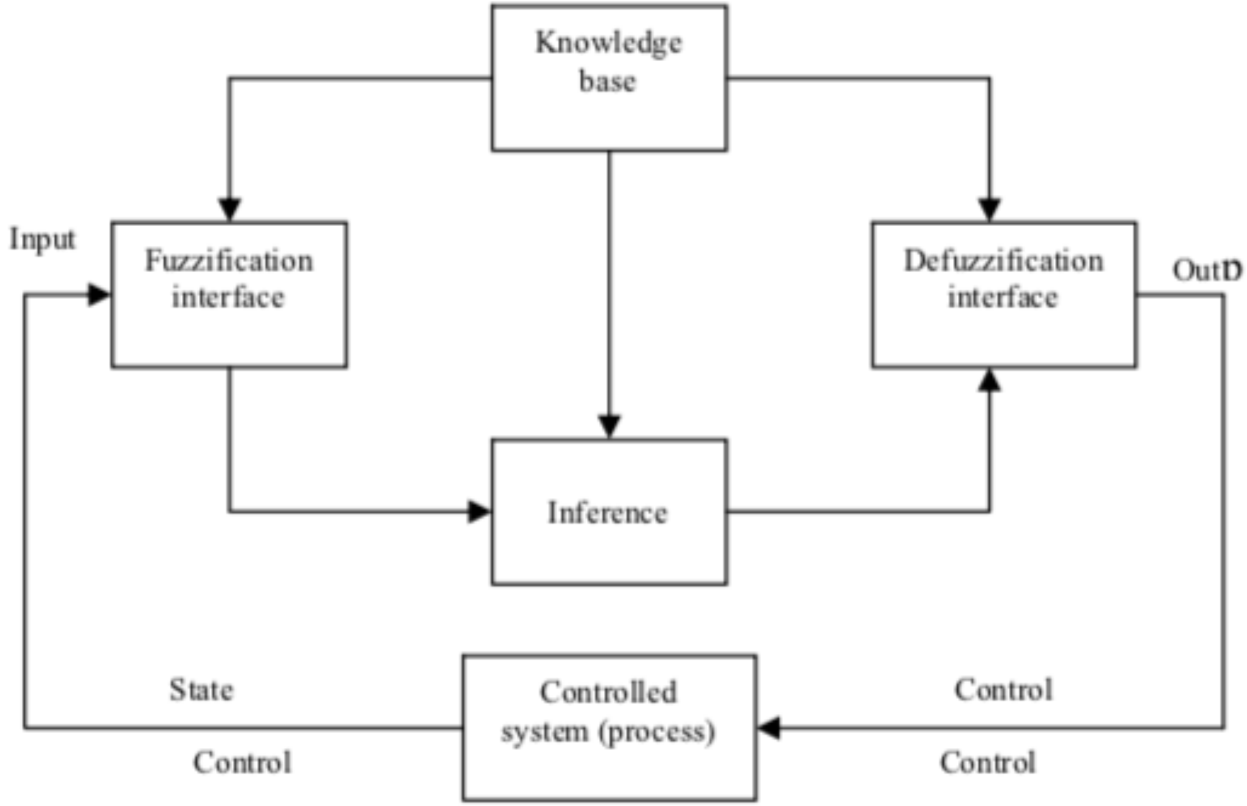
Fuzzy Inference

Murat Osmanoglu

Inference

- most popular form of the representation : if-then rule



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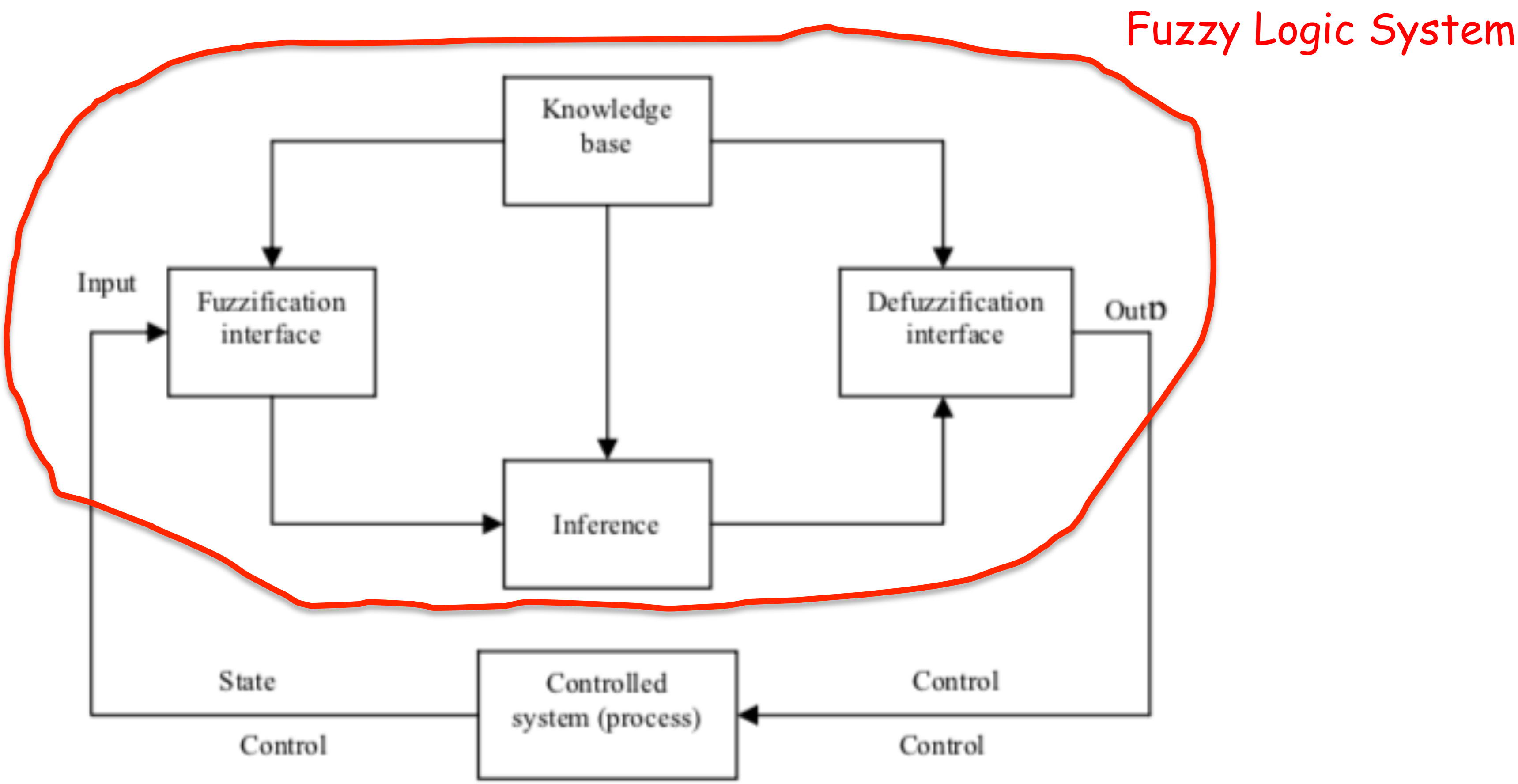

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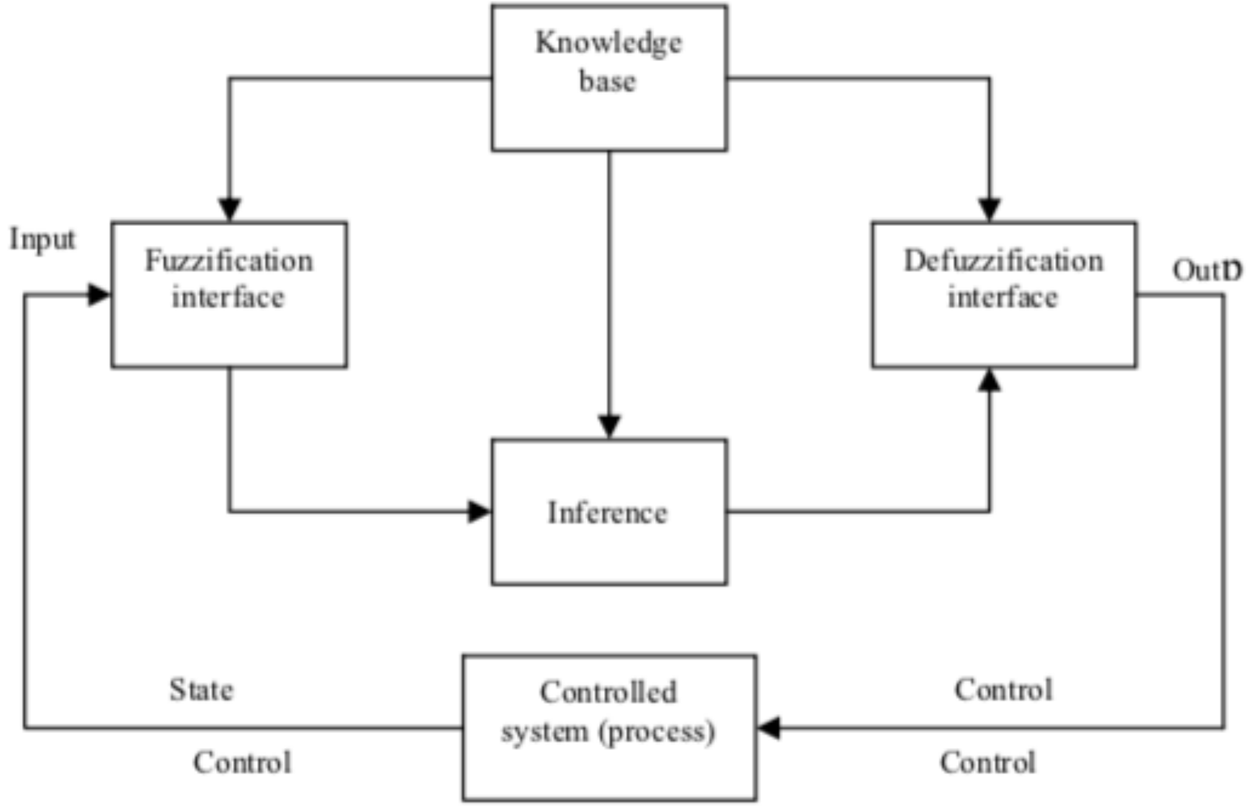

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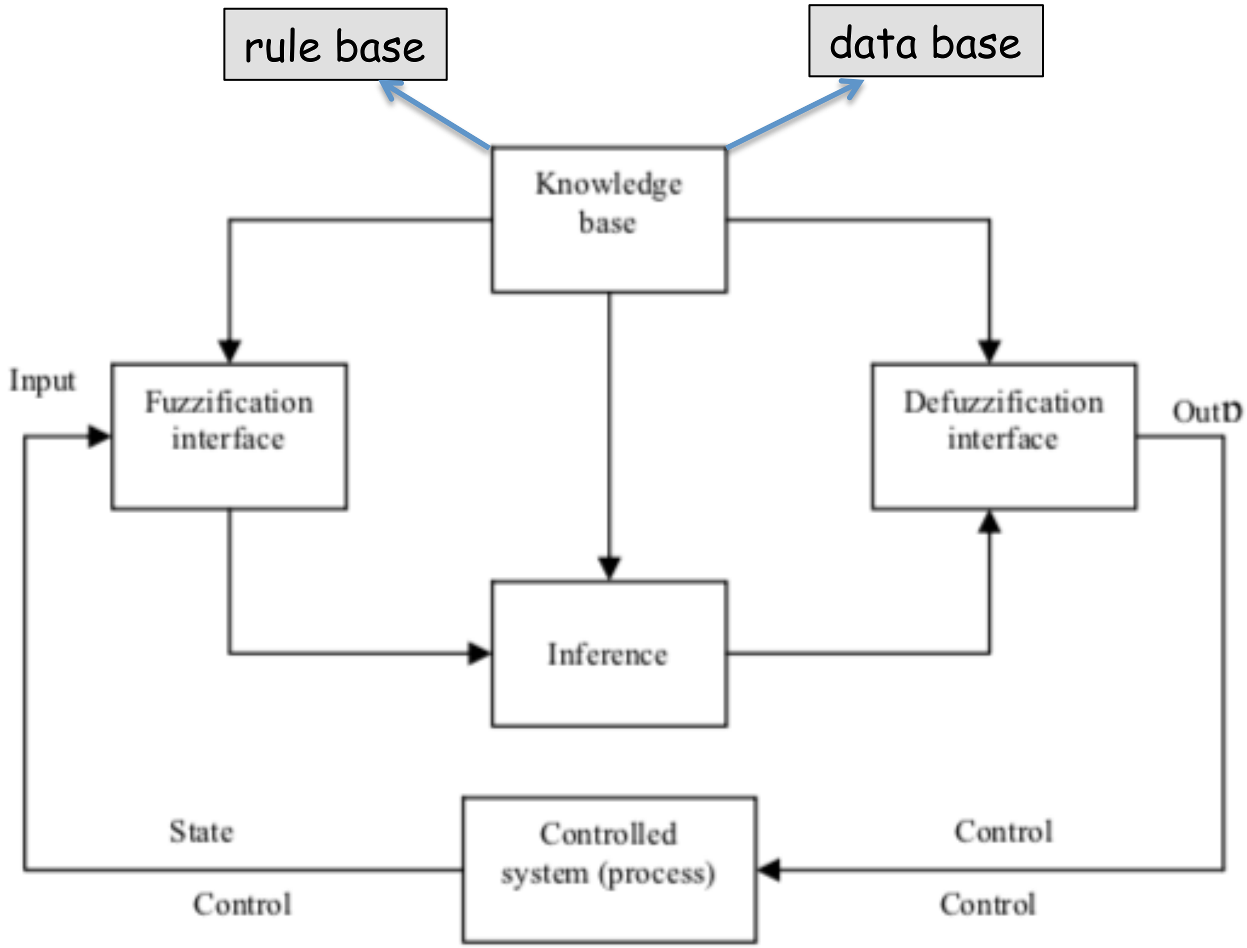

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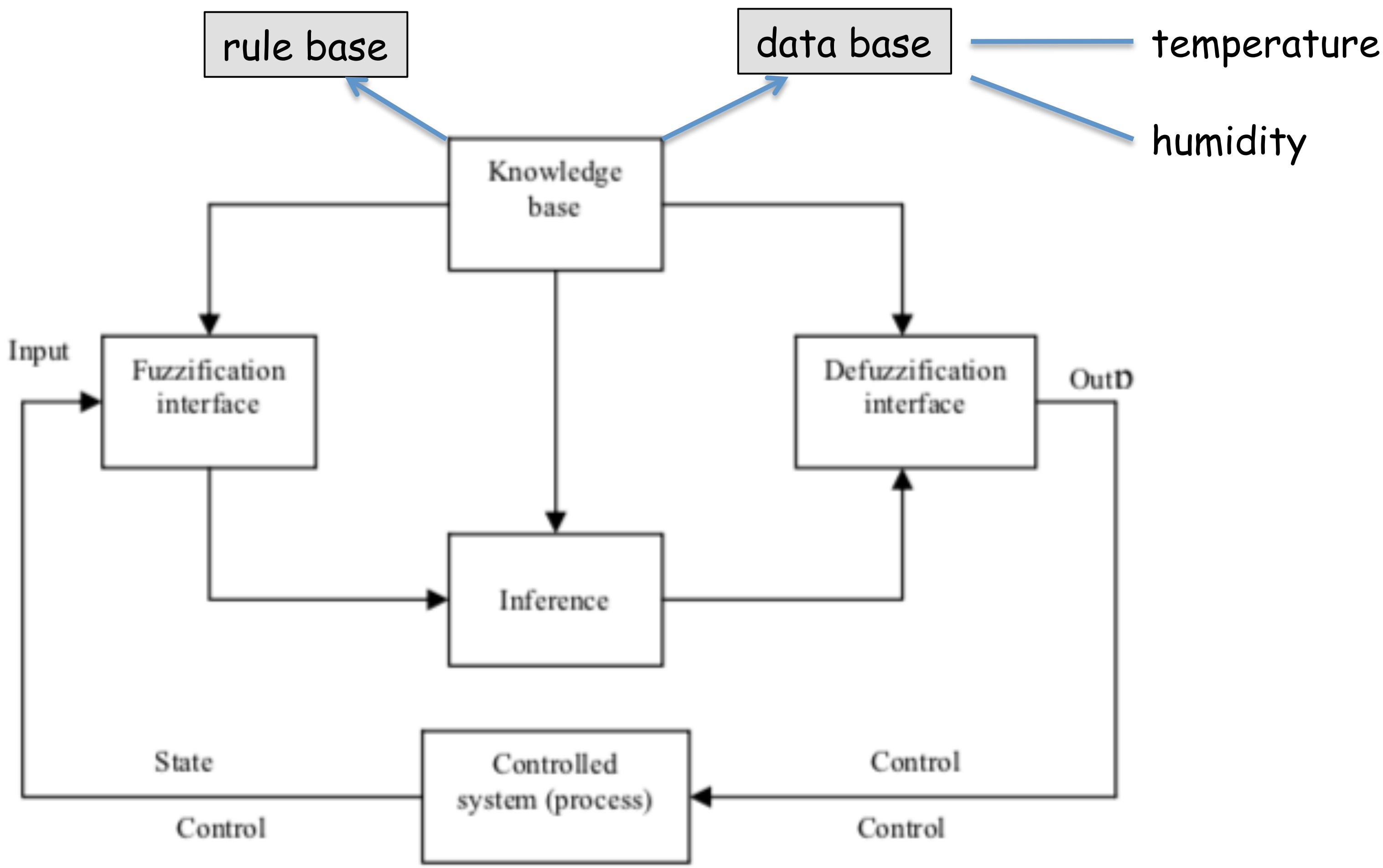

Inference

- most popular form of the representation : if-then rule
- the rule is : If x is a, then y is b

  the fact is : x is a

  the result is : y is b

Inference

- most popular form of the representation : if-then rule
- the rule is : If x is a, then y is b

  the fact is : x is a

  the result is : y is b

- modus ponens

fact : x is a

rule : if x is a, then y is b

result : y is b

- modus tollens

fact : y is not b

rule : if x is a, then y is b

result : a is not b

Inference

- most popular form of the representation : if-then rule
- the rule is : If x is a, then y is b

  the fact is : x is a

  the result is : y is b

- modus ponens

fact : x is a

rule : if x is a, then y is b

result : y is b

p	q	$p \rightarrow q$	$[p \wedge (p \rightarrow q)] \rightarrow q$
1	1	1	1
1	0	0	1
0	1	1	1
0	0	1	1

Inference

- most popular form of the representation : if-then rule
- the rule is : If x is A, then y is B

  the fact is : x is A'

  the result is : y is B'

Inference

- most popular form of the representation : if-then rule
- the rule is : If x is A, then y is B

  If $A(x)$, then $B(y)$

  the fact is : x is A'

  the result is : y is B'

Inference

- most popular form of the representation : if-then rule
- the rule is : If x is A, then y is B

  If $A(x)$, then $B(y)$: $R(x, y)$ ($A(x) \rightarrow B(y)$)

  the fact is : x is A'

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Inference

- most popular form of the representation : if-then rule
- the rule is : If x is A, then y is B
  If $A(x)$, then $B(y)$: $R(x, y)$ $(A(x) \rightarrow B(y))$

  the fact is : x is A'

  the result is : y is B'

'if temperature is high, then humidity is fairly high'

High(x) ➔ Fairly_High(y)

Inference

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- the rule is : If x is A, then y is B
  If $A(x)$, then $B(y)$: $R(x, y)$ ($A(x) \rightarrow B(y)$)

  the fact is : x is A'

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- modus ponens

fact : x is A'

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- most popular form of the representation : if-then rule
- the rule is : If x is A, then y is B
  If $A(x)$, then $B(y)$: $R(x, y)$ ($A(x) \rightarrow B(y)$)

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  the result is : y is B'

- modus ponens

fact	: x is A'	: $R(x)$
rule	: if x is A, then y is B	: $R(x, y)$
result	: y is B'	: $R(y) = R(x) \circ R(x, y)$

Fuzzy Composition

- consider the fuzzy rule and the premise given as :

  'x and y are approximately equal'   and   'x is small'

Fuzzy Composition

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  R(x, y) = ApproximatelyEqual(x, y)

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  $R(x, y) = ApproximatelyEqual(x, y)$

  $R(x) = Small(x)$

Fuzzy Composition

- consider the fuzzy rule and the premise given as :

  'x and y are approximately equal' and 'x is small'

$R(x, y) = \text{ApproximatelyEqual}(x, y)$

$R(x) = \text{Small}(x)$

R(x,y)	1	2	3	4
1	1.0	0.5	0	0
2	0.5	1.0	0.5	0
3	0	0.5	1.0	0.5
4	0	0	0.5	1.0

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R(x)	1	2	3	4
$\mu_R(x)$	1.0	0.7	0.4	0.1

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$$R(y) = R(x) \circ R(x, y)$$

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$$R(y) = R(x) \circ R(x, y)$$

$$\mu_R(y) = \max_x (\min (\mu_R(x), \mu_R(x, y)))$$

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Fuzzy Composition

- consider the fuzzy rule and the premise given as :

  'x and y are approximately equal'   and   'x is 2'

$R(x, y) = \text{ApproximatelyEqual}(x, y)$

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R(x,y)	1	2	3	4
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R(x)	1	2	3	4
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$\mu_R(x)$	0	1.0	0	0

R(y)	1	2	3	4
$\mu_R(y)$	0.5	1.0	0.5	0

$$R(y) = R(x) \circ R(x, y)$$

$$\mu_R(y) = \max_x (\min (\mu_R(x), \mu_R(x, y))$$

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- The rule is : $R(x, y)\ (A(x) \rightarrow B(y))$

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$$\mu_R(x, y) = f(\mu_A(x), \mu_B(y))$$

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$$\mu_R(x, y) = f(\mu_A(x), \mu_B(y))$$

Mamdani

$$f(\mu_A(x), \mu_B(y)) = \mu_A(x) \underline{\triangle} \mu_B(y)$$

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Mamdani

$$f(\mu_A(x), \mu_B(y)) = \mu_A(x) \underline{\triangle} \mu_B(y)$$

Larsen

$$f(\mu_A(x), \mu_B(y)) = \mu_A(x) \cdot \mu_B(y)$$

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  'if temperature is high, then humidity is fairly high'

$R(t, h) = A(t) \rightarrow B(h)$ where A in T and B in H

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A = 'high'

A	10	20	30	40
$\mu_A(t)$	0.1	0.2	0.6	0.9

B = 'fairly high'

B	40	60	80	90
$\mu_B(h)$	0.3	0.5	0.8	1.0

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Mamdani

R(t,h)	40	60	80	90
10				
20				
30				
40				

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Mamdani

R(t,h)	40	60	80	90
10	0.1	0.1	0.1	0.1
20				
30				
40				

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20	0.2	0.2	0.2	0.2
30	0.3	0.5	0.6	0.6
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20	0.2	0.2	0.2	0.2
30	0.3	0.5	0.6	0.6
40	0.3	0.5	0.8	0.9

Larsen

R(t,h)	40	60	80	90
10				
20				
30				
40				

Fuzzy Inference

- consider the fuzzy rule given as :

  'if temperature is high, then humidity is fairly high'

$R(t, h) = A(t) \rightarrow B(h)$ where A in T and B in H

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R(t,h)	40	60	80	90
10	0.1	0.1	0.1	0.1
20	0.2	0.2	0.2	0.2
30	0.3	0.5	0.6	0.6
40	0.3	0.5	0.8	0.9

Larsen

R(t,h)	40	60	80	90
10	0.03			
20				
30				
40				

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R(t,h)	40	60	80	90
10	0.1	0.1	0.1	0.1
20	0.2	0.2	0.2	0.2
30	0.3	0.5	0.6	0.6
40	0.3	0.5	0.8	0.9

Larsen

R(t,h)	40	60	80	90
10	0.03	0.05		
20				
30				
40				

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Mamdani

R(t,h)	40	60	80	90
10	0.1	0.1	0.1	0.1
20	0.2	0.2	0.2	0.2
30	0.3	0.5	0.6	0.6
40	0.3	0.5	0.8	0.9

Larsen

R(t,h)	40	60	80	90
10	0.03	0.05	0.08	
20				
30				
40				

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R(t,h)	40	60	80	90
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20	0.2	0.2	0.2	0.2
30	0.3	0.5	0.6	0.6
40	0.3	0.5	0.8	0.9

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R(t,h)	40	60	80	90
10	0.03	0.05	0.08	0.1
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20	0.2	0.2	0.2	0.2
30	0.3	0.5	0.6	0.6
40	0.3	0.5	0.8	0.9

Larsen

R(t,h)	40	60	80	90
10	0.03	0.05	0.08	0.1
20	0.06	0.1	0.16	0.2
30	0.18	0.3	0.48	0.6
40	0.27	0.45	0.72	0.9

Fuzzy Inference

- consider the fuzzy rule given as :

  R(t,h) = 'if temperature is high, then humidity is fairly high'

  A' = ' temperature is fairly high'

Fuzzy Inference

- consider the fuzzy rule given as :

  R(t,h) = 'if temperature is high, then humidity is fairly high'

  A' = ' temperature is fairly high'

A' = 'fairly high'

A'	10	20	30	40
$\mu_A(t)$	0.02	0.15	0.5	0.8

R(t,h)	40	60	80	90
10	0.1	0.1	0.1	0.1
20	0.2	0.2	0.2	0.2
30	0.3	0.5	0.6	0.6
40	0.3	0.5	0.8	0.9

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- consider the fuzzy rule given as :

  R(t,h) = 'if temperature is high, then humidity is fairly high'

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A'	10	20	30	40
$\mu_A(t)$	0.02	0.15	0.5	0.8

R(t,h)	40	60	80	90
10	0.1	0.1	0.1	0.1
20	0.2	0.2	0.2	0.2
30	0.3	0.5	0.6	0.6
40	0.3	0.5	0.8	0.9

Mamdani

B'	40	60	80	90
$\mu_{B'}(h)$	0.3	0.5	0.8	0.8

Decomposition of Rules

Multiple Input Multiple Output

- R : if x_1 is A_1,x_2 is A_2, …,x_n is A_n, then z_1 is C_1,z_2 is C_2, …,z_m is C_m

Decomposition of Rules

Multiple Input Multiple Output

- R : if x_1 is A_1,x_2 is A_2, …,x_n is A_n, then z_1 is C_1,z_2 is C_2, …,z_m is C_m

R_1 : if x_1 is A_1, x_2 is A_2, …, x_n is A_n, then z_1 is C_1

R_2 : if x_1 is A_1, x_2 is A_2, …, x_n is A_n, then z_2 is C_2

. . .

