STA250 Probability and Statistics

Chapter 7 Notes

Some Continuous Probability Distributions

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STA250 Probability and Statistics

Reference Book

This lecture notes are prepared according to the contents of

“PROBABILITY & STATISTICS FOR ENGINEERS & SCIENTISTS by Walpole, Myers, Myers and Ye”

Normal Distribution Examples

- **The normal distribution is the most important and widely used distribution in statistics. Some examples:**
  - This bell-shaped curve, sometimes called the Gaussian distribution, explains many natural phenomena.
  - Average age of the world's population.
  - Physical measurements like blood pressure.
  - Standardized test scores.
  - Average precipitation levels.
  - Average price of certain stocks in the stock market.
- **The Central Limit Theorem in statistics indicates that random variables that are the sum of a number of component variables follow the normal distribution.**

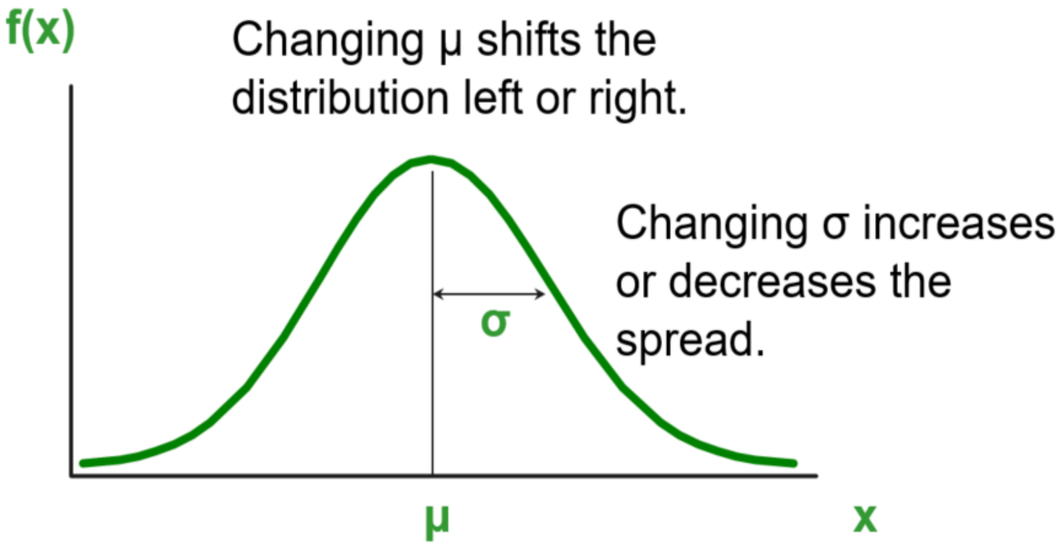
Normal Distribution

- **The density function for a normally distributed random variable with mean μ and standard deviation σ, is**

$$f(x\,;\mu\,,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \quad -\infty \leq x \leq \infty$$

- **As before, μ determines the center of the bell-shaped curve, while σ determines the spread. Notes:**
  - The curve is symmetric about the mean.
  - As expected, the total area under the curve equals 1.
  - The mode , the median and the average are equal.
  - The curve has a point of inflection at $x = \mu \pm \sigma$.

Normal Distribution Shape



Given the mean μ and variance σ we define the normal distribution using the notation $\boxed{X \sim N(\mu, \sigma^2)}$

- Bell Shaped
- Symmetrical
- Mean, Median and Mode are Equal
- Location is determined by the mean, μ
- Spread is determined by the standard deviation, σ

Normal Curves

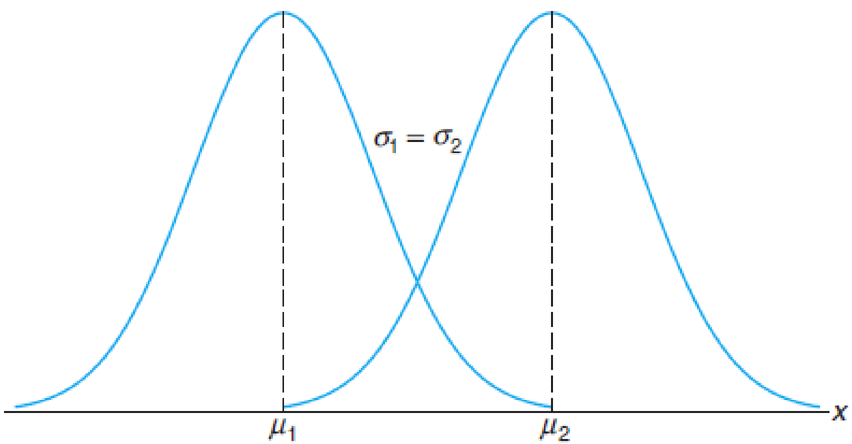


Figure 6.3: Normal curves with $\mu_1 < \mu_2$ and $\sigma_1 = \sigma_2$.

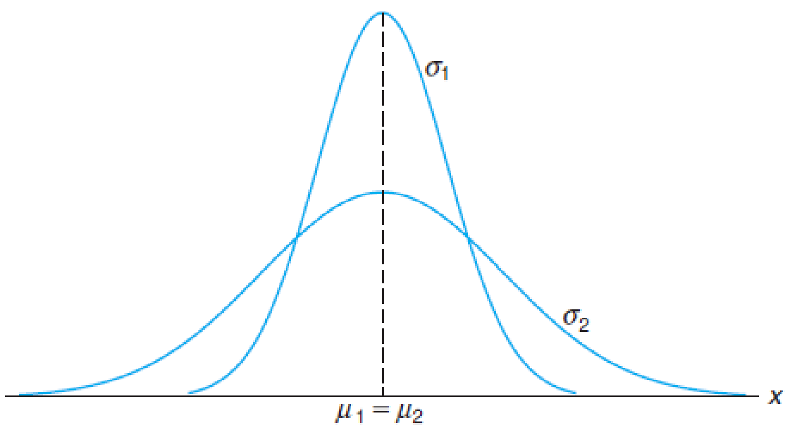


Figure 6.4: Normal curves with $\mu_1 = \mu_2$ and $\sigma_1 < \sigma_2$.

