Uncertainty

Murat Osmanoglu

Extension of Fuzzy Set

$$R \subseteq A \times B$$

crisp relation fuzzy set crisp/fuzzy set

Extension by Crisp Relation

- $B' \subseteq B$ induced by the crisp relation R and the fuzzy set A:

$$B' = \{(y, \mu_{B'}(y)) \mid \mu_{B'}(y) = \max_{x \text{ s.t. } (x,y) \text{ in } R} \mu_A(x)\}$$

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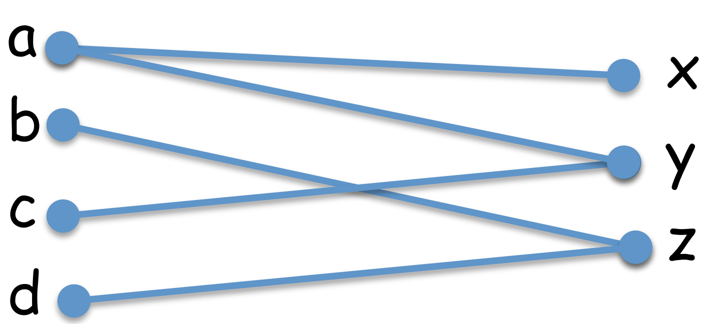

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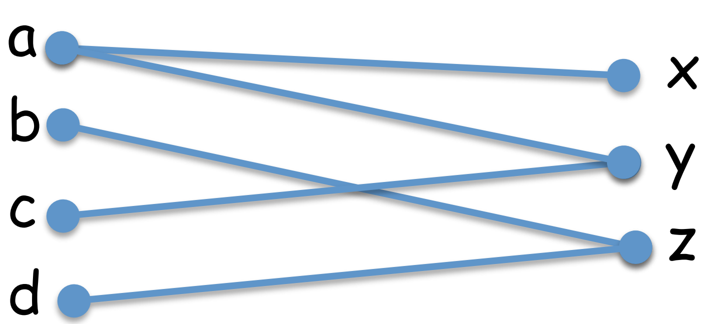

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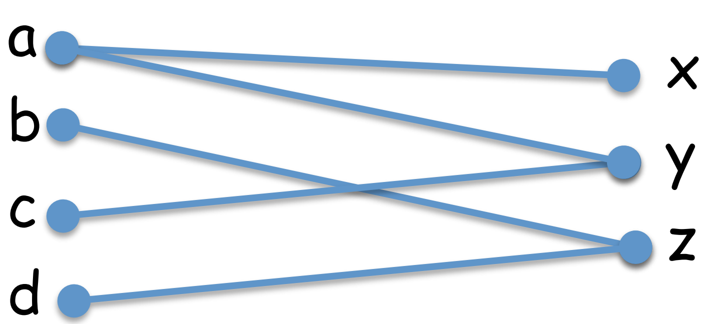

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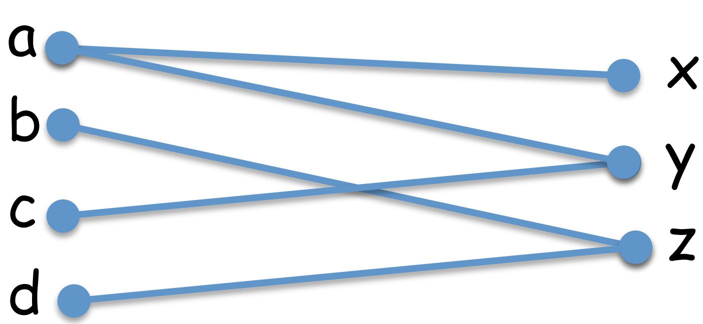