R_m : if x_1 is A_1, x_2 is A_2, …, x_n is A_n, then z_m is C_m

Decomposition of Rules

Multiple Input Multiple Output

- R : if x_1 is A_1,x_2 is A_2, …,x_n is A_n, then z_1 is C_1,z_2 is C_2, …,z_m is C_m

R_1 : if x_1 is A_1, x_2 is A_2, …, x_n is A_n, then z_1 is C_1

R_2 : if x_1 is A_1, x_2 is A_2, …, x_n is A_n, then z_2 is C_2

. . .

R_m : if x_1 is A_1, x_2 is A_2, …, x_n is A_n, then z_m is C_m

- a multiple input multiple output fuzzy system can considered as a collection of multiple input single output fuzzy systems

$$R = \{R_1, R_2, \ldots, R_m\}$$

Decomposition of Rules

Two Input Single Output

- input : x is A' and y is B'

  R_1 : if x is A_1 and y is B_1, then z is C_1

  R_2 : if x is A_2 and y is B_2, then z is C_2

  . . .

  R_m : if x is A_m and y is B_2, then z is C_m

  output : z is C'

Decomposition of Rules

Two Input Single Output

- input : x is A' and y is B'

  R_1 : if x is A_1 and y is B_1, then z is C_1

  R_2 : if x is A_2 and y is B_2, then z is C_2

  . . .

  R_m : if x is A_m and y is B_2, then z is C_m

  output : z is C'

- R_i : if x is A_i and y is B_i, then z is C_i

  R_i : (A_i and B_i) $\rightarrow$ C_i

Decomposition of Rules

Two Input Single Output

- input : x is A' and y is B'

  R_1 : if x is A_1 and y is B_1, then z is C_1

  R_2 : if x is A_2 and y is B_2, then z is C_2

  . . .

  R_m : if x is A_m and y is B_2, then z is C_m

  output : z is C'

- R_i : if x is A_i and y is B_i, then z is C_i

  $R_i : (A_i \text{ and } B_i) \rightarrow C_i$

  $R_i : (A_i \rightarrow C_i) \text{ and } (B_i \rightarrow C_i)$

Decomposition of Rules

Two Input Single Output

- input : x is A' and y is B'

  R_1 : if x is A_1 and y is B_1, then z is C_1

  R_2 : if x is A_2 and y is B_2, then z is C_2

  . . .

  R_m : if x is A_m and y is B_2, then z is C_m

  output : z is C'

- R_i : if x is A_i and y is B_i, then z is C_i

  $R_i : (A_i \text{ and } B_i) \rightarrow C_i$

  $R_i : (A_i \rightarrow C_i) \text{ and } (B_i \rightarrow C_i)$

  $R_i = R_{i,1} \wedge R_{i,2}$

Inference

- most popular form of the representation : if-then rule
- the rule is : If x is A, then z is C

  If $A(x)$, then $C(z)$: $R(x, z)$ $(A(x) \rightarrow C(z))$

  the fact is : x is A'

  the result is : z is C'

Mamdani	Larsen
$R(y) = R(x) \circ R(x, z)$	$R(y) = R(x) \circ R(x, z)$
min for the implication	product for the implication

Mamdani Fuzzy Inference

Fuzzy Input

- the fact is : x is A'

  the rule is : If x is A, then z is C

  the result is : z is C'

Mamdani Fuzzy Inference

Fuzzy Input

- the fact is : x is A'

  the rule is : If x is A, then z is C

  the result is : z is C'

- $C' = A' \circ (A \rightarrow C) = A' \circ R$

Mamdani Fuzzy Inference

Fuzzy Input

- the fact is : x is A'

  the rule is : If x is A, then z is C

  the result is : z is C'

- $C' = A' \circ (A \rightarrow C) = A' \circ R$

$\mu_{C'}(z) = \mu_{A'}(x) \circ (\mu_A(x) \rightarrow \mu_C(z))$

Mamdani Fuzzy Inference

Fuzzy Input

- the fact is : x is A'

  the rule is : If x is A, then z is C

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$\mu_{C'}(z) = \max_x \{\mu_{A'}(x) \wedge \mu_R(x,z)\}$

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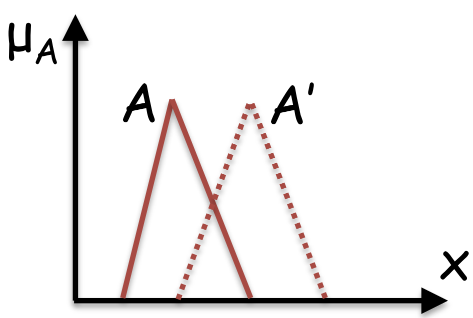

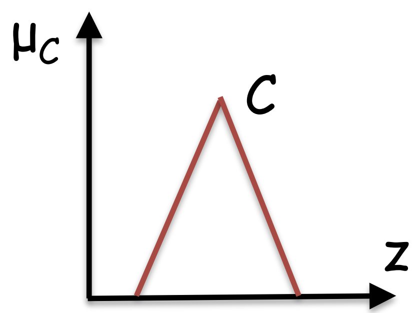

Mamdani Fuzzy Inference

Fuzzy Input

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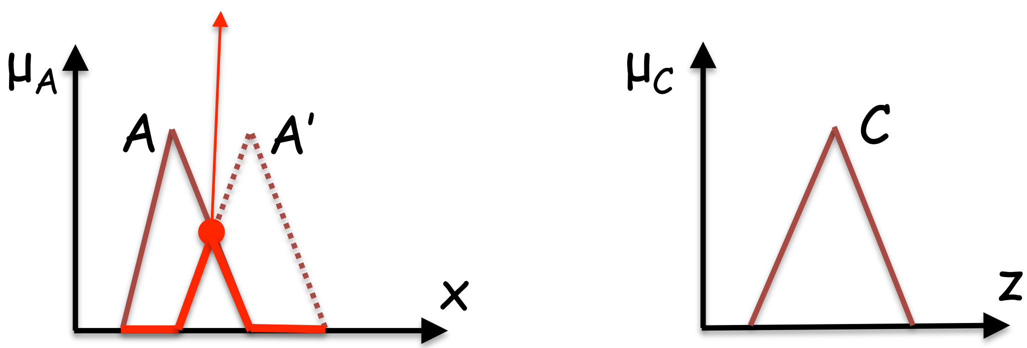

Mamdani Fuzzy Inference

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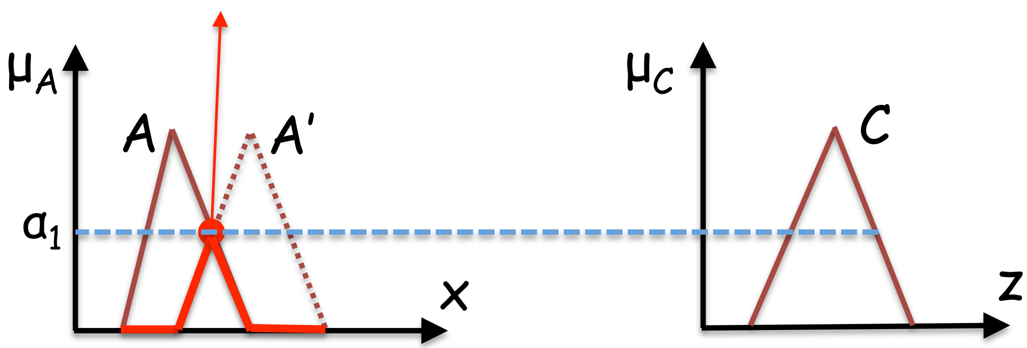

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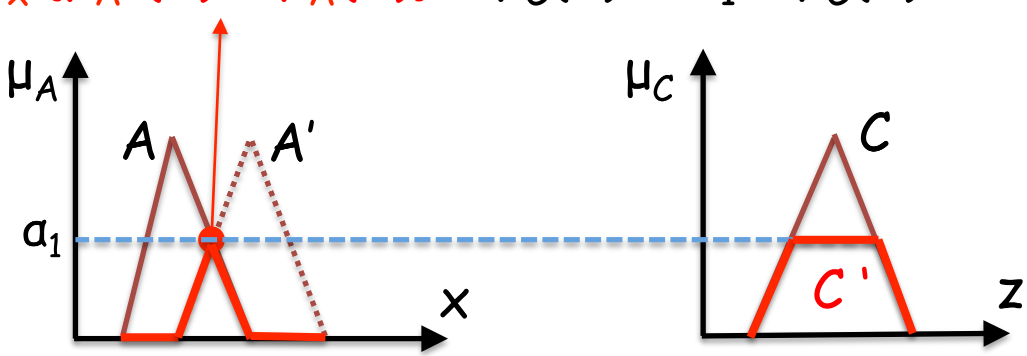

Mamdani Fuzzy Inference

Singleton Input

- the fact is : x is x_0

  the rule is : If x is A, then z is C

  the result is : z is C'

- $C' = x_0 \circ (A \rightarrow C) = A' \circ R$

Mamdani Fuzzy Inference

Singleton Input

- the fact is : x is x_0

  the rule is : If x is A, then z is C

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- $C' = x_0 \circ (A \rightarrow C) = A' \circ R$

$\mu_{C'}(z) = \mu_0 \circ (\mu_A(x) \rightarrow \mu_C(z))$

Mamdani Fuzzy Inference

Singleton Input

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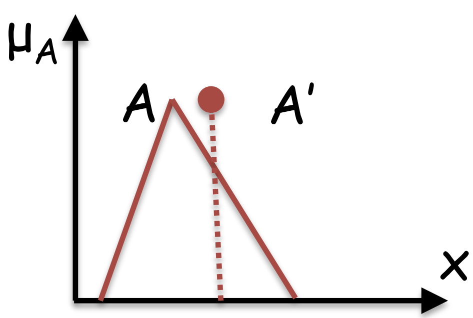

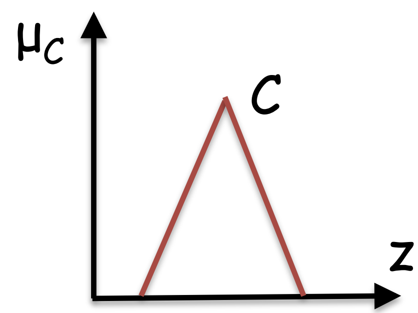

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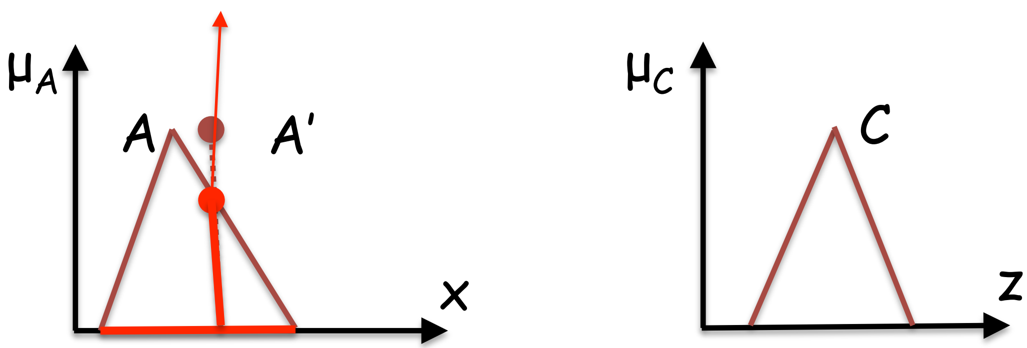

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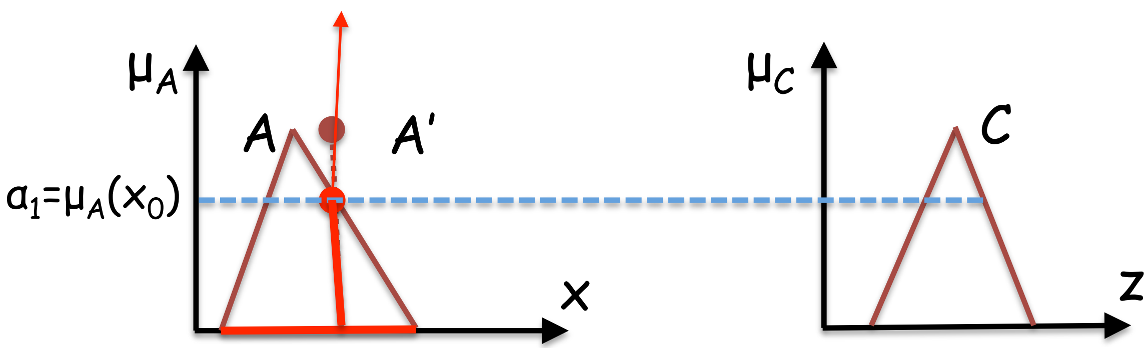

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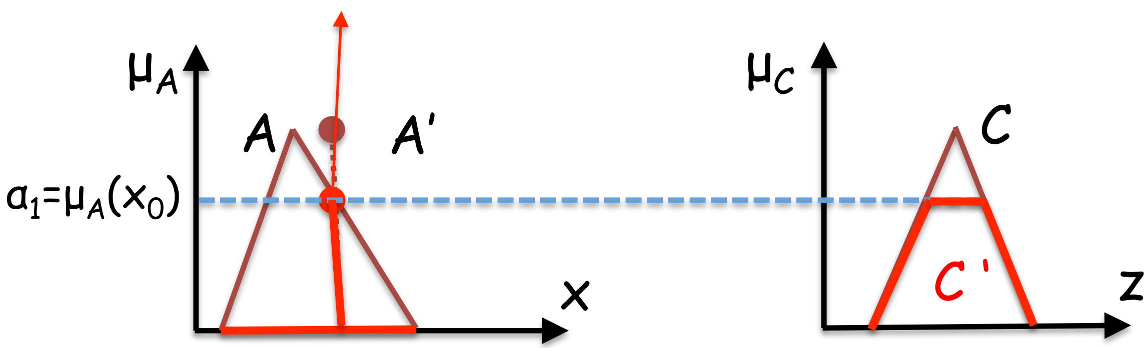

Mamdani Fuzzy Inference

Single Input Single Output

- input : x is A'

  R_1 : if x is A_1, then z is C_1 : $A_1 \rightarrow C_1$

  R_2 : if x is A_2, then z is C_2 : $A_2 \rightarrow C_2$

  output : z is C'

Mamdani Fuzzy Inference

Single Input Single Output

- input : x is A'

  R_1 : if x is A_1, then z is C_1 : $A_1 \rightarrow C_1$

  R_2 : if x is A_2, then z is C_2 : $A_2 \rightarrow C_2$

  output : z is C'

- $C' = A' \circ (R_1 \cup R_2) = A' \circ [(A_1 \rightarrow C_1) \cup (A_2 \rightarrow C_2)]$

'

Mamdani Fuzzy Inference

Single Input Single Output

- input : x is A'

  R_1 : if x is A_1, then z is C_1 : $A_1 \rightarrow C_1$

  R_2 : if x is A_2, then z is C_2 : $A_2 \rightarrow C_2$

  output : z is C'

- $C' = A' \circ (R_1 \cup R_2) = A' \circ [(A_1 \rightarrow C_1) \cup (A_2 \rightarrow C_2)]$

  $C' = [A' \circ (A_1 \rightarrow C_1)] \cup [A' \circ (A_2 \rightarrow C_2)] = C_1' \cup C_2'$

Mamdani Fuzzy Inference

Single Input Single Output

- input : x is A'

  R_1 : if x is A_1, then z is C_1 : $A_1 \rightarrow C_1$

  R_2 : if x is A_2, then z is C_2 : $A_2 \rightarrow C_2$

  output : z is C'

- $C' = A' \circ (R_1 \cup R_2) = A' \circ [(A_1 \rightarrow C_1) \cup (A_2 \rightarrow C_2)]$

  $C' = [A' \circ (A_1 \rightarrow C_1)] \cup [A' \circ (A_2 \rightarrow C_2)] = C_1' \cup C_2'$

  $\mu_{C'}(z) = \max \{ \mu_{C\_1'}(z),\ \mu_{C\_2'}(z) \}$

Mamdani Fuzzy Inference

Single Input Single Output

- input : x is A'

    R_1 : if x is A_1, then z is C_1 : $A_1 \rightarrow C_1$

    R_2 : if x is A_2, then z is C_2 : $A_2 \rightarrow C_2$

  output : z is C'

- $C' = A' o (R_1 \cup R_2) = A' o [(A_1 \rightarrow C_1) \cup (A_2 \rightarrow C_2)]$

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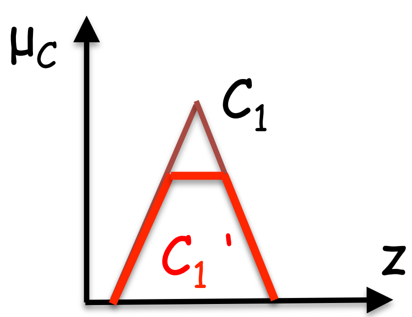

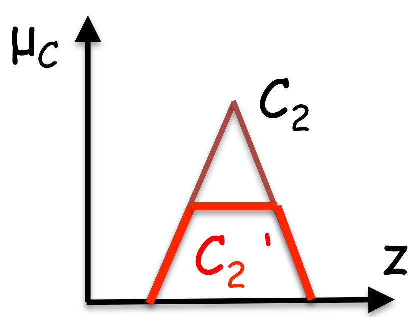

Mamdani Fuzzy Inference

Single Input Single Output

- input : x is A'

  R_1 : if x is A_1, then z is C_1 : $A_1 \rightarrow C_1$

  R_2 : if x is A_2, then z is C_2 : $A_2 \rightarrow C_2$

  output : z is C'

- $C' = A' \circ (R_1 \cup R_2) = A' \circ [(A_1 \rightarrow C_1) \cup (A_2 \rightarrow C_2)]$

  $C' = [A' \circ (A_1 \rightarrow C_1)] \cup [A' \circ (A_2 \rightarrow C_2)] = C_1' \cup C_2'$

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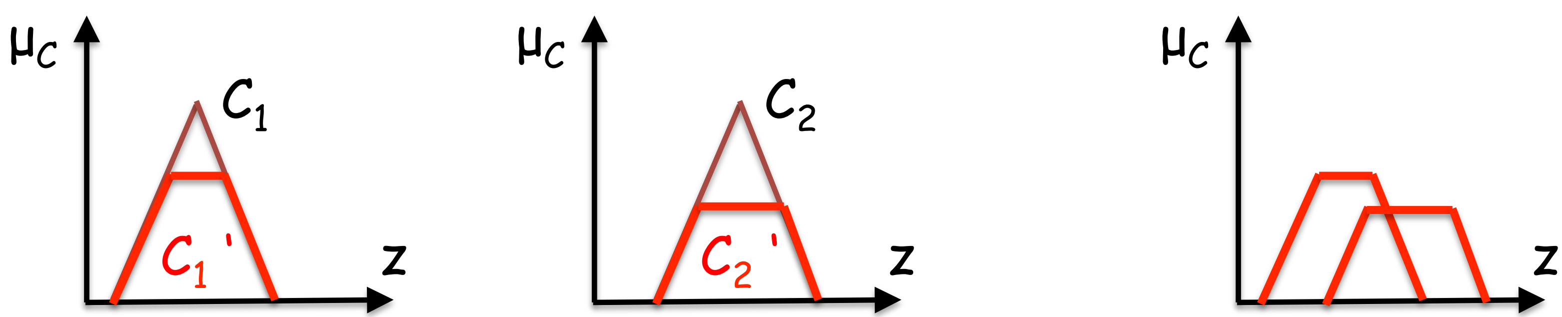

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Single Input Single Output

- input : x is A'

  R_1 : if x is A_1, then z is C_1 : $A_1 \rightarrow C_1$

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  output : z is C'

- $C' = A' \circ (R_1 \cup R_2) = A' \circ [(A_1 \rightarrow C_1) \cup (A_2 \rightarrow C_2)]$

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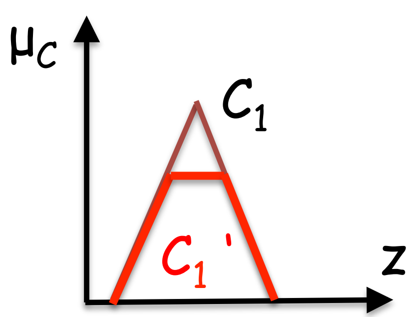

