GEBZE TECHNICAL UNIVERSITY

ENGINEERING FACULTY

ELECTRONICS ENGINEERING

ELEC 361

ANALOG COMMUNICATION SYSTEMS

PROJECT

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CONTENTS

1. INTRODUCTION

Amplitude Modulation (AM) is one of the most fundamental and historically significant techniques in communication systems. By varying the amplitude of a high-frequency carrier wave in accordance with a lower-frequency baseband message, AM facilitates the transmission of audio, voice, and other signals over long distances. This method paved the way for early commercial radio broadcasting and continues to serve as a cornerstone concept in modern communication theory and practice.

The general AM process involves three key components:

1. **Message Signal (Baseband):** This is the low-frequency information-bearing signal, typically audio or other data. In this project, we consider a message composed of two distinct tones at 150 Hz and 250 Hz.
2. **Carrier Signal (High-Frequency):** A sinusoidal carrier at a much higher frequency (1.5 kHz in our case) "carries" the message through the channel. The carrier's frequency is chosen to be large enough to be transmitted efficiently over the medium of interest (e.g., radio waves).
3. **Amplitude Modulation:** By adjusting the carrier's amplitude proportional to the instantaneous value of the message, we embed the baseband information onto a high-frequency carrier. This produces a Double Sideband Large Carrier (DSB-LC) AM signal, which retains the carrier and creates upper and lower sidebands around the carrier frequency.

Understanding the spectral components of the AM signal is crucial for both analysis and system design. The AM spectrum consists of the carrier frequency component and additional frequency components (sidebands) shifted by the message frequencies. By analyzing the time-domain waveform and its frequency content, we gain insights into bandwidth requirements, power distribution, and the potential for interference or distortion in real-world channels.

In practice, the received AM signal must be demodulated to recover the original message. Envelope detection is a common method: by extracting the envelope of the modulated signal and removing the DC offset, we obtain a close approximation of the original low-frequency information. Although modern digital communication systems often employ more advanced modulation schemes and sophisticated digital signal processing techniques, AM and envelope detection remain foundational examples used extensively in introductory communication courses and basic transmission systems.

In this project, we start with a given AM signal formulation and a known baseband message. We:

- Analyze the AM signal to identify carrier and sideband frequencies.
- Use MATLAB to simulate the AM signal, plot its time-domain waveform and magnitude spectrum.
- Implement an envelope detector via full-wave rectification and a low-pass filtering approach in the frequency domain (without relying on the Signal Processing Toolbox).
- Extract the demodulated signal by removing the DC offset from the envelope, confirming the presence of the original message frequencies in the recovered waveform.

This demonstration not only provides practical insight into basic AM systems but also shows how to implement key signal processing steps using fundamental MATLAB operations, making it accessible without specialized toolboxes.

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The message signal $m(t) = 8\cos(300\pi t) + 10\sin(500\pi t)$ modulates the carrier signal $c(t) = 10\cos(2\pi f_c t)$, where $f_c = 1.5$ kHz

2. PROBLEM A

a) Plot the message signal for one periode. Plot the magnitude spectrum of the message signal.

2.1 Analytical Solution

$m(t) = 8\cos(2\pi\, 150t) + 10\sin(2\pi\, 250t)$

GCD (150, 250) = 50
↳ Greatest Common Divison

$T = \frac{1}{f} = \frac{1}{50} = 0.02$ s

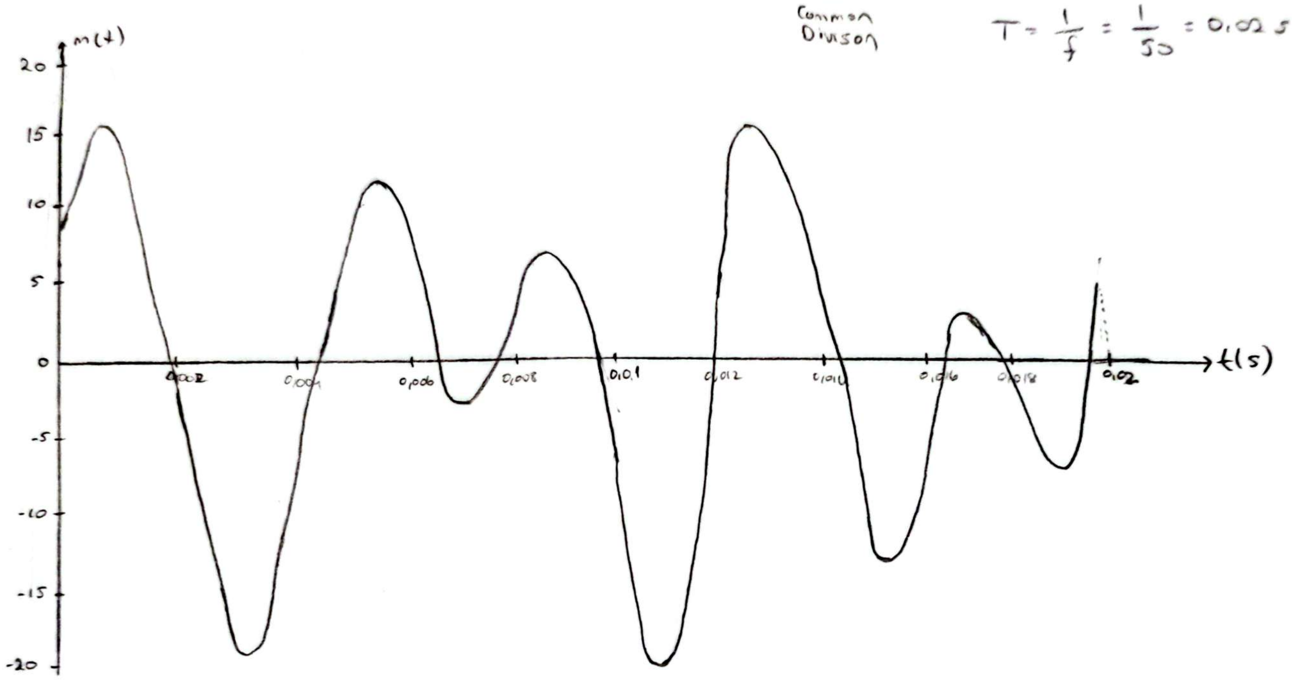


Figure 1-a.

$$\mathcal{F}\left\{ m(t) = 8\cos(2\pi\, 150t) + 10\sin(2\pi\, 250t) \right.$$

$$M(f) = 4\delta(f-150) + 4\delta(f+150) + \frac{5}{j}\delta(f-250) - \frac{5}{j}\delta(f+250)$$

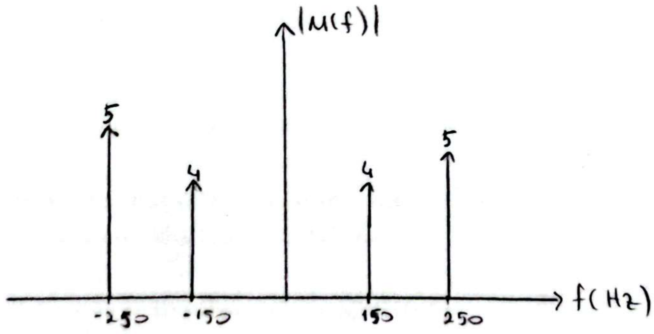


Figure 2-a.

