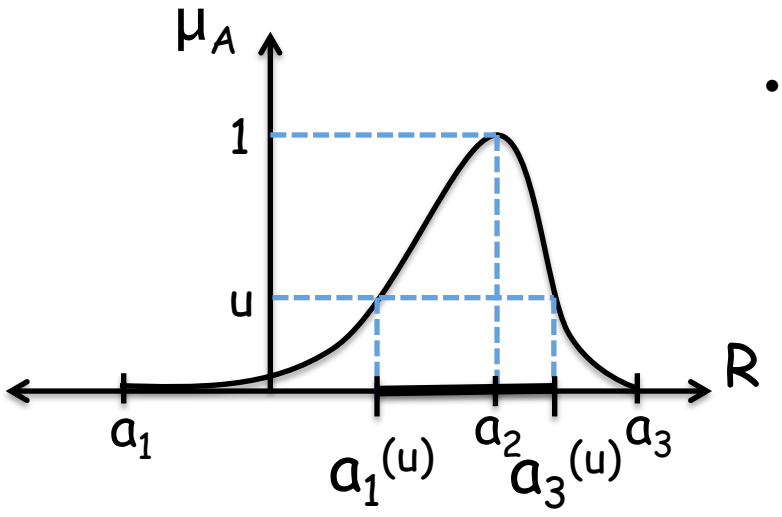

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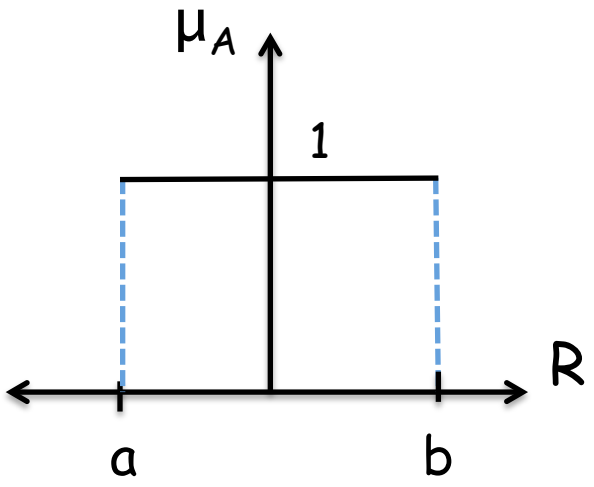

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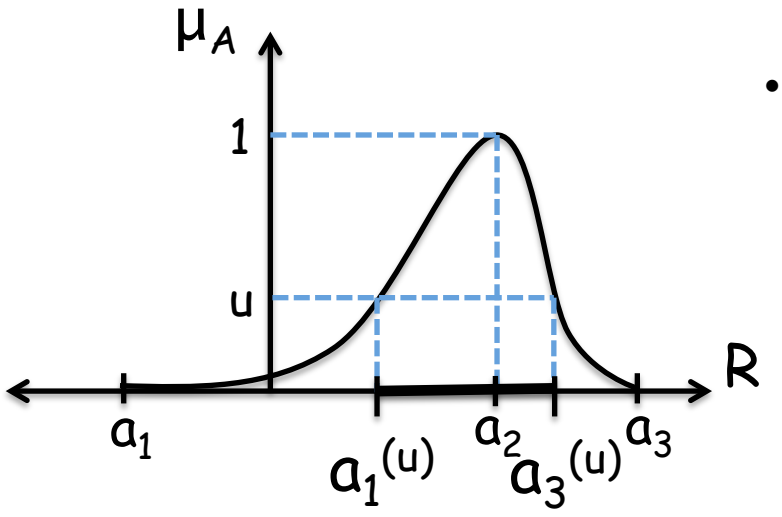

$$A_u = [a_1^{(u)}, a_3^{(u)}]$$

Operations on Intervals

- Given $A = [a_1, a_3]$ and $B = [b_1, b_3]$
  - $[a_1, a_3]$ (+) $[b_1, b_3] = [a_1 + b_1, a_3 + b_3]$
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  - $[a_1, a_3]$ (/) $[b_1, b_3] = [\min \{a_1/b_1, a_1/b_3, a_3/b_1, a_3/b_3\}, \max \{a_1/b_1, a_1/b_3, a_3/b_1, a_3/b_3\}]$
- $A = [-2, 3]$ and $B = [-4, 6]$

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$\mu_C(2)$

$\mu_C(3)$

$\mu_C(4)$

$\mu_C(6)$

Operations on Fuzzy Numbers

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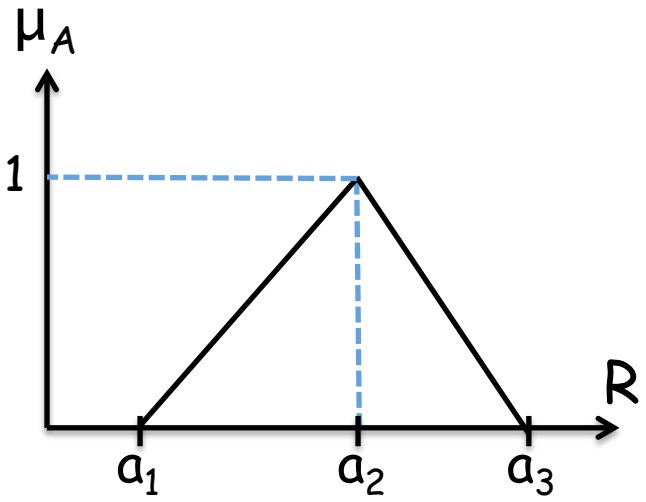
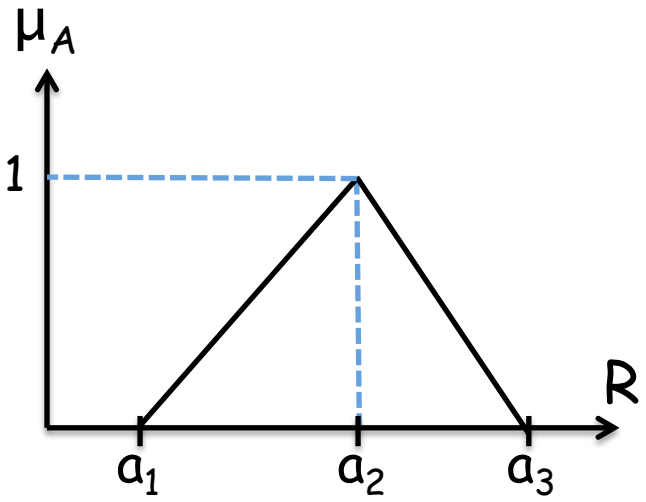
Triangular Fuzzy Number



- $A = (a_1, a_2, a_3)$

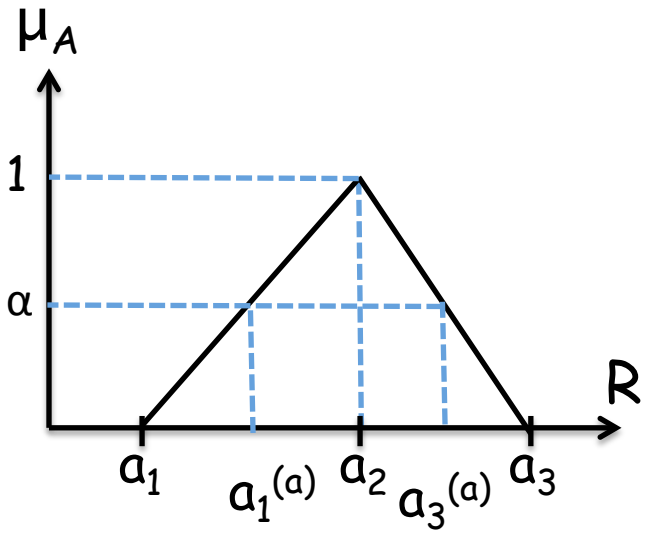
Triangular Fuzzy Number



- $A = (a_1, a_2, a_3)$

$$\mu_A(x) = \begin{cases} 0 & , \text{ if } x < a_1 \\ (x - a_1)/(a_2 - a_1) & , \text{ if } a_1 \le x \le a_2 \\ (a_3 - x)/(a_3 - a_2) & , \text{ if } a_2 \le x \le a_3 \\ 0 & , \text{ if } x > a_3 \end{cases}$$

Triangular Fuzzy Number



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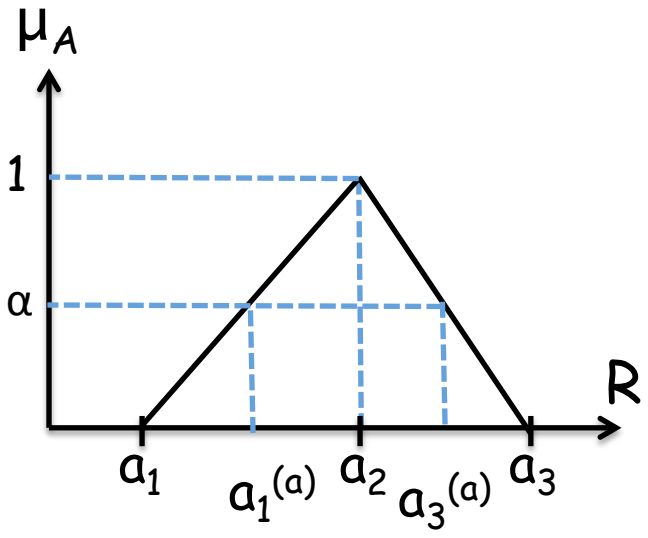