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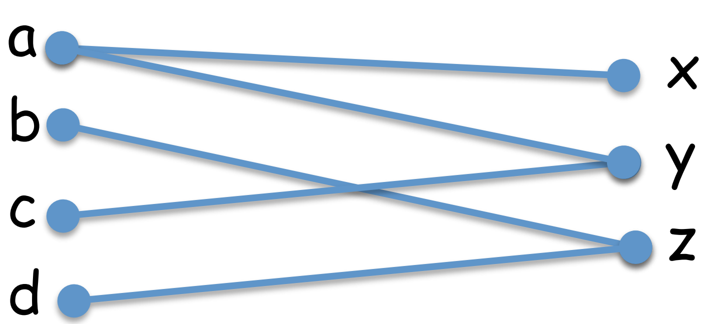

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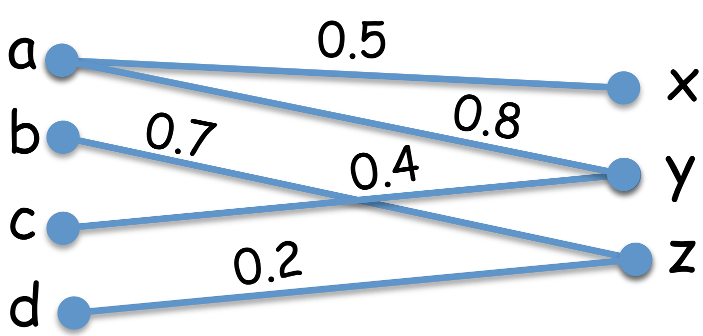

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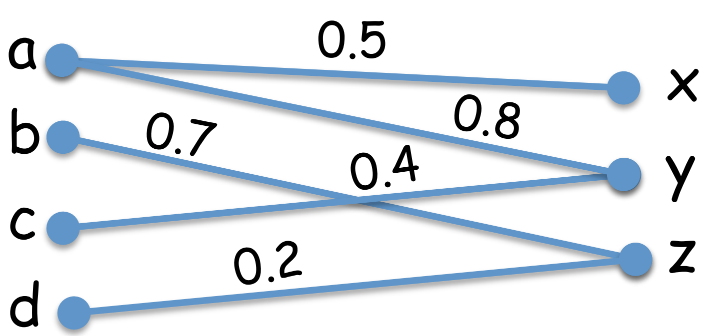

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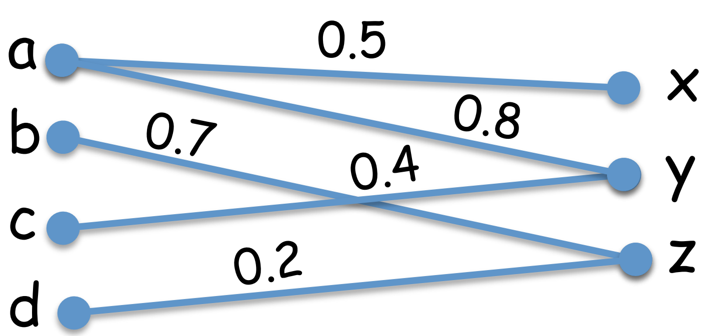

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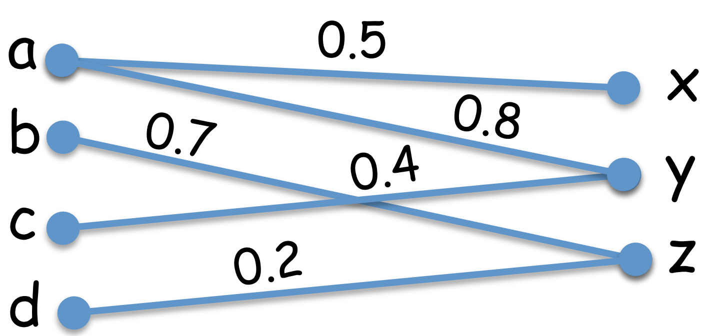

B' = {(x, 0.5), (y, 0.6), (z, 0.7)}

Fuzzy Distance

- calculate the fuzzy distance between the fuzzy sets
  $A = \{(1, 0.5), (2, 1.0), (3, 0.7)\}$ and $B = \{(3, 0.6), (4, 1.0), (5, 0.3)\}$

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Probability vs. Possibility

Probability Distribution

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- no restriction

- A, B, C, D organize a chess tournament. The following table shows the probabilities and the possibilities of the players on the tournament