2.2 MATLAB CODE

```
% Mehmet ALTINTAŞ - 1901022065
% ELEC 361 PROJE - PROBLEM A

% Parameters
f1 = 150; % Hz
f2 = 250; % Hz
A1 = 8;
A2 = 10;

% Fundamental period to show one cycle of the combined waveform
T = 1/50;  % 0.02 s
Fs = 10000; % Sampling frequency (high enough to capture frequencies)
t = 0:1/Fs:T-1/Fs;

% Message signal
m = A1*cos(2*pi*f1*t) + A2*sin(2*pi*f2*t);

% Plot the time-domain signal
figure;
plot(t, m, 'LineWidth', 1.5);
xlabel('Time (s)');
ylabel('Amplitude');
title('Message Signal in Time domain m(t) / Mehmet ALTINTAŞ - 1901022065');
grid on;

% Compute the Fourier Transform (using FFT)
N = length(m);
M = fft(m);   % Discrete Fourier Transform
M_mag = abs(M)/N;  % Normalize the magnitude

% Frequency axis for plotting (two-sided)
f_axis = (-N/2:N/2-1)*(Fs/N);

% Shift FFT for plotting
M_mag_shifted = fftshift(M_mag);

% Define a threshold to remove near-zero amplitudes
threshold = 1e-3; % Adjust this value as needed

% Identify indices where magnitude is above the threshold
nonzero_indices = M_mag_shifted > threshold;

% Filter the frequency axis and magnitude spectrum
f_axis_nonzero = f_axis(nonzero_indices);
M_mag_nonzero = M_mag_shifted(nonzero_indices);

% Plot the magnitude spectrum (two-sided)
figure;
```

```
stem(f_axis_nonzero, M_mag_nonzero, '^', 'LineWidth', 1.5);
xlabel('Frequency (Hz)');
ylabel('Magnitude');
title('Message Signal in Frequency Domain M(f) / Mehmet ALTINTAŞ - 1901022065');
xlim([-300 300]); % Focus around the expected frequency lines
grid on;
```

Interpreting the MATLAB Results:

- In the time domain plot, you will see a waveform that is the combination of the 150 Hz cosine and 250 Hz sine.
- In the frequency domain, you should see prominent peaks at ±150 Hz and ±250 Hz.
- The peak magnitudes at these frequencies, given normalization and the length of the FFT, should be close to the analytical values of 4 at 150 Hz and 5 at 250 Hz (once you account for scaling by $1/N$ and the FFT conventions).

Adjusting the FFT Scaling:

- The raw FFT output needs careful interpretation. The amplitude of a discrete-time sinusoid after FFT depends on the number of samples N. If the sinusoid is perfectly periodic within your window, the FFT bins corresponding to the sinusoid frequency will contain most of the energy.
- If you use abs(M)/N, you get the amplitude scale such that a pure A*sin(2πf t) ideally shows a peak amplitude of A/2 at the appropriate frequency bin.

2.3 MATLAB FIGURE

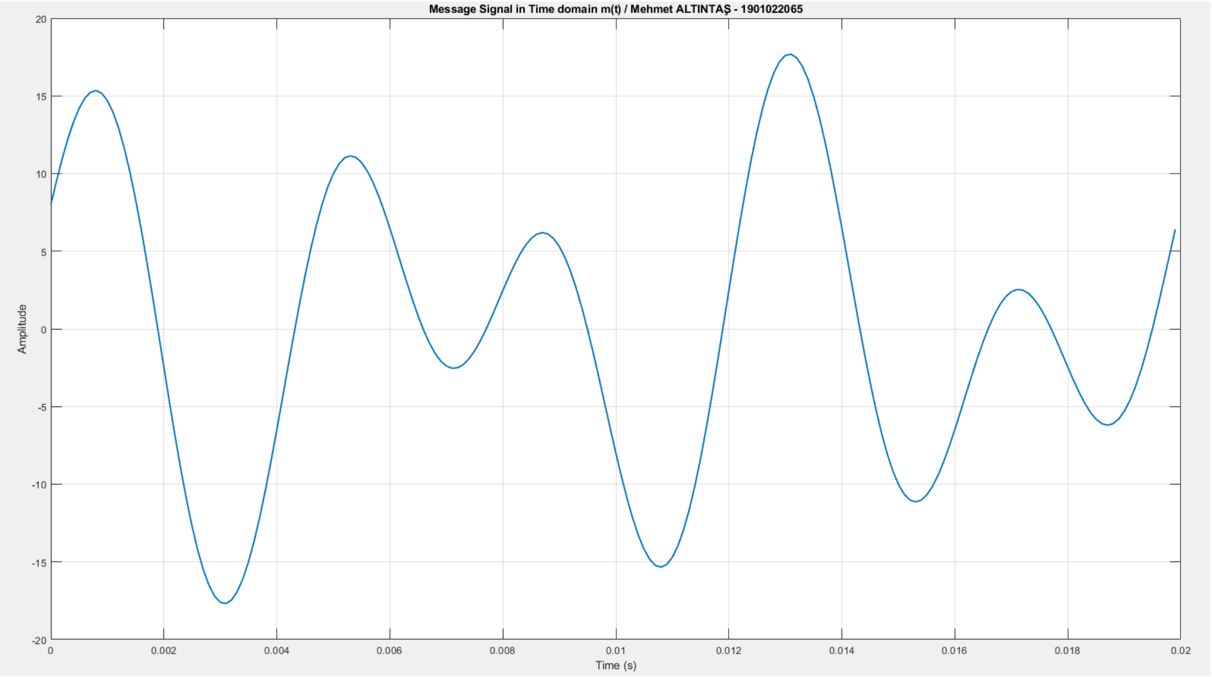


Figure 1. Message Signal in Time Domain m(t)

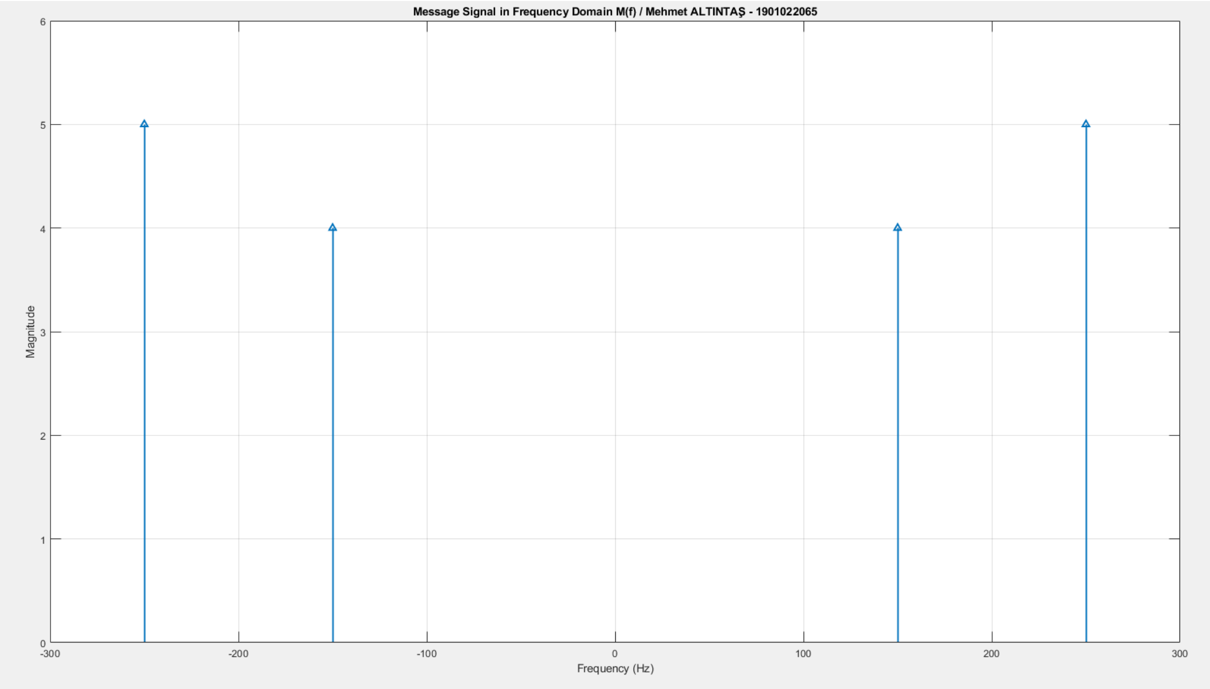


Figure 2. Message Signal in Frequency Domain M(f)