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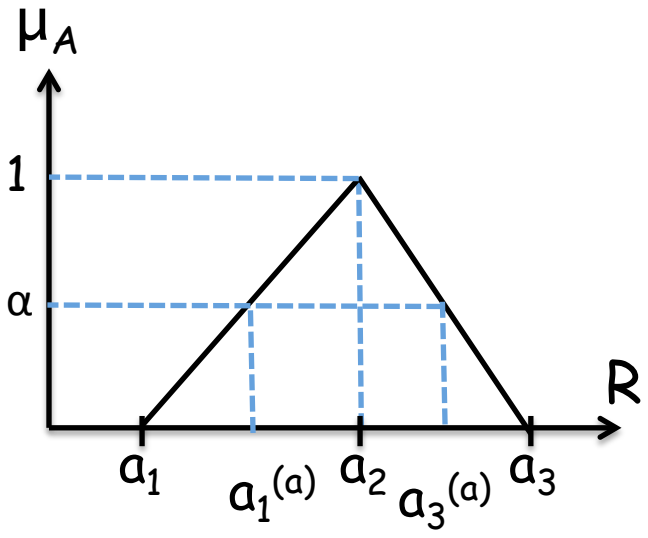
$(a_1^{(\alpha)} - a_1)/(a_2 - a_1) = \alpha$

$(a_3 - a_3^{(\alpha)})/(a_3 - a_2) = \alpha$

Triangular Fuzzy Number



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$$(a_1^{(\alpha)} - a_1)/(a_2 - a_1) = \alpha$$

$$a_1^{(\alpha)} = (a_2 - a_1)\alpha + a_1$$

$$(a_3 - a_3^{(\alpha)})/(a_3 - a_2) = \alpha$$

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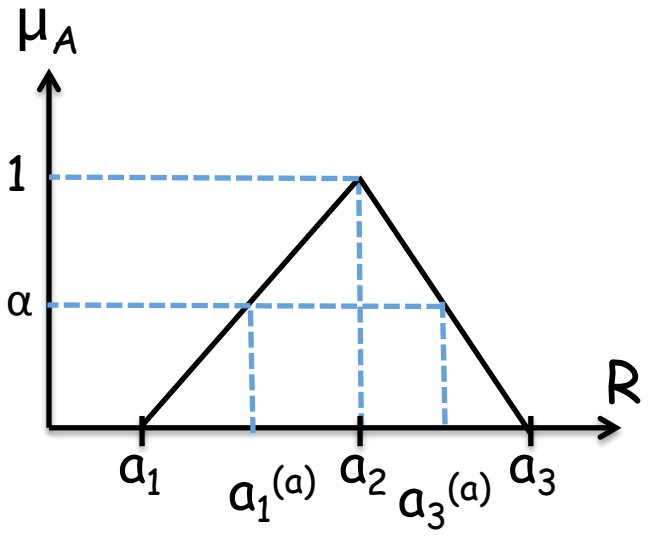
Triangular Fuzzy Number



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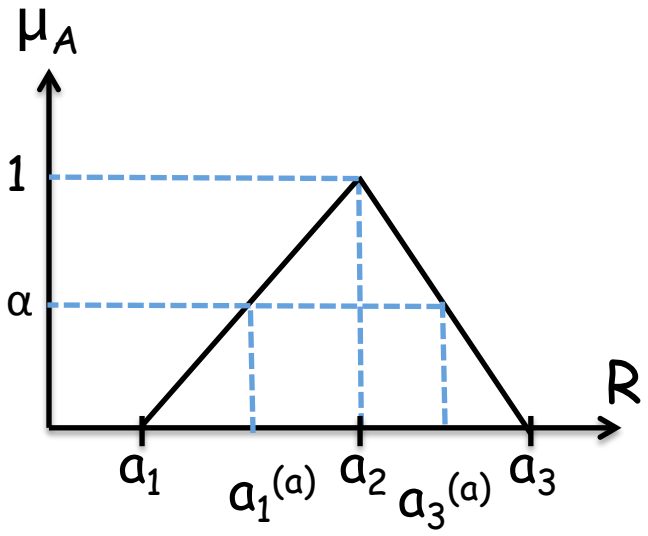

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$$A_0 = [a_1, a_3]$$

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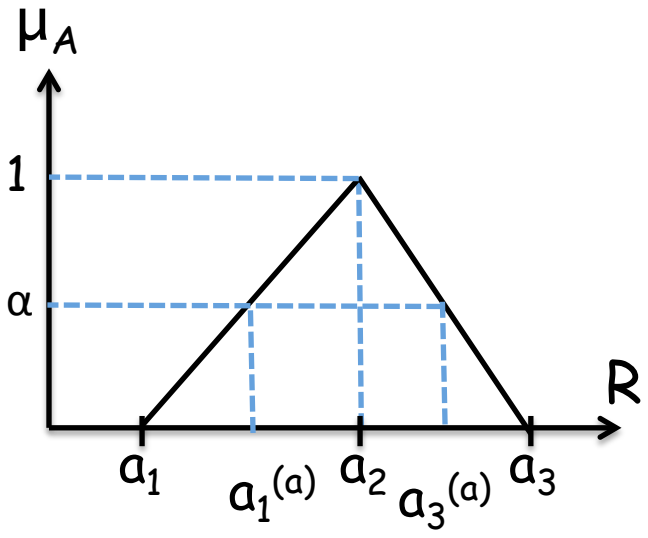

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$$A_0 = [a_1, a_3] \qquad A_1 = [a_2, a_2]$$

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- Given the triangular fuzzy numbers A = (-2, 1, 4) and B = (0, 2, 3), A(+)B = ?

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$$\mu_A(x) = \begin{cases} 0 & , \text{ if } x < -2 \\ (x+2)/3 & , \text{ if } -2 \le x \le 1 \\ (4-x)/3 & , \text{ if } 1 \le x \le 4 \\ 0 & , \text{ if } x > 4 \end{cases}$$

$$\mu_B(x) = \begin{cases} 0 & , \text{ if } x < 0 \\ x/2 & , \text{ if } 0 \le x \le 2 \\ 3-x & , \text{ if } 2 \le x \le 3 \\ 0 & , \text{ if } x > 3 \end{cases}$$

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$$\mu_{A(+)B}(4) = (\mu_A(3) \wedge \mu_B(1)) \vee (\mu_A(2) \wedge \mu_B(2)) \vee (\mu_A(2.5) \wedge \mu_B(1.5))$$
$$\vee (\mu_A(1.5) \wedge \mu_B(2.5)) \vee (\mu_A(1.6) \wedge \mu_B(2.4)) \vee \ldots$$

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$$\mu_{A(+)B}(4) = (\mu_A(3) \wedge \mu_B(1)) \vee (\mu_A(2) \wedge \mu_B(2)) \vee (\mu_A(2.5) \wedge \mu_B(1.5)) \vee (\mu_A(1.5) \wedge \mu_B(2.5)) \vee (\mu_A(1.6) \wedge \mu_B(2.4)) \vee \ldots$$

$$\mu_{A(+)B}(4) = (1/3 \wedge 1/2) \vee (2/3 \wedge 1) \vee (1/2 \wedge 3/4) \vee (5/6 \wedge 1/2) \vee (4/5 \wedge 3/5) \vee \ldots$$

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$A (+) B = (-2, 3, 7)$

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$$\vee (\mu_A(1.5) \wedge \mu_B(2.5)) \vee (\mu_A(1.6) \wedge \mu_B(2.4)) \vee \ldots$$

$\mu_{A(+)B}(4)$ … /4)

$A_\alpha = [3\alpha$ …

- $A = (a_1, a_2, a_3)$ and $B = (b_1, b_2, b_3)$

  $A\ (+)\ B = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$

  $A\ (-)\ B = (a_1 - b_3, a_2 - b_2, a_3 - b_1)$

$A_\alpha\ (+)\ B_\alpha = [5\alpha - 2, 7 - 4\alpha]$ $\quad A_0\ (+)\ B_0 = [-2, 7]$ $\quad A_1\ (+)\ B_1 = [3, 3]$

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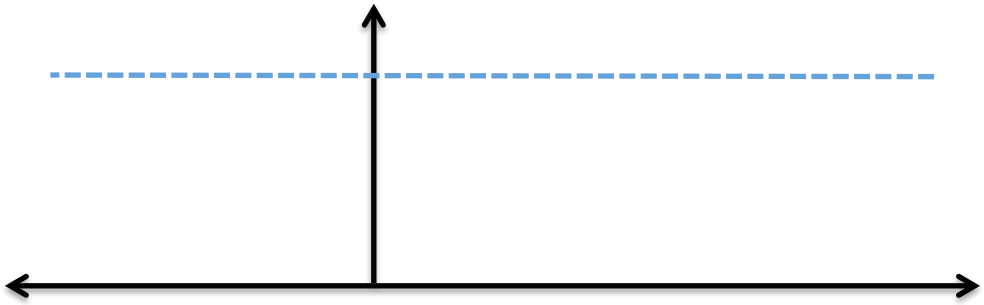
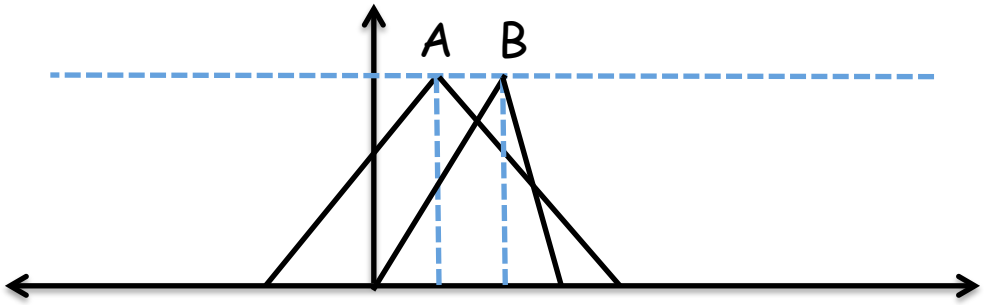
Triangular Fuzzy Number