	A	B	C	D
$P(x)$	0.5	0.3	0.2	0
$\mu(x)$	1.0	0.7	0.4	0.1

$p(x) \le \mu(x)$

Fuzzy Event

Crisp Probability of Fuzzy Event

- consider the sample space $S = \{a, b, c, d\}$ with the probabilities

$$p(a) = 0.4,\ p(b) = 0.2,\ p(c) = 0.1,\ p(d) = 0.3$$

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$$p(A) = 0.4 + 0.2 + 0.1 = 0.7$$

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$$p(A) = 0.4\times0.5 + 0.2\times1.0 + 0.1\times0.3 = 0.43$$

Fuzzy Event

Fuzzy Probability of Fuzzy Event

- consider the sample space $S = \{a, b, c, d\}$ with the probabilities

$$p(a) = 0.4,\ p(b) = 0.2,\ p(c) = 0.1,\ p(d) = 0.3$$

Fuzzy Event

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- consider the sample space $S = \{a, b, c, d\}$ with the probabilities

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$$A_{0.3} = \{a, b, c\},\ A_{0.5} = \{a, b\},\ A_{1.0} = \{b\}$$

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$$p(A_{0.3}) = 0.7,\ \ p(A_{0.5}) = 0.6,\ \ p(A_{1.0}) = 0.2$$

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$$p(A_{0.3}) = 0.7,\ \ p(A_{0.5}) = 0.6,\ \ p(A_{1.0}) = 0.2$$

$$p(A) = \{(0.7, 0.3), (0.6, 0.5), (0.2, 1.0)\}$$

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- A, B, C, D organize a chess tournament. The following table shows the possibilities of the players on the tournament

	A	B	C	D
$\mu(x)$	1.0	0.7	0.4	0.1

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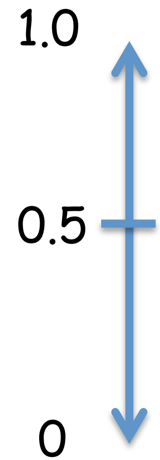

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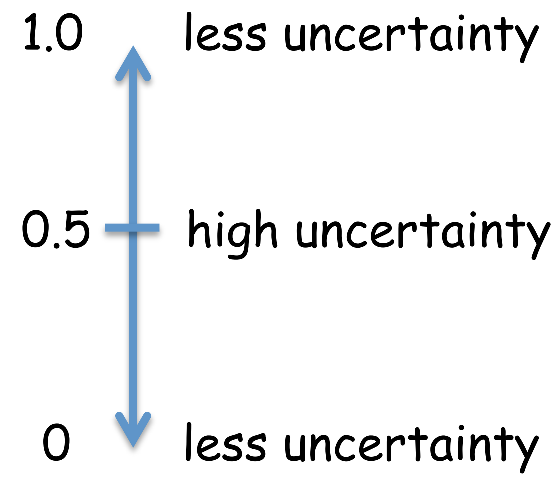

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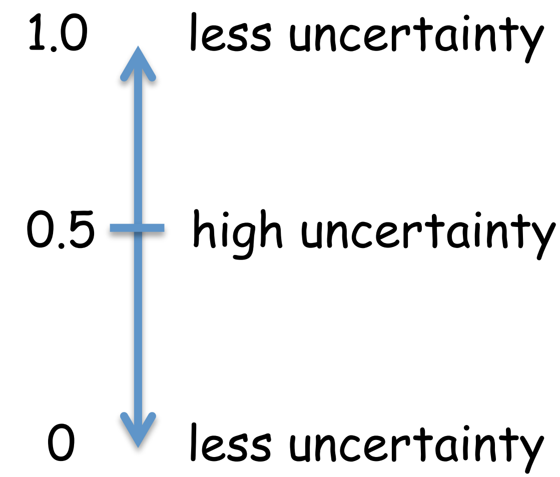