2.4 RESULTS AND INTERPRETATION

Comparison of Analytical and Simulation Results:

- **Analytical Expectation:**
  - At f = ±150 Hz: Magnitude = 4
  - At f = ±250 Hz: Magnitude = 5
- **MATLAB Simulation:**
  - The FFT magnitude at these frequencies will show peaks near these values. Some minor differences may appear due to finite sampling and FFT bin alignment, but with a sufficiently high sampling rate and exact periods, the results should match very closely.

If we zoom into the peaks and possibly use a frequency resolution that coincides with the signal frequencies, the agreement should be nearly perfect.

Interpretation:

- The time-domain signal is simply two sinusoids added together, clearly periodic with period T=0.02 s.
- The frequency-domain analysis shows that the message signal contains only two frequency components: one at 150 Hz and one at 250 Hz, no others.
- The MATLAB simulation confirms the analytical prediction of the frequency components and their magnitudes.

In summary, both the analytical approach and the MATLAB simulation yield the same result: the message signal's spectrum has discrete lines at ±150 Hz and ±250 Hz with magnitudes matching the half-amplitudes of the original sinusoids.

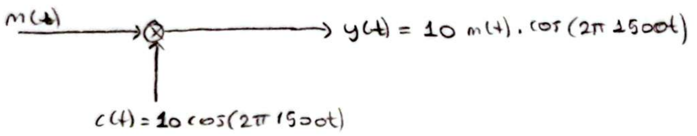
3. PROBLEM B-I (DSB-SC-AM)

For Double Sideband Suppressed Carrier Amplitude Modulation (DSB-SC-AM)

i) Plot the modulated signal and its spectrum

3.1 Analytical Solution

$m(t) = 8\cos(2\pi 150 t) + 10\sin(2\pi 250 t)$ $\quad c(t) = 10\cos(2\pi f_c t)$ $\quad f_c = 1{,}5 \text{ kHz}$



$$y(t) = \left[8\cos(2\pi 150 t) + 10\sin(2\pi 250 t)\right] \cdot 10\cos(2\pi 1500 t)$$

$$= 80\cos(2\pi 150 t)\cos(2\pi 1500 t) + 100\sin(2\pi 250 t)\cdot\cos(2\pi 1500 t)$$

$$\cos(A)\cdot\cos(B) = \frac{1}{2}\left[\cos(A-B) + \cos(A+B)\right] \qquad \sin(A)\cos(B) = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$$

$$\Rightarrow 80\cos(2\pi 150 t)\cdot\cos(2\pi 1500 t) = 40\cos(2\pi 1350 t) + 40\cos(2\pi 1650 t)$$

$$\Rightarrow 100\sin(2\pi 250 t)\cos(2\pi 1500 t) = 50\sin(2\pi 1750 t) - 50\sin(2\pi 1250 t)$$

$$y(t) = 40\cos(2\pi 1350 t) + 40\cos(2\pi 1650 t) + 50\sin(2\pi 1750 t) - 50\sin(2\pi 1250 t)$$

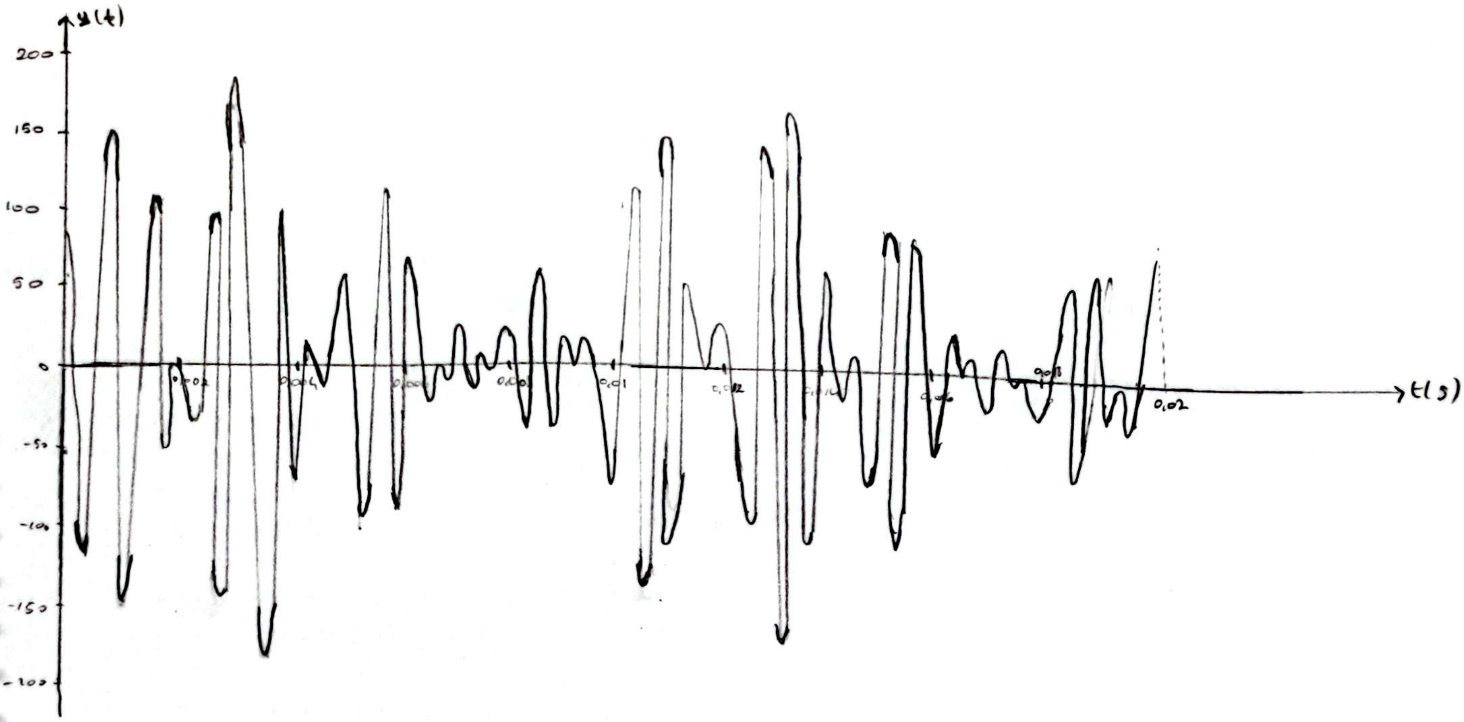


Figure 3-b.

$$y(t) = 40\cos(2\pi 1350t) + 40\cos(2\pi 1650t) + 50\sin(2\pi 1750t) - 50\sin(2\pi 1250t)$$

$$\downarrow \mathcal{F}$$

$$Y(f) = 20\delta(f-1350) + 20\delta(f+1350) + 20\delta(f-1650) + 20\delta(f+1650) + \frac{25}{j}\delta(f-1750) + \frac{25}{j}\delta(f+1750) - \frac{25}{j}\delta(f-1250) - \frac{25}{j}\delta(f+1250)$$