- Given the triangular fuzzy numbers A = (-2, 1, 4) and B = (0, 2, 3)



Triangular Fuzzy Number

- Given the triangular fuzzy numbers A = (-2, 1, 4) and B = (0, 2, 3)



Triangular Fuzzy Number

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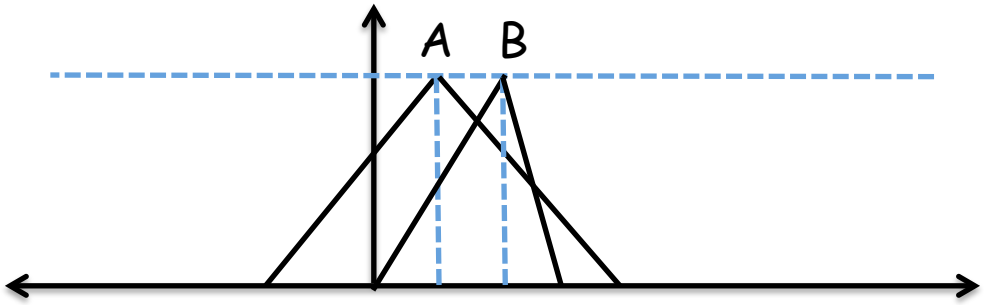

A(+)B = (-2, 3, 7)

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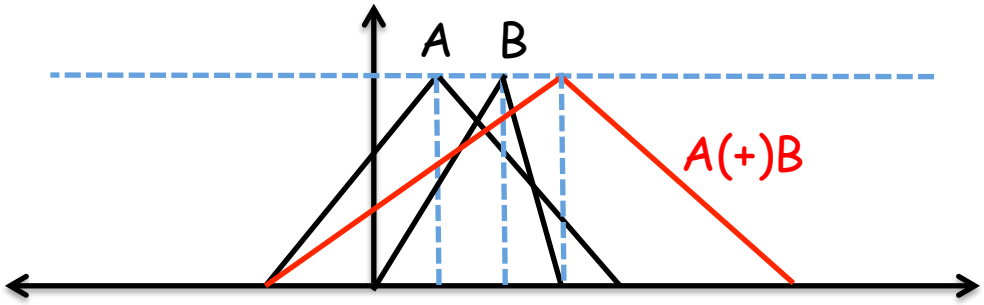

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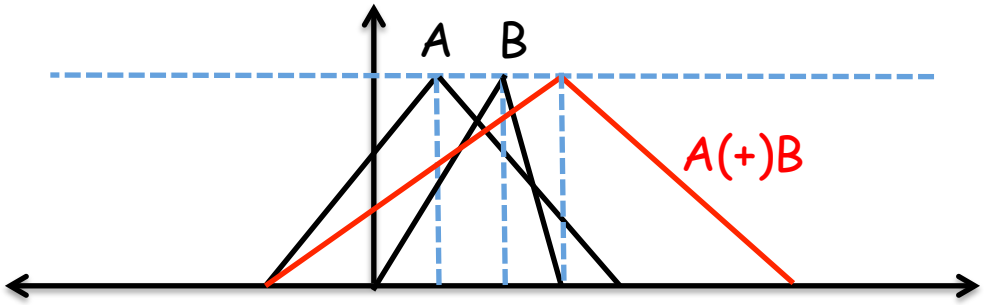

A(+)B = (-2, 3, 7)

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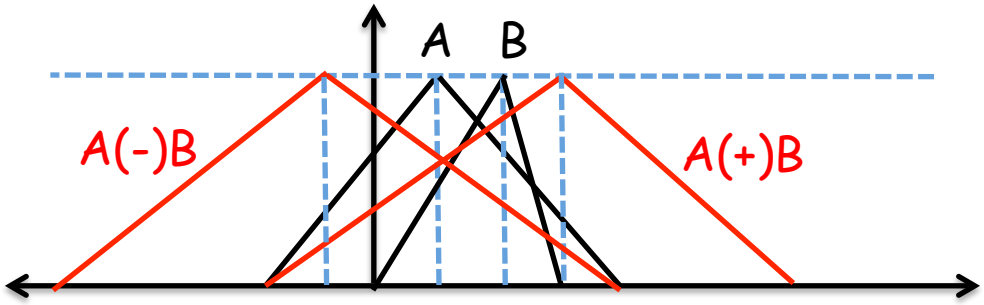
Triangular Fuzzy Number

- Given the triangular fuzzy numbers A = (-2, 1, 4) and B = (0, 2, 3)



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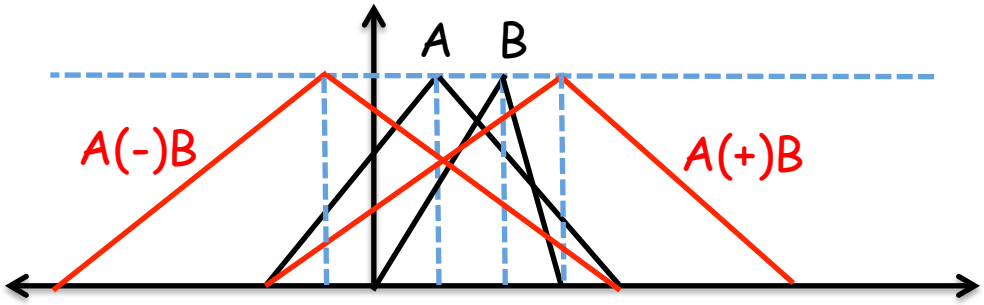

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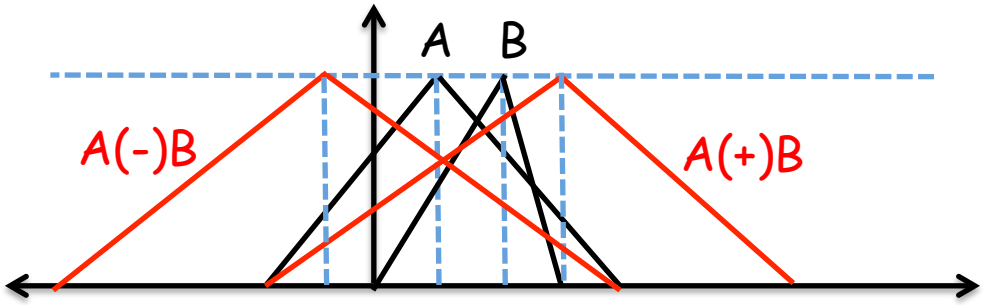

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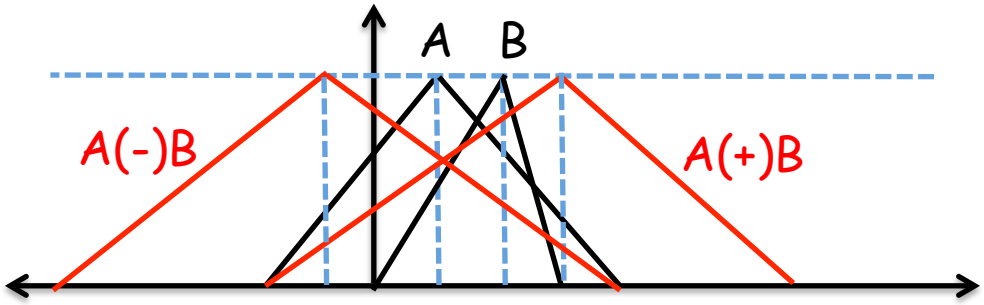

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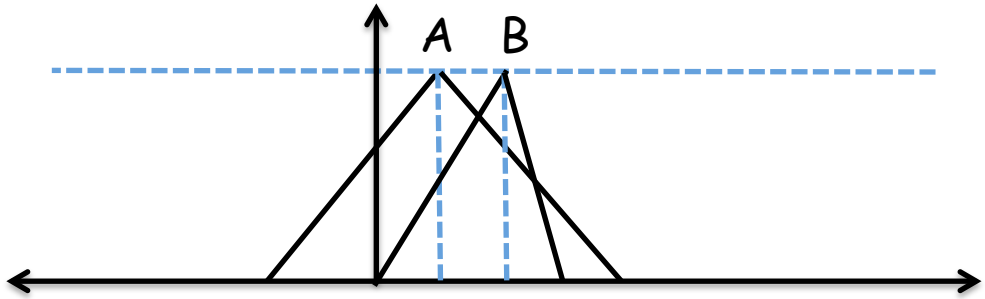

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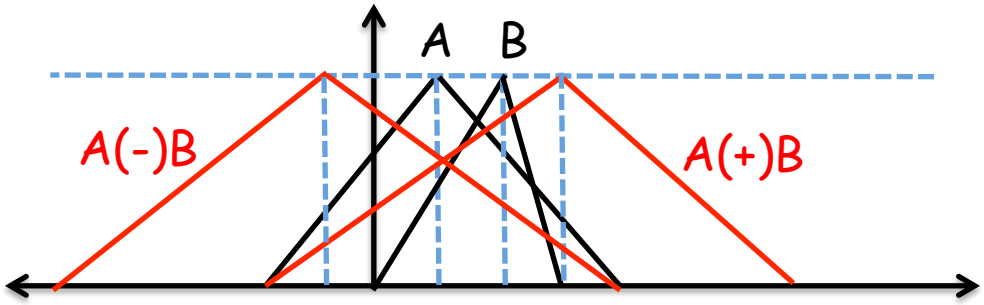