In Figure 6.3,

- There are two normal curves having the same standard deviation but different means.
- The two curves are identical in form but are centered at different positions along the horizontal axis.

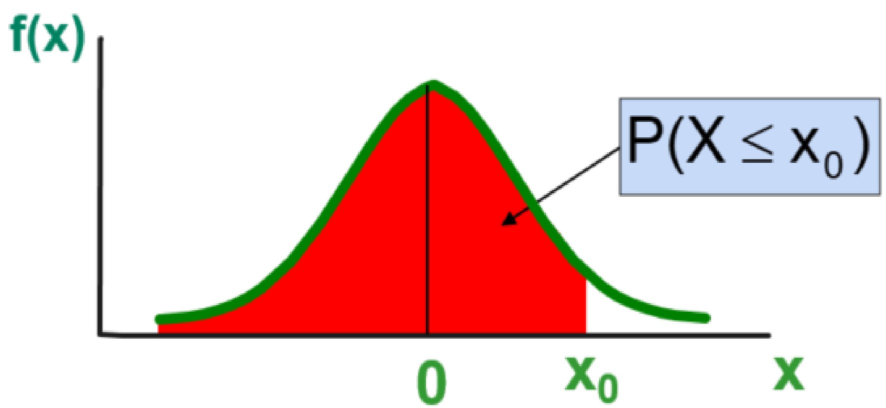
In Figure 6.4,

- There are two normal curves with the same mean but different standard deviations.
- Two curves are centered at exactly the same position on the horizontal axis, but the curve with the larger standard deviation is lower and spreads out farther.

Cumulative Normal Distribution

- For a normal random variable X with mean μ and variance σ2 , i.e., X~N(μ, σ2), the cumulative distribution function is

$$\boxed{F(x_0) = P(X \le x_0)}$$



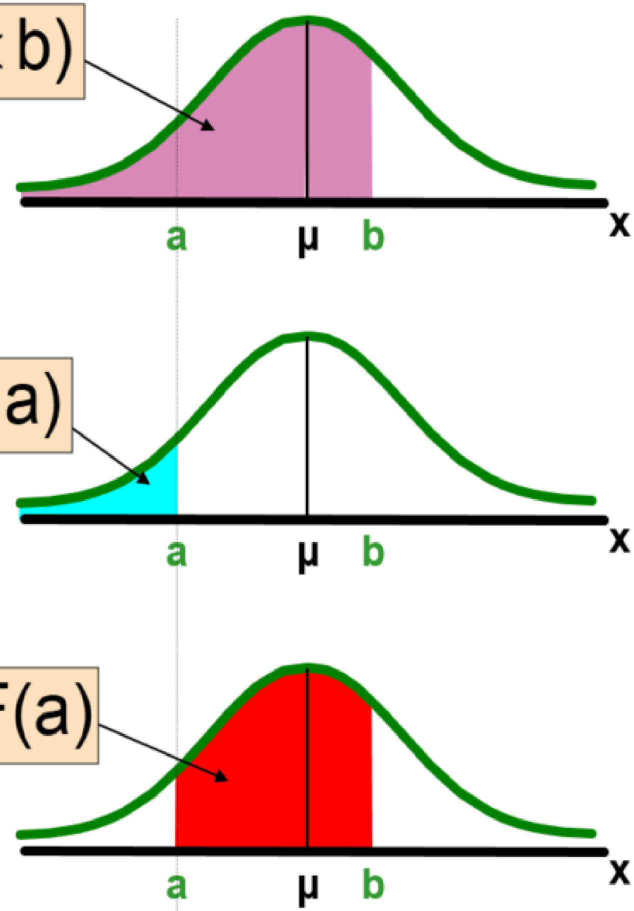
Finding Normal Probabilities

The probability for a range of values is measured by the area under the curve

$$F(b) = P(X < b)$$

$$F(a) = P(X < a)$$

$$P(a < X < b) = F(b) - F(a)$$



Areas Under The Normal Curve

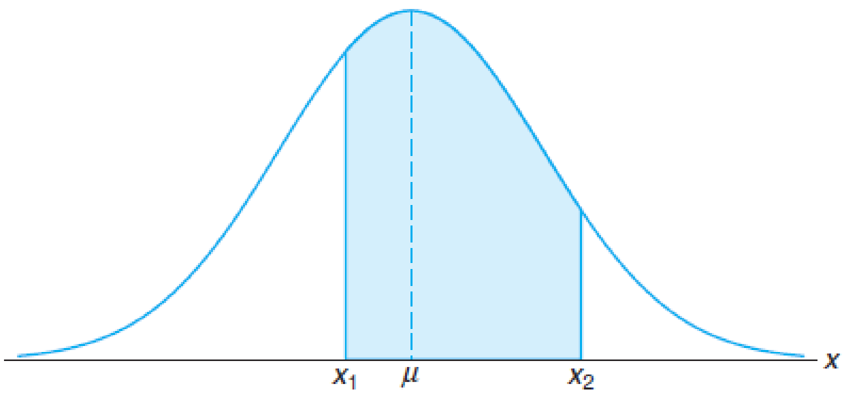


Figure 6.6: $P(x_1 < X < x_2) =$ area of the shaded region.