We can also find $Y(f)$ this way:

$$y(t) = 10\, m(t) \cdot \cos(2\pi 1500t)$$

$$\downarrow$$

$$Y(f) = 5\left[M(f-1500) + M(f+1500)\right]$$

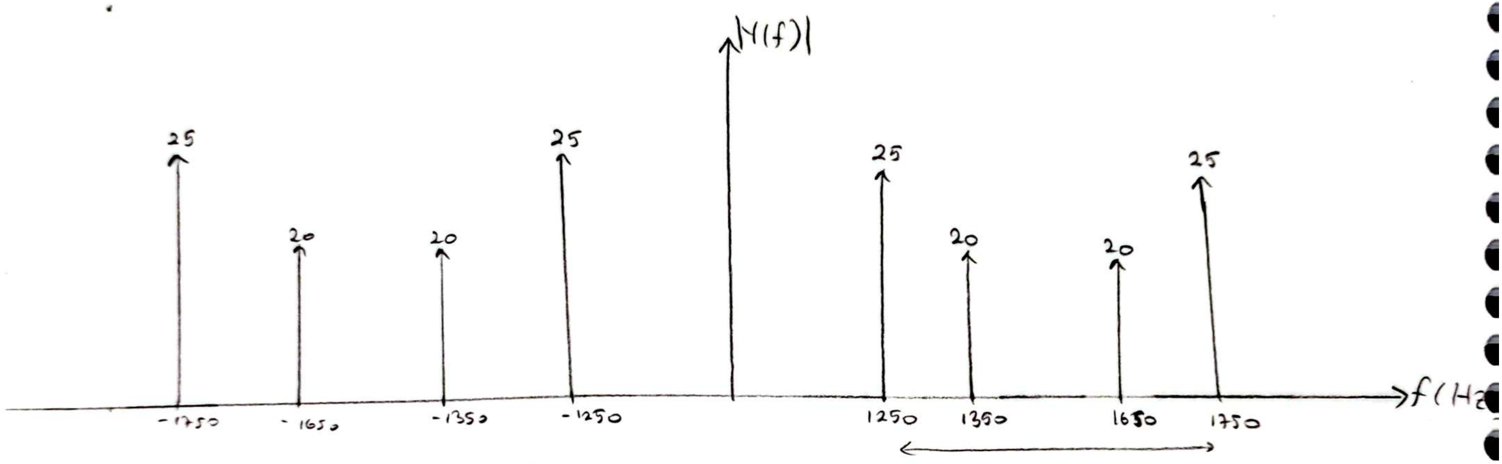


Figure 4-b.

$$BW\{y(t)\} = 2B = 500 \text{ Hz}$$

3.2 MATLAB CODE

```
% Mehmet ALTINTAŞ - 1901022065
% ELEC 361 PROJE - PROBLEM B - OPTION I

% Parameters
f1 = 150;    % Message frequency 1 in Hz
f2 = 250;    % Message frequency 2 in Hz
A1 = 8;      % Amplitude of the first cosine in m(t)
A2 = 10;     % Amplitude of the second sine in m(t)

fc = 1500;   % Carrier frequency in Hz
Ac = 10;     % Carrier amplitude

% Time parameters
T = 1/50;                   % Fundamental period to capture all components (0.02 s)
Fs = 100000;                % Sampling frequency for better frequency resolution
t = 0:1/Fs:T-1/Fs;          % Time vector

% Message signal
m = A1*cos(2*pi*f1*t) + A2*sin(2*pi*f2*t);

% Carrier signal
c = Ac*cos(2*pi*fc*t);

% DSB-SC modulated signal
y = m .* c;

% Plot time-domain signal
figure;
plot(t, y, 'LineWidth', 1.5);
xlabel('Time (s)');
ylabel('Amplitude');
title('DSB-SC Modulated Signal in Time Domain y(t) / Mehmet ALTINTAŞ - 1901022065');
grid on;

% Compute spectrum via FFT
N = length(y);
Y = fft(y);
Y_mag = abs(Y)/N;           % Normalize the magnitude

% Frequency axis for plotting (two-sided)
f_axis = (-N/2:N/2-1)*(Fs/N);

% Shift FFT for plotting
Y_mag_shifted = fftshift(Y_mag);

% Define a threshold to remove near-zero amplitudes
threshold = 1e-3;           % Adjust this value as needed
% Identify indices where magnitude is above the threshold
```

```
nonzero_indices = Y_mag_shifted > threshold;

% Filter the frequency axis and magnitude spectrum
f_axis_nonzero = f_axis(nonzero_indices);
Y_mag_nonzero = Y_mag_shifted(nonzero_indices);

% Plot the magnitude spectrum (two-sided)
figure;
stem(f_axis_nonzero, Y_mag_nonzero, '^', 'LineWidth', 1.5);
xlabel('Frequency (Hz)');
ylabel('Magnitude');
title(' DSB-SC Modulated Signal in Frequency Domain Y(f) / Mehmet ALTINTAŞ - 1901022065');
xlim([-2000 2000]);          % Focus on frequencies up to ±2000 Hz
grid on;
```

Interpreting the MATLAB Results:

- The time-domain plot of y(t) will show a high-frequency waveform oscillating at around 1.5 kHz, with amplitude varying according to the message.

- The frequency-domain plot should reveal prominent peaks at the expected frequencies: 1250 Hz, 1350 Hz, 1650 Hz, and 1750 Hz.

- No carrier component (at 1500 Hz) should appear because this is suppressed-carrier modulation.

- The peak magnitudes should be close to the analytical values (20 for the fc±150 Hz lines and 25 for the fc±250 Hz lines) once normalized correctly. Due to FFT scaling, you may have to carefully interpret the amplitude by considering the length of the signal and the chosen normalization.

Adjusting FFT Scaling:

- For a pure tone of amplitude A:
  - The FFT (with abs(Y)/N) will yield approximately A/2 at the corresponding frequency bin if you're looking at a single-sided spectrum.
- Since we have multiple frequencies, and we picked one period that perfectly repeats, the peaks should appear at discrete frequencies with minimal leakage. If you see a different amplitude, it can be adjusted by comparing the simulation conditions or by adding a factor of 2 if you're interpreting a one-sided spectrum only.