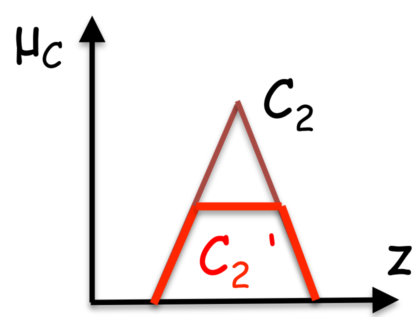

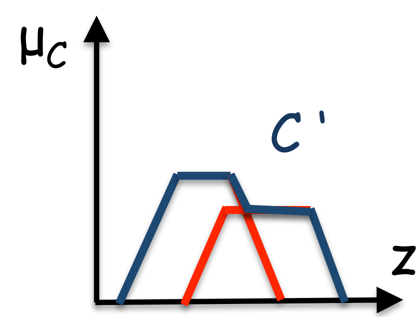

Mamdani Fuzzy Inference

Two Input Single Output

- input : x is A' and y is B'

  R : if x is A and y is B, then z is C : (A and B) ➔ C

  output : z is C'

Mamdani Fuzzy Inference

Two Input Single Output

- input : x is A' and y is B'

  R : if x is A and y is B, then z is C : (A and B) $\rightarrow$ C

  output : z is C'

- $C' = A' \circ R = A' \circ [(A \text{ and } B) \rightarrow C] = A' \circ [(A \rightarrow C) \cap (B \rightarrow C)]$

Mamdani Fuzzy Inference

Two Input Single Output

- input : x is A' and y is B'

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  R : if x is A and y is B, then z is C   : (A and B) $\rightarrow$ C

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  $C' = [A' \circ (A \rightarrow C)] \cap [A' \circ (B \rightarrow C)] = C_1' \cap C_2'$

  $\mu_{C'}(z) = \min\{ \mu_{C\_1'}(z),\ \mu_{C\_2'}(z) \}$

Mamdani Fuzzy Inference

Two Input Single Output

- input : x is A' and y is B'

  R : if x is A and y is B, then z is C : (A and B) $\rightarrow$ C

  output : z is C'

- $C' = A' \circ R = A' \circ [(A \text{ and } B) \rightarrow C] = A' \circ [(A \rightarrow C) \cap (B \rightarrow C)]$

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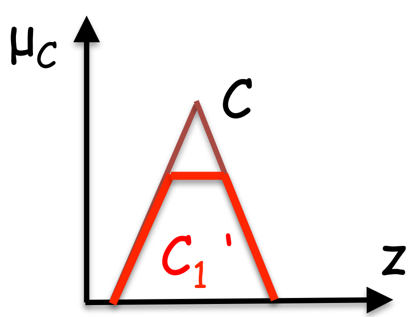

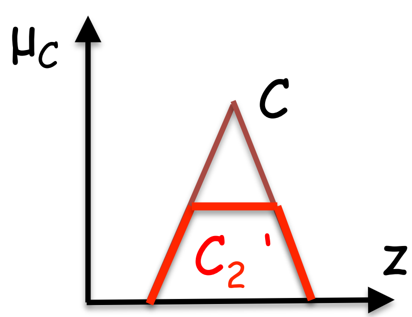

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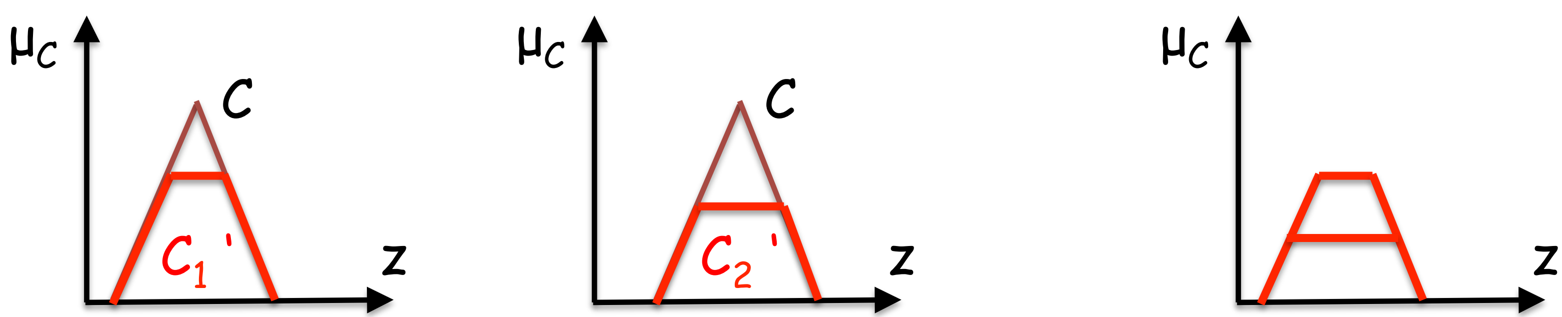

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Two Input Single Output

- input : x is A' and y is B'

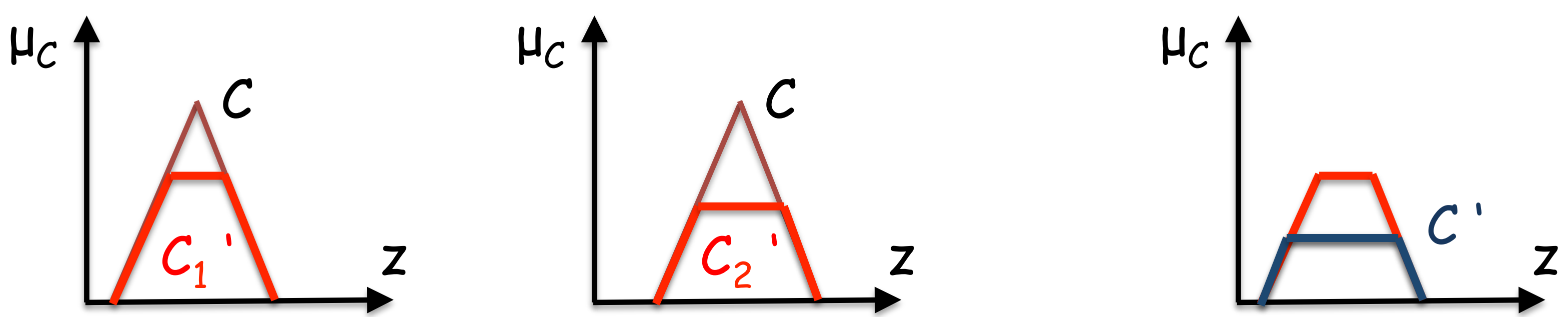
  R : if x is A and y is B, then z is C : (A and B) → C

  output : z is C'

- $C' = A' \circ R = A' \circ [(A \text{ and } B) \rightarrow C] = A' \circ [(A \rightarrow C) \cap (B \rightarrow C)]$

  $C' = [A' \circ (A \rightarrow C)] \cap [A' \circ (B \rightarrow C)] = C_1' \cap C_2'$

  $\mu_{C'}(z) = \min\{\, \mu_{C\_1'}(z),\ \mu_{C\_2'}(z) \,\}$



Mamdani Fuzzy Inference

Singleton Input

- the fact is : x is 3 and y is 4

  the rule is : If x is A and y is B, then z is C

  the result is : z is C'

where $A = (0, 2, 5)$, $B = (3, 5, 6)$, and $C = (1, 3, 5)$

Mamdani Fuzzy Inference

Singleton Input

- the fact is : x is 3 and y is 4
  the rule is : If x is A and y is B, then z is C
  the result is : z is $C\,'$

where A = (0, 2, 5), B = (3, 5, 6), and C = (1, 3, 5)

- $\mu_{C\_1'}(z) = \alpha_1 \wedge \mu_C(z)$ where $\alpha_1 = \mu_A(x_0)$

Mamdani Fuzzy Inference

Singleton Input

- the fact is : x is 3 and y is 4

  the rule is : If x is A and y is B, then z is C

  the result is : z is C'

where $A = (0, 2, 5)$, $B = (3, 5, 6)$, and $C = (1, 3, 5)$

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  $\mu_{C'}(z) = \min \{ \mu_{C\_1'}(z),\ \mu_{C\_2'}(z) \} = (\alpha_1 \wedge \alpha_2) \wedge \mu_C(z)$

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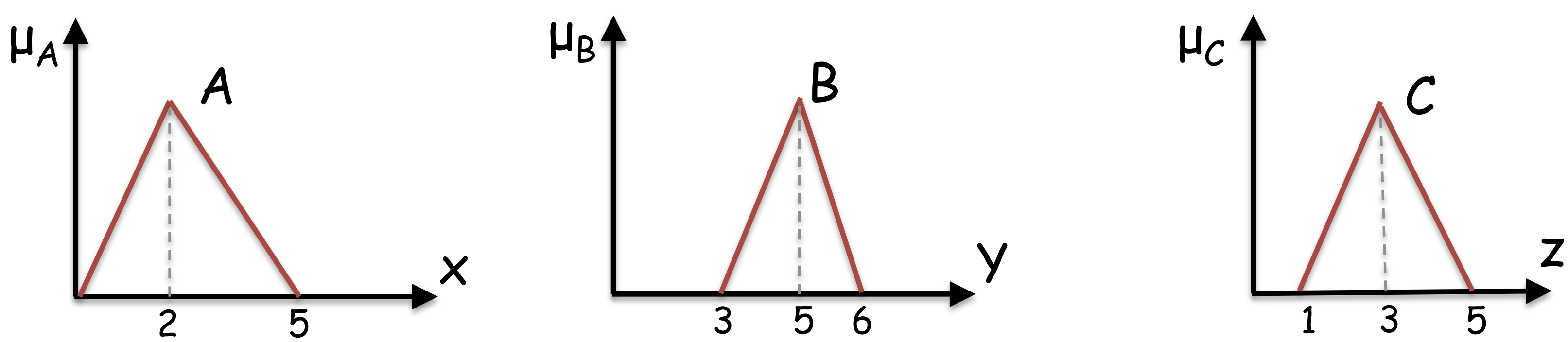

Mamdani Fuzzy Inference

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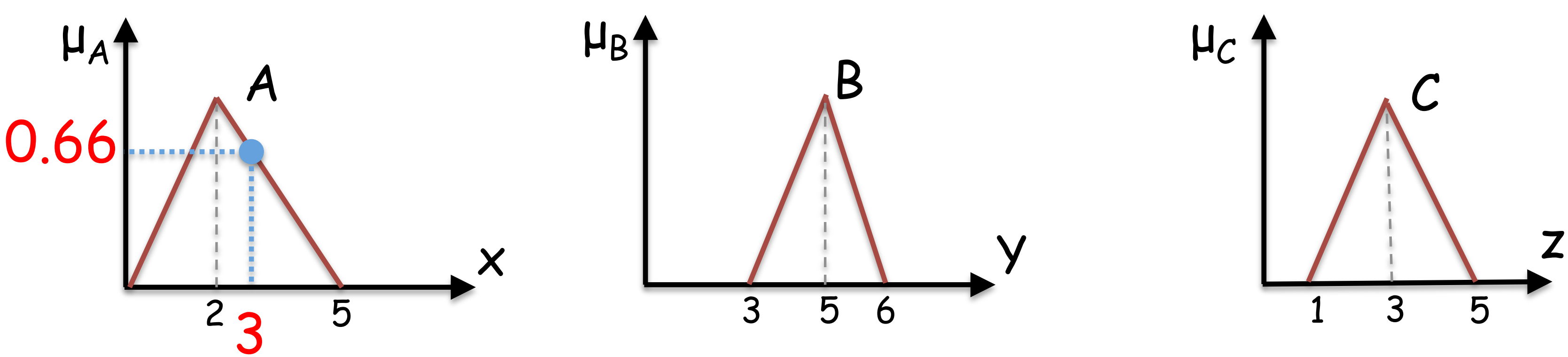

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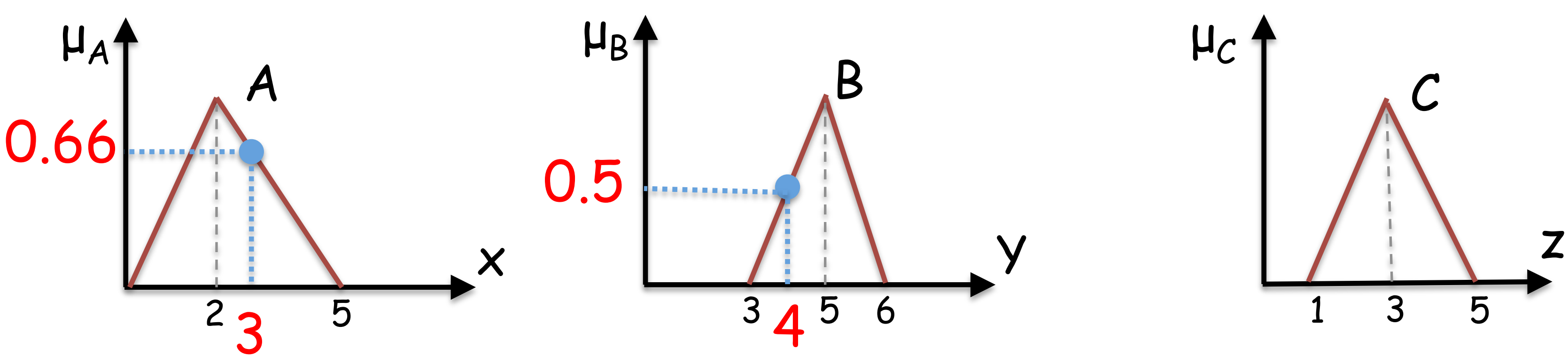

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- the fact is : x is 3 and y is 4

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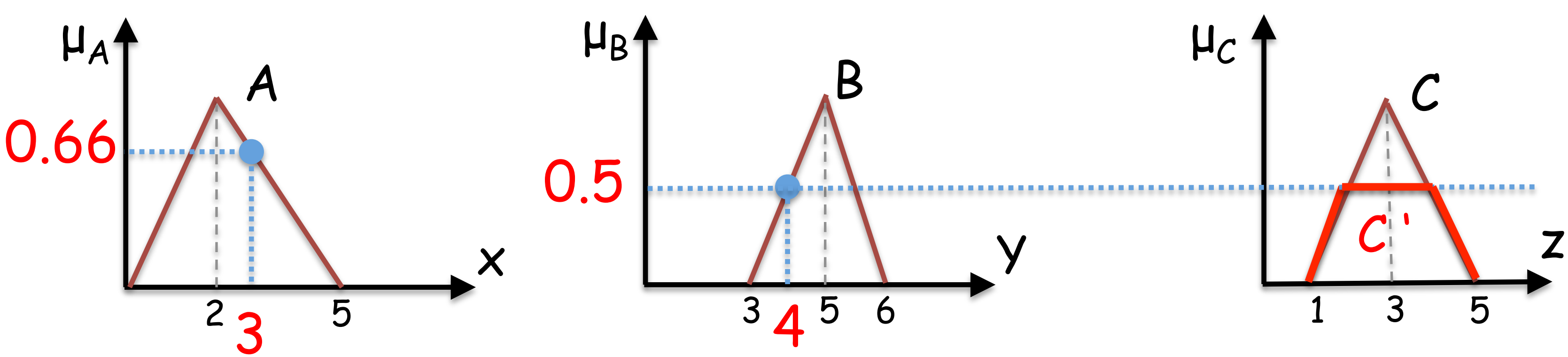

Mamdani Fuzzy Inference

Fuzzy Input

- the fact is : x is A' and y is B' --A'=(2, 4, 6) and B'=(2, 3, 5)--

  the rule is : If x is A and y is B, then z is C

  the result is : z is C'

where A = (0, 2, 5), B = (3, 5, 6), and C = (1, 3, 5)

Mamdani Fuzzy Inference

Fuzzy Input

- the fact is : x is A' and y is B′ --A'=(2, 4, 6) and B′=(2, 3, 5)--

  the rule is : If x is A and y is B, then z is C

  the result is : z is C'

where A = (0, 2, 5), B = (3, 5, 6), and C = (1, 3, 5)

- $\mu_{C\_1'}(z) = \alpha_1 \wedge \mu_C(z)$ where $\alpha_1 = \max_x \{\min(\mu_A(x), \mu_{A'}(x))\}$

  $\mu_{C\_2'}(z) = \alpha_2 \wedge \mu_C(z)$ where $\alpha_2 = \max_y \{\min(\mu_B(y), \mu_{B'}(y))\}$

Mamdani Fuzzy Inference

Fuzzy Input

- the fact is : x is A' and y is B' --A'=(2, 4, 6) and B'=(2, 3, 5)--

  the rule is : If x is A and y is B, then z is C

  the result is : z is C'

where A = (0, 2, 5), B = (3, 5, 6), and C = (1, 3, 5)

- $\mu_{C\_1'}(z) = \alpha_1 \wedge \mu_C(z)$ where $\alpha_1 = \max_x \{\min(\mu_A(x), \mu_{A'}(x))\}$

  $\mu_{C\_2'}(z) = \alpha_2 \wedge \mu_C(z)$ where $\alpha_2 = \max_y \{\min(\mu_B(y), \mu_{B'}(y))\}$

  $\mu_{C'}(z) = \min\{\ \mu_{C\_1'}(z),\ \ \mu_{C\_2'}(z)\ \} = (\alpha_1 \wedge \alpha_2) \wedge \mu_C(z)$

Mamdani Fuzzy Inference

Fuzzy Input

- the fact is : x is A' and y is B' --A'=(2, 4, 6) and B'=(2, 3, 5)--

  the rule is : If x is A and y is B, then z is C

  the result is : z is C'

where A = (0, 2, 5), B = (3, 5, 6), and C = (1, 3, 5)

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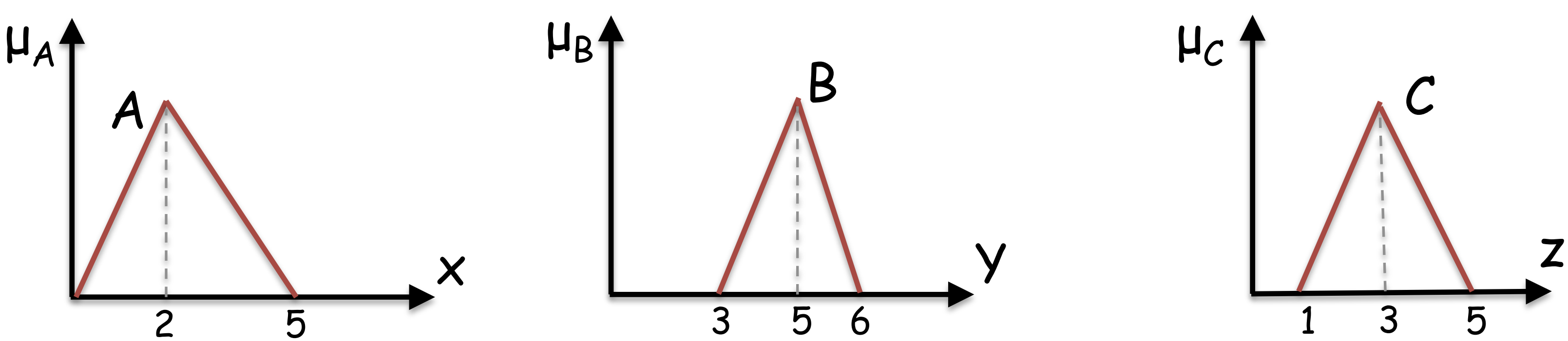
  $\mu_{C\_2'}(z) = \alpha_2 \wedge \mu_C(z)$ where $\alpha_2 = \max_y \{\min(\mu_B(y), \mu_{B'}(y))\}$

  $\mu_{C'}(z) = \min\{\mu_{C\_1'}(z), \mu_{C\_2'}(z)\} = (\alpha_1 \wedge \alpha_2) \wedge \mu_C(z)$



Mamdani Fuzzy Inference

Fuzzy Input

- the fact is : x is A' and y is B' --A'=(2, 4, 6) and B'=(2, 3, 5)--
  the rule is : If x is A and y is B, then z is C
  the result is : z is C'

where A = (0, 2, 5), B = (3, 5, 6), and C = (1, 3, 5)

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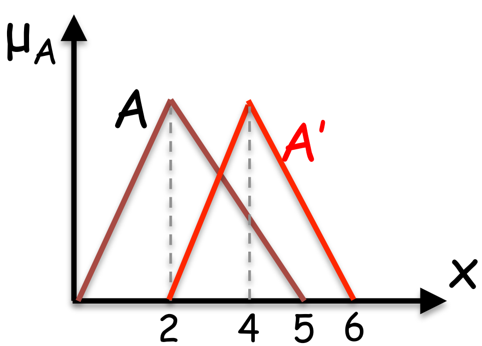
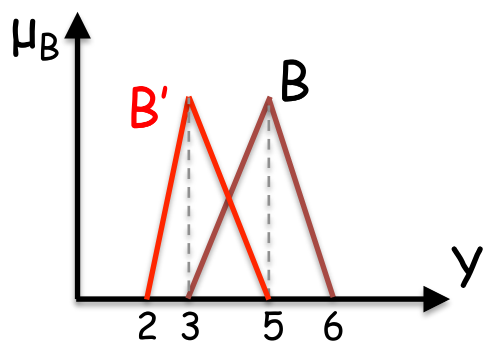
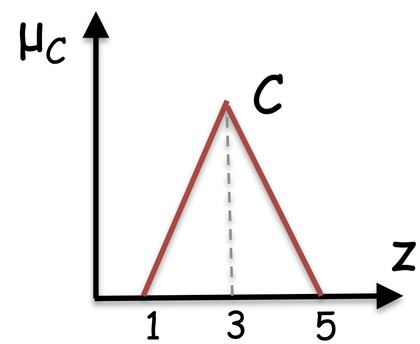






Mamdani Fuzzy Inference

Fuzzy Input

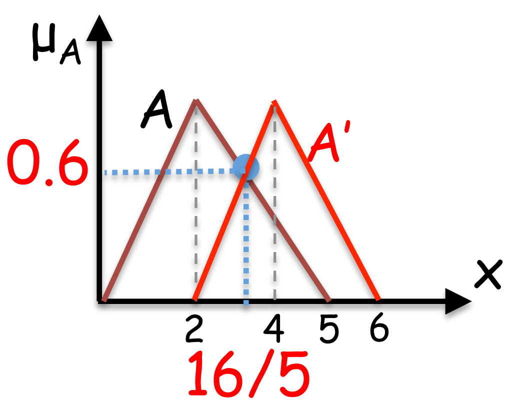
- the fact is : x is A' and y is B' --A'=(2, 4, 6) and B'=(2, 3, 5)--
  the rule is : If x is A and y is B, then z is C
  the result is : z is C'

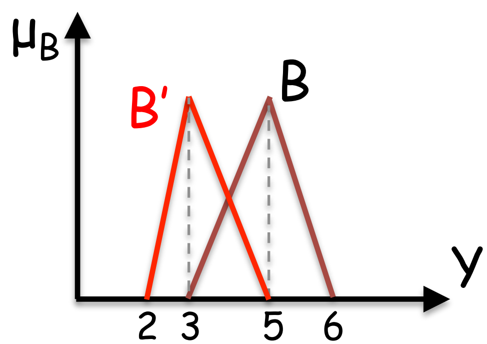
where A = (0, 2, 5), B = (3, 5, 6), and C = (1, 3, 5)