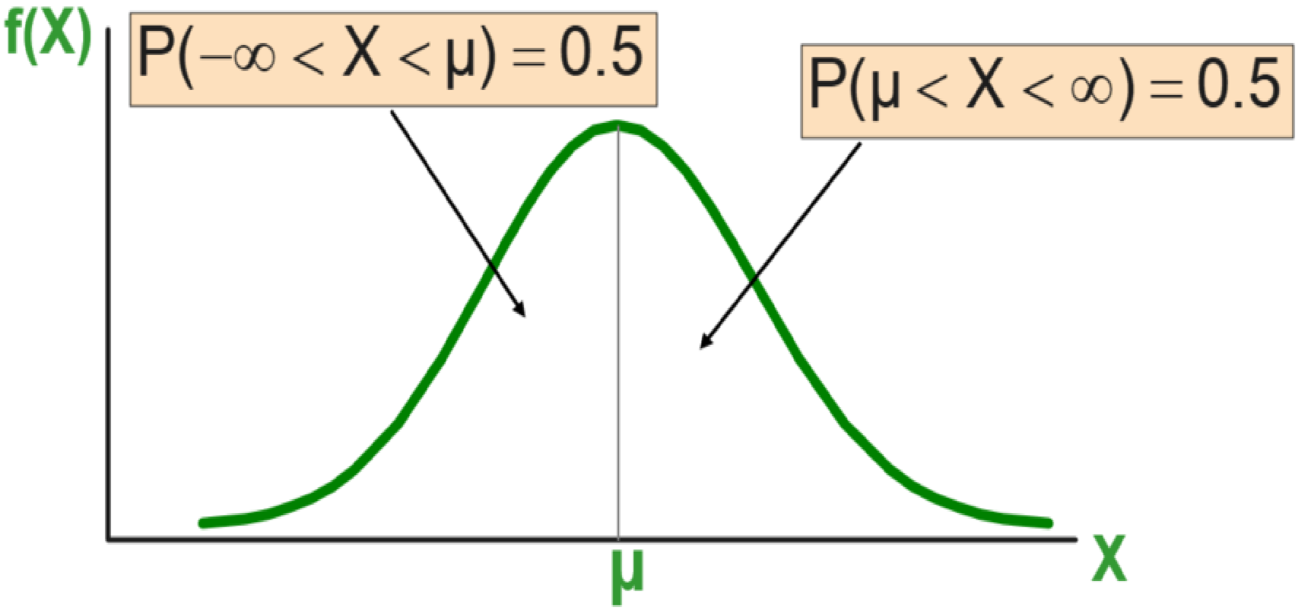
In Figure 6.6 is represented by the area of the shaded region.

$$P(x_1 < X < x_2) = \int_{x_1}^{x_2} n(x;\mu,\sigma)\ dx = \frac{1}{\sqrt{2\pi}\sigma}\int_{x_1}^{x_2} e^{-\frac{1}{2\sigma^2}(x-\mu)^2} dx$$

Unfortunately, a closed-form expression for this integral does not exist; hence, its evaluation requires the use of numerical integration techniques.

Area Under The Curve

The total area under the curve is 1.0, and the curve is symmetric, so half is above the mean, half is below



$$P(-\infty < X < \infty) = 1.0$$

Standard Normal Distribution

- Any normal distribution (with any mean and variance combination) can be transformed into the standardized normal distribution (Z), with mean 0 and variance 1, Z~N(0,1).
- A standard normal random variable can be from any normal random variable with a simple transformation:

$$\boxed{Z = \frac{X - \mu}{\sigma}}$$

- Intuitively, Z shows the number of standard deviations above or below (if negative) the mean.

Standard Normal Distribution Table

Table entry for z is the area under the standard normal curve to the left of z.

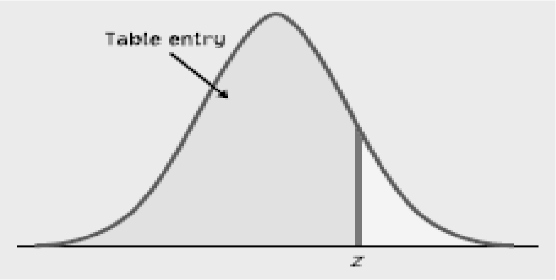


TABLE A Standard normal probabilities (*continued*)

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

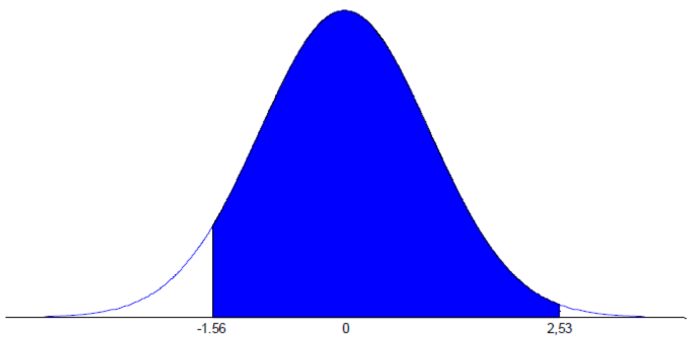
Example-Reading Table

TABLE A Standard normal probabilities (*continued*)

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986

Example-Reading Table

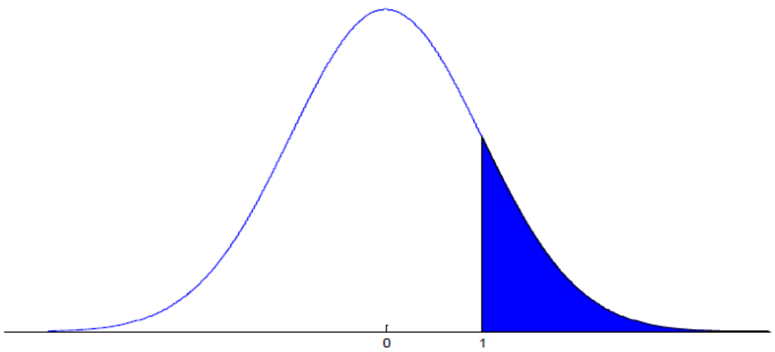
Find the area of shaded region?