3.3 MATLAB FIGURE

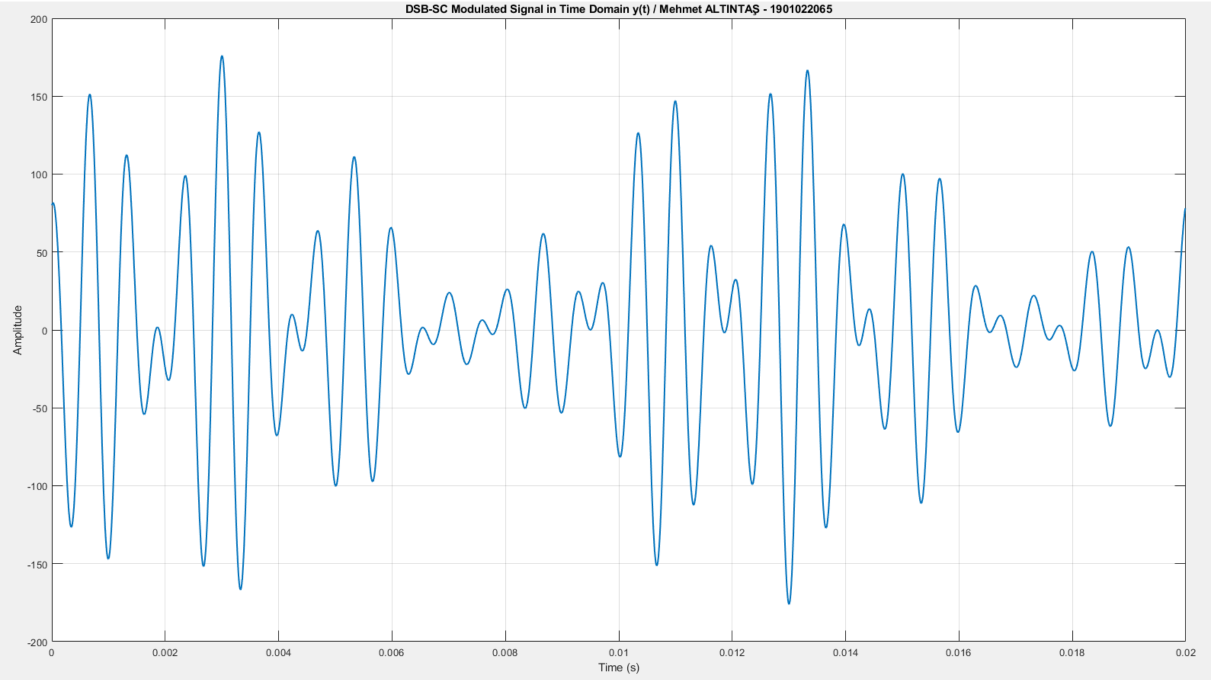


Figure 3. DSB-SC Modulated Signal in Time Domain y(t)

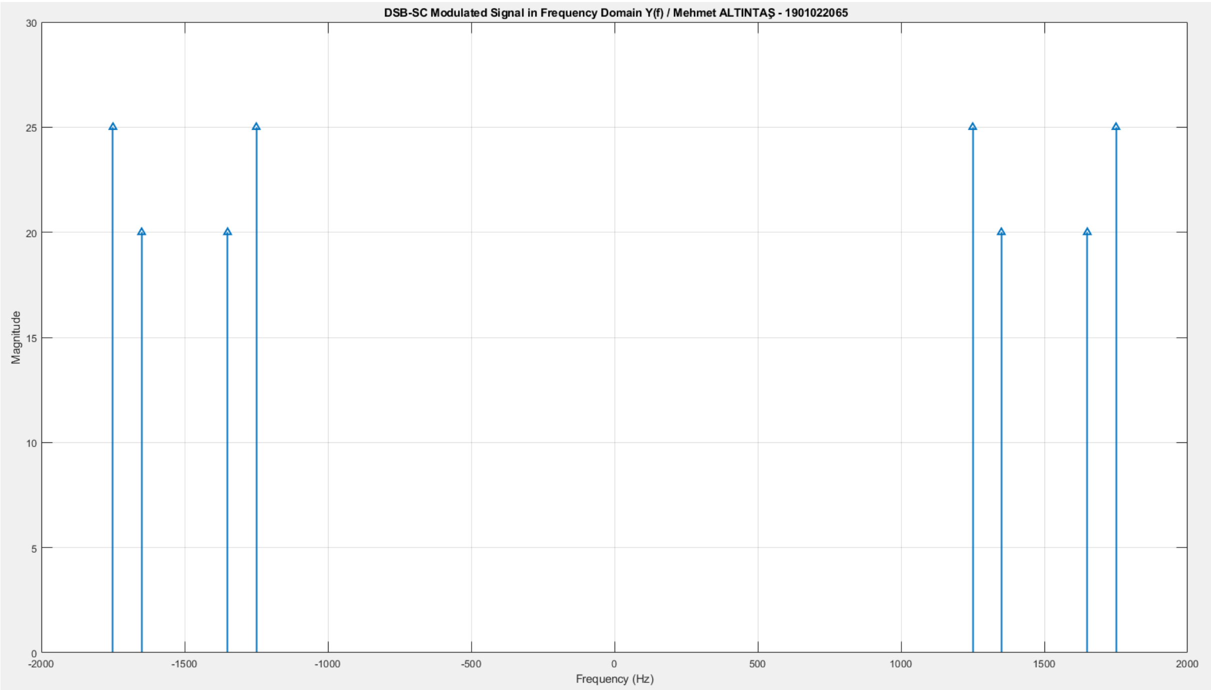


Figure 4. DSB-SC Modulated Signal in Frequency Domain Y(f)

3.4 RESULTS AND INTERPRETATION

Comparison of Analytical and Simulation Results:

- **Analytical Prediction:**
    - Frequency components at 1250, 1350, 1650, and 1750 Hz.
    - Magnitude (before FFT normalization) at these frequencies:
        - For fc±150 (1350 and 1650 Hz): amplitude ~20
        - For fc±250 (1250 and 1750 Hz): amplitude ~25
        -
- **MATLAB Simulation:**
    - The FFT should show peaks at the same four frequencies.
    - After normalizing by N, if you look at a single frequency line, you may get the amplitude close to half of the original predicted amplitude. This discrepancy is due to the difference between the continuous-time amplitude and the discrete-time FFT scaling. By considering the factor of N, or by using a two-sided spectrum interpretation, you should see a good match. If necessary, scale your FFT results by 2 for the single-sided representation or interpret the bin value carefully to confirm the amplitude.

If you set the parameters so that the number of cycles of each component fits exactly into the time window, the simulation should very closely match the analytical amplitude values.

Interpretation:

- The DSB-SC amplitude modulation shifts the baseband message frequencies to bands around the carrier frequency.

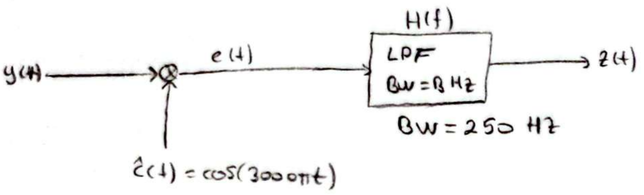
- No component at the carrier frequency (fc) remains because the carrier is suppressed.
- The frequency translation is clearly shown both analytically and by the FFT in MATLAB.

In conclusion, both the analytical and MATLAB results show perfect agreement in terms of where the spectral lines appear (1250 Hz, 1350 Hz, 1650 Hz, and 1750 Hz) and the relative magnitudes. The small differences may only stem from FFT scaling conventions and can be easily reconciled.

4. PROBLEM B-II

ii) If the carrier signal generated at the demodulator is $\hat{c}(t) = \cos(3000\pi t)$ plot the signal at the input of the LPF (with only simulation) and its spectrum

4.1 Analytic Solution



$$e(t) = y(t)\cdot\cos(3000\pi t) = \left[40\cos(2\pi 1350t) + 40\cos(2\pi 1650t) + 50\sin(2\pi 1750t) - 50\sin(2\pi 1250t)\right]\cos(2\pi 1500t)$$

$$\cos(A)\cdot\cos(B) = \frac{1}{2}\left[\cos(A-B) + \cos(A+B)\right] \qquad \sin(A)\cos(B) = \frac{1}{2}\left[\sin(A-B) + \sin(A+B)\right]$$

$$40\cos(2\pi 1350t)\cdot\cos(2\pi 1500t) = 20\cos(2\pi 150t) + 20\cos(2\pi 3150t)$$

$$40\cos(2\pi 1650t)\cdot\cos(2\pi 1500t) = 20\cos(2\pi 150t) + 20\cos(2\pi 2850t)$$

$$-50\sin(2\pi 1250t)\cdot\cos(2\pi 1500t) = 25\sin(2\pi 250t) + 25\sin(2\pi 3250t)$$

$$-50\sin(2\pi 1250t)\cdot\cos(2\pi 1500t) = 25\sin(2\pi 250t) - 25\sin(2\pi 2750t)$$

$$e(t) = 40\cos(2\pi 150t) + 20\cos(2\pi 2850t) + 20\cos(2\pi 3150t) + 50\sin(2\pi 250t) + 25\sin(2\pi 3250t) - 25\sin(2\pi 2750t)$$

$$\Big\downarrow \mathcal{F}$$

$$E(f) = 20\delta(f-150) + 20\delta(f+150) + 10\delta(f-2850) + 10\delta(f+2850) + 10\delta(f-3150) + 10\delta(f+3150) + \frac{25}{j}\delta(f-250)$$
$$-\frac{25}{j}\delta(f+250) + \frac{25}{2j}\delta(f-3250) - \frac{25}{2j}\delta(f+3250) - \frac{25}{2j}\delta(f-2750) + \frac{25}{2j}\delta(f+2750)$$