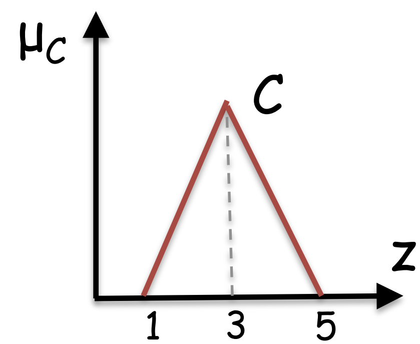
- $\mu_{C\_1'}(z) = \alpha_1 \wedge \mu_C(z)$ where $\alpha_1 = \max_x \{\min(\mu_A(x), \mu_{A'}(x))\}$

  $\mu_{C\_2'}(z) = \alpha_2 \wedge \mu_C(z)$ where $\alpha_2 = \max_y \{\min(\mu_B(y), \mu_{B'}(y))\}$

  $\mu_{C'}(z) = \min\{\mu_{C\_1'}(z), \mu_{C\_2'}(z)\} = (\alpha_1 \wedge \alpha_2) \wedge \mu_C(z)$







Mamdani Fuzzy Inference

Fuzzy Input

- the fact is : x is A' and y is B' --A'=(2, 4, 6) and B'=(2, 3, 5)--

  the rule is : If x is A and y is B, then z is C

  the result is : z is C'

where A = (0, 2, 5), B = (3, 5, 6), and C = (1, 3, 5)

- $\mu_{C\_1'}(z) = \alpha_1 \wedge \mu_C(z)$ where $\alpha_1 = \max_x \{\min(\mu_A(x), \mu_{A'}(x))\}$

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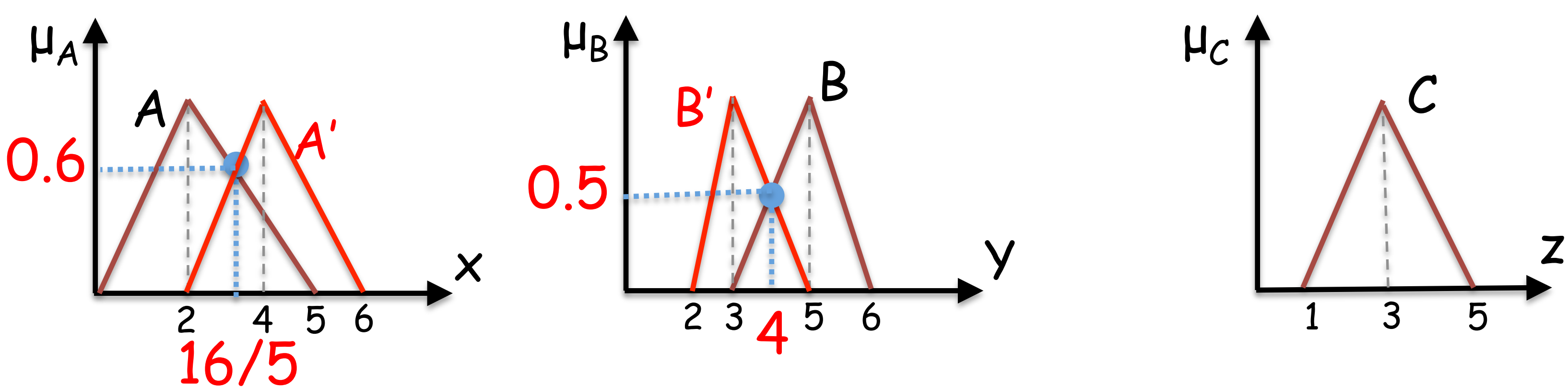

Mamdani Fuzzy Inference

Fuzzy Input

- the fact is : x is A' and y is B' --A'=(2, 4, 6) and B'=(2, 3, 5)--

  the rule is : If x is A and y is B, then z is C

  the result is : z is C'

where A = (0, 2, 5), B = (3, 5, 6), and C = (1, 3, 5)

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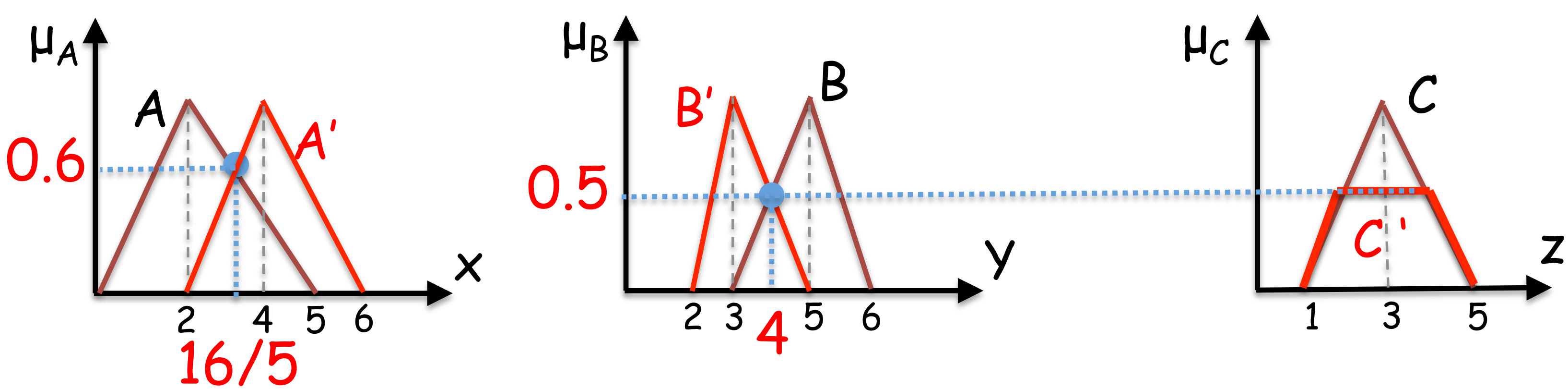

Larsen Fuzzy Inference

Singleton Input

- the fact is : x is 3 and y is 4
  the rule is : If x is A and y is B, then z is C
  the result is : z is C'

where A = (0, 2, 6), B = (3, 6, 7), and C = (1, 3, 5)

Larsen Fuzzy Inference

Singleton Input

- the fact is : x is 3 and y is 4

  the rule is : If x is *A* and y is B, then z is *C*

  the result is : z is *C* '

where *A* = (0, 2, 6), B = (3, 6, 7), and *C* = (1, 3, 5)

- $\mu_{C\_1'}(z) = \alpha_1 \,.\, \mu_C(z)$ where $\alpha_1 = \mu_A(x_0)$

  $\mu_{C\_2'}(z) = \alpha_2 \,.\, \mu_C(z)$ where $\alpha_2 = \mu_B(y_0)$

Larsen Fuzzy Inference

Singleton Input

- the fact is : x is 3 and y is 4

  the rule is : If x is A and y is B, then z is C

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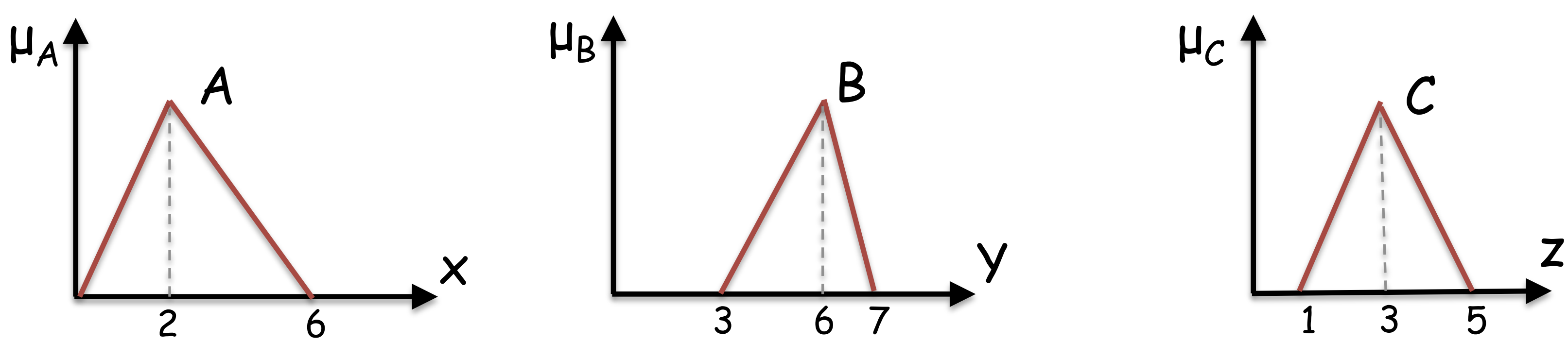

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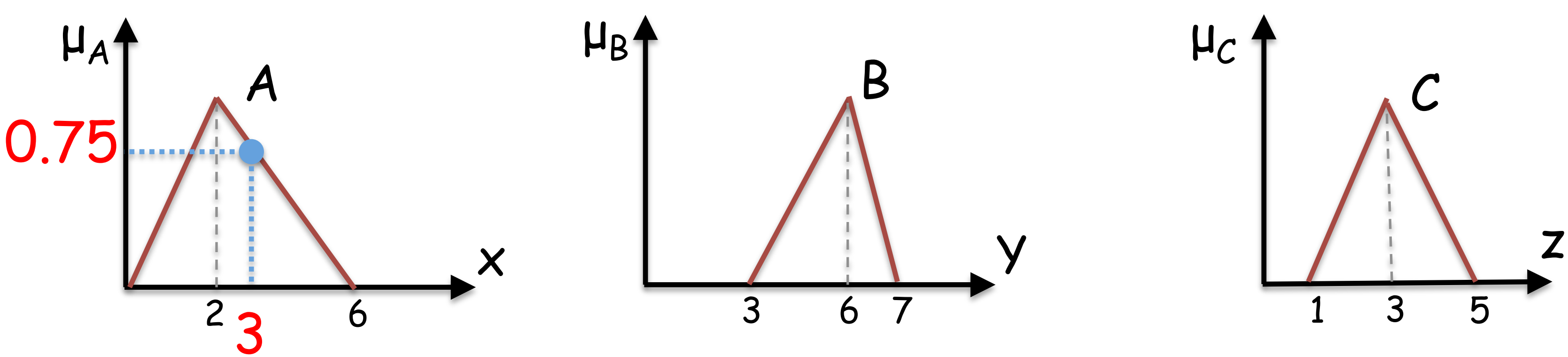

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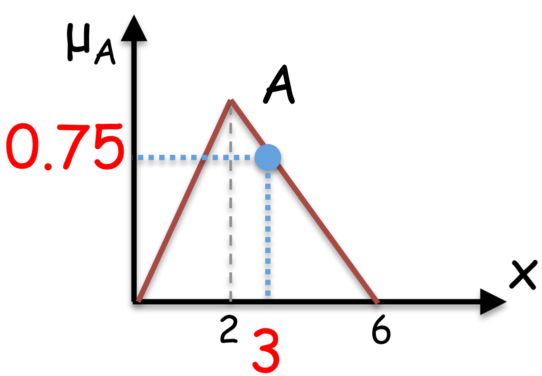

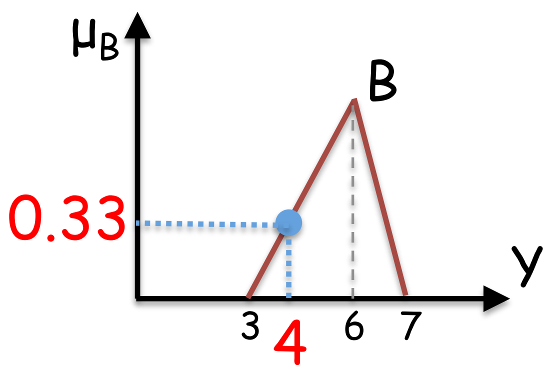

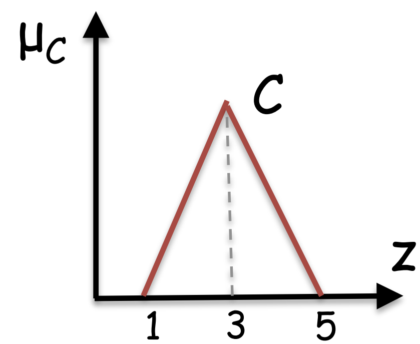

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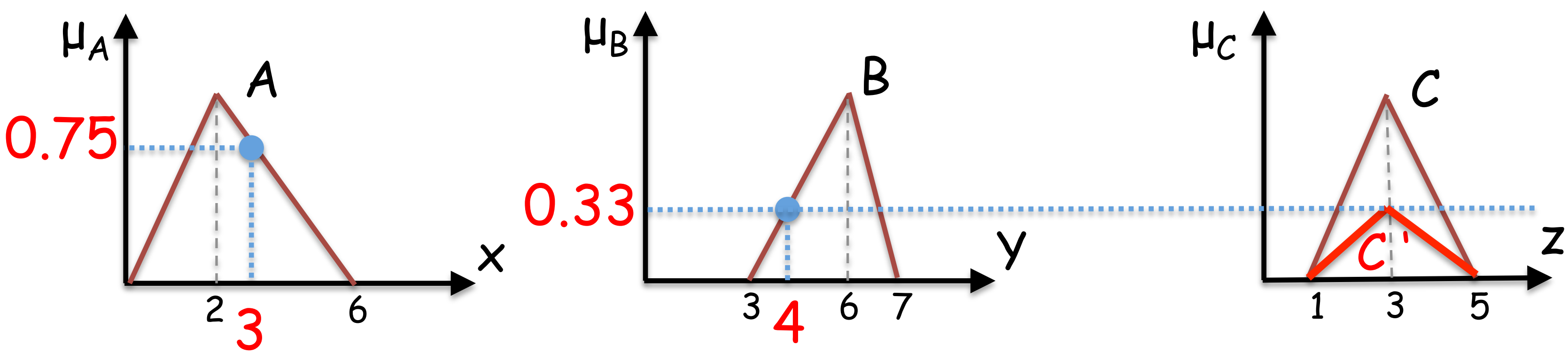
  $\mu_{C\_2'}(z) = \alpha_2 \cdot \mu_C(z)$ where $\alpha_2 = \mu_B(y_0)$

  $\mu_{C'}(z) = \min\{\mu_{C\_1'}(z), \mu_{C\_2'}(z)\} = (\alpha_1 \wedge \alpha_2) \cdot \mu_C(z)$



Larsen Fuzzy Inference

Fuzzy Input

- the fact is : x is A' and y is B' --A'=(2, 4, 5) and B'=(2, 3, 5)--

  the rule is : If x is A and y is B, then z is C

  the result is : z is C'

where A = (0, 2, 6), B = (3, 6, 7), and C = (1, 3, 5)

Larsen Fuzzy Inference

Fuzzy Input

- the fact is : x is A' and y is B' --A'=(2, 4, 5) and B'=(2, 3, 5)--
  the rule is : If x is A and y is B, then z is C
  the result is : z is C'

where A = (0, 2, 6), B = (3, 6, 7), and C = (1, 3, 5)

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  $\mu_{C\_2'}(z) = \alpha_2 \,.\, \mu_C(z)$ where $\alpha_2 = \max_y \{\min(\mu_B(y), \mu_{B'}(y))\}$

Larsen Fuzzy Inference

Fuzzy Input

- the fact is : x is A' and y is B' --A'=(2, 4, 5) and B'=(2, 3, 5)--

  the rule is : If x is A and y is B, then z is C

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  $\mu_{C\_2'}(z) = \alpha_2 \,.\, \mu_C(z)$ where $\alpha_2 = \max_y \{\min(\mu_B(y), \mu_{B'}(y))\}$

  $\mu_{C'}(z) = \min\{\, \mu_{C\_1'}(z),\ \mu_{C\_2'}(z)\,\} = (\alpha_1 \wedge \alpha_2) \,.\, \mu_C(z)$

Larsen Fuzzy Inference

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- the fact is : x is A' and y is B' --A'=(2, 4, 5) and B'=(2, 3, 5)--

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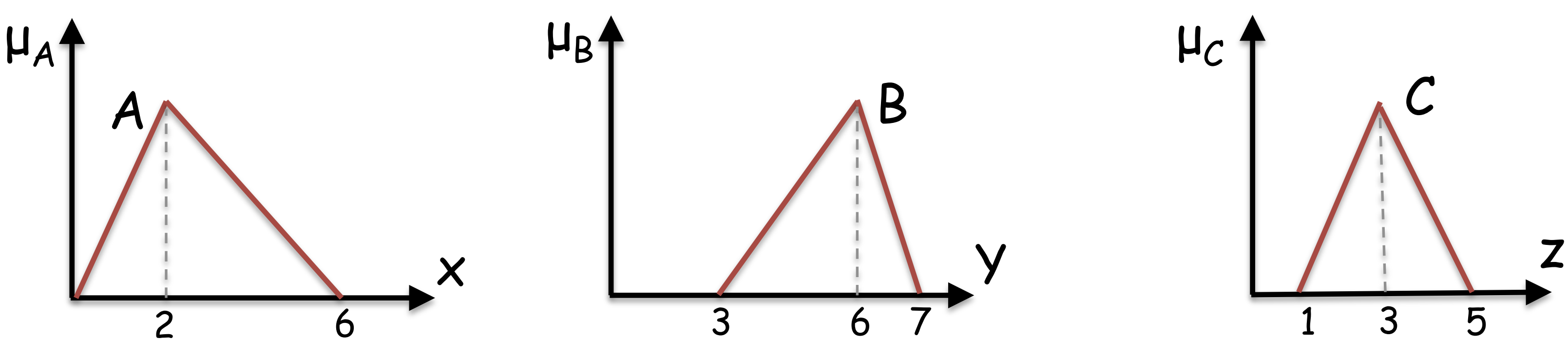


Larsen Fuzzy Inference

Fuzzy Input

- the fact is : x is A' and y is B' --A'=(2, 4, 5) and B'=(2, 3, 5)--

  the rule is : If x is A and y is B, then z is C

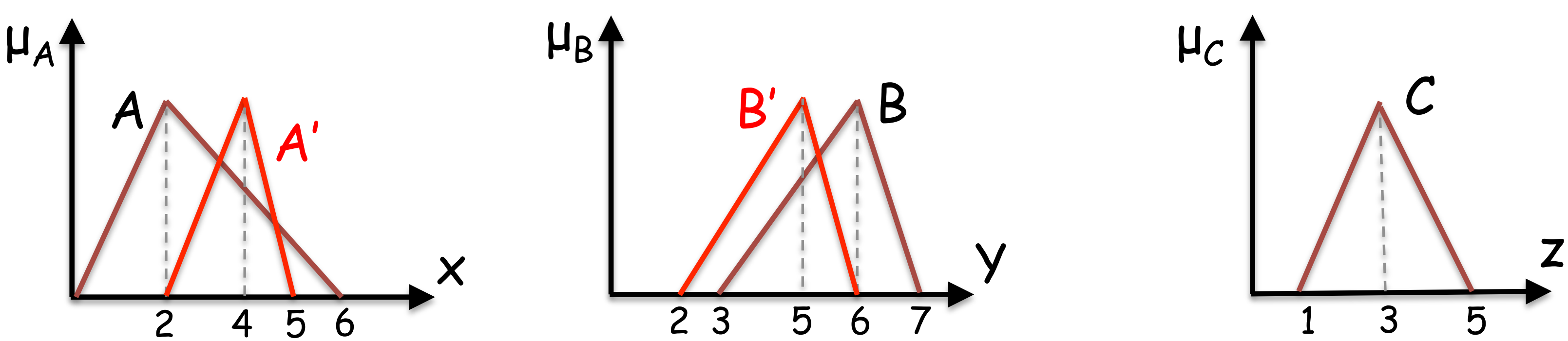
  the result is : z is C'

where A = (0, 2, 6), B = (3, 6, 7), and C = (1, 3, 5)

- $\mu_{C\_1'}(z) = \alpha_1 . \mu_C(z)$ where $\alpha_1 = \max_x \{\min(\mu_A(x), \mu_{A'}(x))\}$

  $\mu_{C\_2'}(z) = \alpha_2 . \mu_C(z)$ where $\alpha_2 = \max_y \{\min(\mu_B(y), \mu_{B'}(y))\}$

  $\mu_{C'}(z) = \min\{ \mu_{C\_1'}(z), \mu_{C\_2'}(z) \} = (\alpha_1 \wedge \alpha_2) . \mu_C(z)$



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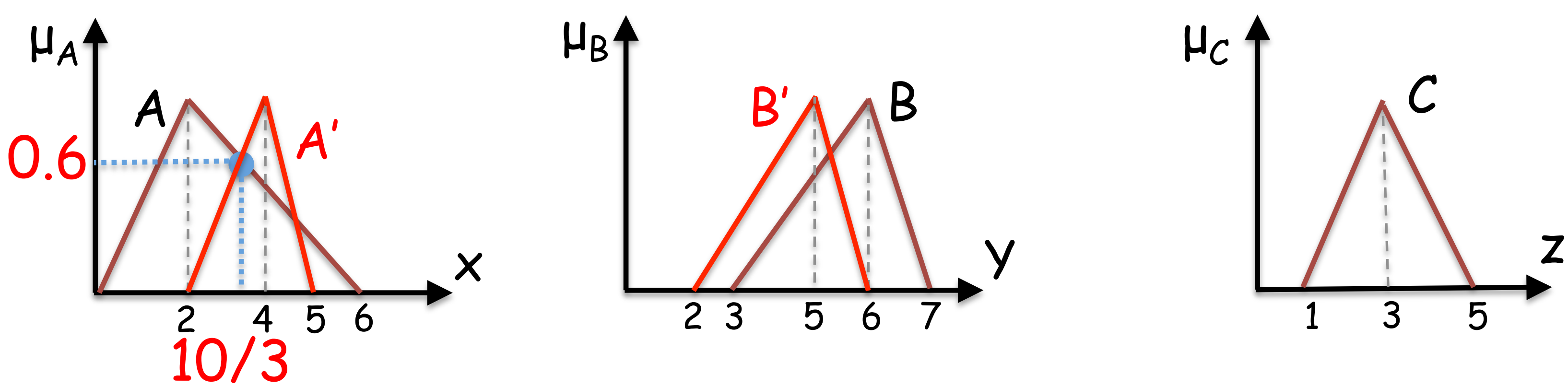

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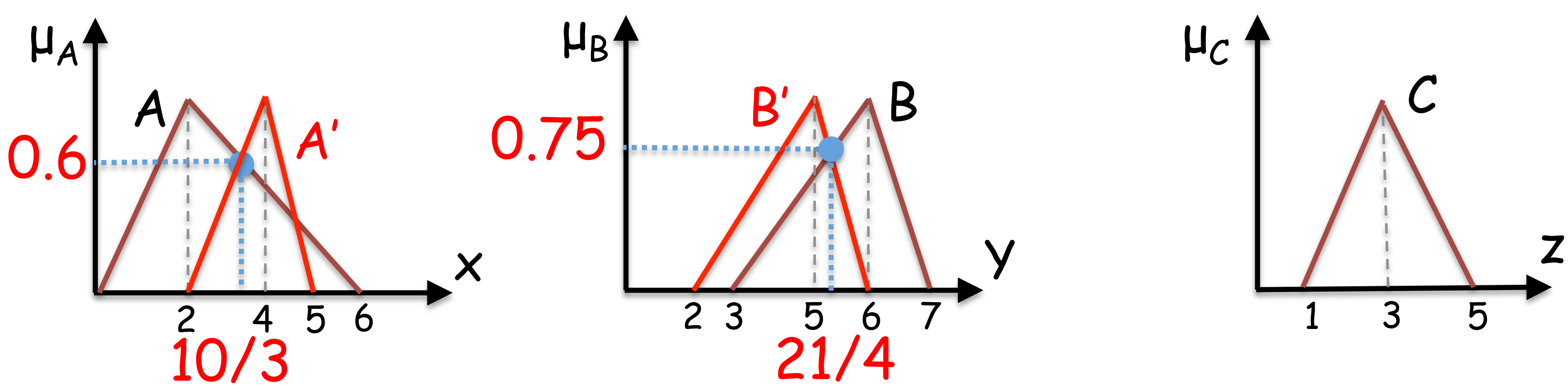