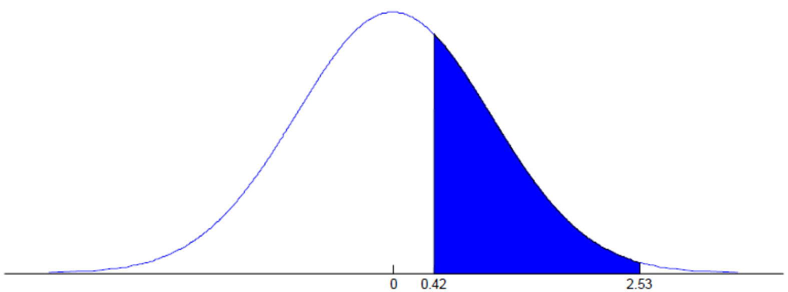
$P(-1.56 < Z < 2.53) = P(Z < 2.53) - P(Z < -1.56)$

$= P(Z < 2.53) - (1 - P(Z < 1.56))$

$= 0.9943 - (1 - 0.9406) = 0.9349$



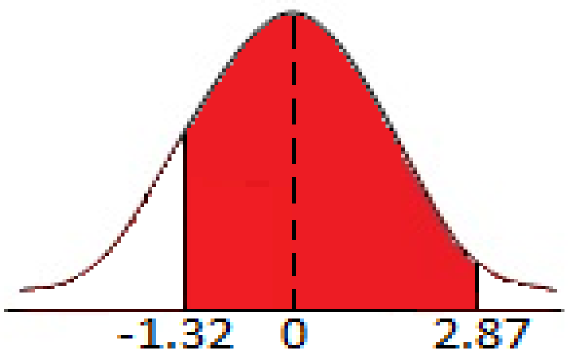
$P(Z > 1) = 1 - P(Z < 1) = 1 - 0.8413 = 0.1587$



$P(0.42 < Z < 2.53) = P(Z < 2.53) - P(Z < 0.42)$

$= 0.9943 - 0.6628$

$= 0.3315$



$P(-1.32 < Z < 2.87) = P(Z < 2.87) - P(Z < -1.32)$

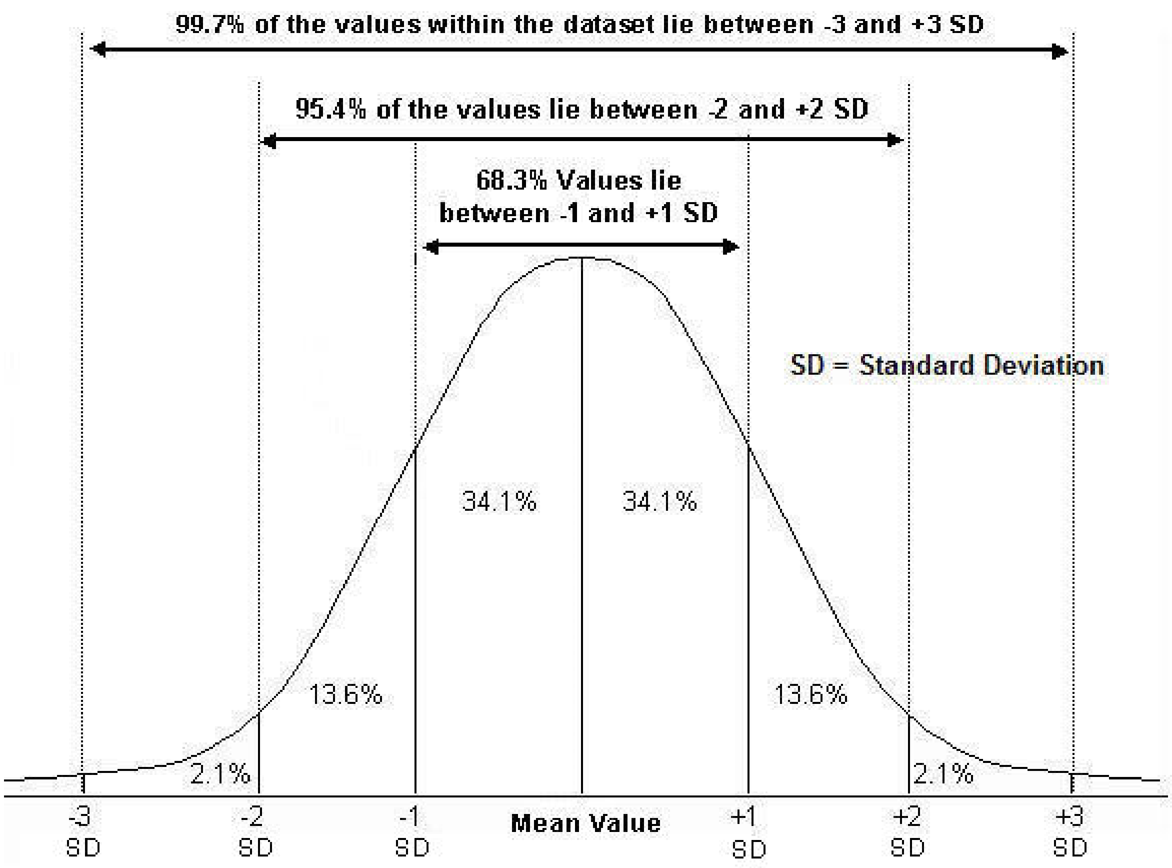
$=P(Z < 2.87) - P(Z > 1.32)$

$= P(Z < 2.87) - \big(1 - P(Z < 1.32)\big)$

$= P(Z < 1.32) + P(Z < 2.87) - 1$

=0.9066+0.9980-1=0.9046

Empirical Rule



To verify the Empirical Rule:

z of 1.00 = .3413 so .3413 * 2 = .6826 or about 68%

z of 2.00 = .4772 so .4772 * 2 = .9544 or about 95%

z of 3.00 = .4987 so .4987 * 2 = .9974 or about 99.7%

Empirical Rule-Example 1

- A random sample of data has a mean of 75 and variance of 25. Use the normal probability rule to find the approximate percent of observations between 65 and 85.

$$P(65 \leq X \leq 85) = P\left(\frac{65-75}{5} \leq Z \leq \frac{85-75}{5}\right) = P(-2 \leq Z \leq 2)$$

$$= 0.4772+0.4772 \cong 95\%$$

Empirical Rule-Example (2 of 1)

As part of its quality assurance program, the Autolite Battery Company conducts tests on battery life. For a particular D-cell alkaline battery, the mean life is 19 hours. The useful life of the battery follows a normal distribution with a standard deviation of 1.2 hours.

1. About 68% of the batteries failed between what two values?

$$\boldsymbol{\mu} \pm \mathbf{1} \times (\boldsymbol{standard\ deviation});$$

$$\mathbf{19} \pm \mathbf{1}(\mathbf{1.2})\ \boldsymbol{hours};$$

About 68% of batteries will fail between 17.8 and 20.2 hours.