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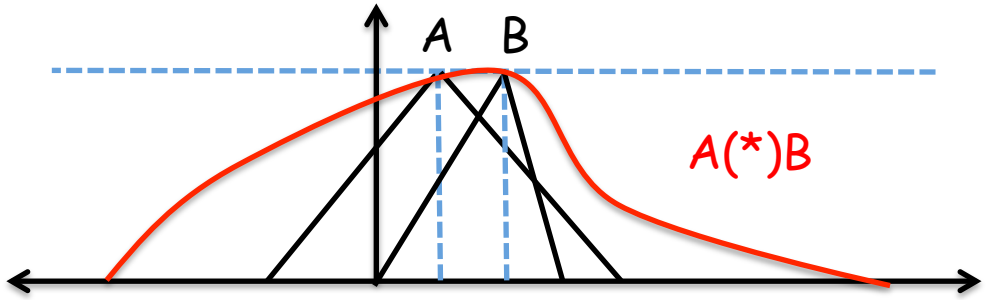

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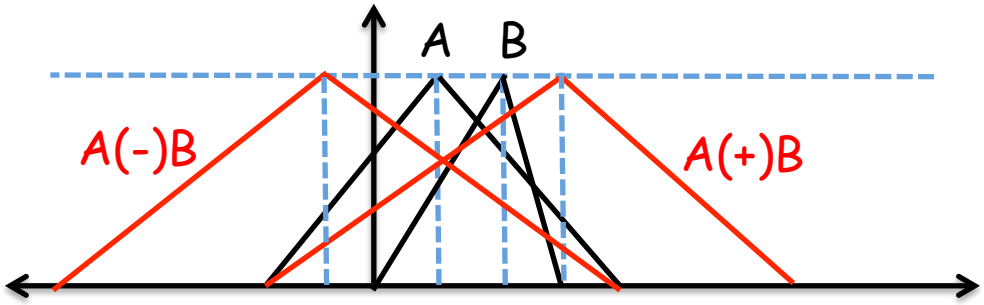

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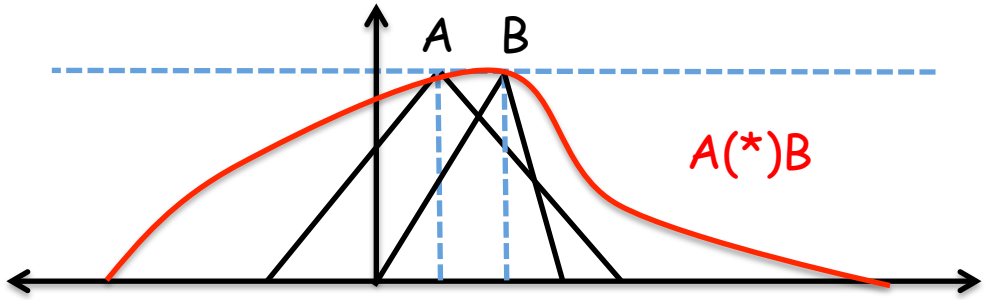

A(*)B ≈

A(/)B ≈

Triangular Fuzzy Number

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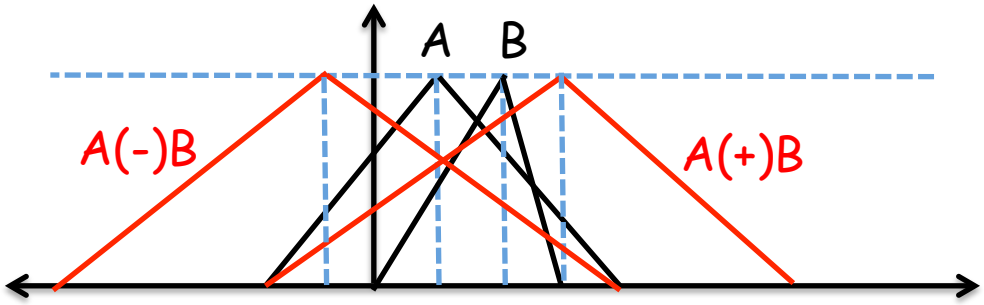

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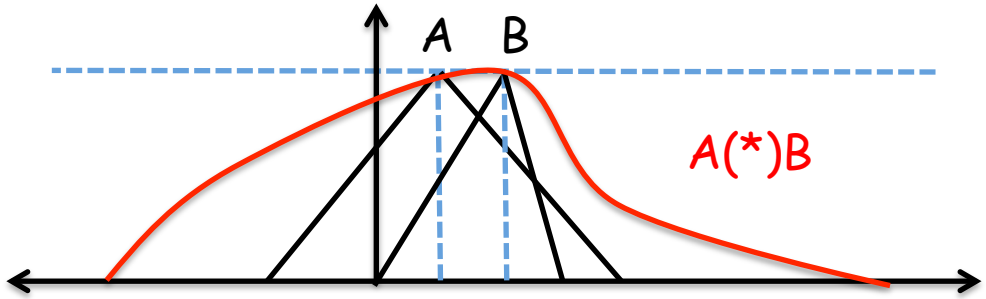
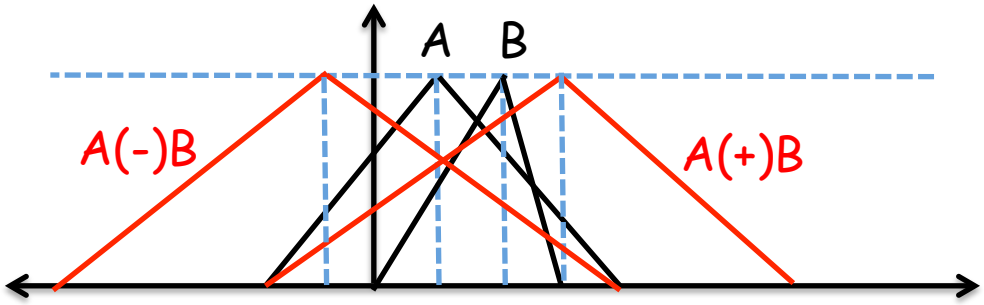


A(*)B ≈ (0 ∧ -6 ∧ 0 ∧ 12, 2, 0 ∨ -6 ∨ 0 ∨ 12)

A(/)B ≈

Triangular Fuzzy Number

- Given the triangular fuzzy numbers A = (-2, 1, 4) and B = (0, 2, 3)



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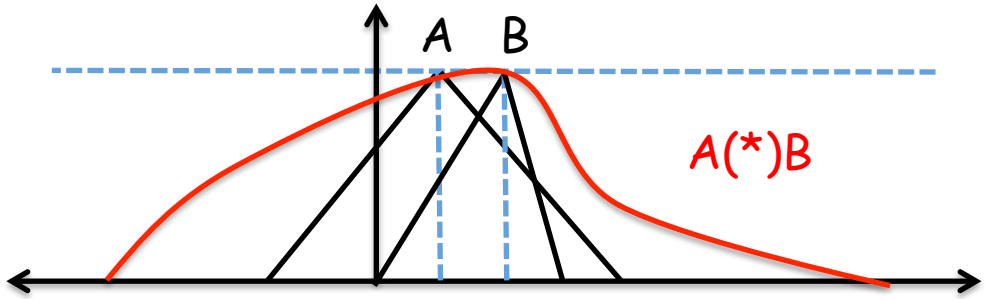

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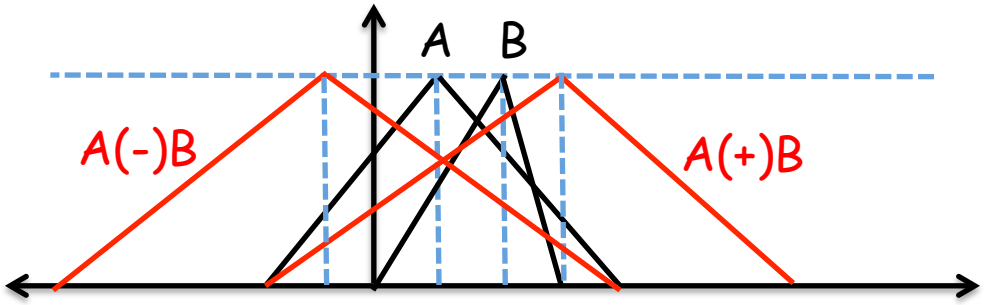

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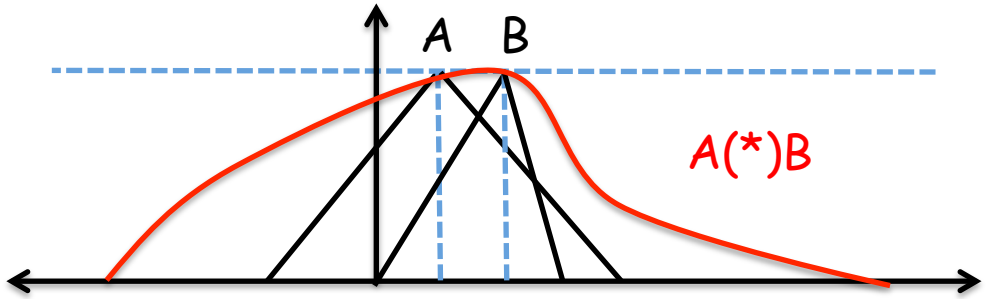
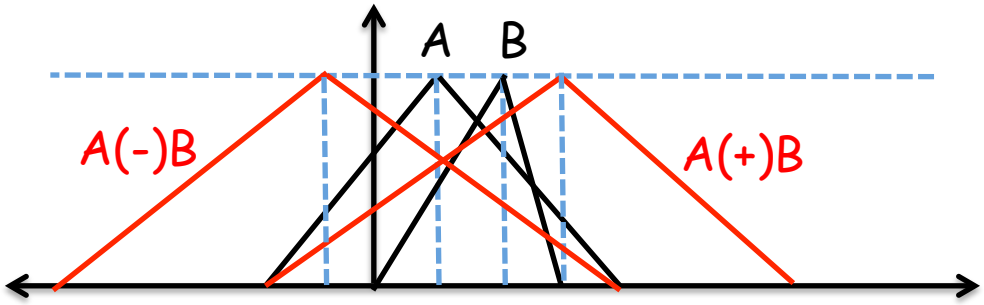


