STA250 Probability and Statistics

Chapter 10 Notes

Hypothesis Testing

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STA250 Probability and Statistics

Reference Book

This lecture notes are prepared according to the contents of

“**PROBABILITY & STATISTICS FOR ENGINEERS & SCIENTISTS** by Walpole, Myers, Myers and Ye”

Definition

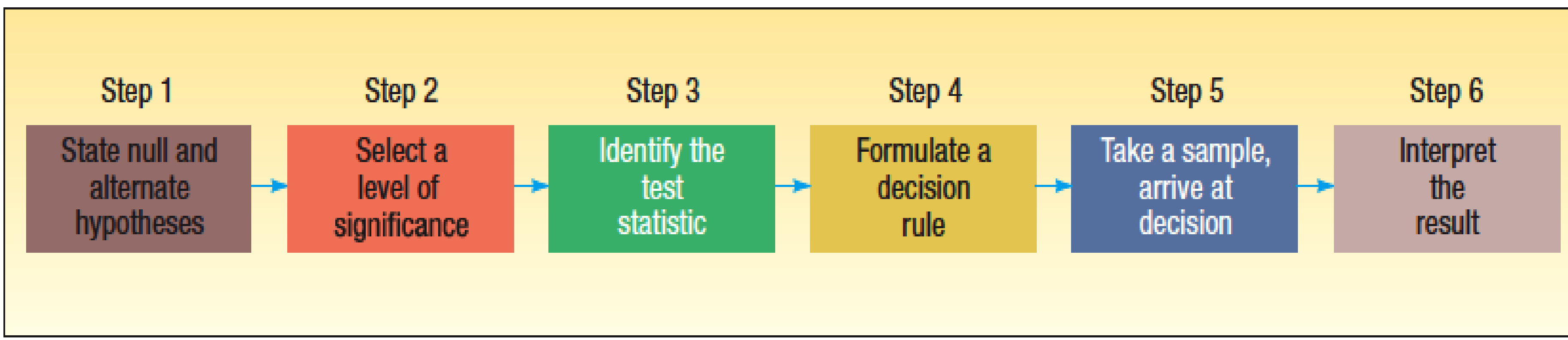
- **A hypothesis is a claim (assumption) about a population parameter.**

- **Examples:**
  - **<u>Population mean</u>**
  - The mean monthly cell phone bill of this city is $\mu = 100TL$

  - **<u>Population proportion</u>**
  - The proportion of adults in this city with cell phones is $p = 0.68$

Hypothesis Testing

- The objective of hypothesis testing is to verify the validity of a statement about a population parameter
- A procedure based on sample evidence and probability theory to determine whether the hypothesis is a reasonable statement.



Hypothesis Testing-Step 1

- State the null hypothesis (H_0) and the alternate hypothesis (H_1)
- Null Hypothesis: A statement about the value of a population parameter developed for the purpose of testing numerical evidence.
  - The null hypothesis is always includes the equal sign
    - For example: =, ≥, or ≤ will be used in H0
- Alternative Hypothesis: A statement that is accepted if the sample data provide sufficient evidence that the null hypothesis is false.
  - The alternate hypothesis never includes the equal sign
    - For example; ≠, <, or > is used in H1

Null Hypothesis, H0

- Begin with the assumption that the null hypothesis is true
  - Similar to the notion of innocent until proven guilty
- Always contains "=" , "≤" or "≥" sign
- May or may not be rejected
  - Examples:
    - The average number of TV sets in U.S. Homes is equal to three ($H_0: \mu = 3$)
    - STA 249 course midterm grade average is 50 ($H_0: \mu = 50$).
- Is always about a population parameter, not about a sample statistic

$$H_0: \mu = 3 \qquad \cancel{H_0: \bar{\boldsymbol{x}} = 3}$$

Alternative Hypothesis, H_1

- Is the opposite of the null hypothesis

  - Examples:
    - The average number of TV sets in U.S. homes is not equal to 3 $(H_1: \mu \neq 3)$.
    - STA 249 course midterm grade average is smaller than 50 $(H_1: \mu > 50)$.