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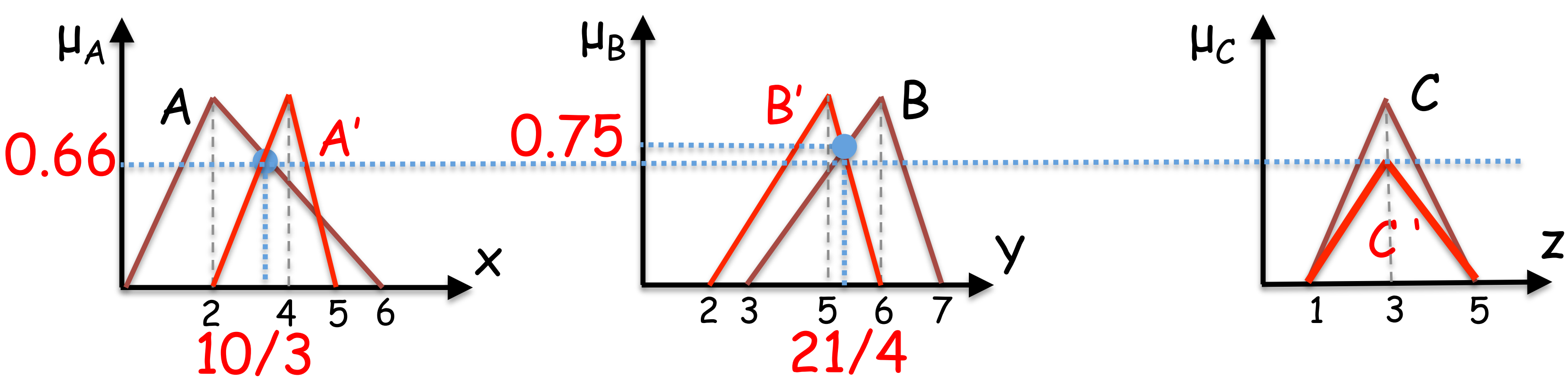

Tsukamoto Fuzzy Inference

Singleton Input

- the fact is : x is 3 and y is 4

  the rule is : If x is A and y is B, then z is C

  the result is : $z = z_0$

where A = (0, 2, 6) and B = (3, 6, 7)

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- the consequence of the fuzzy rule is represented by a fuzzy set with a monotonic membership function
- the output for each rule will be a crisp value induced by the rule's matching degree

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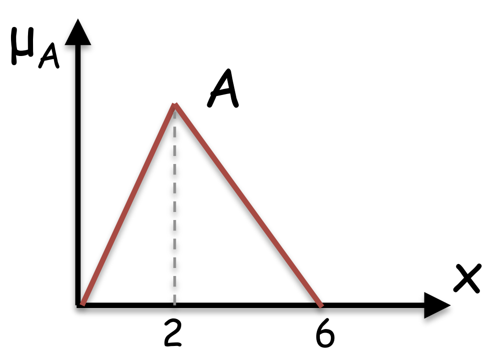

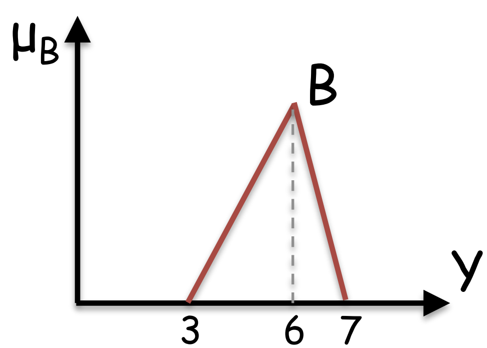

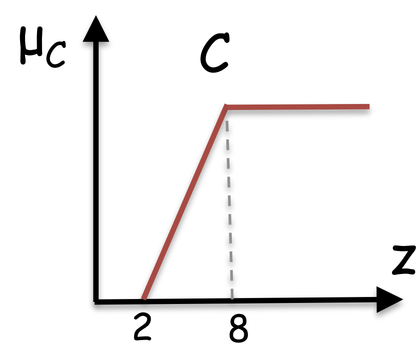

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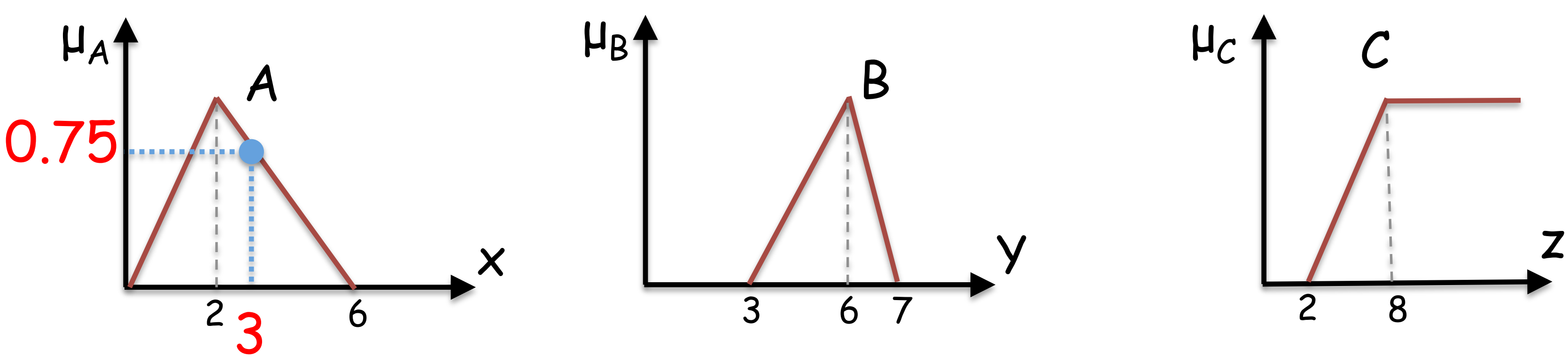

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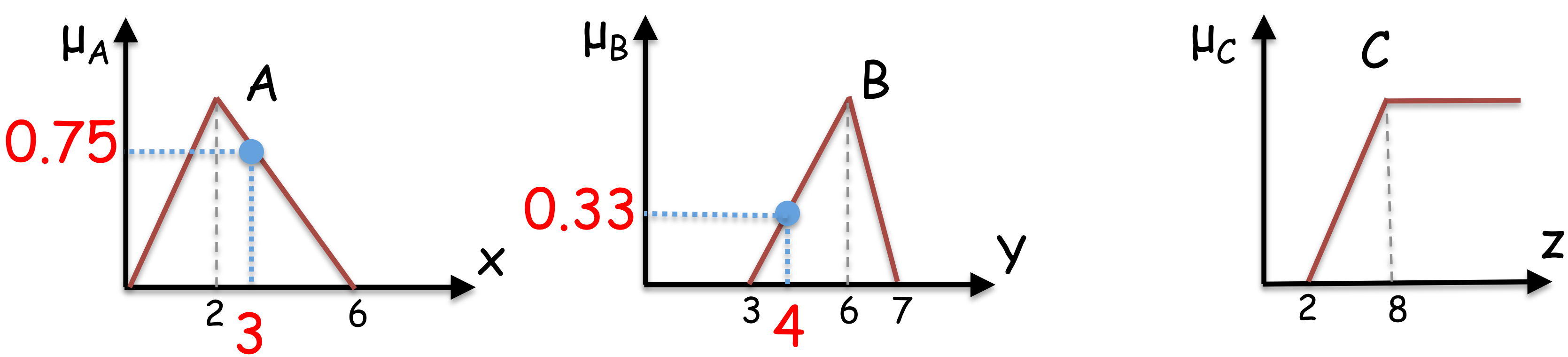

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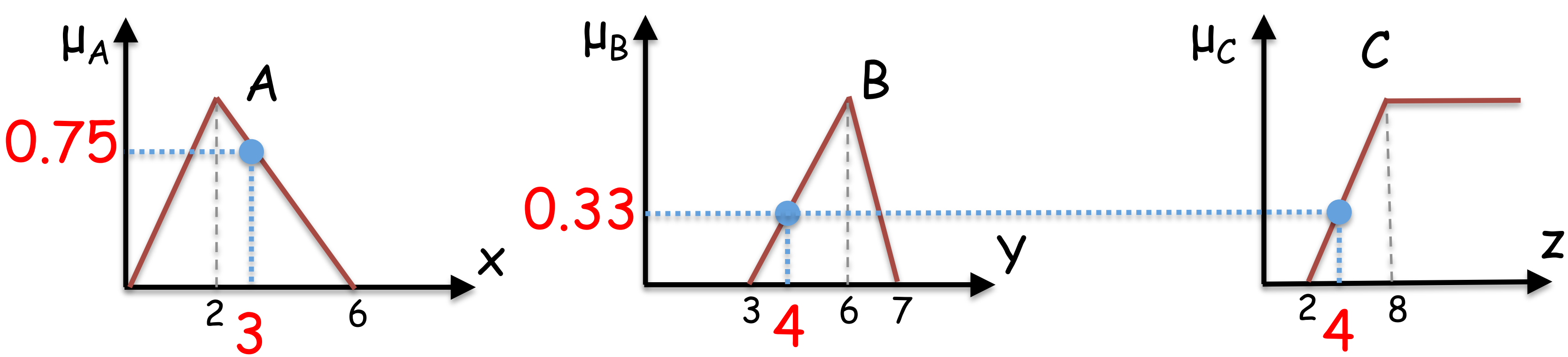

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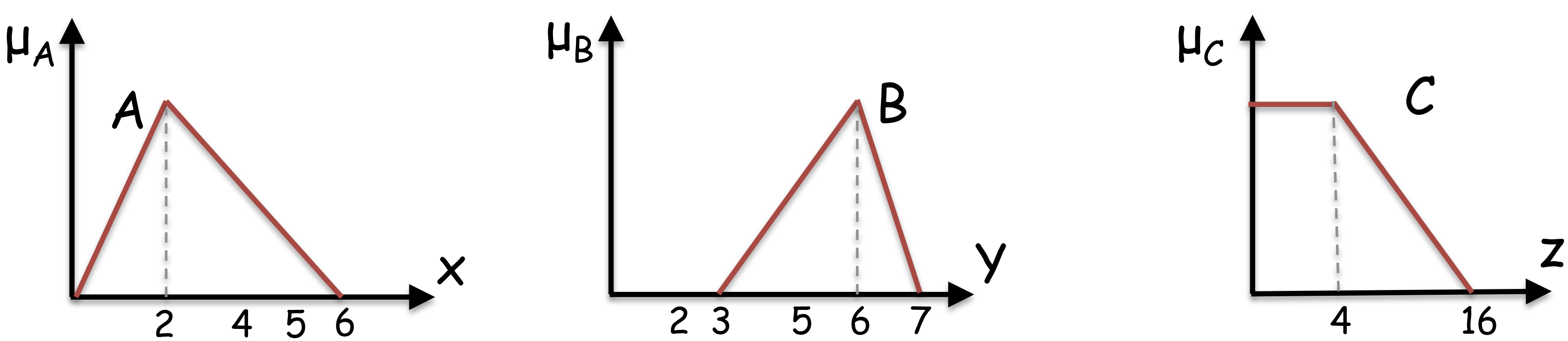

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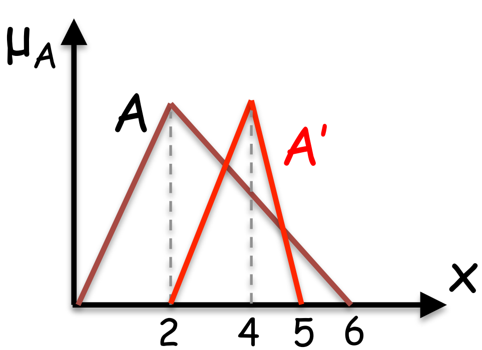

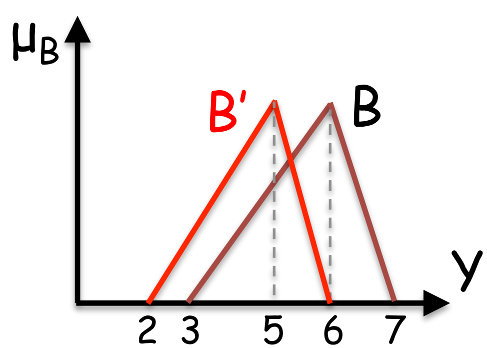

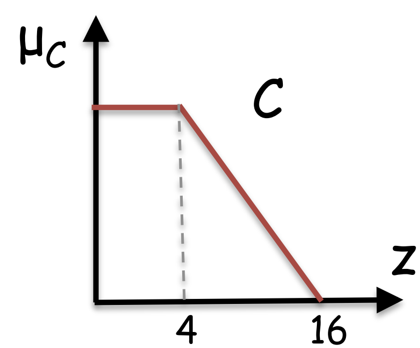

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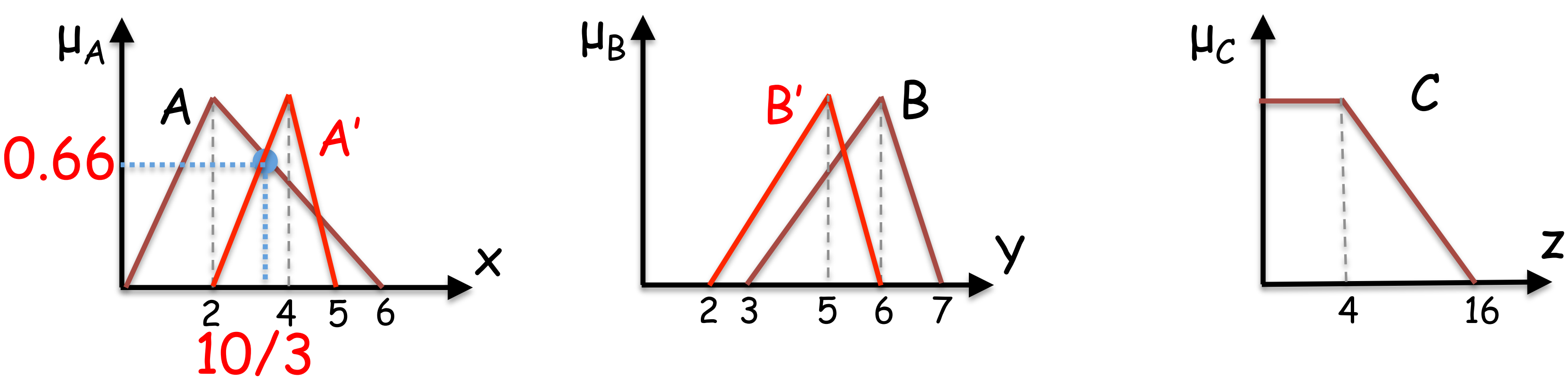

Tsukamoto Fuzzy Inference

Fuzzy Input

- the fact is : x is A′ and y is B′ --A′=(2, 4, 5) and B′=(2, 3, 5)--
  the rule is : If x is A and y is B, then z is C
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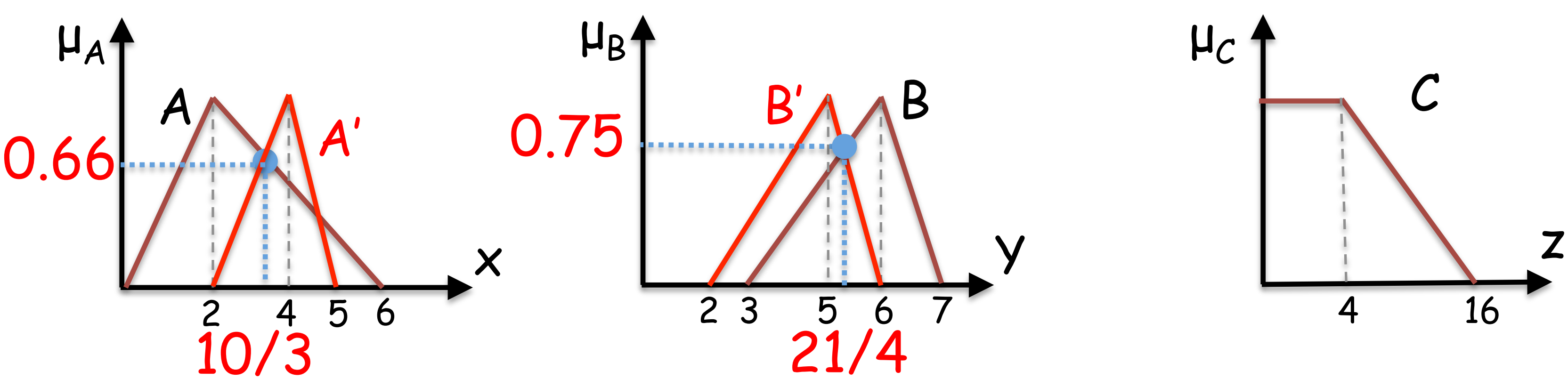

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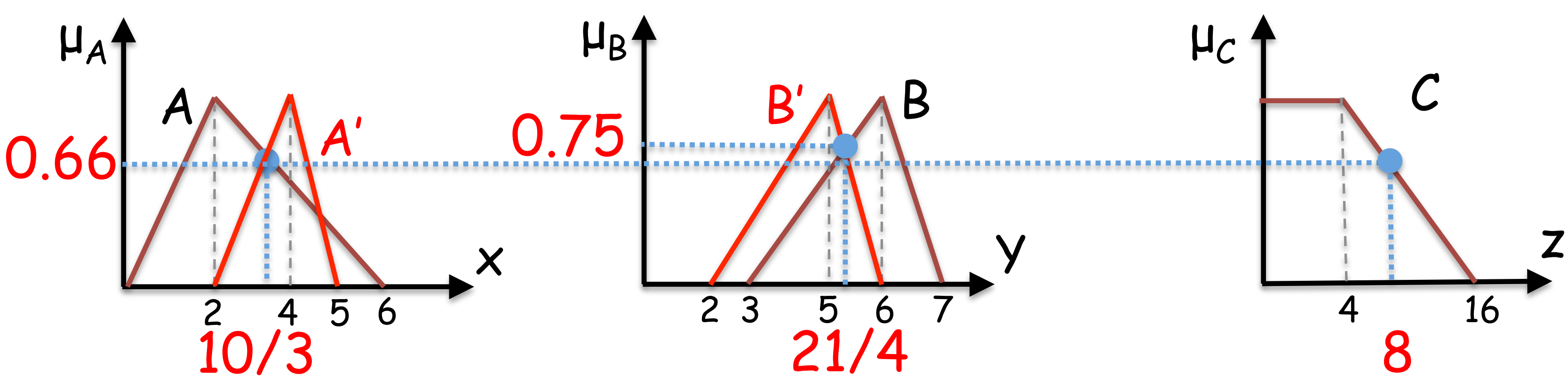

Defuzzification

Mean of Maximum

$z^* = (a + b) / 2$ where the membership function gets the maximum value at the interval $[a, b]$

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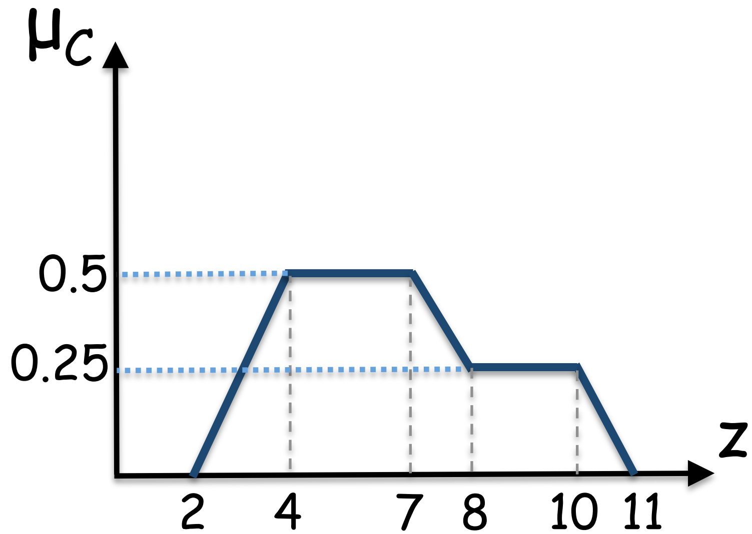

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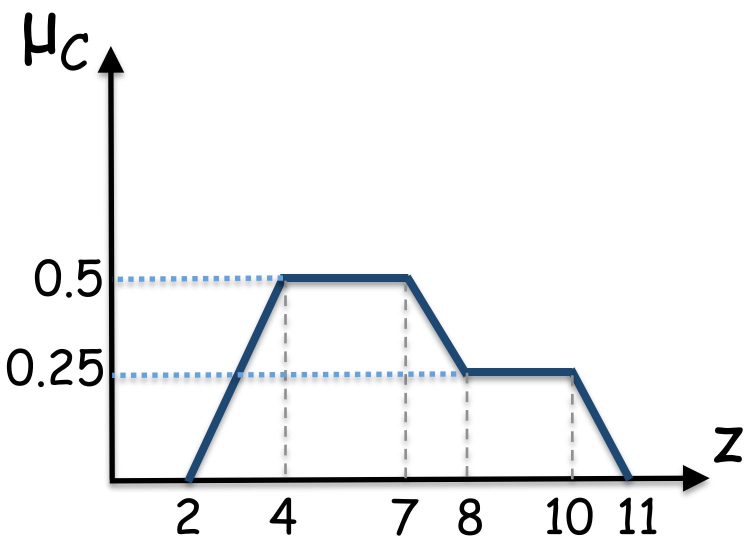

$z^* = (4 + 7) / 2 = 5.5$

Defuzzification

Center of Area

$$z^* = (\Sigma\ \mu_C(z_i).z_i)/(\Sigma\ \mu_C(z_i))$$

Defuzzification

Center of Area

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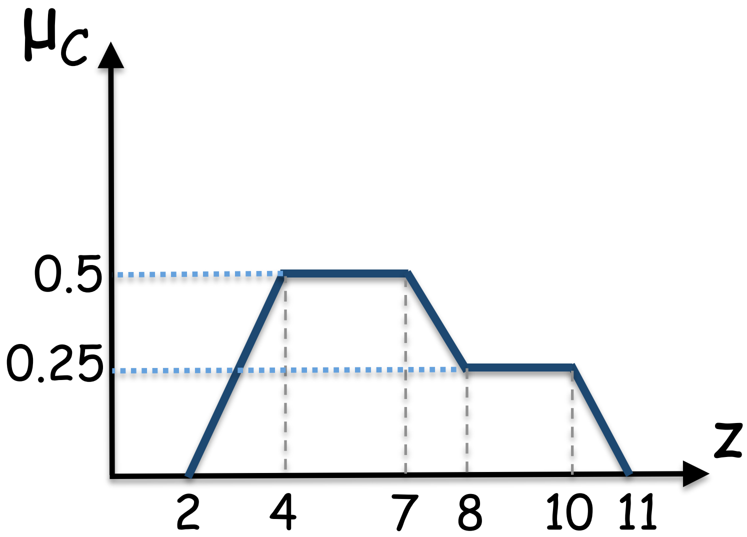