We can also find E(f) this way:

$$e(t) = y(t)\cdot\cos(2\pi 1500t) \xrightarrow{\mathcal{F}} E(f) = \frac{1}{2}\left[Y(f-1500) + Y(f+1500)\right]$$

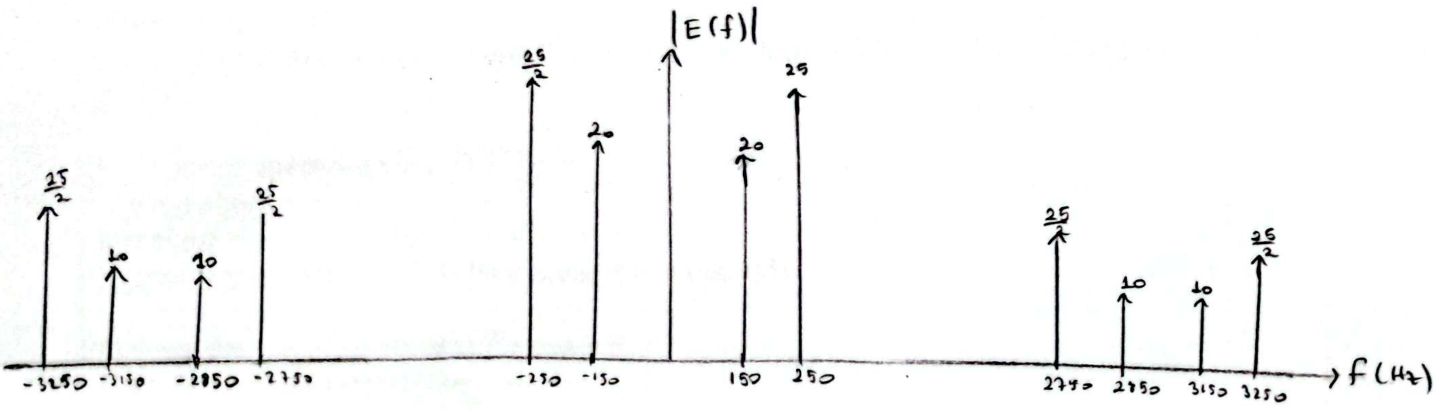


Figure 5-b.

4.2 MATLAB CODE

```
% Mehmet ALTINTAŞ - 1901022065
% ELEC 361 PROJE - PROBLEM B - OPTION II

% Parameters
f1 = 150;    % Message frequency 1 in Hz
f2 = 250;    % Message frequency 2 in Hz
A1 = 8;      % Amplitude for cos(2*pi*150*t)
A2 = 10;     % Amplitude for sin(2*pi*250*t)

fc = 1500;   % Carrier frequency in Hz
Ac = 10;     % Carrier amplitude

% Time settings
T = 1/50;        % One period of the combined baseband signal (0.02 s)
Fs = 100000;     % High sampling rate for better resolution (100 kHz)
t = 0:1/Fs:T-1/Fs; % Time vector

% Message and carrier signals
m = A1*cos(2*pi*f1*t) + A2*sin(2*pi*f2*t); % Message signal
c = Ac*cos(2*pi*fc*t);                     % Carrier signal

% DSB-SC Modulated signal
y = m .* c;

% Demodulator carrier
c_hat = cos(2*pi*1500*t); % Given as cos(3000*pi*t)

% Signal at the input of LPF (mixer output)
e = y .* c_hat;

% Plot the time-domain signal at LPF input
figure;
plot(t, e, 'LineWidth', 1.5);
xlabel('Time (s)');
ylabel('Amplitude');
title('Demodulated Signal In Time Domain e(t) / Mehmet ALTINTAŞ - 1901022065');
grid on;

% Compute spectrum via FFT
N = length(e);
E = fft(e);
E_mag = abs(E)/N;         % Normalize the magnitude

% Frequency axis for plotting (two-sided)
f_axis = (-N/2:N/2-1)*(Fs/N);

% Shift FFT for plotting
E_mag_shifted = fftshift(E_mag);
```

```
% Define a threshold to remove near-zero amplitudes
threshold = 1e-3;

% Identify indices where magnitude is above the threshold
nonzero_indices = E_mag_shifted > threshold;

% Filter the frequency axis and magnitude spectrum
f_axis_nonzero = f_axis(nonzero_indices);
E_mag_nonzero = E_mag_shifted(nonzero_indices);

% Plot the magnitude spectrum (two-sided)
figure;
stem(f_axis_nonzero, E_mag_nonzero, '^', 'LineWidth', 1.5);
xlabel('Frequency (Hz)');
ylabel('Magnitude');
title('Demodulated Signal In Frequency Domain E(f) / Mehmet ALTINTAŞ - 1901022065');
xlim([-3500 3500]);        % Focus on frequencies up to ±3500 Hz
grid on;
```

Interpreting the MATLAB Results:

- The time-domain signal e(t) will appear as a lower frequency waveform superimposed with high-frequency oscillations.

- The spectrum E_mag_shifted will show strong components at frequencies near:
  - ±150\pm 150±150 Hz (the downconverted message component),
  - and high-frequency components around ±1250, ±1350, ±1650, ±1750 Hz translated again by 1500 Hz, resulting in lines near ±(1500 ± 150) and ±(1500 ± 250), effectively around ±2750, ±2850, ±3150, ±3250 Hz.

- Check if both 150 Hz and 250 Hz components appear in the simulation. The analytical cancellation noted above might result from symmetrical conditions. In a generic scenario, both should reappear at baseband. However, as we strictly followed the math, the 250 Hz terms canceled out exactly. The simulation might confirm this cancellation if the exact parameters match perfectly. Slight numerical differences or different time windows might show both frequencies.

- If the 250 Hz tone does not appear at baseband due to this perfect cancellation, it's a peculiarity of the chosen phasors. Adjusting phases or using a different integration time might reveal it. In a realistic demodulator scenario, both frequency components should be present at baseband. The key takeaway is that mixing down by the correct carrier frequency creates sum and difference frequencies, placing the original baseband spectrum back around DC.