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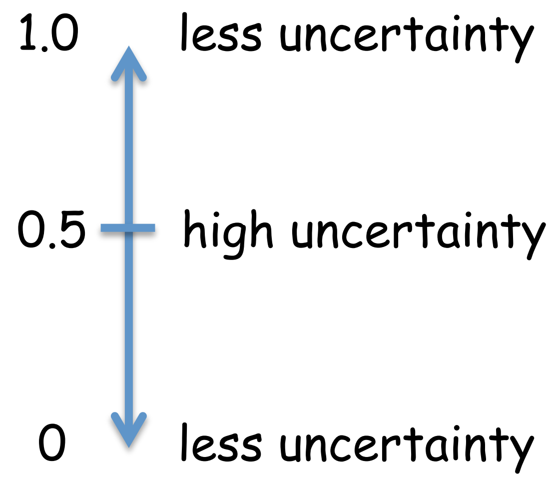
Uncertainty

- A, B, C, D organize a chess tournament. The following table shows the possibilities of the players on the tournament

	A	B	C	D
$\mu(x)$	1.0	0.7	0.4	0.1



- Given fuzzy set $A = \{(a, 1.0), (b, 0.7), (c, 0.4), (d, 0.1)\}$,

  How do we measure the fuzziness of the fuzzy set A ?

Uncertainty

- A, B, C, D organize a chess tournament. The following table shows the possibilities of the players on the tournament

	A	B	C	D
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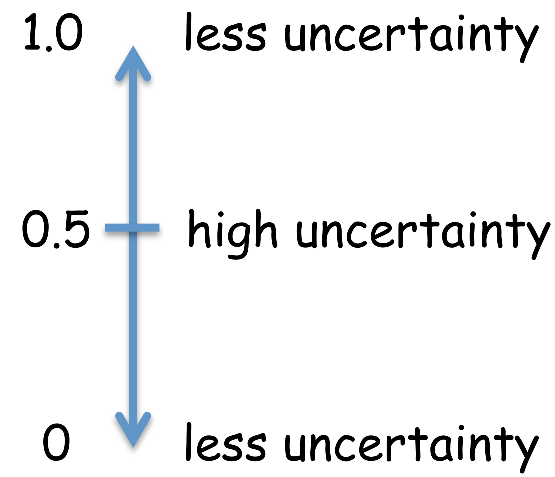

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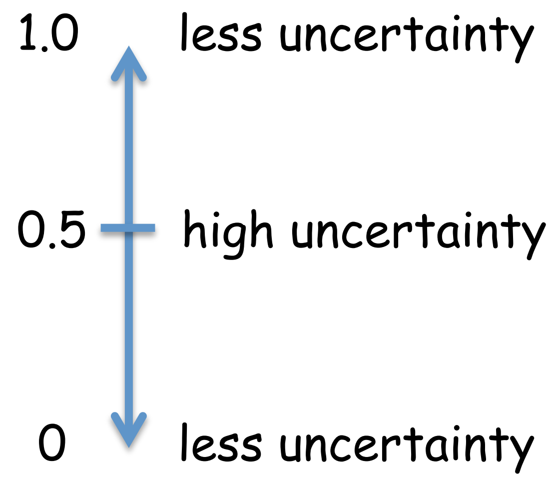

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  How do we compare the fuzziness of two fuzzy sets ?

  How do we decide which one is more uncertain ?

Measure of Fuzziness

$$f : P(X) \rightarrow R$$

all subsets of the universal set

real numbers

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- Given fuzzy sets

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- if the uncertainty is the maximum for a fuzzy set B, then $f(B)$ should have the maximum value
- Given fuzzy sets

$A = \{(a, 1.0), (b, 0.7), (c, 0.4)\}$, $B = \{(a, 0.5), (b, 0.5), (c, 0.5)\}$

$f(A) < f(B)$ and $f(B)$ should be maximum

Measure of Fuzziness

Measure with Entropy

Measure of Fuzziness

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- Shannon's entropy is used to measure the amount of uncertainty