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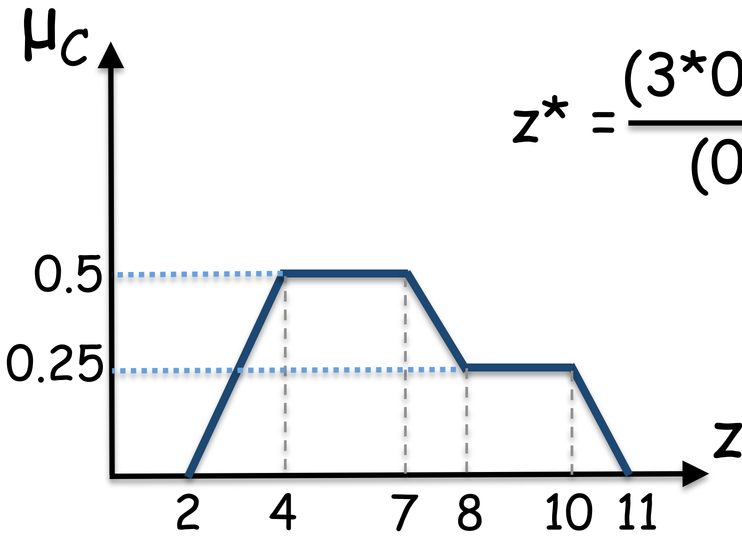
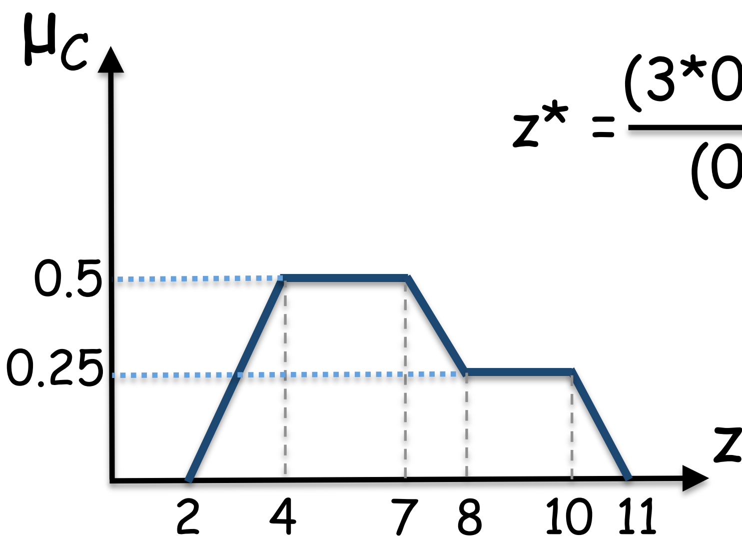


$$z^* = \frac{(3*0.25+22*0.5+27*0.25)}{(0.25+4*0.5+3*0.25)}$$

Defuzzification

Center of Area

$$z^* = (\Sigma\ \mu_C(z_i).z_i)/(\Sigma\ \mu_C(z_i))$$



$$z^* = \frac{(3*0.25+22*0.5+27*0.25)}{(0.25+4*0.5+3*0.25)} = 6.33$$

Defuzzification

Bisector of Area

z^* such that $I(a, z^*) = I(z^*, b)$ where the membership function gets the nonzero value at the interval $[a, b]$

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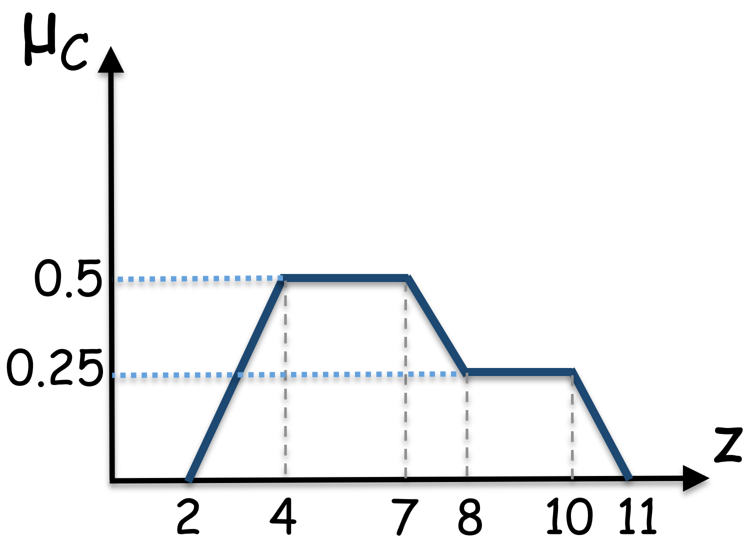

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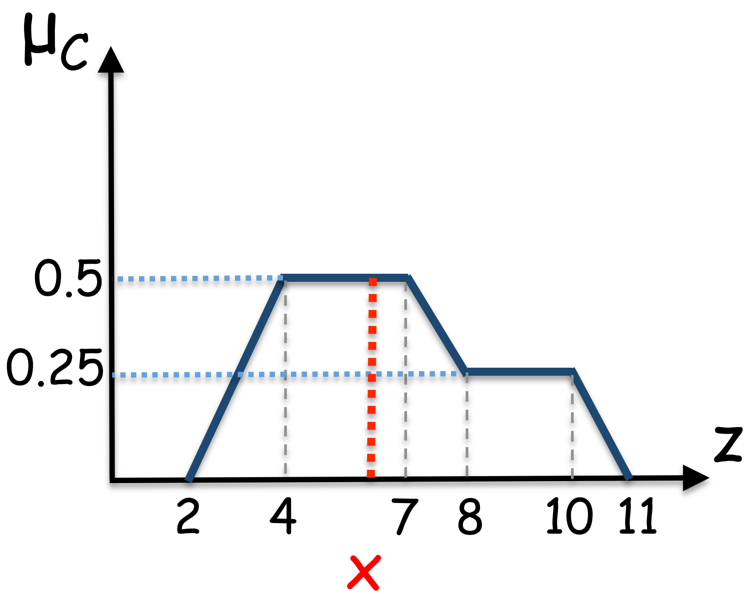

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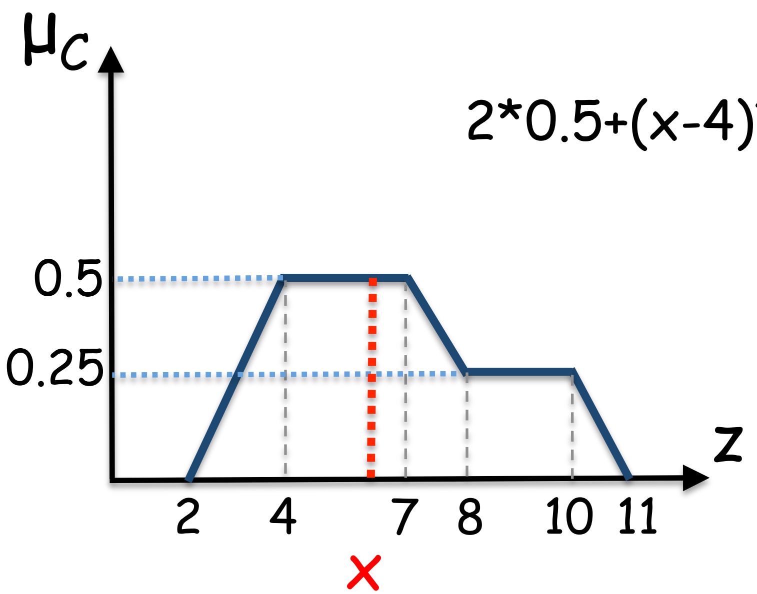

$$2*0.5+(x-4)*0.5 = (7-x)*0.5+0.375+2*0.25+0.125$$

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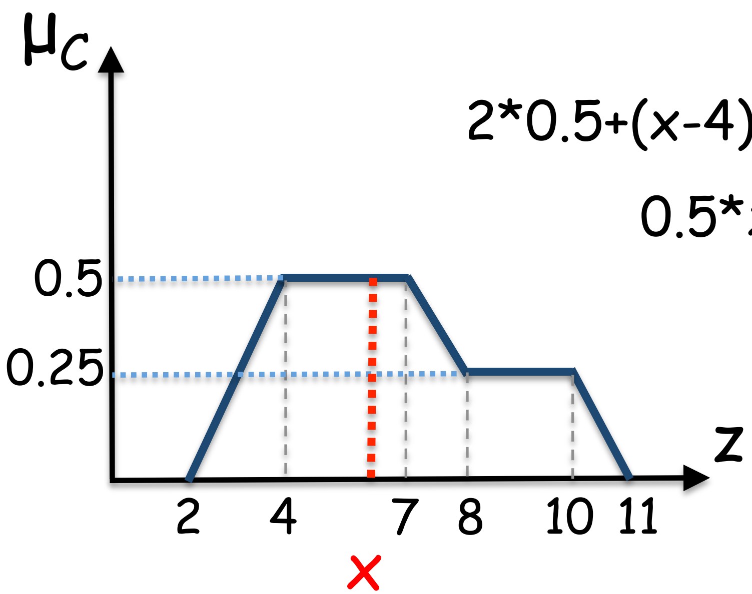

$$2*0.5+(x-4)*0.5 = (7-x)*0.5+0.375+2*0.25+0.125$$

$$0.5*x - 1 = 3.5 - 0.5*x + 1$$

Defuzzification

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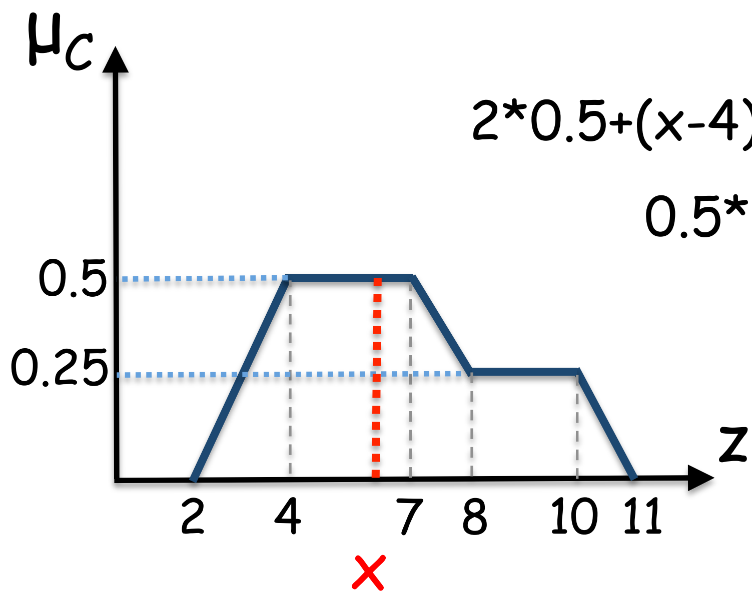
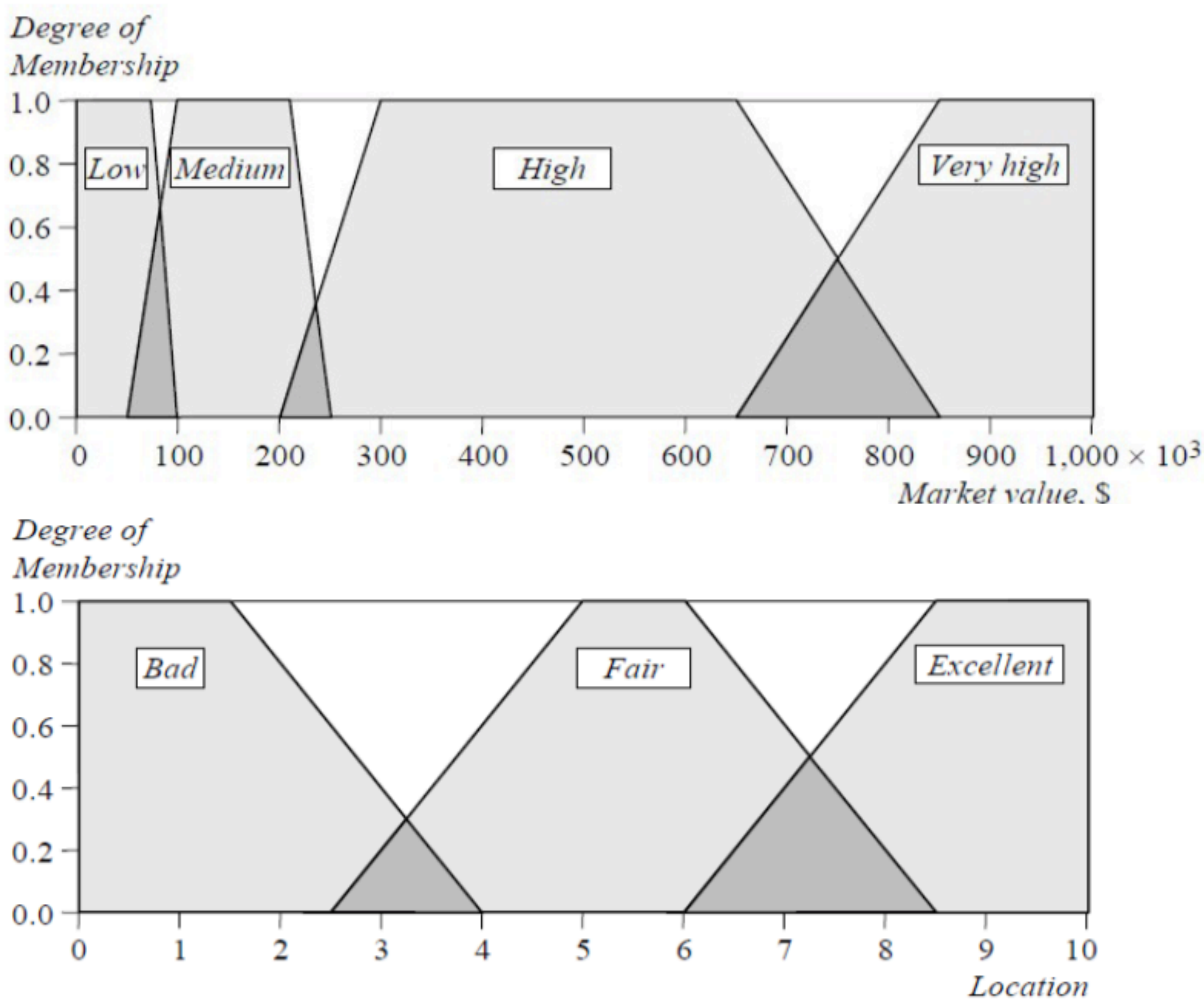