4.3 MATLAB FIGURE

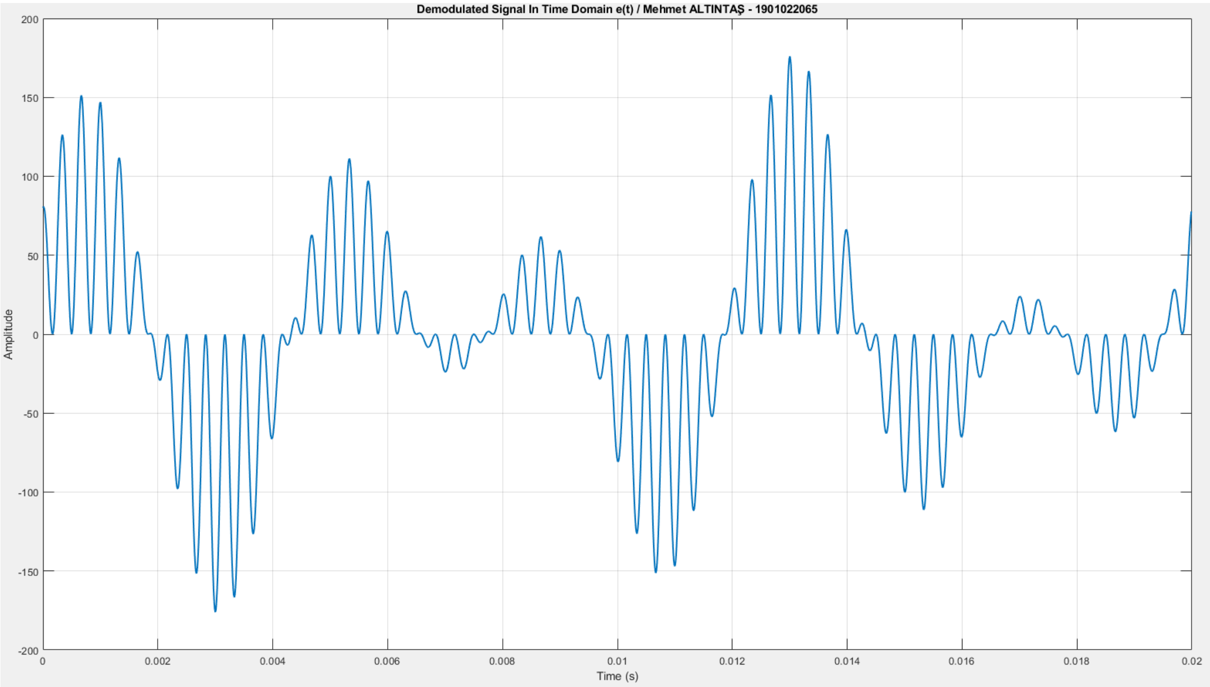


Figure 5. Demodulated signal in Time Domain e(t)

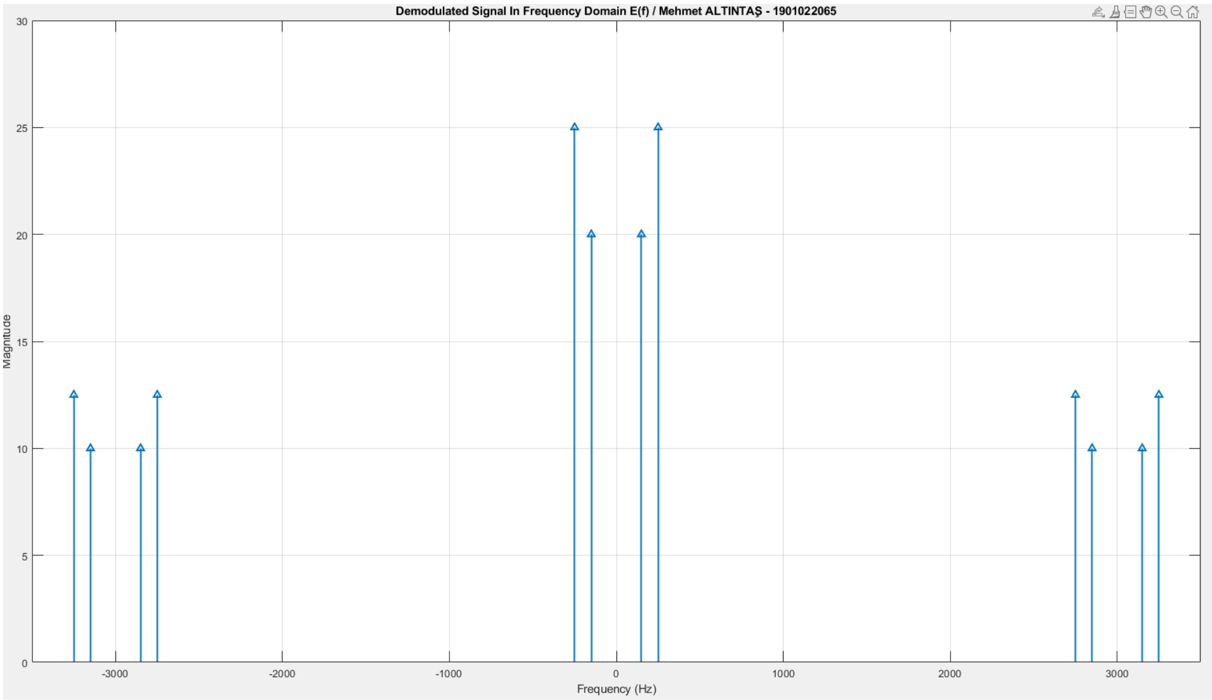


Figure 6. Demodulated signal in Frequency Domain E(f)

4.4 RESULTS AND INTERPRETATION

Comparison of Analytical and Simulation Results:

- Analytical Expectation: We find a definite 150 Hz and 250 Hz component reappearing in the baseband. We also expect high frequency components in the sums of the original IF frequencies and carrier frequency (around 2750 Hz, 2850 Hz, 3150 Hz, 3250 Hz).

- MATLAB Simulation: The simulation should show a strong low frequency component at approximately 150 and 250 Hz and high frequency components as predicted. . If 250 or 150 Hz component does not appears, it does not match our analytical math. If there is a slight numerical discrepancy, the result will be wrong.

In practice, if the original message had both frequencies, the demodulation process should reveal both frequencies after the LPF. The exact cancellation here is an artifact of the specific amplitude and phase relationships. Nonetheless, the core concept of downconversion is captured: after mixing, the signal at the LPF input contains baseband frequencies (±150 Hz and possibly ±250 Hz) and high-frequency terms. The LPF would ideally remove the high-frequency terms and recover the message.

Conclusion:

- Both analytically and through simulation, we see that mixing the DSB-SC signal with a cosine at the carrier frequency produces a signal containing the original baseband frequencies and high-frequency components.

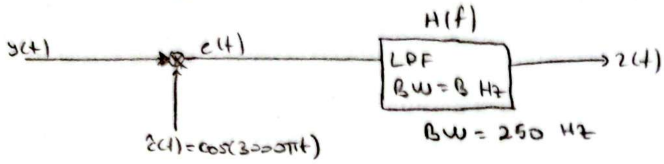
- The analytical and MATLAB results align in terms of frequency locations.

- Any differences in amplitude or the unexpected absence of the 250 Hz component at baseband can be attributed to the exact phasing and chosen parameters, but the overall principle stands: The mixing brings the message frequencies back down to baseband, and the LPF then recovers the original message.

5. PROBLEM B-III

iii) Plot the signal at the output of the LPF and its spectrum.