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Measure with Entropy

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- $X = \{00, 01, 10, 11\}$

  $p(00) = 3/4$, $p(01) = 1/8$, $p(10) = 1/16$, $p(11) = 1/16$

Measure of Fuzziness

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Measure of Fuzziness

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$p(00) = 3/4,\ p(01) = 1/8,\ p(10) = 1/16,\ p(11) = 1/16$

$$H(p(x)) = - (3/4) \log (3/4) - (1/8) \log (1/8) - (1/16) \log (1/16) - (1/16) \log (1/16)$$

$$H(p(x)) = (3/2 + 3/8 + 1/4 + 1/4) - \log 3$$

$$H(p(x)) \approx 0.791$$

Measure of Fuzziness

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$$H(p(x)) = - (1/4) \log (1/4) - (1/4) \log (1/4) - (1/4) \log (1/4) - (1/4) \log (1/4)$$

Measure of Fuzziness

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$$H(p(x)) = - (1/4) \log (1/4) - (1/4) \log (1/4) - (1/4) \log (1/4) - (1/4) \log (1/4)$$

$$H(p(x)) = 1/2 + 1/2 + 1/2 + 1/2$$

$$H(p(x)) = 2$$

Measure of Fuzziness

Measure with Entropy

- $f(A) = - \Sigma_{x \text{ in } X} [\mu_A(x).\log \mu_A(x) + (1 - \mu_A(x)).\log (1 - \mu_A(x))]$

Measure of Fuzziness

Measure with Entropy

- $f(A) = - \Sigma_{x \text{ in } X} [\mu_A(x).\log \mu_A(x) + (1 - \mu_A(x)).\log (1 - \mu_A(x))]$
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- $A = \{(a, 0.6), (b, 0.5), (c, 0.2)\}$

  $f(A) = - (0.6 \log 0.6 + 0.5 \log 0.5 + 0.2 \log 0.2$

  $0.4 \log 0.4 + 0.5 \log 0.5 + 0.8 \log 0.8)$

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  $0.4 \log 0.4 + 0.5 \log 0.5 + 0.8 \log 0.8)$

  $f(A) = 2.686$

Measure of Fuzziness

Measure with Entropy

- $f(A) = - \Sigma_{x \text{ in } X} [\mu_A(x).\log \mu_A(x) + (1 - \mu_A(x)).\log (1 - \mu_A(x))]$
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- $A = \{(a, 0.6), (b, 0.5), (c, 0.2)\}$

$f(A) = - (0.6 \log 0.6 + 0.5 \log 0.5 + 0.2 \log 0.2$

$0.4 \log 0.4 + 0.5 \log 0.5 + 0.8 \log 0.8)$

$f(A) = 2.686$, $f'(A) = 2.686/3 = 0.89$

Measure of Fuzziness

Measure with Entropy

- $f(A) = - \Sigma_{x \text{ in } X} [\mu_A(x).\log \mu_A(x) + (1 - \mu_A(x)).\log (1 - \mu_A(x))]$
- the normalized measure $f'(A) = f(A) / |X|$
- $A = \{(a, 0.6), (b, 0.5), (c, 0.2)\}$

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  $0.4 \log 0.4 + 0.5 \log 0.5 + 0.8 \log 0.8)$

  $f(A) = 2.686$, $f'(A) = 2.686/3 = 0.89$
- $B = \{(a, 0.5), (b, 0.5), (c, 0.5)\}$

Measure of Fuzziness

Measure with Entropy

- $f(A) = - \Sigma_{x \text{ in } X} [\mu_A(x).\log \mu_A(x) + (1 - \mu_A(x)).\log (1 - \mu_A(x))]$
- the normalized measure $f'(A) = f(A) / |X|$
- $A = \{(a, 0.6), (b, 0.5), (c, 0.2)\}$

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  $0.4 \log 0.4 + 0.5 \log 0.5 + 0.8 \log 0.8)$

  $f(A) = 2.686$, $f'(A) = 2.686/3 = 0.89$
- $B = \{(a, 0.5), (b, 0.5), (c, 0.5)\}$