$$2*0.5+(x-4)*0.5 = (7-x)*0.5+0.375+2*0.25+0.125$$

$$0.5*x - 1 = 3.5 - 0.5*x + 1$$

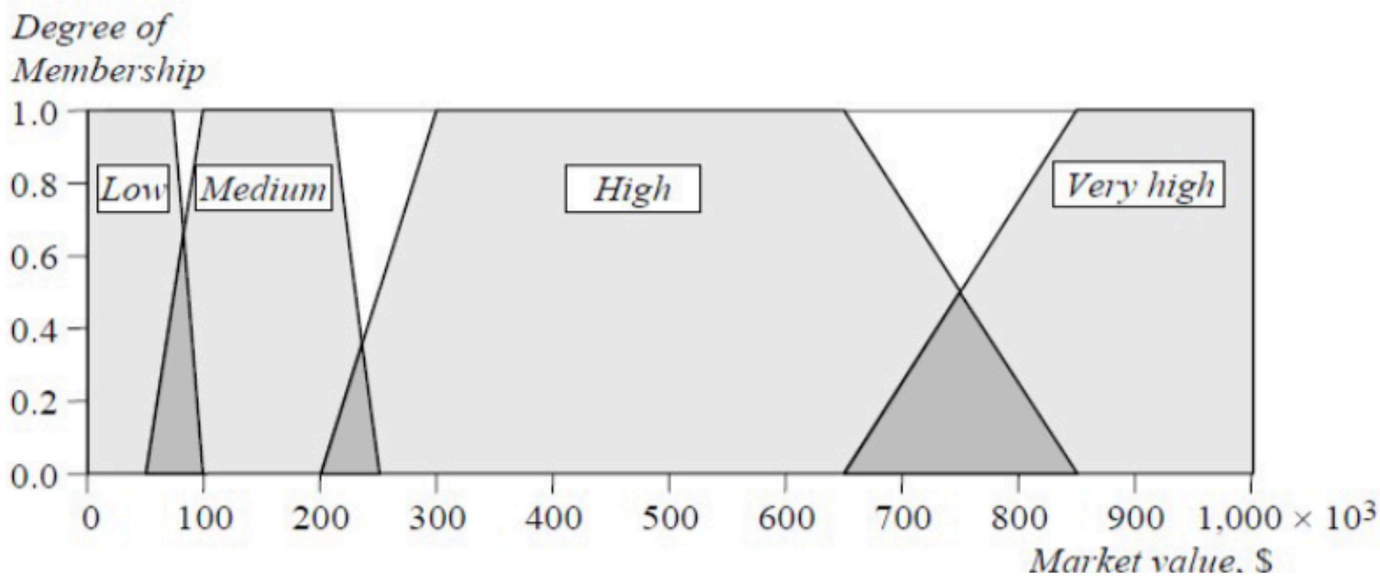
$$x = 5.5$$

Second Project

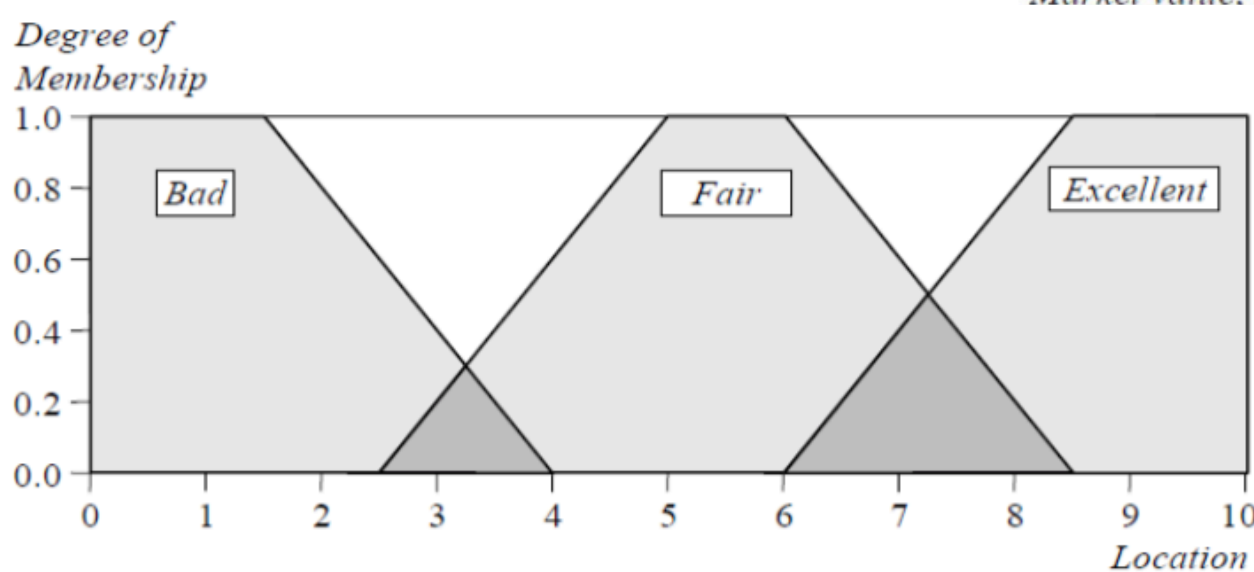


Second Project

market value = 700



location = 6

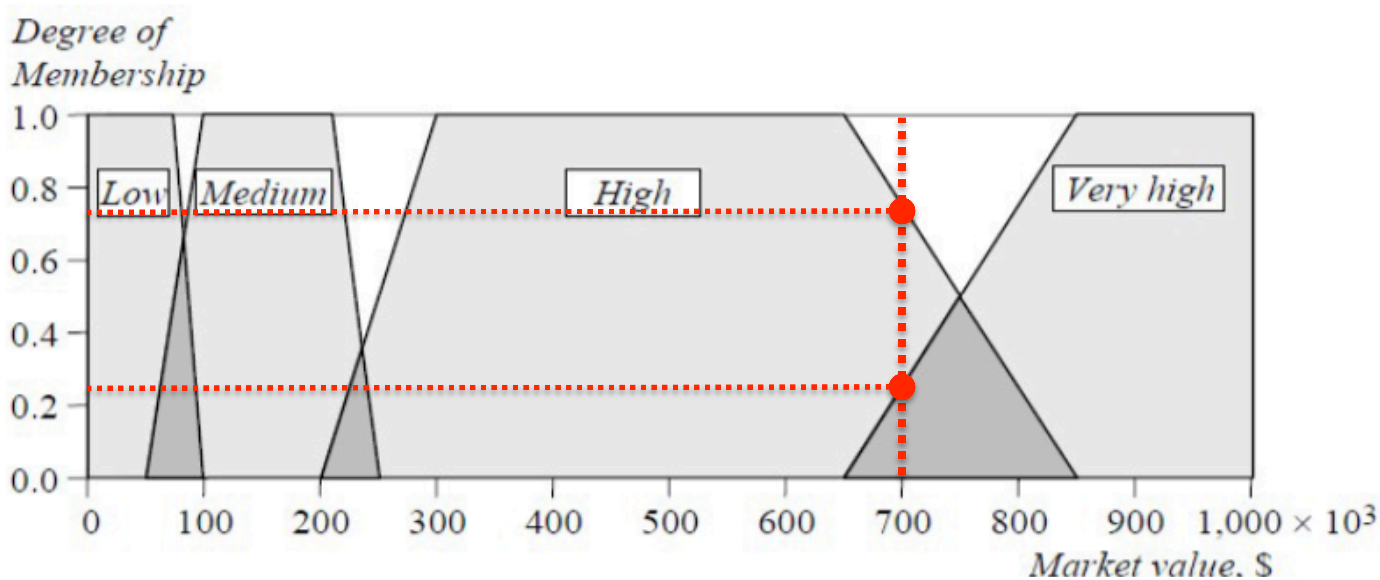


Second Project

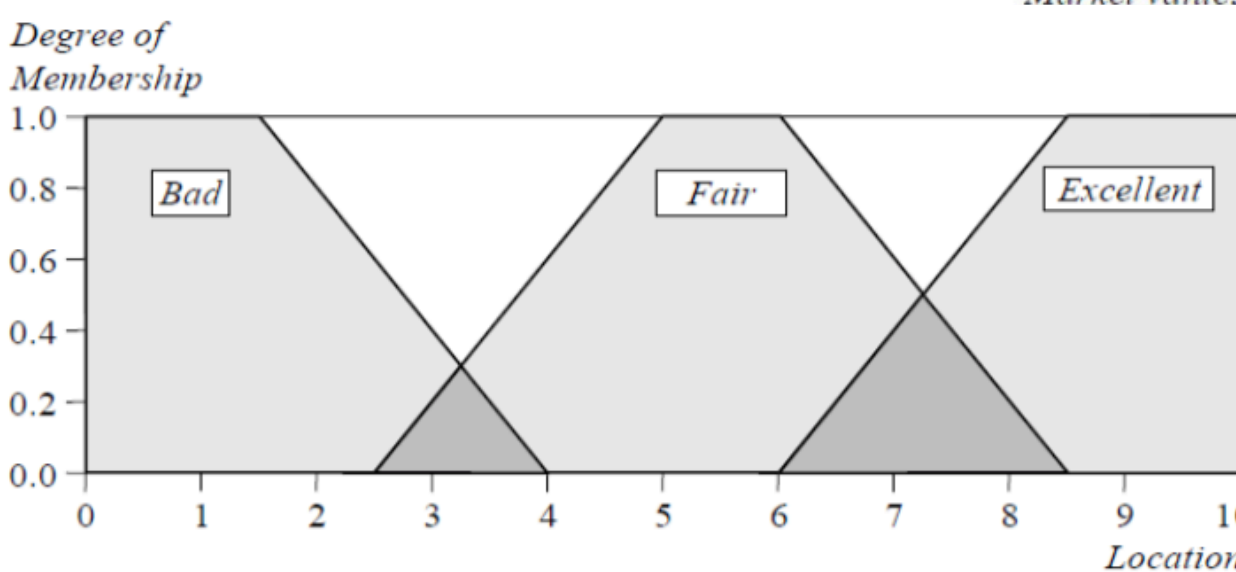
market value = 700

$\alpha_1 = 0.75$

$\alpha_2 = 0.25$



location = 6



Second Project

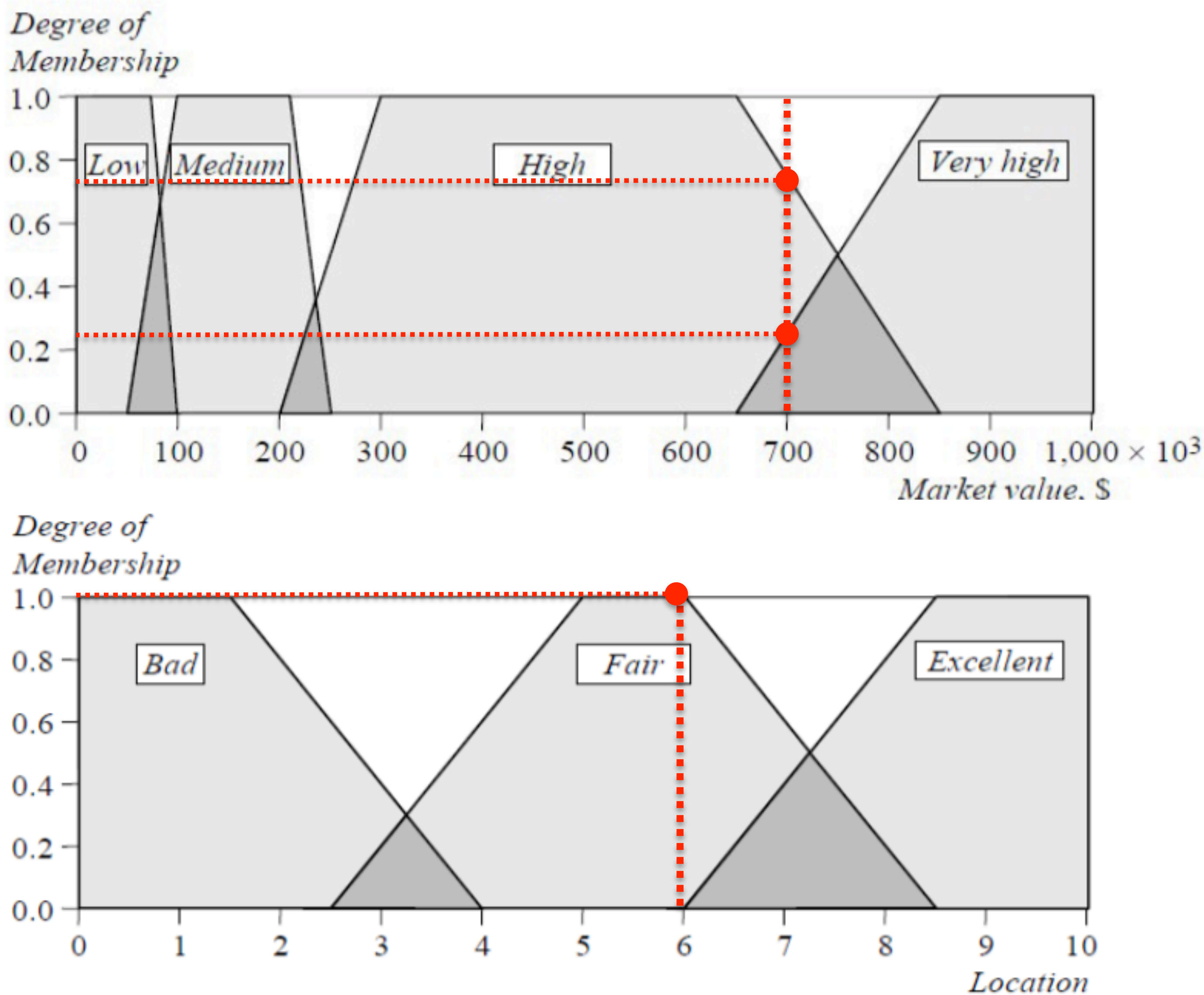
market value = 700

$\alpha_1 = 0.75$

$\alpha_2 = 0.25$

location = 6

$\alpha_3 = 1.0$

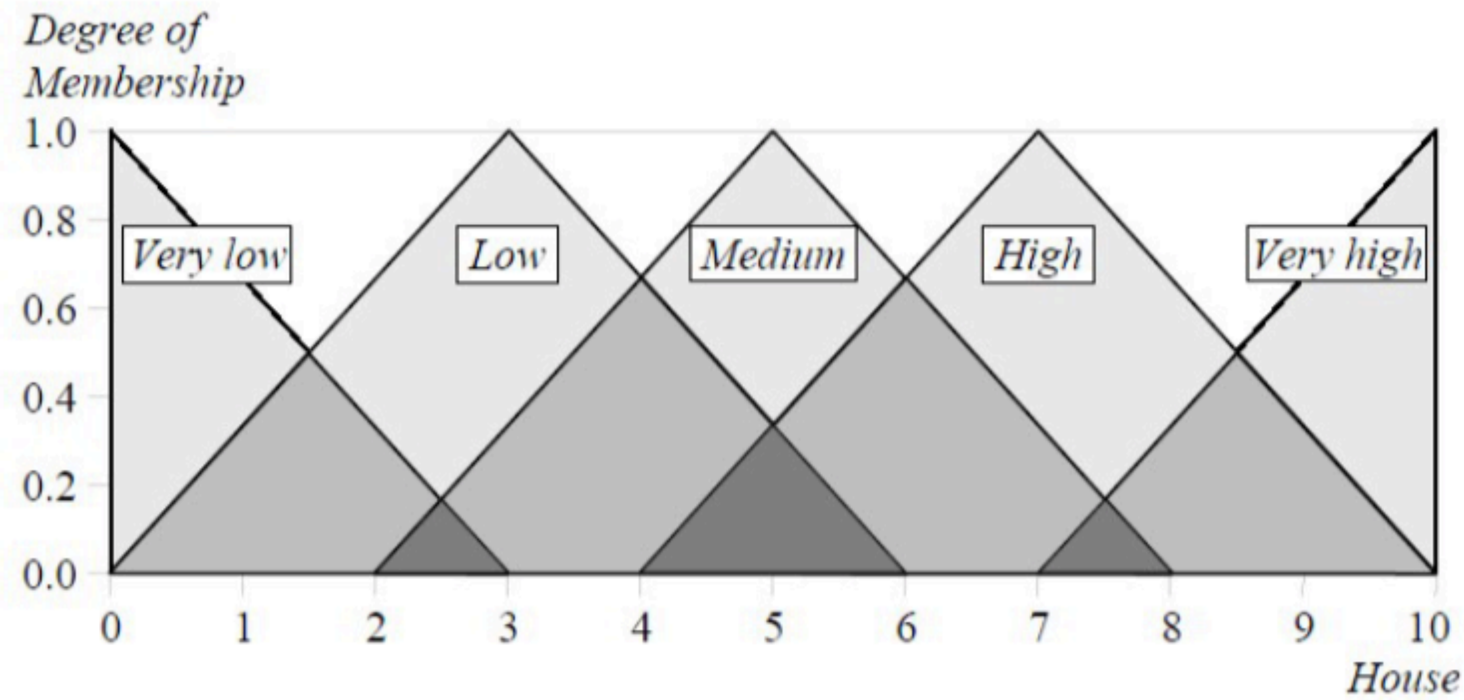


Second Project

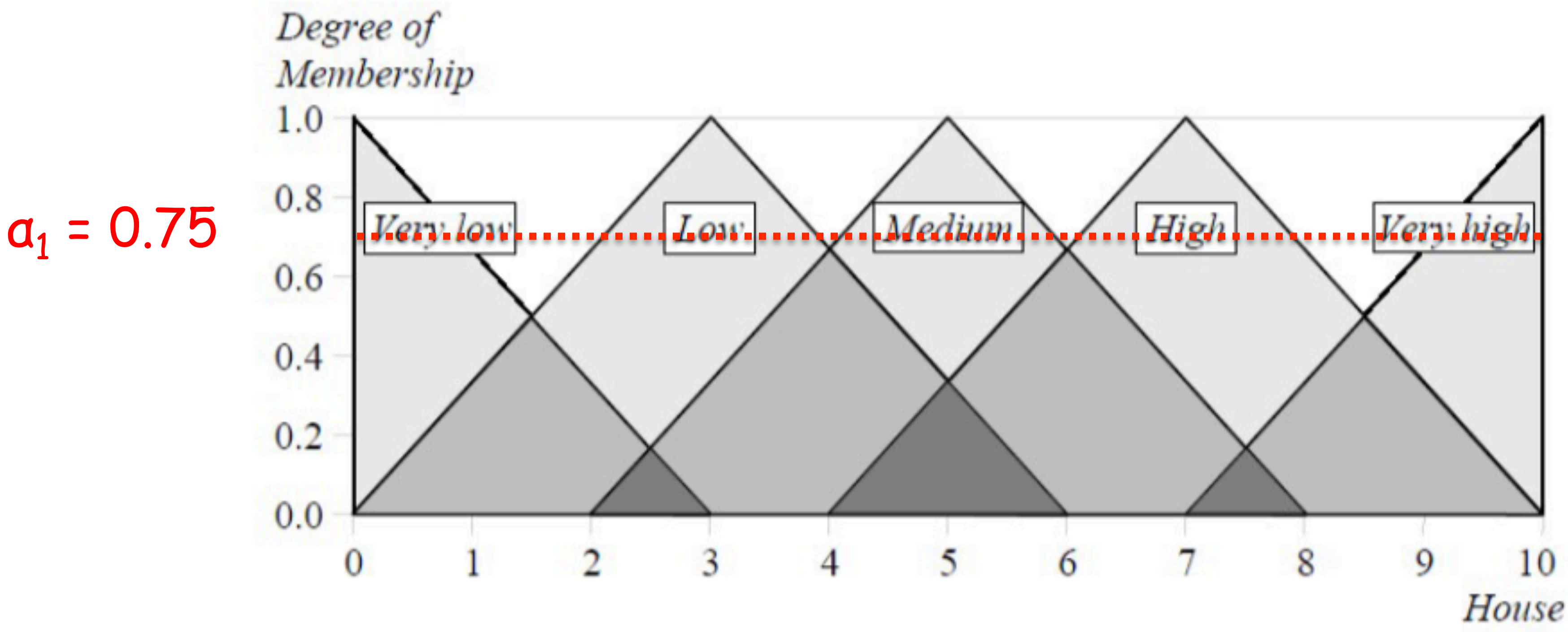
1. House Evaluation

1. If (Market_value is Low) then (House is Low)
2. If (Location is Bad) then (House is Low)
3. If (Location is Bad) and (Market_value is Low) then (House is Very_low)
4. If (Location is Bad) and (Market_value is Medium) then (House is Low)
5. If (Location is Bad) and (Market_value is High) then (House is Medium)
6. If (Location is Bad) and (Market_value is Very_high) then (House is High)
7. If (Location is Fair) and (Market_value is Low) then (House is Low)
8. If (Location is Fair) and (Market_value is Medium) then (House is Medium)
9. If (Location is Fair) and (Market_value is High) then (House is High)
10. If (Location is Fair) and (Market_value is Very_high) then (House is Very_high)
11. If (Location is Excellent) and (Market_value is Low) then (House is Medium)
12. If (Location is Excellent) and (Market_value is Medium) then (House is High)
13. If (Location is Excellent) and (Market_value is High) then (House is Very_high)
14. If (Location is Excellent) and (Market_value is Very_high) then (House is Very_high)

Second Project



Second Project



Second Project

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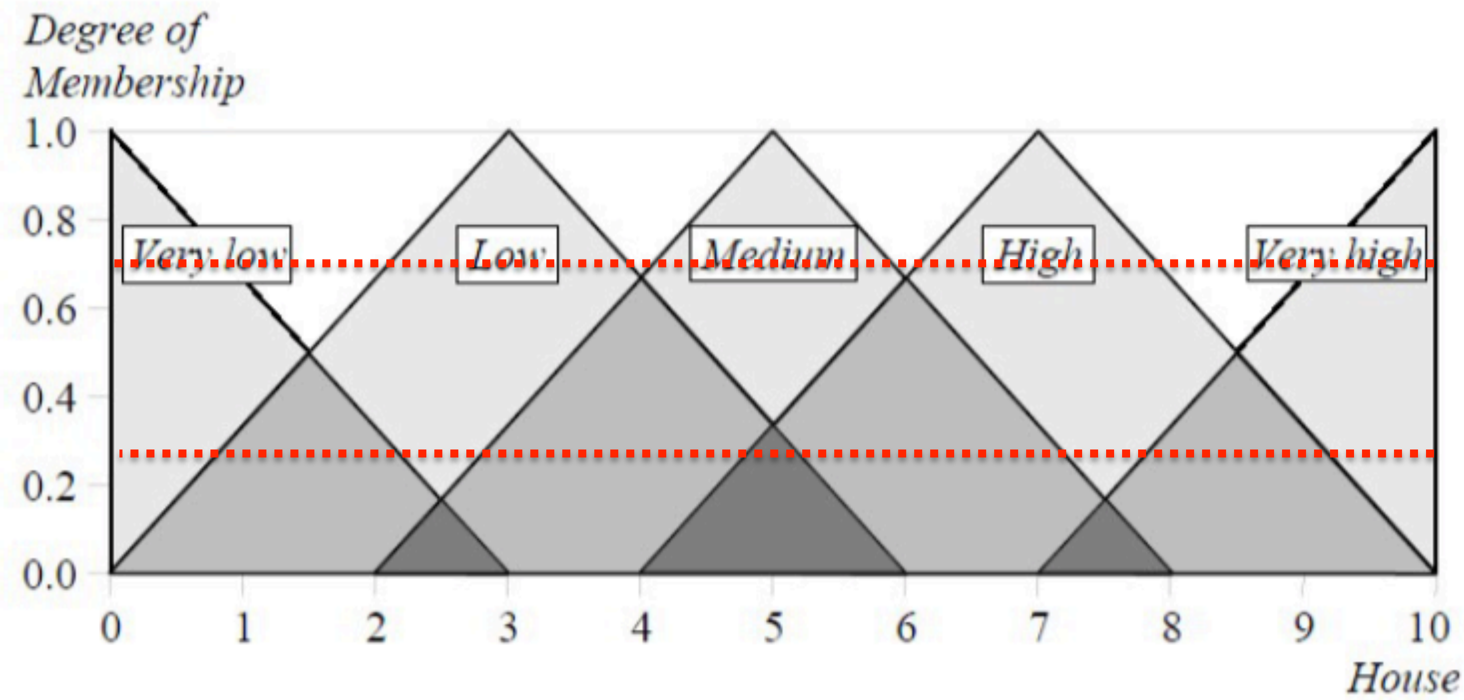
$\alpha_2 = 0.25$



Second Project

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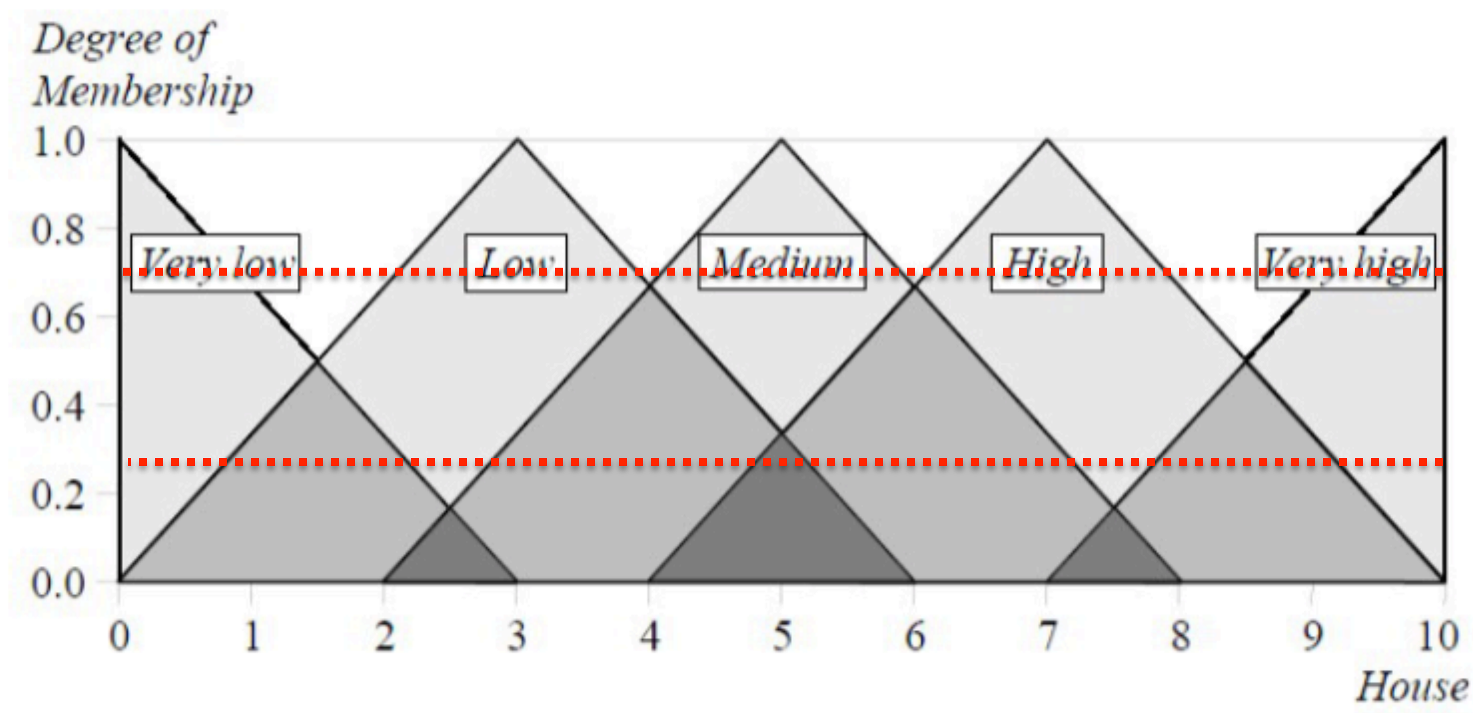
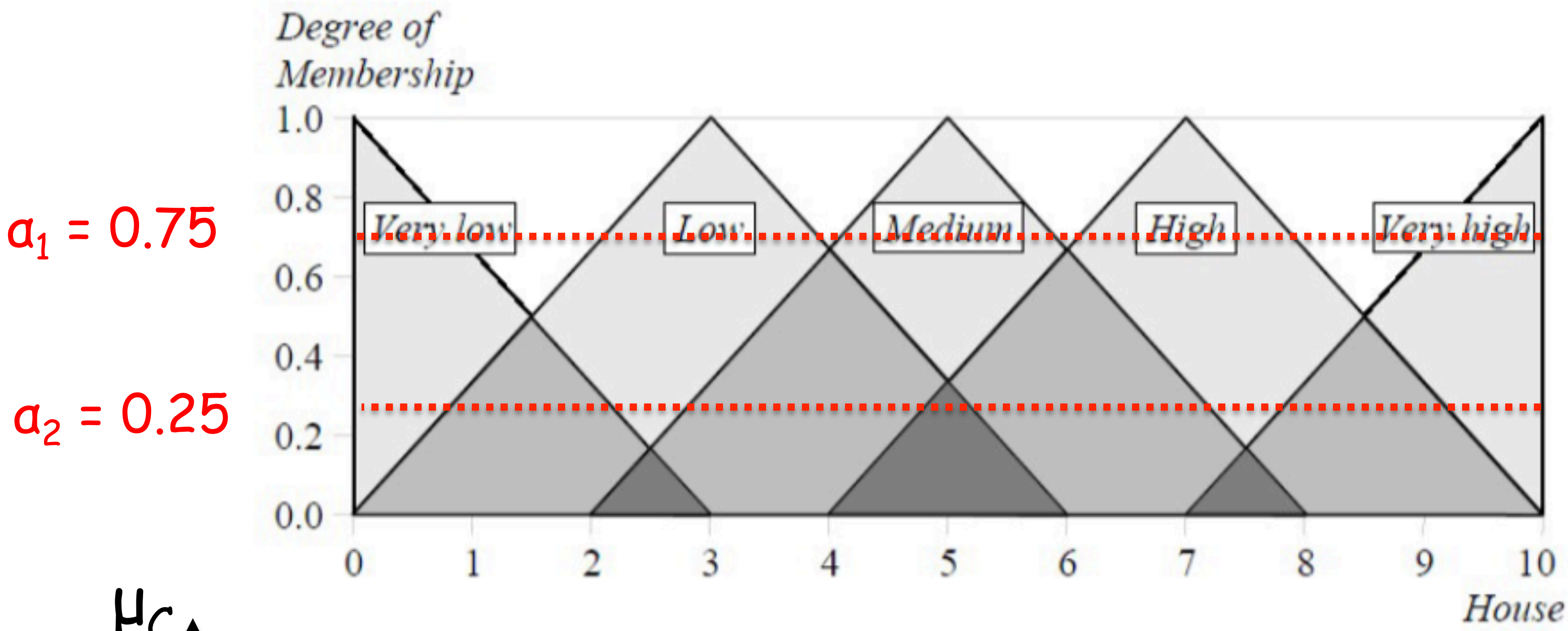


Mamdani

Second Project



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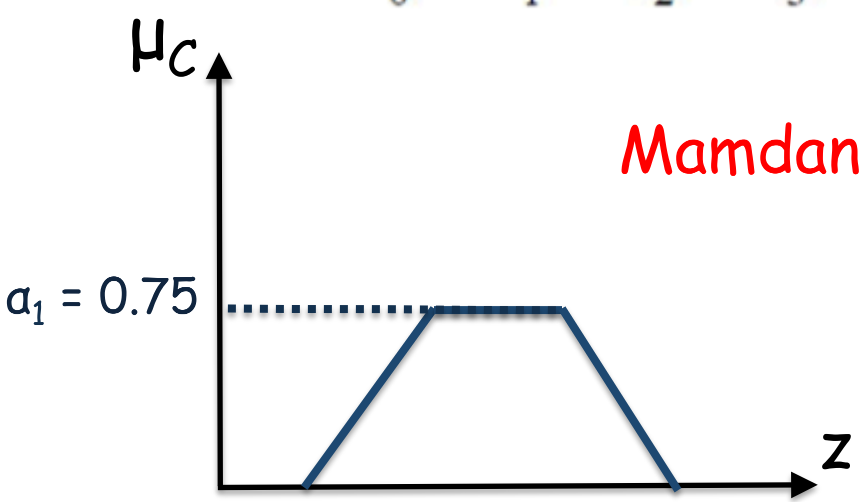