- Never contains the "=" , "≤" or "≥" sign.
- May or may not be supported
- Is generally the hypothesis that the researcher is trying to support

Level of Significance, α- Step 2

- **Defines the unlikely values of the sample statistic if the null hypothesis is true**
  - Defines rejection region of the sampling distribution
- Is designated by α, (level of significance)
- Typical values are .01, .05, or .10
- Is selected by the researcher at the beginning
- Provides the critical value(s) of the test

Level of Significance and Rejection Region

Level of significance = α

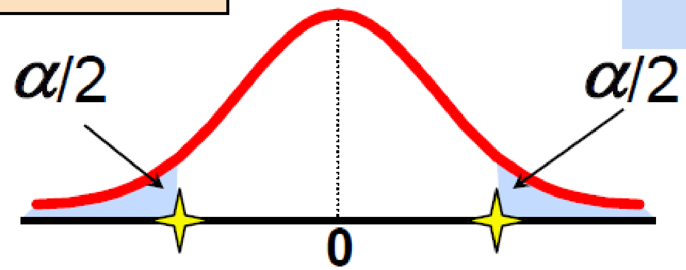
Represents critical value

Rejection region is shaded

H_0: $\mu = 3$

H_1: $\mu \neq 3$

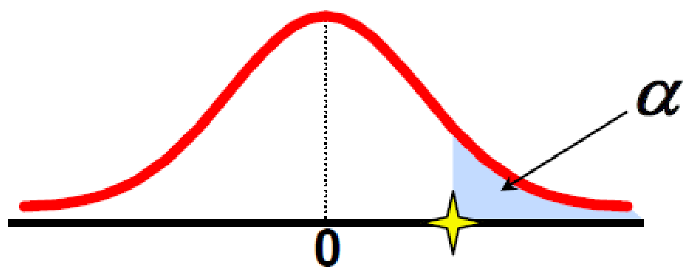
Two-tail test



H_0: $\mu \leq 3$

H_1: $\mu > 3$

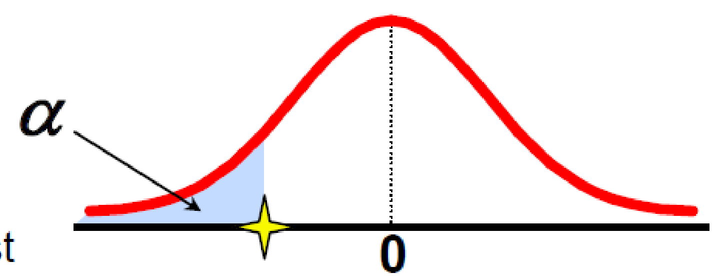
Upper-tail test



H_0: $\mu \geq 3$

H_1: $\mu < 3$

Lower-tail test

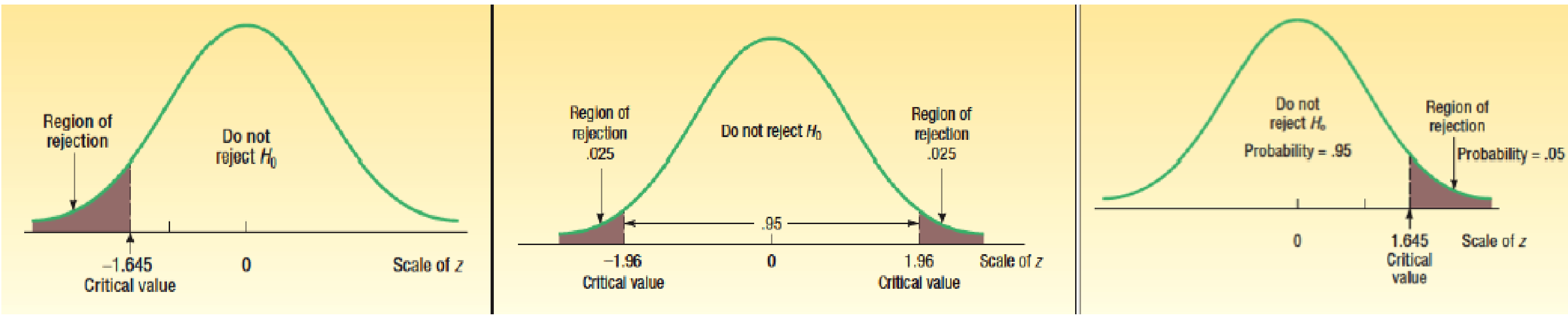


One-Tailed and Two-Tailed Tests

H_0: ≥ 60,000 miles
H_1: < 60,000 miles
with an $\alpha = .05$
Left-tailed test

H_0: = \$65,000 per year
H_1: ≠ \$65,000 per year
with an $\alpha = .05$
Two-tailed test

H_0: ≤ 453 grams
H_1: > 453 grams
with an $\alpha = .05$
Right-tailed test



Note that the total area in the normal distribution is 1.0000.