Empirical Rule-Example (2 of 2)

2. **About 95% of the batteries failed between what two values?**

$$\mu \pm 2\, standard\ deviation;$$

$$19 \pm 2(1.2)\ hours;$$

About 95% of batteries will fail between 16.6 and 21.4 hours.

3. **Virtually all of the batteries failed between what two values?**

$$\mu \pm 3\, standard\ deviation;$$

$$19 \pm 3(1.2) hours;$$

Practically all of the batteries will fail between 15.4 and 22.6 hours.

Standard Normal Probability Example 1

Example 6.4: Given a random variable X having a normal distribution with $\mu = 50$ and $\sigma = 10$, find the probability that X assumes a value between 45 and 62.

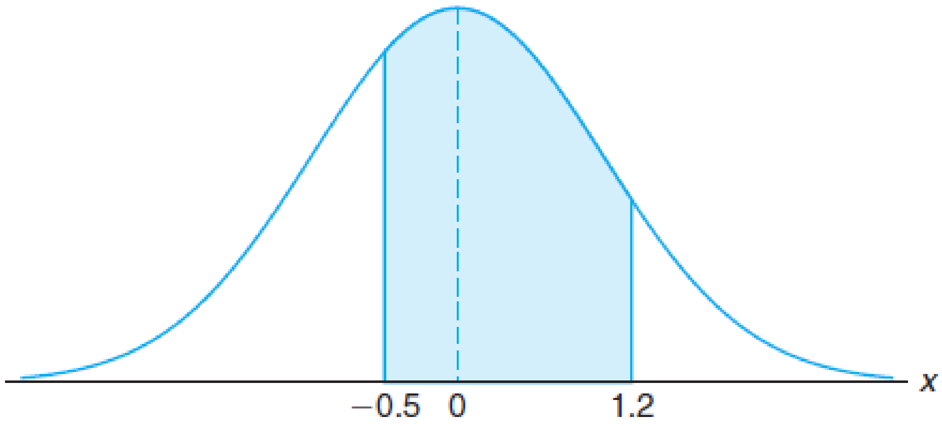


Figure 6.11: Area for Example 6.4.

***Solution*:** The z values corresponding to $x_1 = 45$ and $x_2 = 62$ are

$$z_1 = \frac{45 - 50}{10} = -0.5 \text{ and } z_2 = \frac{62 - 50}{10} = 1.2.$$

Standard Normal Probability Example (1 of 2)

Therefore,

$$P(45 < X < 62) = P(-0.5 < Z < 1.2).$$

$P(-0.5 < Z < 1.2)$ is shown by the area of the shaded region in Figure 6.11. This area may be found by subtracting the area to the left of the ordinate $z = -0.5$ from the entire area to the left of $z = 1.2$. Using Table A.3, we have

$$\begin{aligned} P(45 < X < 62) &= P(-0.5 < Z < 1.2) = P(Z < 1.2) - P(Z < -0.5) \\ &= 0.8849 - 0.3085 = 0.5764. \end{aligned}$$

Standard Normal Probability Example (2)

Example 6.5: Given that X has a normal distribution with $\mu = 300$ and $\sigma = 50$, find the probability that X assumes a value greater than 362.

***Solution*:** The normal probability distribution with the desired area shaded is shown in Figure 6.12. To find $P(X > 362)$, we need to evaluate the area under the normal curve to the right of $x = 362$. This can be done by transforming $x = 362$ to the corresponding z value, obtaining the area to the left of z from Table A.3, and then subtracting this area from 1. We find that

$$z = \frac{362 - 300}{50} = 1.24.$$

Hence,

$$P(X > 362) = P(Z > 1.24) = 1 - P(Z < 1.24) = 1 - 0.8925 = 0.1075.$$

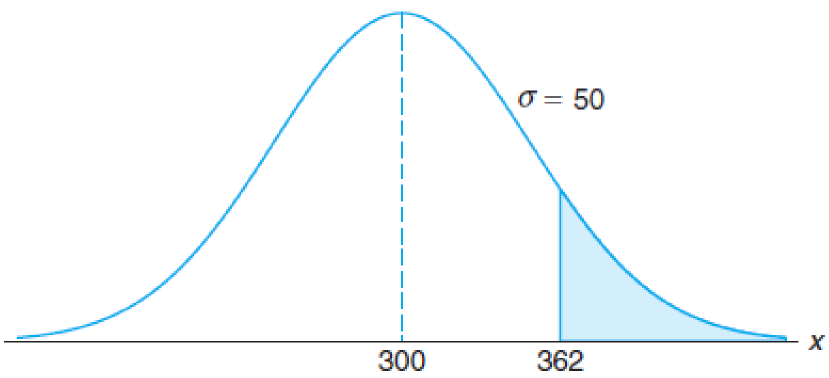


Figure 6.12: Area for Example 6.5.

Standard Normal Probability Example (3)

Example 6.7: A certain type of storage battery lasts, on average, 3.0 years with a standard deviation of 0.5 year. Assuming that battery life is normally distributed, find the probability that a given battery will last less than 2.3 years.

***Solution*:** First construct a diagram such as Figure 6.14, showing the given distribution of battery lives and the desired area. To find $P(X < 2.3)$, we need to evaluate the area under the normal curve to the left of 2.3. This is accomplished by finding the area to the left of the corresponding z value. Hence, we find that

$$z = \frac{2.3 - 3}{0.5} = -1.4,$$

and then, using Table A.3, we have

$$P(X < 2.3) = P(Z < -1.4) = 0.0808.$$

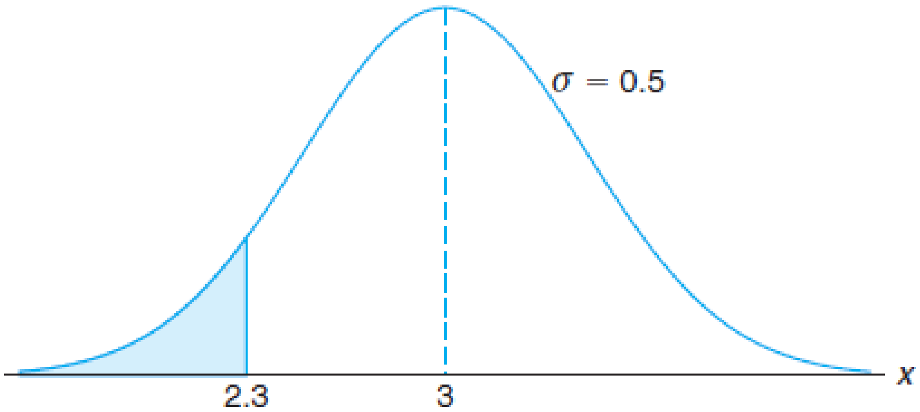


Figure 6.14: Area for Example 6.7.