$A(*)B \approx (0 \wedge -6 \wedge 0 \wedge 12, 2, 0 \vee -6 \vee 0 \vee 12) = (-6, 2, 12)$

$A(/)B \approx (-2/3 \wedge 4/3, 1/2, -2/3 \vee 4/3)$

Triangular Fuzzy Number

- Given the triangular fuzzy numbers A = (-2, 1, 4) and B = (0, 2, 3)



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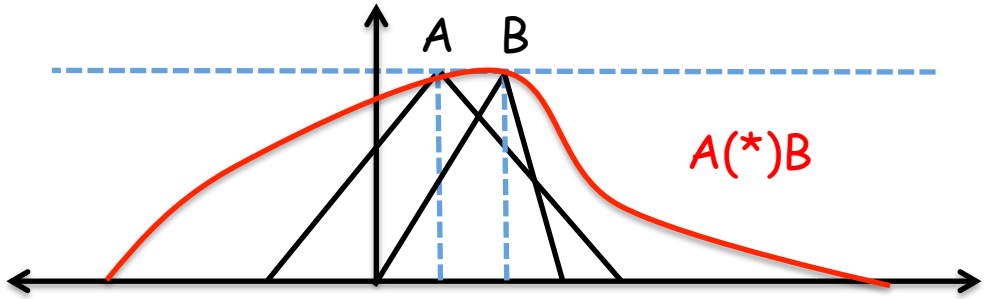
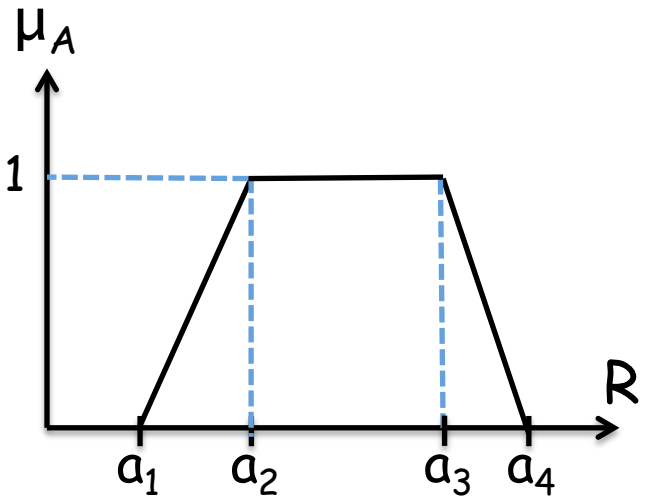


$A(*)B \approx (0 \wedge -6 \wedge 0 \wedge 12, 2, 0 \vee -6 \vee 0 \vee 12) = (-6, 2, 12)$

$A(/)B \approx (-2/3 \wedge 4/3, 1/2, -2/3 \vee 4/3) = (-2/3, 1/2, 4/3)$

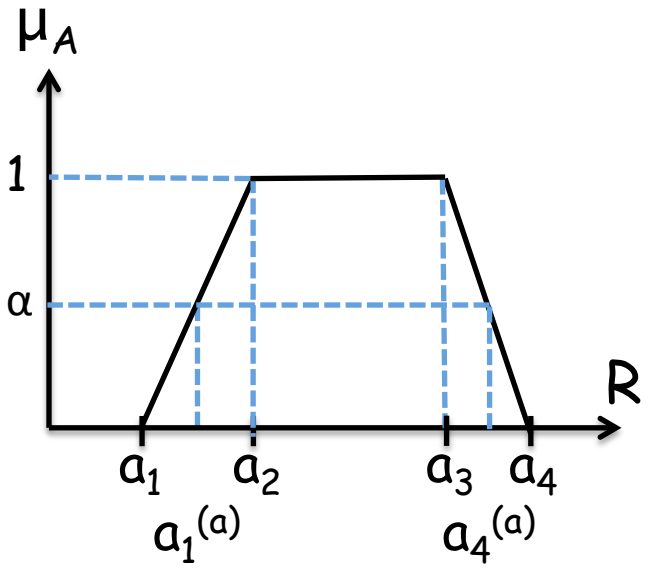
Trapezoidal Fuzzy Number



- $A = (a_1, a_2, a_3, a_4)$

$$\mu_A(x) = \begin{cases} 0 & , \text{ if } x < a_1 \\ (x - a_1)/(a_2 - a_1) & , \text{ if } a_1 \le x \le a_2 \\ 1 & , \text{ if } a_2 \le x \le a_3 \\ (a_4 - x)/(a_4 - a_3) & , \text{ if } a_3 \le x \le a_4 \\ 0 & , \text{ if } x > a_4 \end{cases}$$

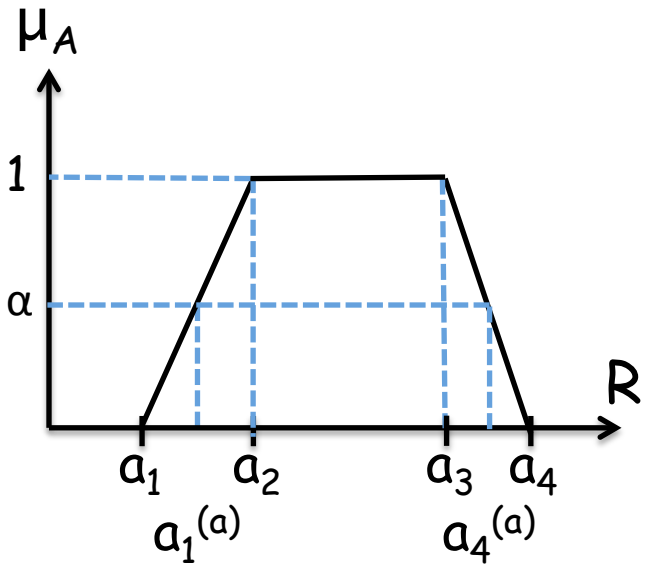
Trapezoidal Fuzzy Number



- $A = (a_1, a_2, a_3, a_4)$

$$\mu_A(x) = \begin{cases} 0 & , \text{ if } x < a_1 \\ (x - a_1)/(a_2 - a_1) & , \text{ if } a_1 \le x \le a_2 \\ 1 & , \text{ if } a_2 \le x \le a_3 \\ (a_4 - x)/(a_4 - a_3) & , \text{ if } a_3 \le x \le a_4 \\ 0 & , \text{ if } x > a_4 \end{cases}$$

Trapezoidal Fuzzy Number



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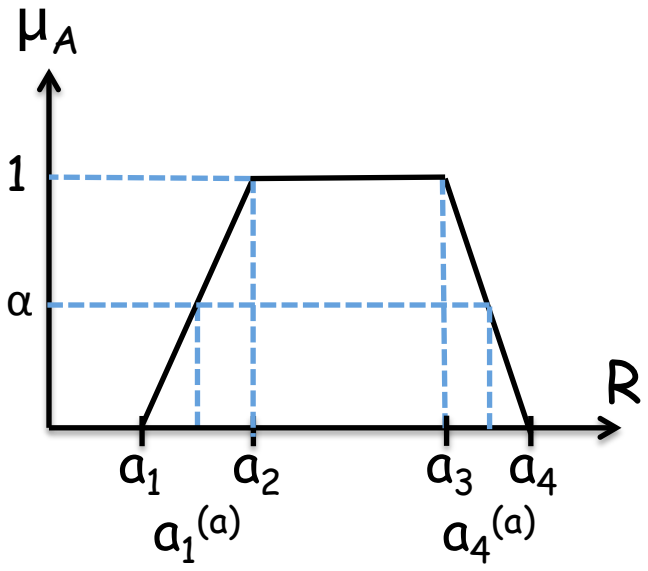
$(a_1^{(\alpha)} - a_1)/(a_2 - a_1) = \alpha$

$(a_4 - a_4^{(\alpha)})/(a_4 - a_3) = \alpha$

Trapezoidal Fuzzy Number



- $A = (a_1, a_2, a_3, a_4)$

$$\mu_A(x) = \begin{cases} 0, & \text{if } x < a_1 \\ (x - a_1)/(a_2 - a_1), & \text{if } a_1 \le x \le a_2 \\ 1, & \text{if } a_2 \le x \le a_3 \\ (a_4 - x)/(a_4 - a_3), & \text{if } a_3 \le x \le a_4 \\ 0, & \text{if } x > a_4 \end{cases}$$

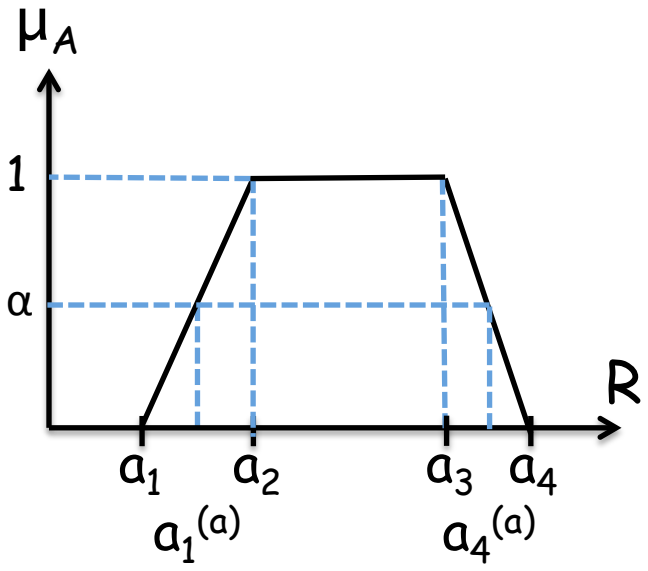
$(a_1^{(\alpha)} - a_1)/(a_2 - a_1) = \alpha$