Test Statistic, Step 3

- Then, select the test statistic

TEST STATISTIC A value, determined from sample information, used to determine whether to reject the null hypothesis.

- In hypothesis testing for the mean, μ, when σ is known, the test statistic z is computed with the following formula

TESTING A MEAN, σ KNOWN

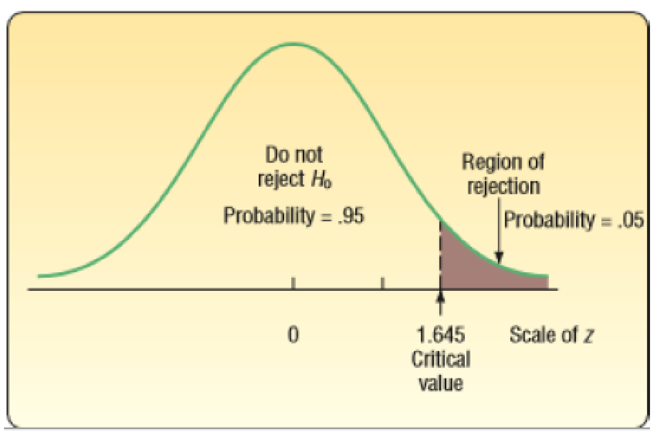
$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} \qquad (10\text{–}1)$$

- We can determine whether the distance between $\bar{x}$ and μ is statistically significant by finding the number of standard deviations $\bar{x}$ is from μ

Step 4 of the Process

- **Formulate the decision rule**
- **Critical Value :** The dividing point between the region where the null hypothesis is rejected and the region where it is not rejected.

- The sampling distribution of the statistic z follows the normal distribution
- Here, an α of .05 is used in a one-tailed test
- The value 1.645 separates the regions where the null hypothesis is rejected and where it is not rejected
- The value 1.645 is the critical value



Step 5 and Step 6 of the Six-Step Process

- **Step 5 Make a decision**
  - First, select a sample and compute the value of the test statistic
  - Compare the value of the test statistic to the critical value
  - Then, make the decision regarding the null hypothesis

- **Step 6 Conclude(H_1) & Interpret the results**
  - What can we say or report based on the results of the statistical test?

Error is Making Decisions

- **There are two types of error that can be made when testing a null hypothesis.**
  - Type I error: Rejecting the null hypothesis when it is true.
  - Type II error: Accepting the null hypothesis when it is false.
  - We define α = P(type I error), and β = P(type II error).
- **In hypothesis testing, we generally want to minimize α, the probability of making a type I error.**