  $f(B) = - (0.5 \log 0.5 + 0.5 \log 0.5 + 0.5 \log 0.5$

  $0.5 \log 0.5 + 0.5 \log 0.5 + 0.5 \log 0.5)$

Measure of Fuzziness

Measure with Entropy

- $f(A) = - \Sigma_{x \text{ in } X} [\mu_A(x).\log \mu_A(x) + (1 - \mu_A(x)).\log (1 - \mu_A(x))]$
- the normalized measure $f'(A) = f(A) / |X|$
- $A = \{(a, 0.6), (b, 0.5), (c, 0.2)\}$

  $f(A) = - (0.6 \log 0.6 + 0.5 \log 0.5 + 0.2 \log 0.2$

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  $f(A) = 2.686$, $f'(A) = 2.686/3 = 0.89$
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  $f(B) = - (0.5 \log 0.5 + 0.5 \log 0.5 + 0.5 \log 0.5$

  $0.5 \log 0.5 + 0.5 \log 0.5 + 0.5 \log 0.5)$

  $f(B) = 3$

Measure of Fuzziness

Measure with Entropy

- $f(A) = - \Sigma_{x \text{ in } X} [\mu_A(x).\log \mu_A(x) + (1 - \mu_A(x)).\log (1 - \mu_A(x))]$
- the normalized measure $f'(A) = f(A) / |X|$
- $A = \{(a, 0.6), (b, 0.5), (c, 0.2)\}$

  $f(A) = - (0.6 \log 0.6 + 0.5 \log 0.5 + 0.2 \log 0.2$

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  $f(A) = 2.686$, $f'(A) = 2.686/3 = 0.89$
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  $f(B) = - (0.5 \log 0.5 + 0.5 \log 0.5 + 0.5 \log 0.5$

  $0.5 \log 0.5 + 0.5 \log 0.5 + 0.5 \log 0.5)$

  $f(B) = 3$, $f'(B) = 1$

Measure of Fuzziness

Measure with Metric Distance

Measure of Fuzziness

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- $f(A) = \Sigma_{x \text{ in } X} (0.5 - |\mu_A(x) - 0.5|)$

Measure of Fuzziness

Measure with Metric Distance

- $f(A) = \Sigma_{x \text{ in } X} (0.5 - |\mu_A(x) - 0.5|)$
- the normalized measure $f'(A) = f(A) / (0.5\ |X|)$

Measure of Fuzziness

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- $A = \{(a, 0.6), (b, 0.5), (c, 0.2)\}$

Measure of Fuzziness

<u>Measure with Metric Distance</u>

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- $A = \{(a, 0.6), (b, 0.5), (c, 0.2)\}$

  $f(A) = (0.5 - |0.6 - 0.5|) + (0.5 - |0.5 - 0.5|) + (0.5 - |0.2 - 0.5|)$

Measure of Fuzziness

Measure with Metric Distance

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- $A = \{(a, 0.6), (b, 0.5), (c, 0.2)\}$

  $f(A) = (0.5 - |0.6 - 0.5|) + (0.5 - |0.5 - 0.5|) + (0.5 - |0.2 - 0.5|)$

  $f(A) = 0.4 + 0.5 + 0.2 = 1.1$

Measure of Fuzziness

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  $f(A) = 0.4 + 0.5 + 0.2 = 1.1$, $f'(A) = 1.1/1.5 = 0.73$

Measure of Fuzziness

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- $B = \{(a, 0.5), (b, 0.5), (c, 0.5)\}$

Measure of Fuzziness

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  $f(B) = (0.5 - |0.5 - 0.5|) + (0.5 - |0.5 - 0.5|) + (0.5 - |0.5 - 0.5|)$

Measure of Fuzziness

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  $f(A) = 0.4 + 0.5 + 0.2 = 1.1$, $f'(A) = 1.1/1.5 = 0.73$

- $B = \{(a, 0.5), (b, 0.5), (c, 0.5)\}$

  $f(B) = (0.5 - |0.5 - 0.5|) + (0.5 - |0.5 - 0.5|) + (0.5 - |0.5 - 0.5|)$

  $f(B) = 0.5 + 0.5 + 0.5 = 1.5$

Measure of Fuzziness

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