$a_1^{(\alpha)} = (a_2 - a_1)\alpha + a_1$

$(a_4 - a_4^{(\alpha)})/(a_4 - a_3) = \alpha$

$a_4^{(\alpha)} = (a_3 - a_4)\alpha + a_4$

Trapezoidal Fuzzy Number



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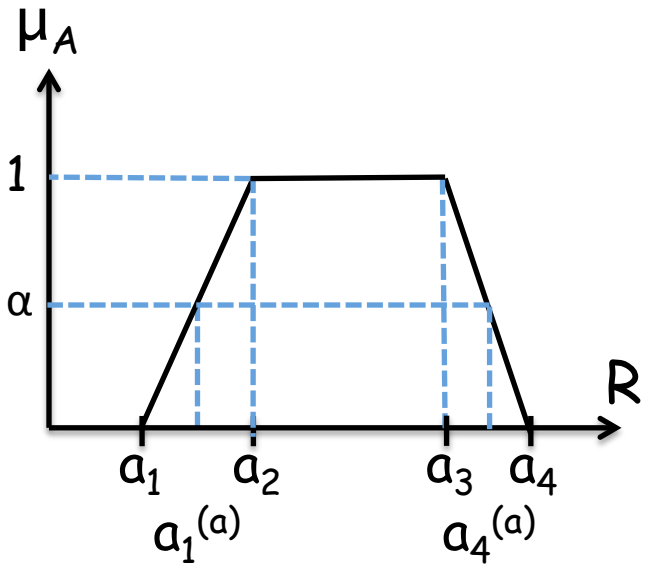
$$(a_1^{(\alpha)} - a_1)/(a_2 - a_1) = \alpha \qquad (a_4 - a_4^{(\alpha)})/(a_4 - a_3) = \alpha$$

$$a_1^{(\alpha)} = (a_2 - a_1)\alpha + a_1 \qquad a_4^{(\alpha)} = (a_3 - a_4)\alpha + a_4$$

$$A_\alpha = [a_1^{(\alpha)}, a_4^{(\alpha)}] = [(a_2 - a_1)\alpha + a_1, (a_3 - a_4)\alpha + a_4]$$

Trapezoidal Fuzzy Number



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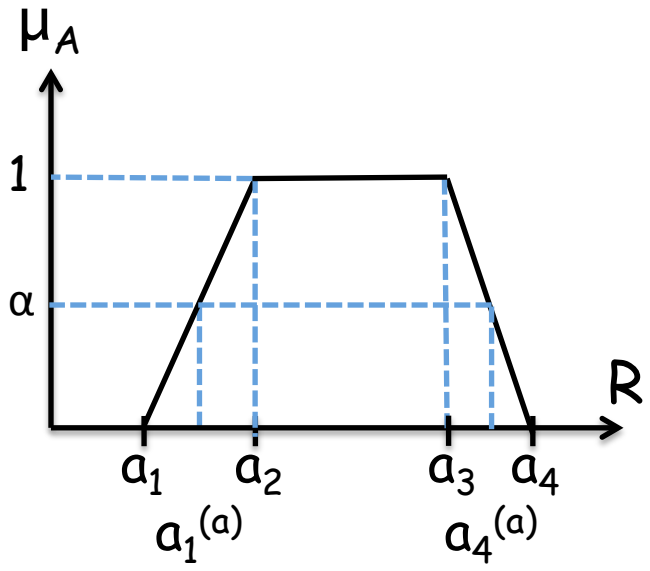
$A_0 = [a_1, a_4]$

$A_1 = [a_2, a_3]$

Trapezoidal Fuzzy Number



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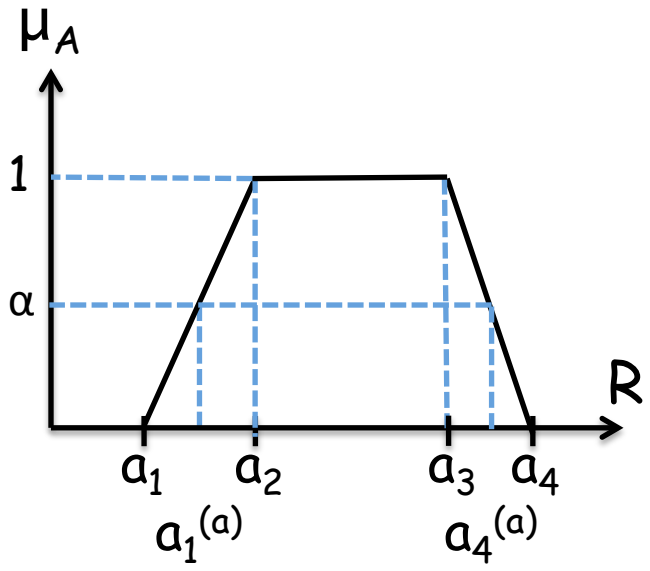
- Given the trapezoidal fuzzy numbers $A = (1, 2, 4, 7)$ and $B = (2, 4, 5, 6)$

Trapezoidal Fuzzy Number



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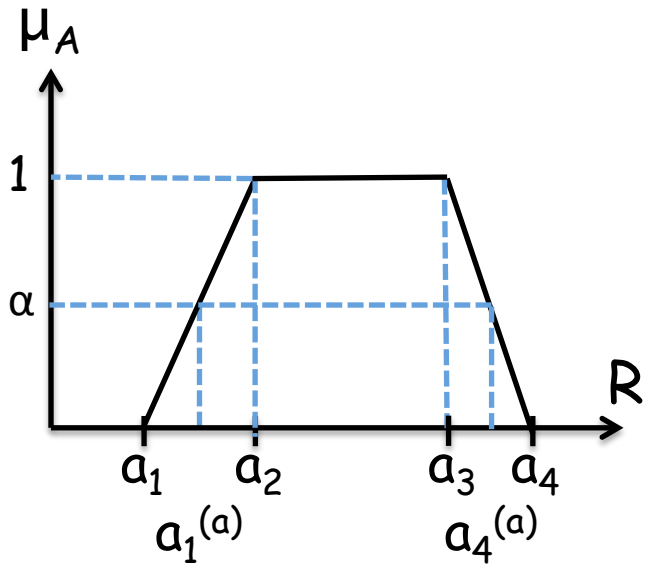
  A(+)B = (1+2, 2+4, 4+5, 7+6) = (3, 6, 9, 13)

Trapezoidal Fuzzy Number



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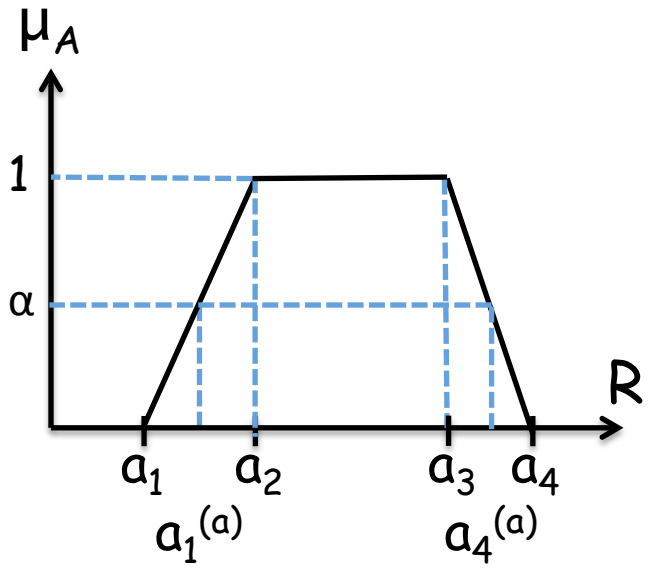
A(-)B = (1-5, 2-5, 4-4, 7-2) = (-4, -3, 0, 5)

Trapezoidal Fuzzy Number



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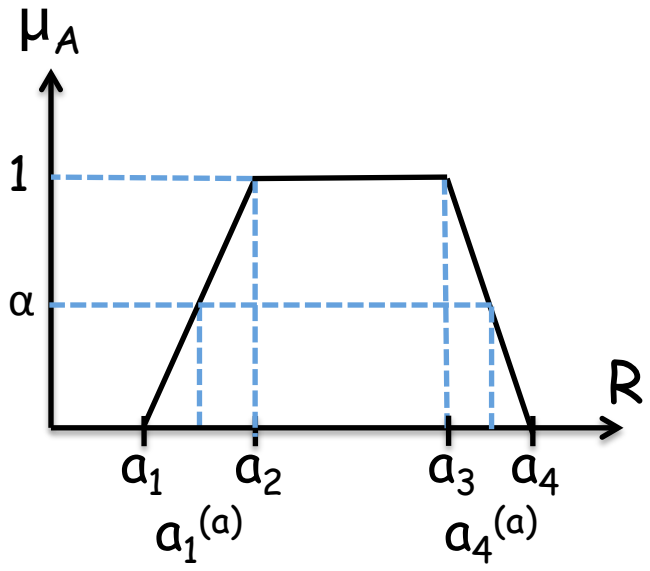
A(-)B = (1-5, 2-5, 4-4, 7-2) = (-4, -3, 0, 5)

A(*)B ≈ (min{2,6,14,42}, min{8,10,16,20}, max{8,10,16,20}, max{2,6,14,42})