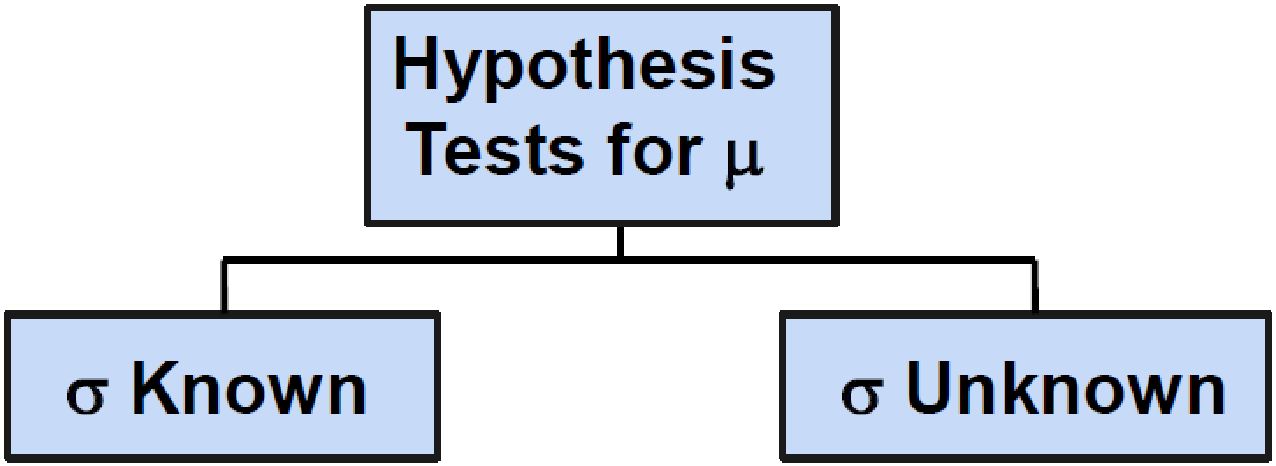
Power of Test

- **The power of a test** is the probability of rejecting a null hypothesis that is false.
  - Power = P(Reject H_0 | H_1 is true)
- Power of the test increases as the sample size increases

Summary of hypothesis test outcomes:

Reality ⇒ Decision ⇓	H_0 true	H_1 true
Do Not Reject H_0	No Error (1 - α)	Type II Error (β)
Reject H_0	Type I Error (α)	No Error (power = 1 - β)

Hypothesis Tests for the Mean



Hypothesis Tests for the Mean (σ Known)

- Convert sample result ($\bar{x}$) to a z value

Hypothesis Tests for μ

σ Known | **σ Unknown**

Consider the test

$$H_0 : \mu = \mu_0$$
$$H_1 : \mu > \mu_0$$

(Assume the population is normal)

The decision rule is:

$$\text{Reject } H_0 \text{ if } z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}} > z_\alpha$$

p-value Approach to Testing

- **A p-value** is the lowest level (of significance) at which the observed value of the test statistic is significant.
- The approach is designed to give the user an alternative (in terms of a probability) to a mere “reject” or “do not reject” conclusion.
- Convert sample result (e.g.,) to test statistic (e.g., z statistic)
- Obtain the p-value

  For an upper tail test:

$$\text{p-value} = P\left(Z > \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}, \text{ given that } H_0 \text{ is true}\right)$$

$$= P\left(Z > \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} \mid \mu = \mu_0\right)$$

- Decision rule: compare the p-value to α
  - If p-value $\leq \alpha$, reject H_0
  - If p-value $> \alpha$, do not reject H_0

Example 1-1

Example 10.3: A random sample of 100 recorded deaths in the United States during the past year showed an average life span of 71.8 years. Assuming a population standard deviation of 8.9 years, does this seem to indicate that the mean life span today is greater than 70 years? Use a 0.05 level of significance.

Solution:

1. H_0: $\mu = 70$ years.
2. H_1: $\mu > 70$ years.
3. $\alpha = 0.05$.
4. Critical region: $z > 1.645$, where $z = \frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}}$.
5. Computations: $\bar{x} = 71.8$ years, $\sigma = 8.9$ years, and hence $z = \frac{71.8-70}{8.9/\sqrt{100}} = 2.02$.
6. Decision: Reject H_0 and conclude that the mean life span today is greater than 70 years.

The P-value corresponding to $z = 2.02$ is given by the area of the shaded region in Figure 10.10.

Using Table A.3, we have

$$P = P(Z > 2.02) = 0.0217.$$

As a result, the evidence in favor of H_1 is even stronger than that suggested by a 0.05 level of significance.