Standard Normal Probability Example (4)

Example 6.8: An electrical firm manufactures light bulbs that have a life, before burn-out, that is normally distributed with mean equal to 800 hours and a standard deviation of 40 hours. Find the probability that a bulb burns between 778 and 834 hours.

Solution: The distribution of light bulb life is illustrated in Figure 6.15. The z values corresponding to $x_1 = 778$ and $x_2 = 834$ are

$$z_1 = \frac{778 - 800}{40} = -0.55 \text{ and } z_2 = \frac{834 - 800}{40} = 0.85.$$

Hence,

$$\begin{aligned} P(778 < X < 834) &= P(-0.55 < Z < 0.85) = P(Z < 0.85) - P(Z < -0.55) \\ &= 0.8023 - 0.2912 = 0.5111. \end{aligned}$$

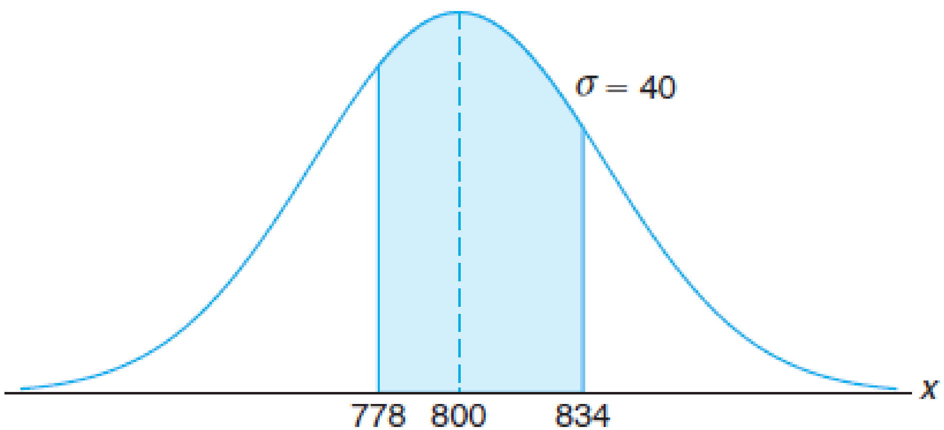


Figure 6.15: Area for Example 6.8.

Reverse calculation: starting with a probability, find X.

- **Steps to find the X value for a known probability:**
- 1. Find the z value for the known probability
- 2. Convert to X units using the formula:

$$x = \mu - z\sigma$$

Finding a Value for x Using z Example (1)

Example 6.13: The average grade for an exam is 74, and the standard deviation is 7. If 12% of the class is given As, and the grades are curved to follow a normal distribution, what is the lowest possible A and the highest possible B?

Solution: In this example, we begin with a known area of probability, find the z value, and then determine x from the formula $x = \sigma z + \mu$. An area of 0.12, corresponding to the fraction of students receiving As, is shaded in Figure 6.20. We require a z value that leaves 0.12 of the area to the right and, hence, an area of 0.88 to the left. From Table A.3, $P(Z < 1.18)$ has the closest value to 0.88, so the desired z value is 1.18. Hence,

$$x = (7)(1.18) + 74 = 82.26.$$

Therefore, the lowest A is 83 and the highest B is 82.

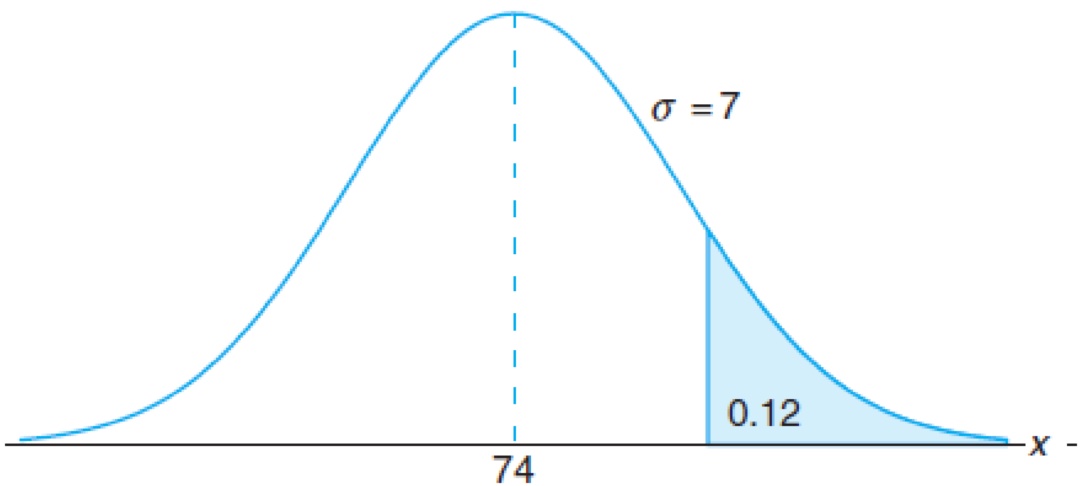


Figure 6.20: Area for Example 6.13.