Trapezoidal Fuzzy Number



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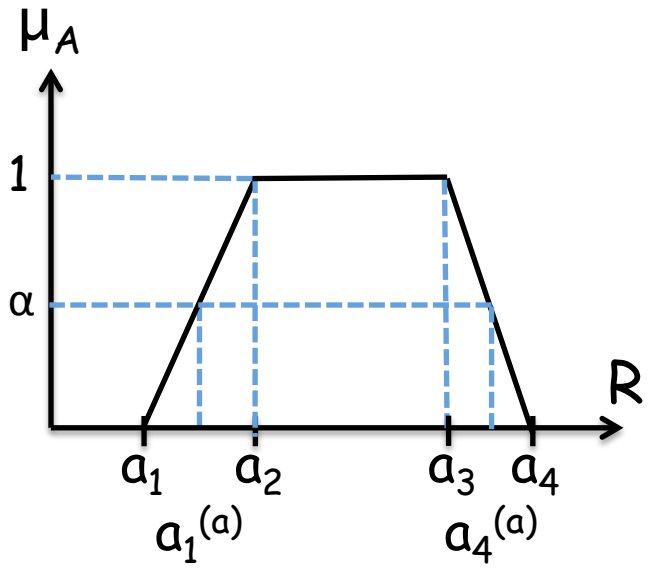
A(*)B ≈ (2, 8, 20, 42)

Trapezoidal Fuzzy Number



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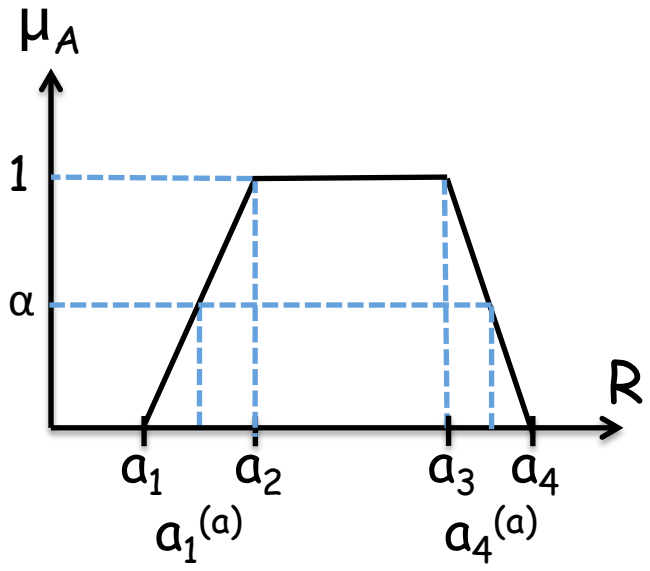
A(/)B ≈ (min{1/2,1/6,7/2,7/6}, min{1/2,2/5,1,4/5}, max{1/2,2/5,1,4/5}, max{1/2,1/6,7/2,7/6})

Trapezoidal Fuzzy Number



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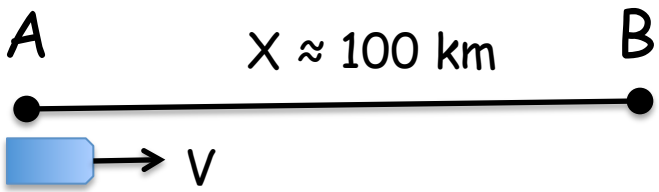
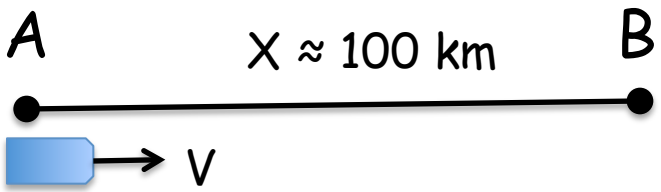
A(/)B ≈ (1/6, 2/5, 1, 7/2)

Fuzzy Numbers

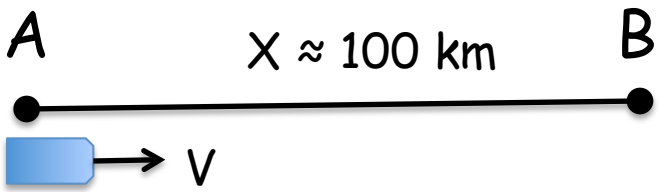


Fuzzy Numbers



- the speed of the bus cannot exceed 120 km/h
- there are some toll booths, and the speed decreases at the booths
- the bus usually leaves the city A late, but the lateness never exceeds more than 30 min

Fuzzy Numbers



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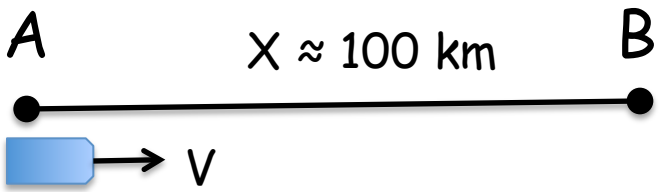
Fuzzy Numbers



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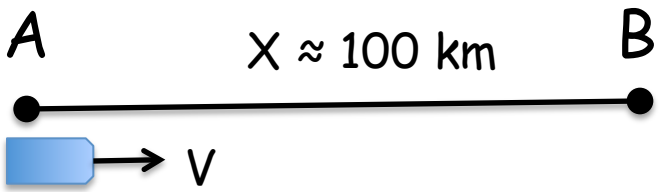
- if the speed is 90 km/h in average, and the bus left A 15 min late,

Fuzzy Numbers



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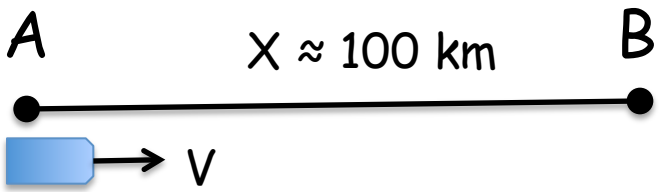
$$T = 100h/90 + 1/4 = 1.36\ h$$

Fuzzy Numbers



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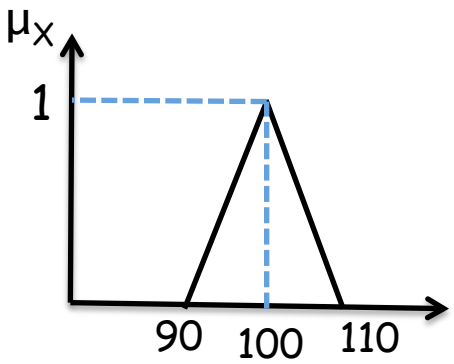

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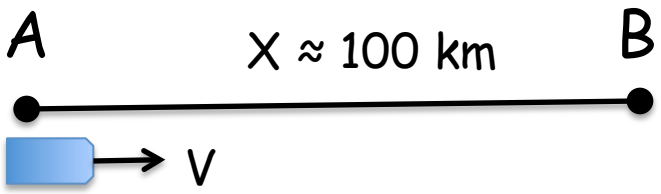

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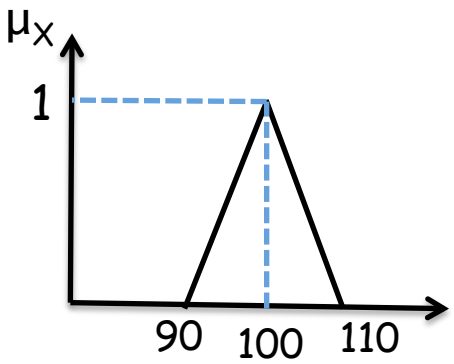
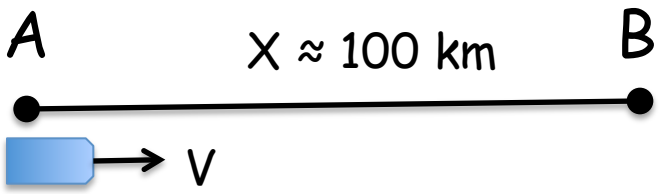


X = (90, 100, 110)

Fuzzy Numbers



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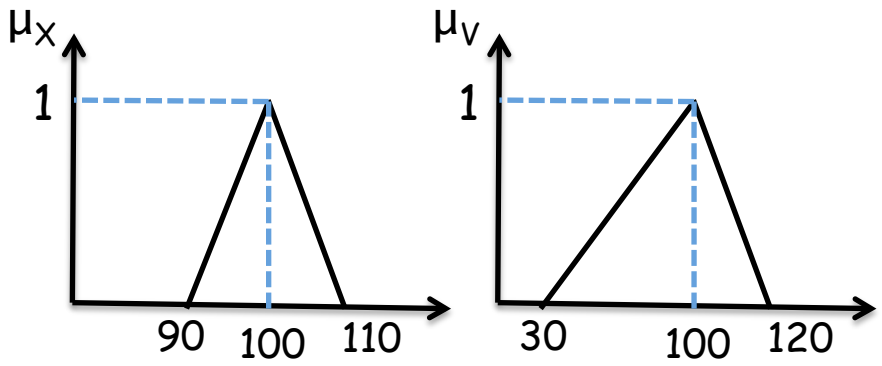

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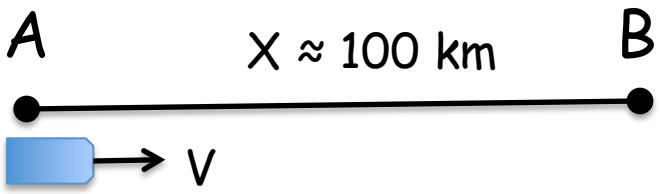

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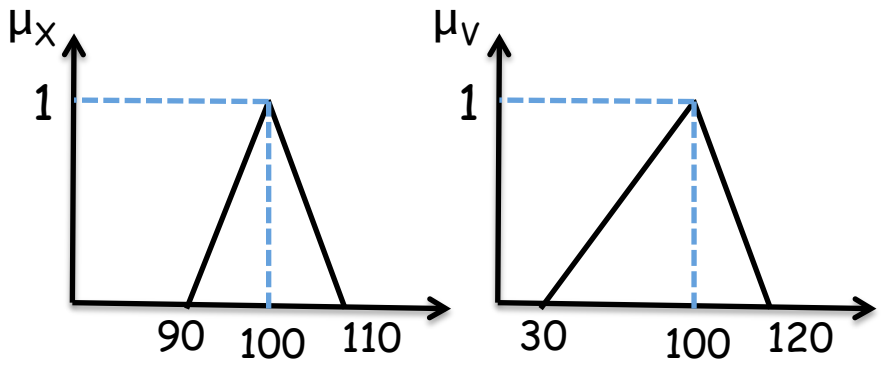
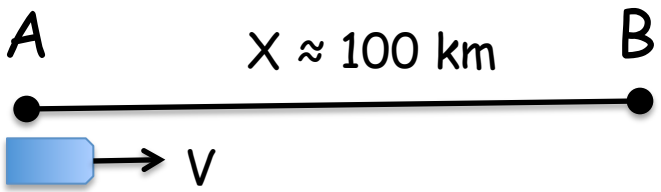