Example 1-2

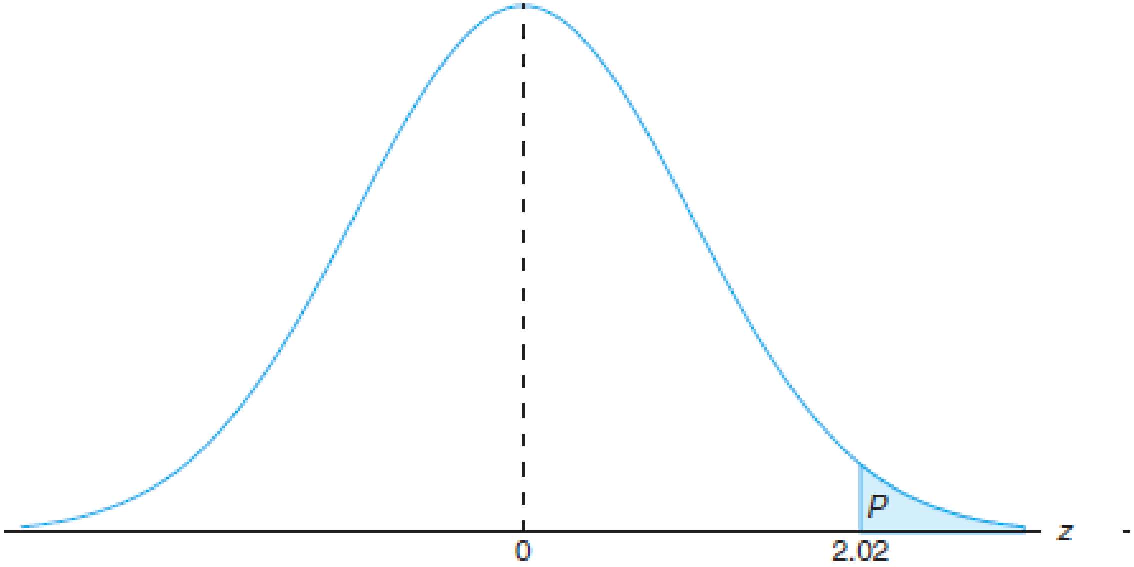


Figure 10.10: P-value for Example 10.3.

Example 2-1

Example 10.4: A manufacturer of sports equipment has developed a new synthetic fishing line that the company claims has a mean breaking strength of 8 kilograms with a standard deviation of 0.5 kilogram. Test the hypothesis that $\mu = 8$ kilograms against the alternative that $\mu \neq 8$ kilograms if a random sample of 50 lines is tested and found to have a mean breaking strength of 7.8 kilograms. Use a 0.01 level of significance.

Solution:

1. H_0: $\mu = 8$ kilograms.
2. H_1: $\mu \neq 8$ kilograms.
3. $\alpha = 0.01$.
4. Critical region: $z < -2.575$ and $z > 2.575$, where $z = \frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}}$.
5. Computations: $\bar{x} = 7.8$ kilograms, $n = 50$, and hence $z = \frac{7.8-8}{0.5/\sqrt{50}} = -2.83$.
6. Decision: Reject H_0 and conclude that the average breaking strength is not equal to 8 but is, in fact, less than 8 kilograms.

Since the test in this example is two tailed, the desired P-value is twice the area of the shaded region in Figure 10.11 to the left of $z = -2.83$. Therefore, using Table A.3, we have

$$P = P(|Z| > 2.83) = 2P(Z < -2.83) = 0.0046,$$

which allows us to reject the null hypothesis that $\mu = 8$ kilograms at a level of significance smaller than 0.01.

Example 2-2

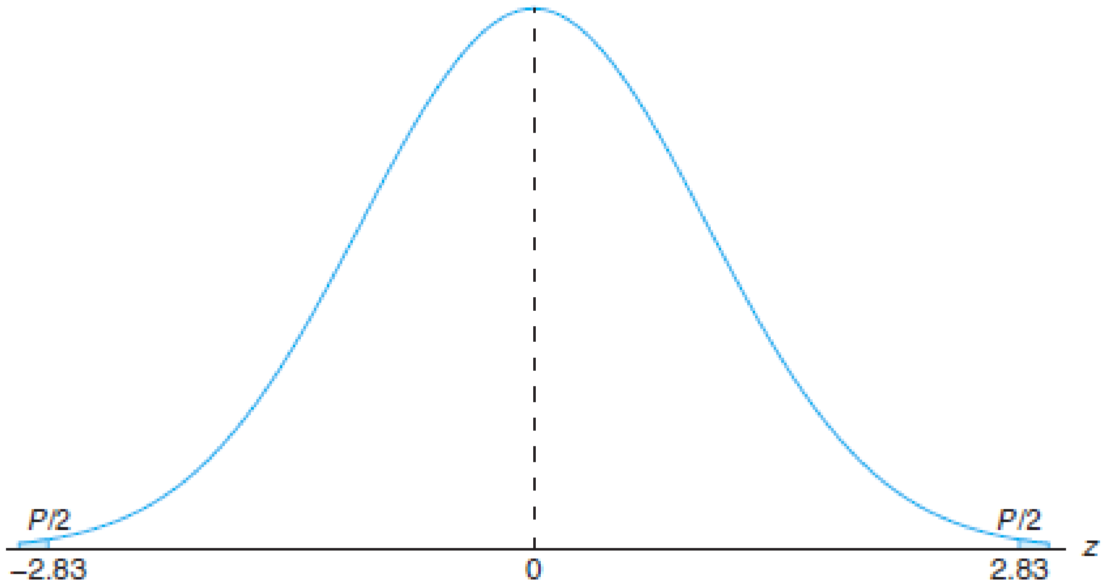


Figure 10.11: P-value for Example 10.4.