Finding a Value for x Using z Example (1)

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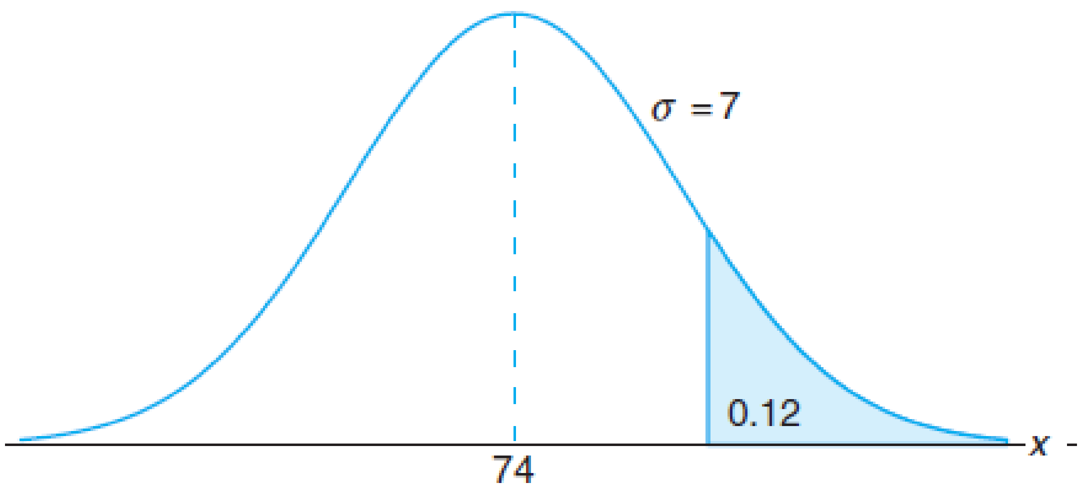


Figure 6.20: Area for Example 6.13.

Finding a Value for x Using z Example (1)

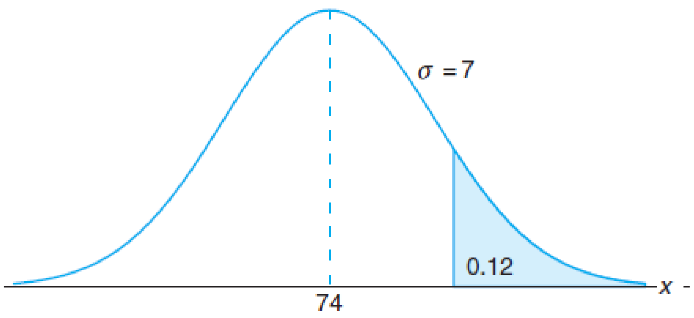


Figure 6.20: Area for Example 6.13.

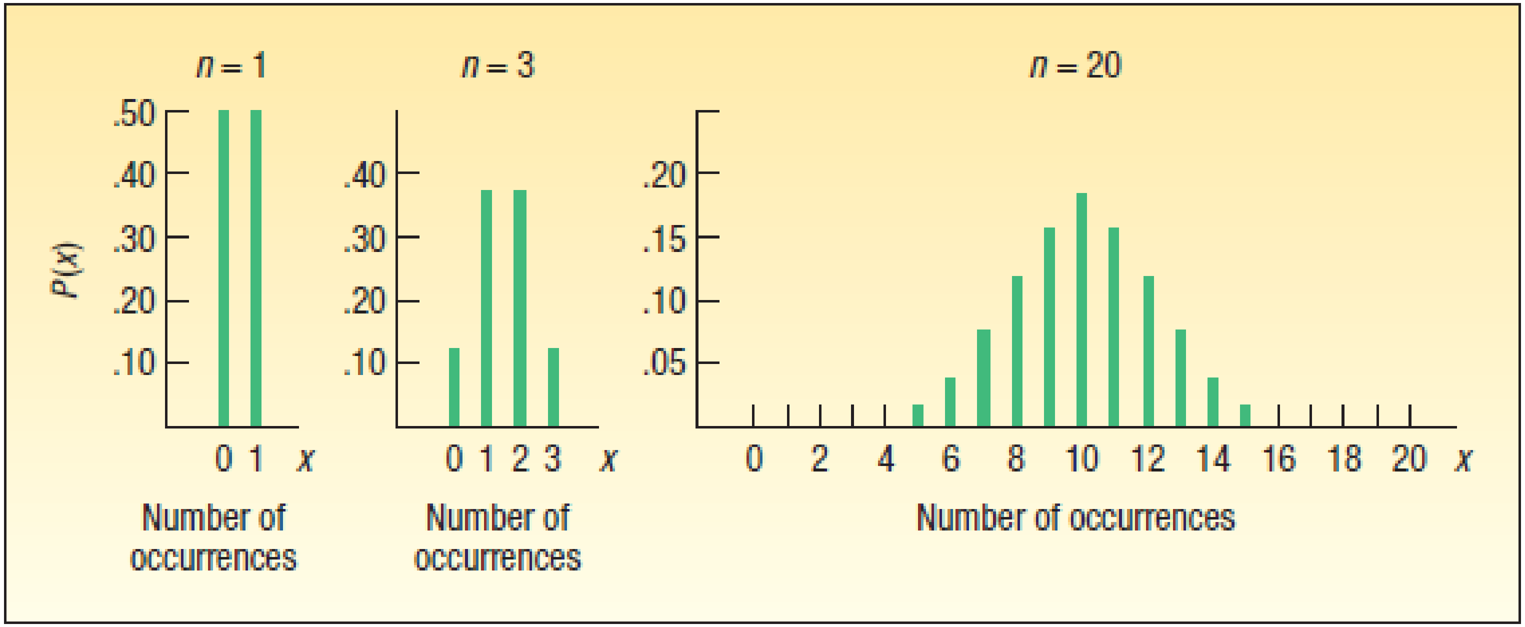
TABLE A Standard normal probabilities (*continued*)

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319

Normal Approximation to Binomial

- **The Normal probability distribution can approximate a binomial distribution IF n is large and p is not too close to 0 or 1.**
- $\boldsymbol{np\ and\ np(1-p)}$ **must both be at least 5**

Notice how the distribution approximates the shape of a normal distribution as n becomes larger



Binomial distribution review

- **The four conditions for a binomial probability distribution are**
  - There are only two possible outcomes (success or failure)
  - p remains the same from trial to trial
  - The trials are independent
  - The distribution results from a count of the number of successes in a fixed number of trials

- **The binomial probability distribution function is,**

$$b(x; n, p) = \binom{n}{x} p^x q^{n-x}, x = 0,1,2,\ldots,n.$$

$$\mu = np \qquad\qquad \sigma^2 = npq$$

Normal Approximation to Binomial

- **If X binomial with parameters n and p, then**

$$P(X \leq x) \approx P\left(Z \leq \frac{x + 0.5 - np}{\sqrt{npq}}\right)$$

for $n \to \infty$. This is a good approximation:

CONTINUITY CORRECTION FACTOR The value 0.5 subtracted or added, depending on the question, to a selected value when a discrete probability distribution is approximated by a continuous probability distribution.