X = (90, 100, 110)

V = (30, 100, 120)

Fuzzy Numbers



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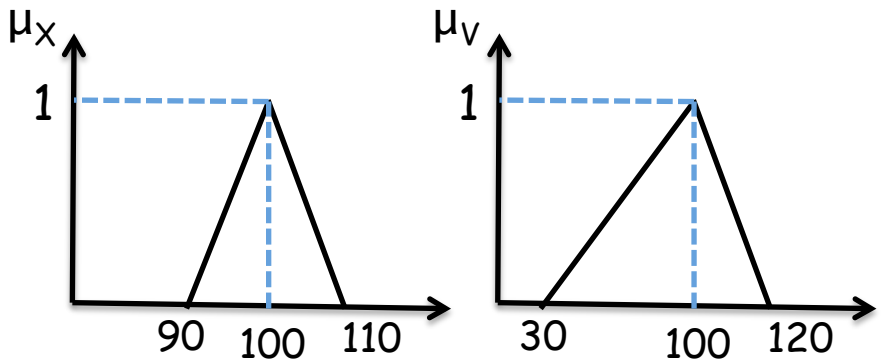


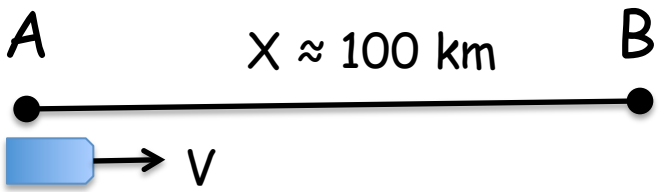
X = (90, 100, 110)

$X_\alpha = [10\alpha + 90, 110 - 10\alpha]$

V = (30, 100, 120)

$V_\alpha = [70\alpha + 30, 120 - 20\alpha]$

Fuzzy Numbers



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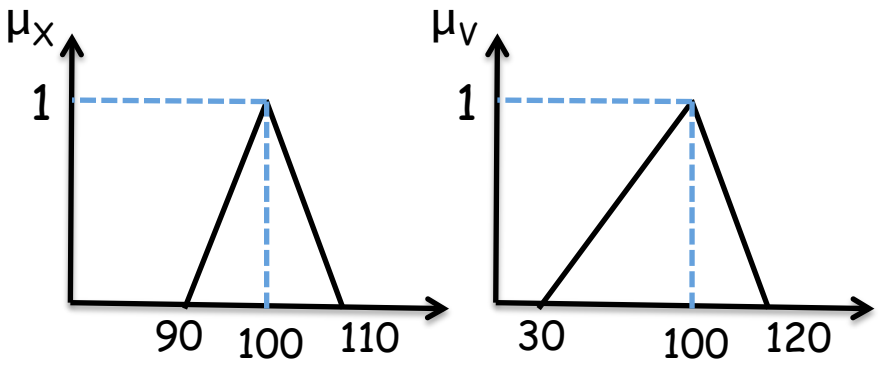

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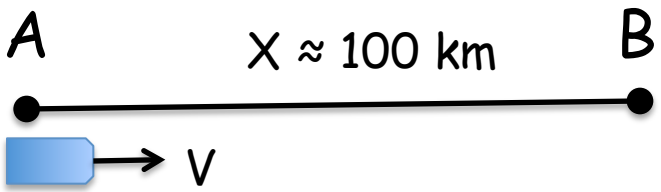
Fuzzy Numbers



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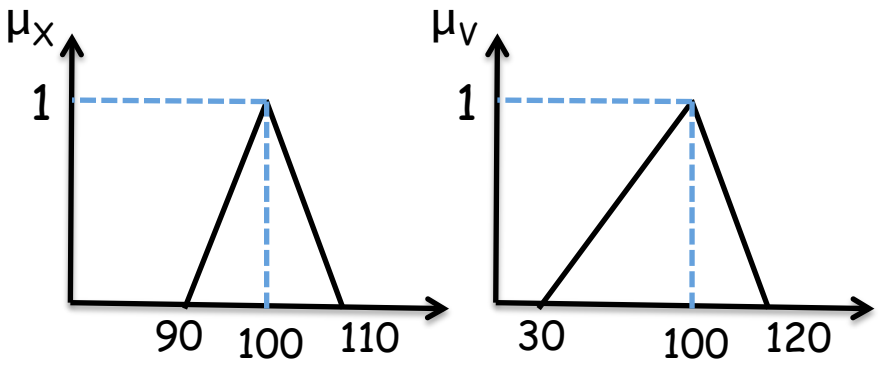


X = (90, 100, 110)

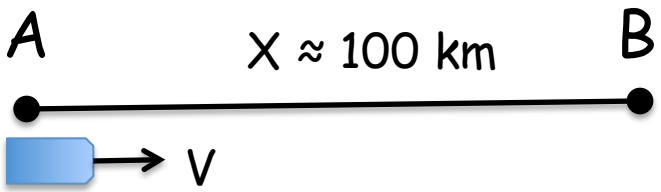
X_α = [10α + 90, 110 - 10α]

V = (30, 100, 120)

V_α = [70α + 30, 120 - 20α]

- T ' = (0, 0, 0.5); defined as the delay time
- T_α ' = [0, (1 – α)/2]

Fuzzy Numbers



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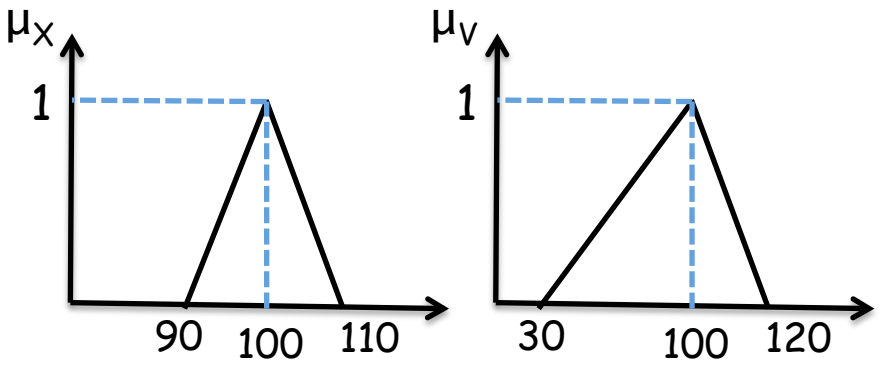
- there are some toll booths, and the speed decreases at the booths
- the bus usually leaves the city A late, but the lateness never exceeds more than 30 min
- What is the total time spent on the trip from A to B ?



X = (90, 100, 110)

X_α = [10α + 90, 110 - 10α]

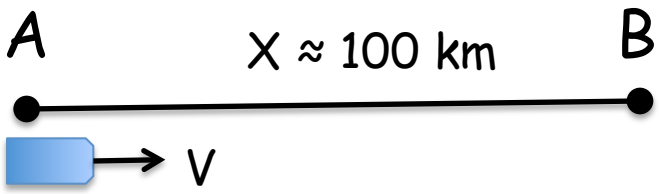
V = (30, 100, 120)

V_α = [70α + 30, 120 - 20α]

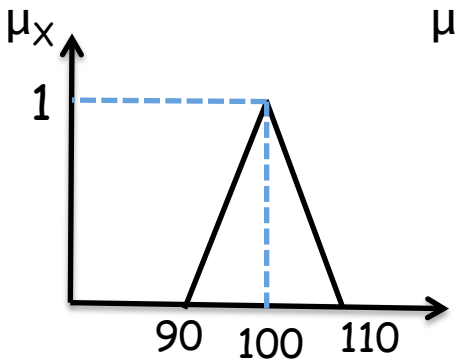
- T ' = (0, 0, 0.5); defined as the delay time
- T_α ' = [0, (1 – α)/2]
- T_α '' = [(10α+90)/(120-20α), (110-10α)/(70α+30)]

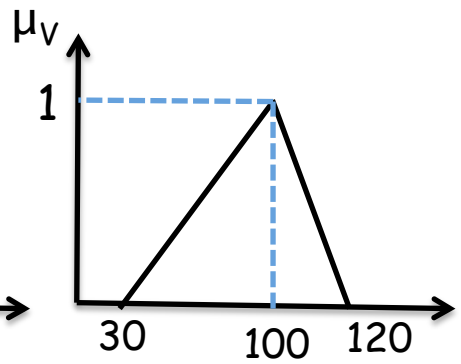
where T '' = X / V

Fuzzy Numbers



- the speed of the bus cannot exceed 120 km/h
- there are some toll booths, and the speed decreases at the booths
- the bus usually leaves the city A late, but the lateness never exceeds more than 30 min
- What is the total time spent on the trip from A to B ?





- $T' = (0, 0, 0.5)$; defined as the delay time
- $T_\alpha' = [0, (1 - \alpha)/2]$
- $T_\alpha'' = [(10\alpha+90)/(120-20\alpha), (110-10\alpha)/(70\alpha+30)]$

where $T'' = X / V$

$X = (90, 100, 110)$ $V = (30, 100, 120)$ $T = T'' + T'$

$X_\alpha = [10\alpha + 90, 110 - 10\alpha]$ $V_\alpha = [70\alpha + 30, 120 - 20\alpha]$