Example 3-1

A phone industry manager thinks that customer monthly cell phone bill have increased, and now average over $52 per month. The company wishes to test this claim. (Assume $\sigma = 10$ is known)

Form hypothesis test:

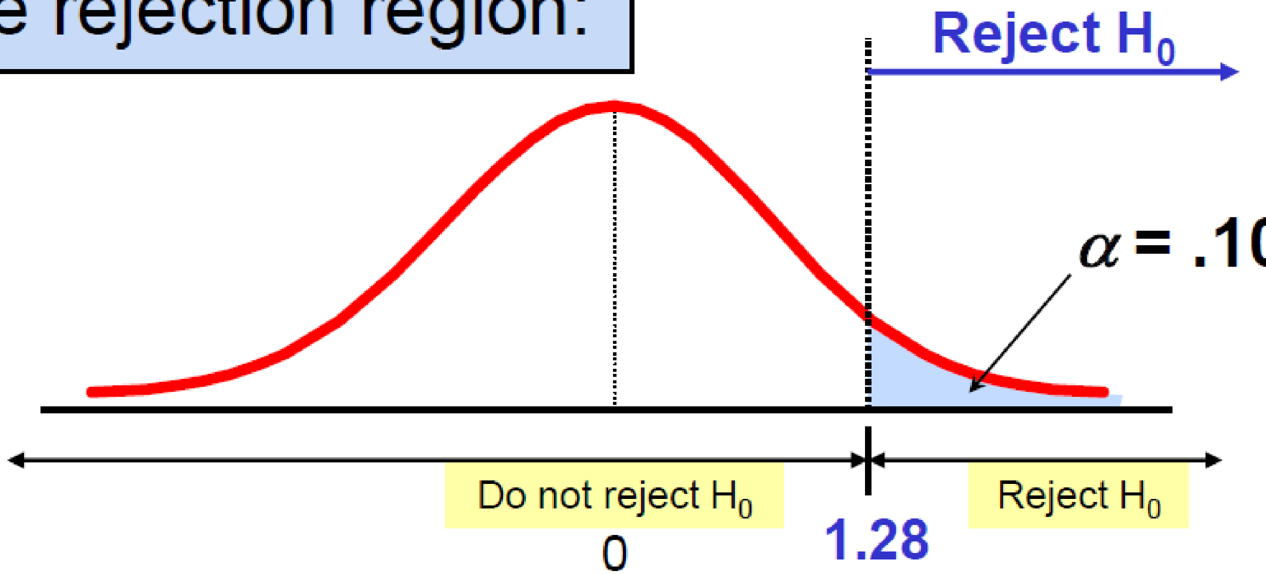
H_0: $\mu \leq 52$ the average is not over $52 per month

H_1: $\mu > 52$ the average **is** greater than $52 per month (i.e., sufficient evidence exists to support the manager's claim)

Example 3-2

- Suppose that α = .10 is chosen for this test

Find the rejection region:



$$\text{Reject } H_0 \text{ if } z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} > 1.28$$