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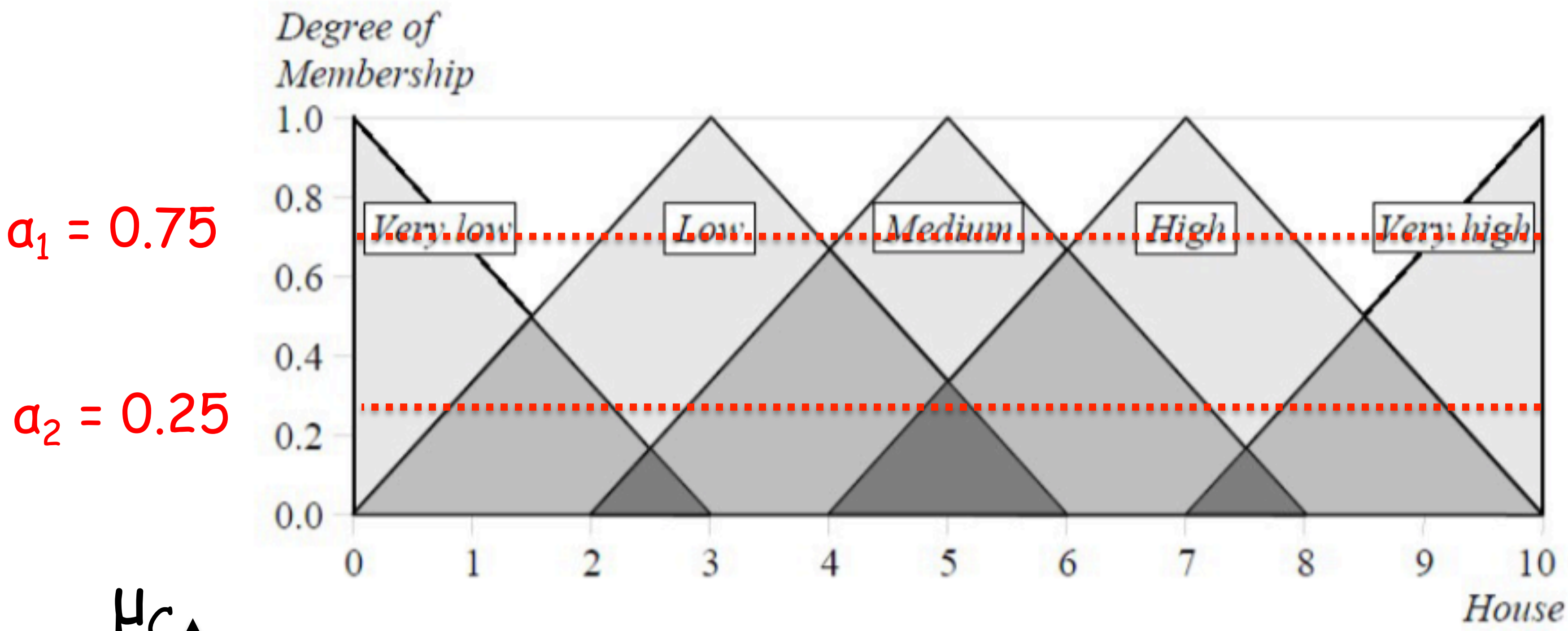

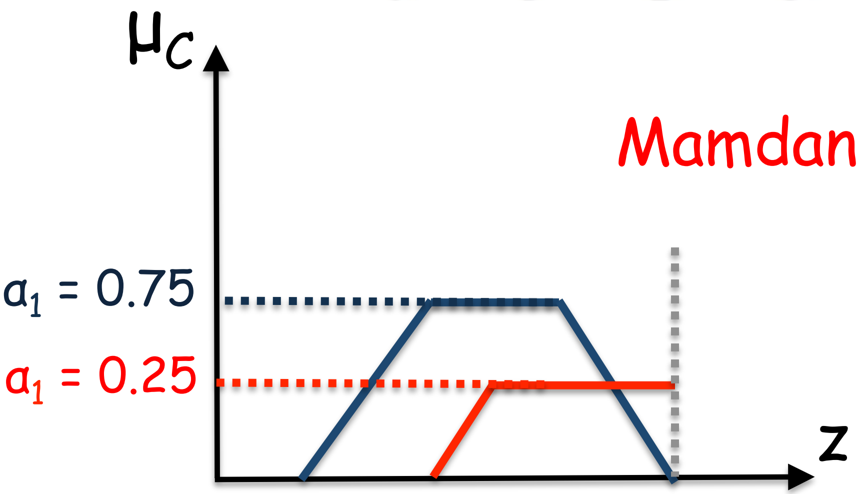

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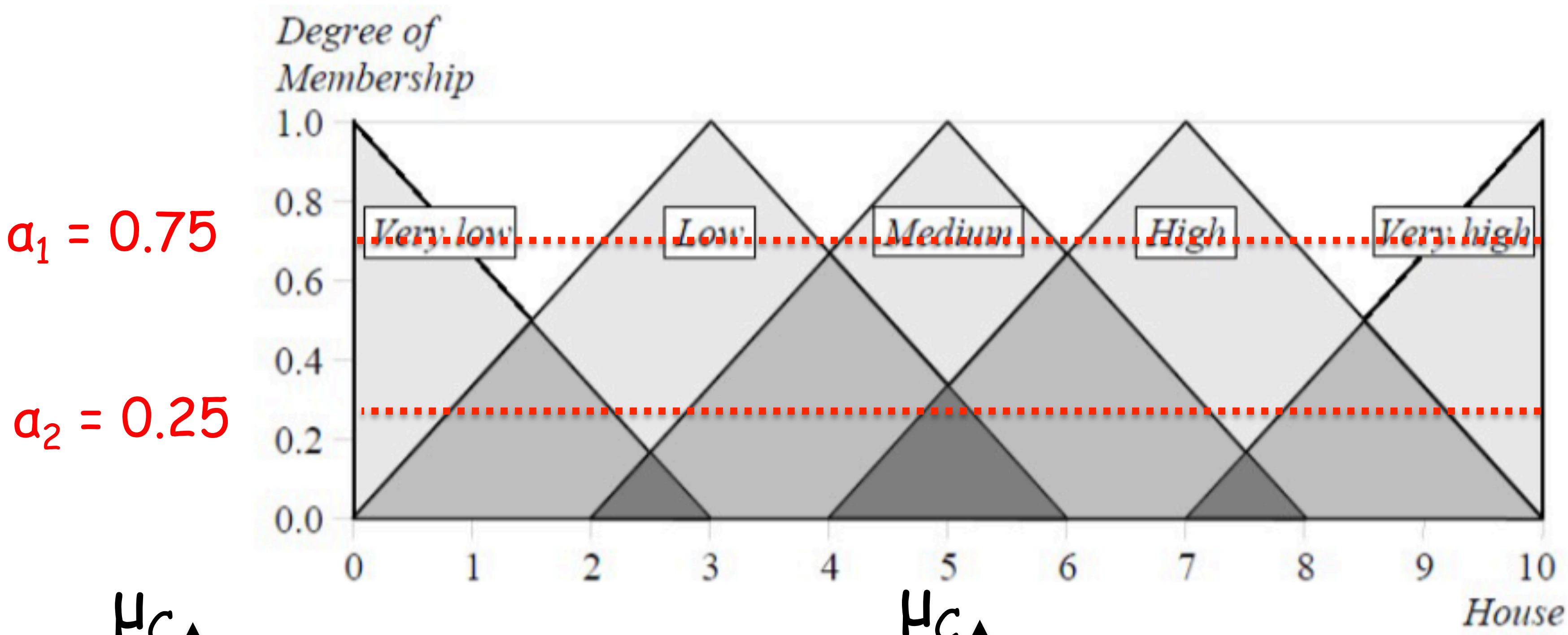

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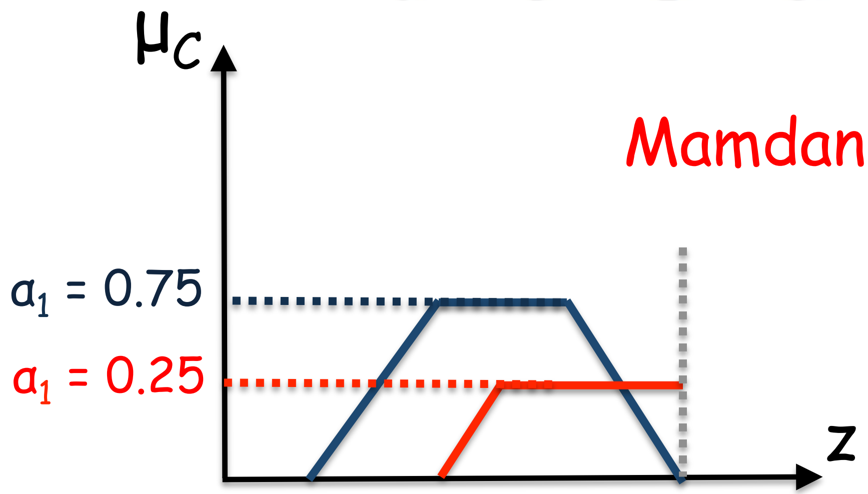

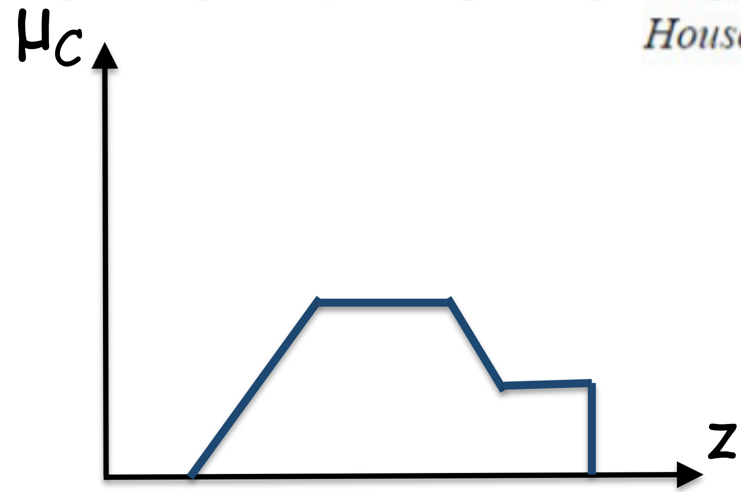

Second Project

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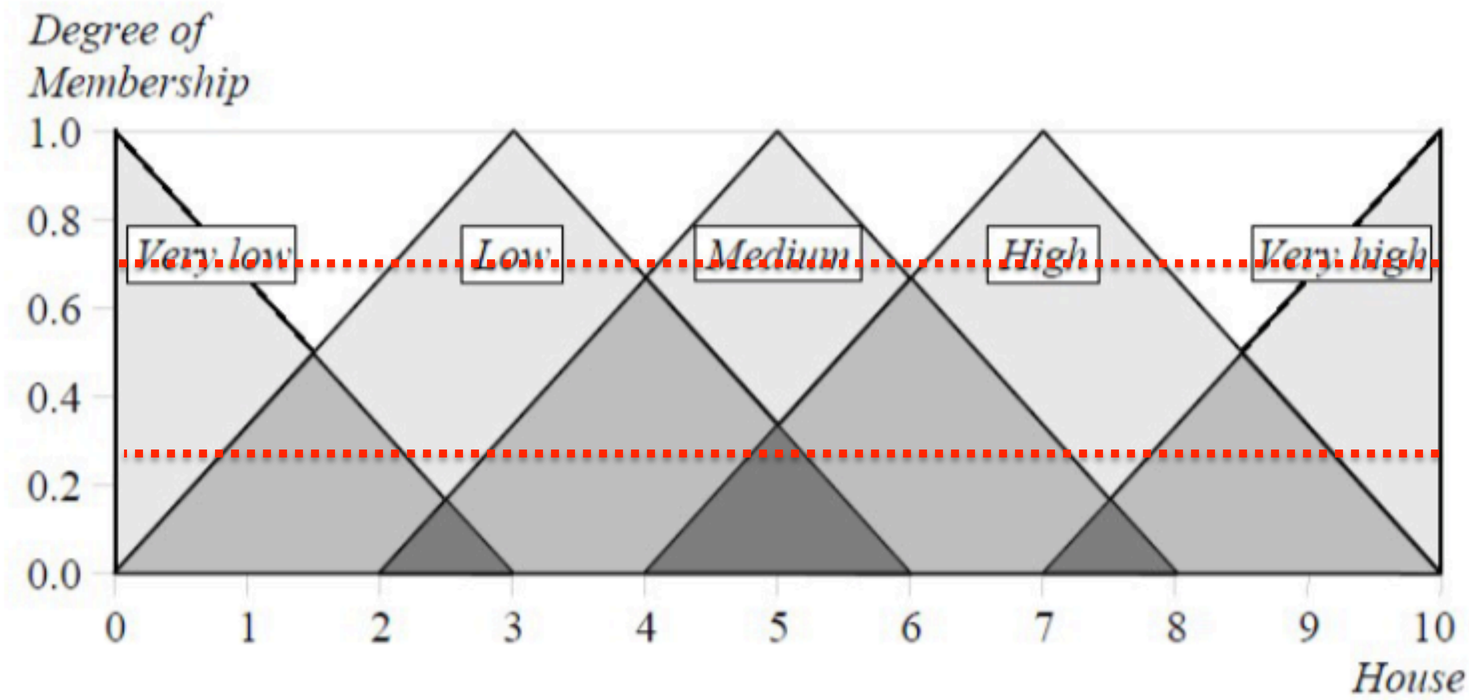
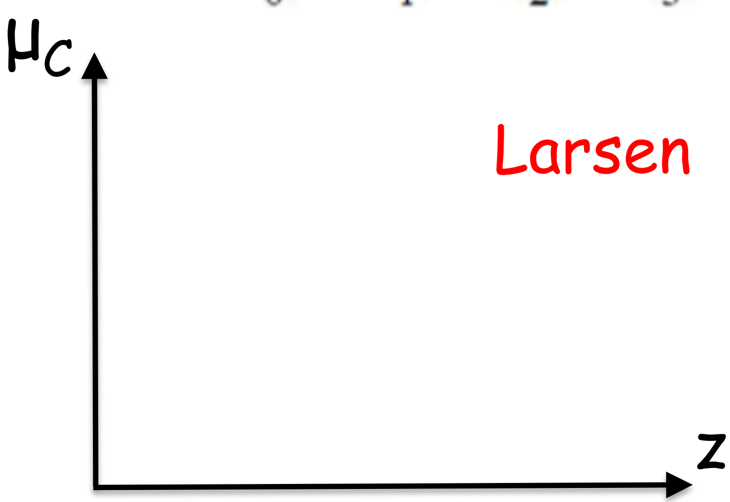


Larsen



Second Project

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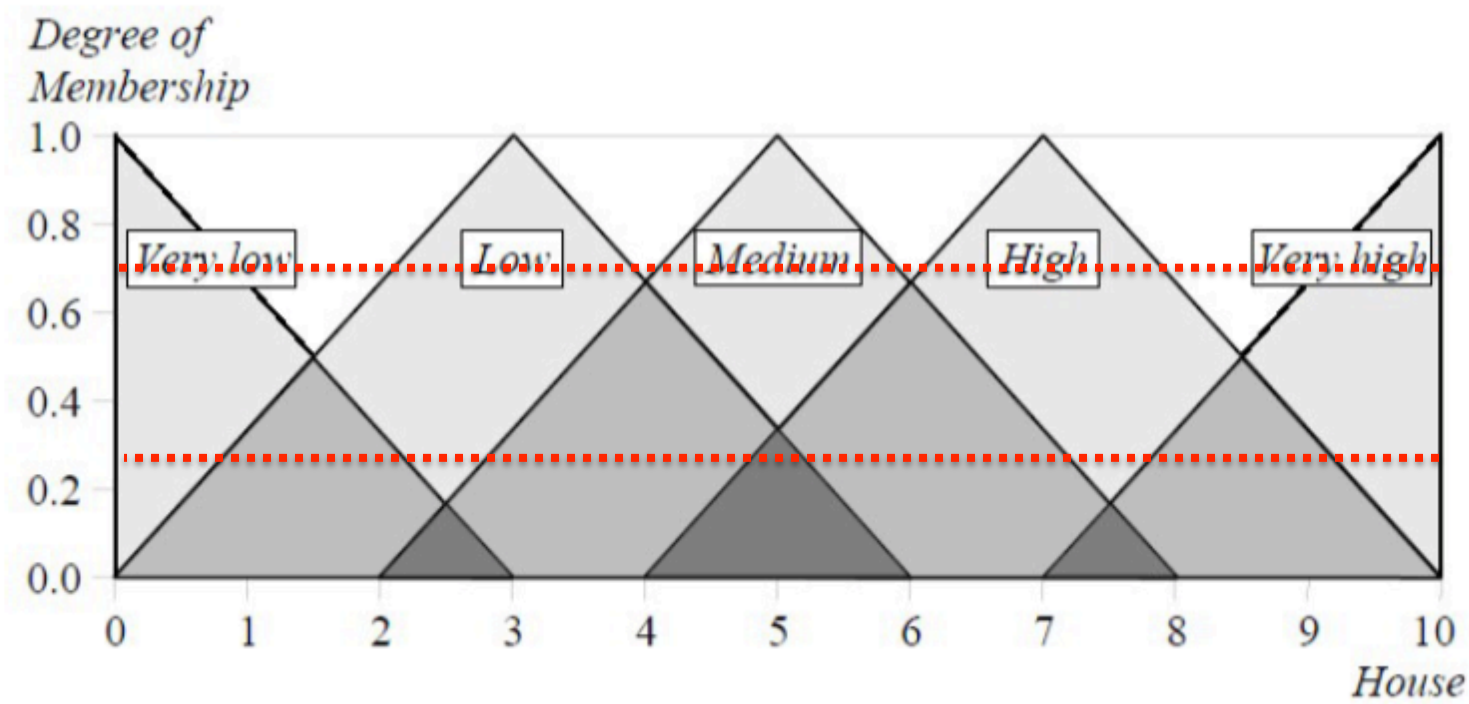

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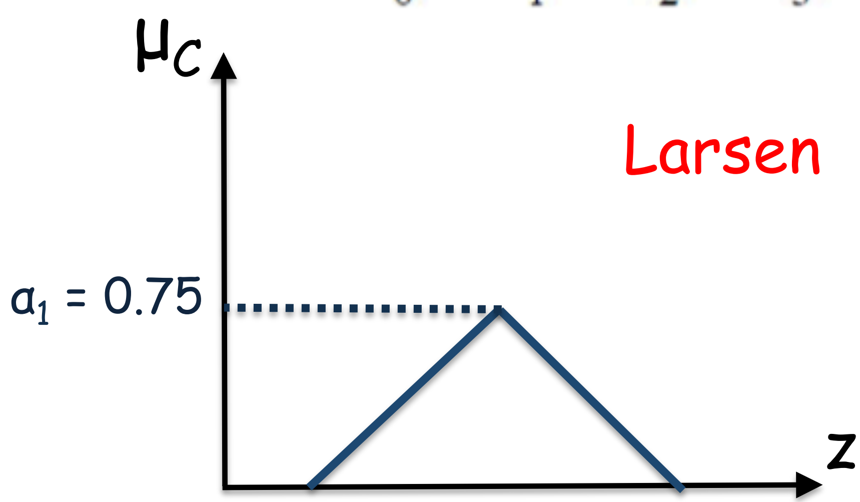
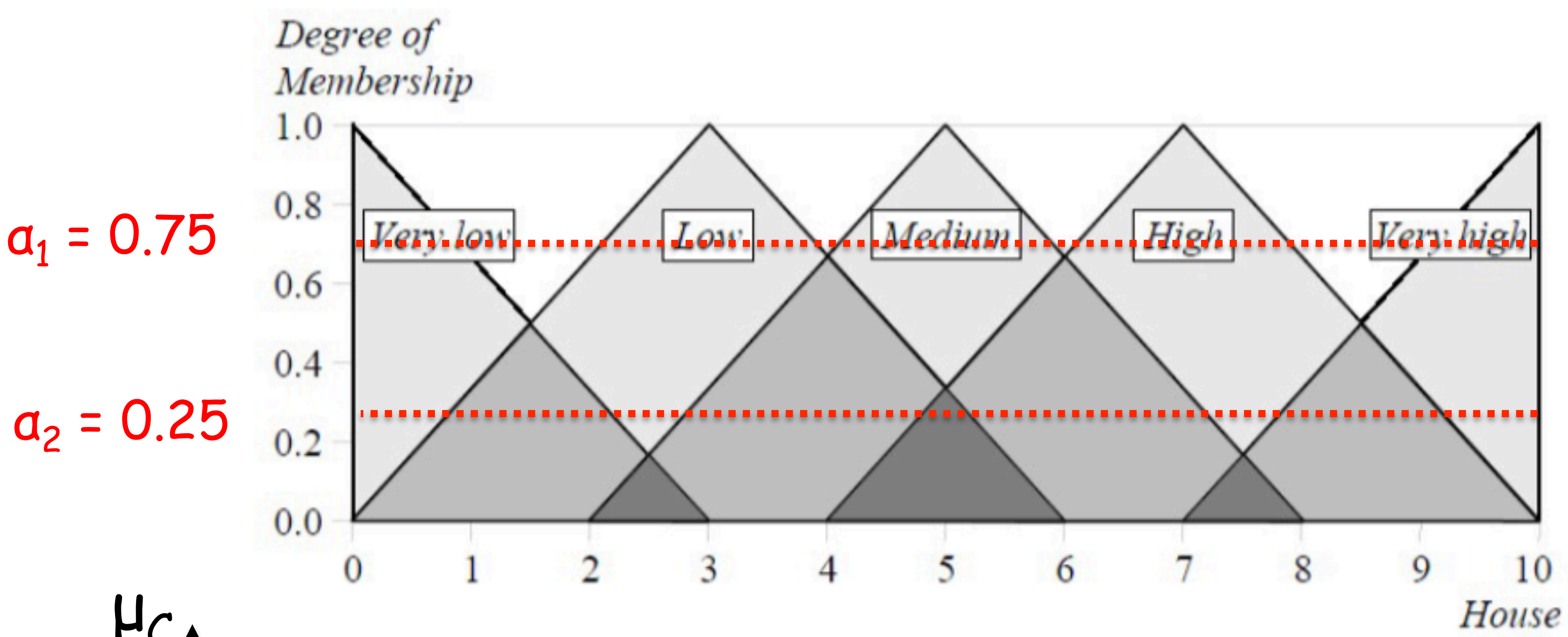
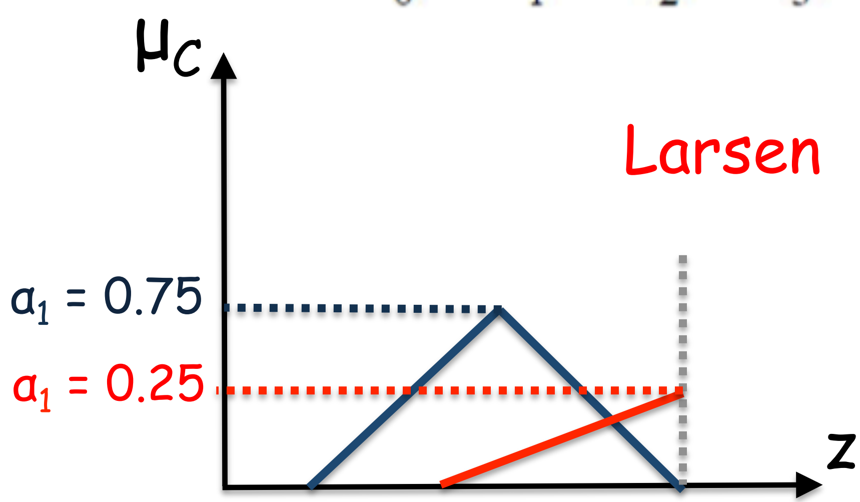


Second Project





Second Project