Normal Approximation to Binomial

- **Only four cases may arise**
  - For the probability *at least x* occur,
    use the area above (x - .5)
  - For the probability that *more than x* occur,
    use the area above (x + .5)
  - For the probability that *x or fewer* occur,
    use the area below (x + .5)
  - For the probability that *fewer than x* occur,
    use the area below (x - .5)

Normal Approximation to Binomial-Example (1 of 1)

Example 6.15: The probability that a patient recovers from a rare blood disease is 0.4. If 100 people are known to have contracted this disease, what is the probability that fewer than 30 survive?

Solution: Let the binomial variable X represent the number of patients who survive. Since $n = 100$, we should obtain fairly accurate results using the normal-curve approximation with

$$\mu = np = (100)(0.4) = 40 \text{ and } \sigma = \sqrt{npq} = \sqrt{(100)(0.4)(0.6)} = 4.899.$$

To obtain the desired probability, we have to find the area to the left of $x = 29.5$.

Normal Approximation to Binomial-Example (1 of 2)

The z value corresponding to 29.5 is

$$z = \frac{29.5 - 40}{4.899} = -2.14,$$

and the probability of fewer than 30 of the 100 patients surviving is given by the shaded region in Figure 6.26. Hence,

$$P(X < 30) \approx P(Z < -2.14) = 0.0162.$$

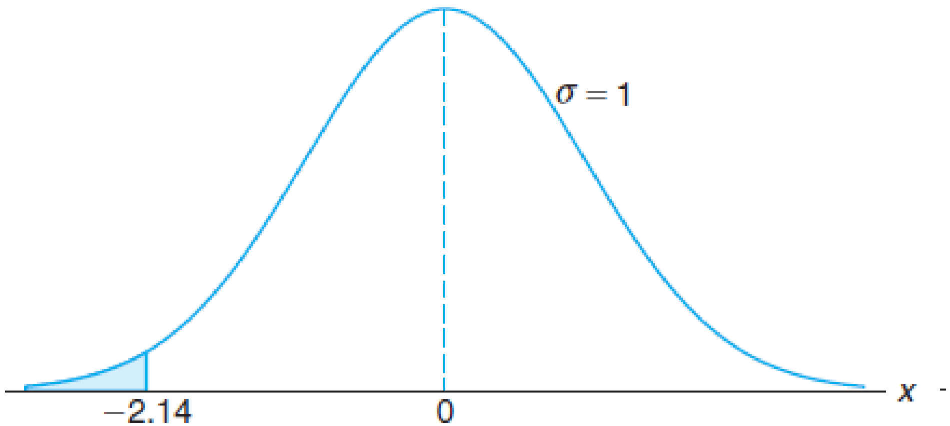


Figure 6.26: Area for Example 6.15.

Normal Approximation to Binomial-Example (2)

A salesman makes initial phone contact with potential customers in an effort to assess whether a follow-up visit to their homes is worthwhile. His experience suggests that %40 of the initial contacts lead to follow-up visit. If he contacts 100 people by phone, what is the probability that between 45 and 50 home visits will result?

X= # of follow up visits

$X \sim \text{Bin}\ (n=100,\ \pi=0.4)$ → $\mu = n\pi = 40$ → $\sigma^2 = n\pi(1-\pi) = 24$

Since $\min(n\pi, n(1-\pi)) = \min(40,60) = 40 \geq 5$ → we can use **normal approximation to binomial**

$P(45 \leq X \leq 50) = P(44.5 \leq X \leq 50.5)$

$= P(\frac{44.5-40}{\sqrt{24}} \leq Z \leq \frac{50.5-40}{\sqrt{24}}) = P(0.92 \leq Z \leq 2.14)$

$= 0.4838 - 0.3212 = 0.1626 \cong 16\%$

Other Continuous Distributions

- **The <u>Gamma Distribution</u> with parameters α and β, is defined as**

$$f(x) = \frac{1}{\beta^{\alpha}\Gamma(\alpha)} x^{\alpha-1} e^{-\frac{x}{\beta}} \quad for \quad x > 0$$
$$= 0 \qquad elsewhere$$

where $\alpha > 0, \beta > 0$**, and** $\Gamma(\alpha) = \int_0^{\infty} x^{\alpha-1} e^{-x} dx$

- Note that if α is an integer, Γ(α) simplifies to (α − 1)!

then $\mu = \alpha\beta$

and $\sigma^2 = \alpha\beta^2$

Other Continuous Distributions

- **An <u>Exponential Distribution</u> with parameter λ, is defined as**

$$f(t) = \lambda e^{-\lambda t} \quad for \quad t > 0$$
$$= 0 \qquad elsewhere$$

then

$$\mu = \frac{1}{\lambda}$$

and

$$\sigma^2 = \frac{1}{\lambda^2}$$

- **Exponential distribution notes:**
  - The mean equals the standard deviation.
  - Note that <u>λ = (1/β)</u>, where <u>β</u> is the parameter used in the book for the exponential distribution.

Other Continuous Distributions

- **Chi-Squared distribution. Used for statistical inference. We will use the chi-squared table in the appendix later in the semester.**

- **Weibull distribution. Has many applications in reliability and life testing and is used to model failure rate. We're skipping this distribution this semester.**
  - The failure rate at time t for the Weibull distribution is

$$Z(t) = \alpha\beta t^{\beta-1}, \qquad for\ t > 0$$

  - For $\beta = 1$ failure rate is constant (memoryless).
  - For $\beta > 1$ failure rate increases over time.
  - For $\beta < 1$ failure rate decreases over time.

Next Lesson

- **Sampling Distributions…**

See you☺