Example 3-3

Obtain sample and compute the test statistic

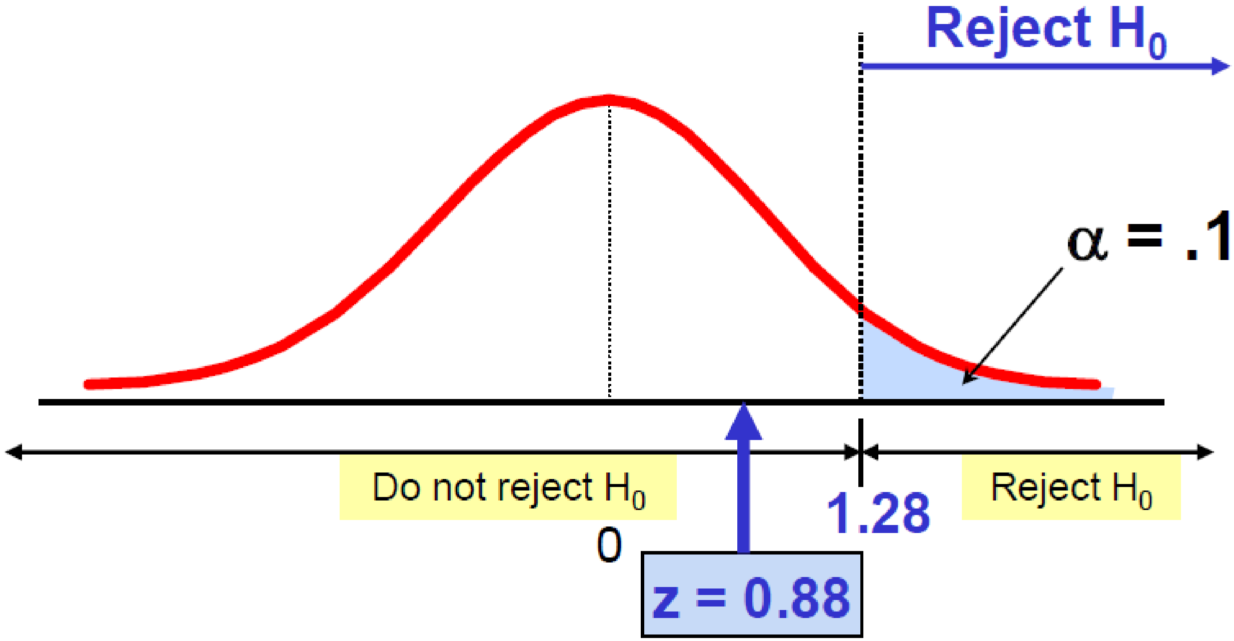
Suppose a sample is taken with the following results: $n = 64$, $\bar{x} = 53.1$ (σ=10 was assumed known)

- Using the sample results,

$$z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{53.1 - 52}{\frac{10}{\sqrt{64}}} = 0.88$$

Example 3-4

Reach a decision and interpret the result:



Do not reject H_0 since $z = 0.88 < 1.28$

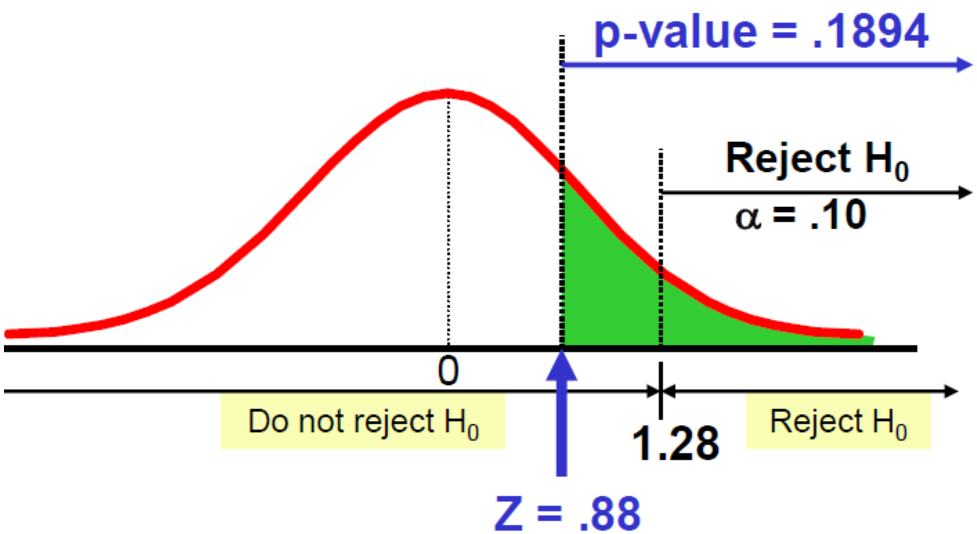
i.e.: there is not sufficient evidence that the mean bill is over \$52

Example: p-Value Solution

(continued)

Calculate the p-value and compare to α

(assuming that μ = 52.0)



$$P(\bar{x} \geq 53.1 \mid \mu = 52.0)$$

$$= P\left(z \geq \frac{53.1 - 52.0}{10/\sqrt{64}} \right)$$

$$= P(z \geq 0.88) = 1 - .8106$$

$$= .1894$$

Do not reject H_0 since p-value = .1894 > α = .10

Next Lesson

- **Hypothesis Testing**

See you☺