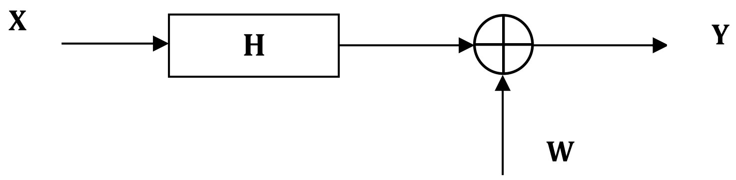
ELEC 462/562 Statistical Signal Processing
Project
Due: 16.12.2024

Electrocardiogram (ECG) Signal Restoration

Assume that we observe the electrocardiogram (ECG) signal $\mathbf{Y} = \mathbf{HX} + \mathbf{W}$ over an LTI noncausal system given in the following figure. Here $\mathbf{H}$ is a convolution matrix constituted by the system impulse response. The noise $\mathbf{W} \in \mathbb{R}^{300}$ is a white Gaussian vector with zero mean and variance 16×10^{-4}. $\mathbf{X}$ and $\mathbf{W}$ are assumed to be uncorrelated.



We assume that the signal $\mathbf{X}$ is a Gaussian random vector with zero mean and covariance matrix $(\lambda \mathbf{L}^{\boldsymbol{T}}\mathbf{L})^{-\mathbf{1}}$ where $\mathbf{L}$ is a Tikhonov regularization matrix. The parameter λ is the regularization parameter that controls the smoothing level. The observation vector $\mathbf{Y} = \mathbf{y} \in \mathbb{R}^{300}$, the ground-truth signal $\mathbf{x} \in \mathbb{R}^{300}$, the system matrix $\mathbf{H} \in \mathbb{R}^{300\times300}$ and the matrix $\mathbf{L} \in \mathbb{R}^{300\times300}$ are given in the attached mat files. In this project, your task is to estimate the signal $\mathbf{X}$ using two different estimators and to compare their performances.

a) Derive the maximum likelihood (ML) estimator for $\mathbf{X}$.
b) For given observation vector $\mathbf{Y} = \mathbf{y}$, obtain the ML estimate $\hat{\boldsymbol{x}}_{\mathrm{ML}}$. Plot the observation $\mathbf{y}$. Plot the estimate $\hat{\boldsymbol{x}}_{\mathrm{ML}}$ along with the ground-truth $\mathbf{x}$ in the same figure. Interpret the result.
c) Derive the maximum-a-posteriori (MAP) estimator for $\mathbf{X}$.
d) For given observation vector $\mathbf{Y} = \mathbf{y}$, obtain the linear estimate $\hat{\mathbf{x}}_{\mathrm{MAP}}(\lambda)$ for $\lambda = 1, 10,\ 100,\ 1000$. Plot each $\hat{\mathbf{x}}_{\mathrm{MAP}}(\lambda)$ for different values of λ along with the ground-truth $\mathbf{x}$. Interpret the effect of λ on estimation.
e) Calculate the mean square error of each estimate such that

$$\mathrm{MSE} = \frac{1}{300}\,||\mathbf{x} - \hat{\mathbf{x}}||^2$$

List the results in a table such that

	ML	MAP ($\lambda = 1$)	MAP ($\lambda = 10$)	MAP ($\lambda = 100$)	MAP ($\lambda = 1000$)
MSE					