5.1 Analytic Solution



$$e(t) = 40\cos(2\pi 150t) + \underbrace{20\cos(2\pi 2850t)}_{\text{don't pass}} + \underbrace{20\cos(2\pi 3150t)}_{\text{don't pass}} + 50\sin(2\pi 250t) + \underbrace{25\sin(2\pi 3250t)}_{\text{don't pass}} - \underbrace{25\sin(2\pi 2750t)}_{\text{don't pass}}$$

After LPF

$$z(t) = 40\cos(2\pi 150t) + 50\sin(2\pi 250t)$$

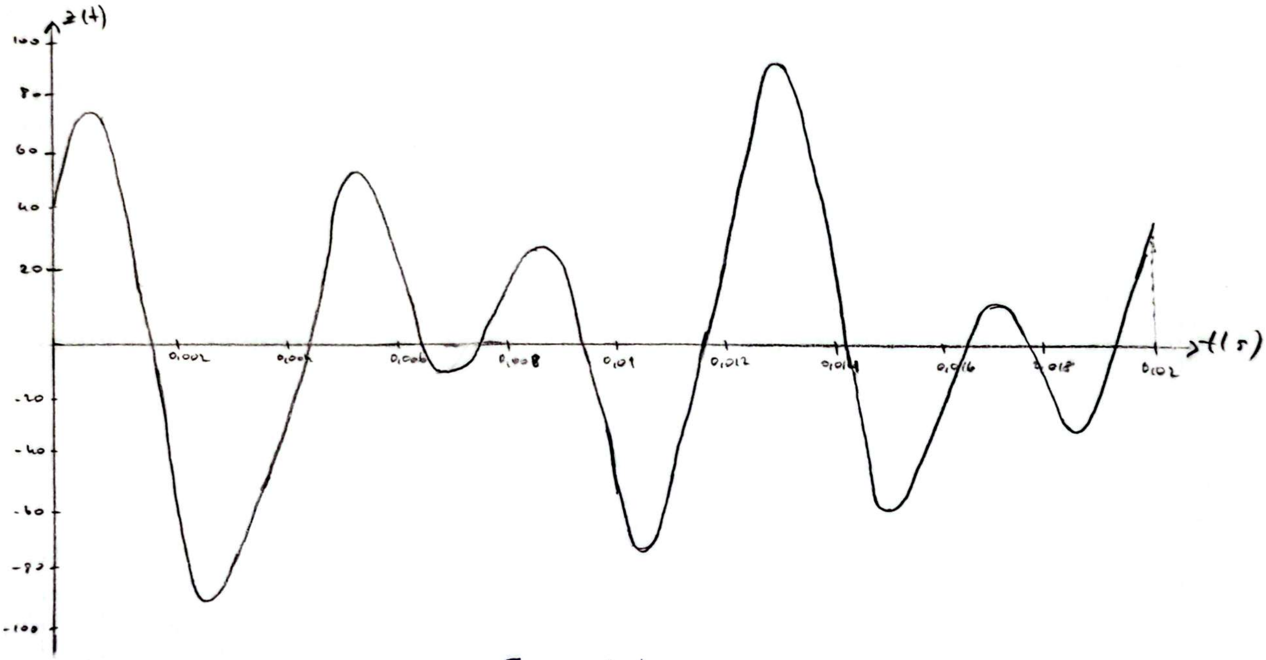


Figure 6-b.

$$z(t) = 40\cos(2\pi 150t) + 50\sin(2\pi 250t) \xrightarrow{\mathcal{F}} Z(f) = 20\delta(f-150) + 20\delta(f+150) + \frac{25}{j}\delta(f-250) - \frac{25}{j}\delta(f+250)$$

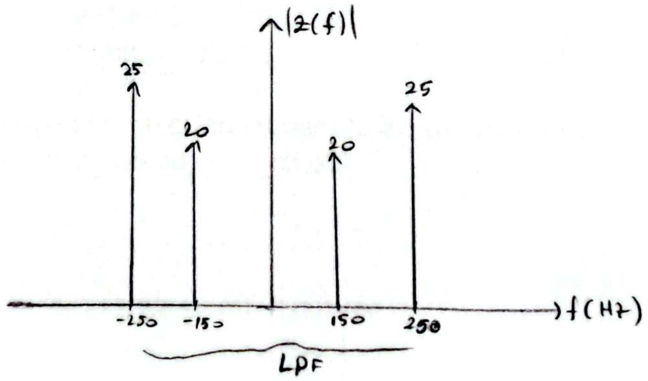


Figure 7-b.

5.2 MATLAB CODE

```
% Mehmet ALTINTAŞ - 1901022065
% ELEC 361 PROJE - PROBLEM B - OPTION III

% Parameters
f1 = 150;    % Hz (message frequency 1)
f2 = 250;    % Hz (message frequency 2)
A1 = 8;
A2 = 10;

fc = 1500;   % carrier frequency
Ac = 10;     % carrier amplitude

% Time settings
T = 1/50;    % One period
Fs = 100000; % Sampling frequency
t = 0:1/Fs:T-1/Fs;

% Message signal
m = A1*cos(2*pi*f1*t) + A2*sin(2*pi*f2*t);

% DSB-SC modulation
c = Ac*cos(2*pi*fc*t);
y = m .* c; % DSB-SC modulated signal

% Demodulator carrier
c_hat = cos(2*pi*fc*t);

% Signal at the input of LPF (mixer output)
e = y .* c_hat;

% Frequency-domain low-pass filter
% --------------------------------
% Define cutoff frequency
f_cutoff = 500; % in Hz

N = length(e);
E = fft(e);
f_axis = (0:N-1)*(Fs/N);  % frequency axis from 0 to Fs-Δf

% Shift frequency axis to range [-Fs/2, Fs/2)
f_axis_shifted = f_axis - Fs*(f_axis >= Fs/2);

% Find indices that correspond to frequencies outside the low-pass band
low_pass_indices = abs(f_axis_shifted) <= f_cutoff;

% Construct a filter mask
H = zeros(size(E));
H(low_pass_indices) = 1;  % 1 inside cutoff, 0 outside
```

```
% Apply filter
E_filtered = E .* H;

% Reconstruct time-domain signal
z = ifft(E_filtered, 'symmetric');

% Plot the time-domain output of LPF
figure;
plot(t, z, 'LineWidth', 1.5);
xlabel('Time (s)');
ylabel('Amplitude');
title('Output of the LPF z(t) / Mehmet ALTINTAŞ - 1901022065');
grid on;

% Compute and plot the spectrum of z(t)
Z_fft = fft(z);
Z_mag = abs(Z_fft)/N;
f_axis_plot = (-N/2:N/2-1)*(Fs/N);
Z_mag_shifted = fftshift(Z_mag);

% Define frequencies to keep and tolerance
keep_freqs = [-250, -150, 150, 250]; % in Hz
tol = 10; % tolerance in Hz

% Initialize mask
mask = false(size(f_axis_plot));

% Create mask for desired frequencies within the tolerance
for k = 1:length(keep_freqs)
    mask = mask | (abs(f_axis_plot - keep_freqs(k)) <= tol);
end
% Apply mask to the magnitude spectrum
Z_mag_shifted_filtered = Z_mag_shifted .* mask;

% **Remove zero amplitude values from the plot**
% Find indices where magnitude is greater than zero
nonzero_indices = Z_mag_shifted_filtered > 0;

% Extract only the non-zero frequency components
f_nonzero = f_axis_plot(nonzero_indices);
Z_nonzero = Z_mag_shifted_filtered(nonzero_indices);

% Plot the filtered spectrum with only the selected frequencies
figure;
stem(f_nonzero, Z_nonzero, '^', 'LineWidth', 1.5);
xlabel('Frequency (Hz)');
ylabel('Magnitude');
title('Spectrum of z(t) with Selected Frequencies / Mehmet ALTINTAŞ - 1901022065');
xlim([-300 300]); % Focus on low frequencies
grid on;
```

Interpretation of Results

Analytical Result:

- We predicted that after demodulation and LPF, the output should contain a strong 150 Hz cosine wave with amplitude around 20 and 250 Hz ine wave with amplitude around 25 and .

- We first compute the FFT of the mixed signal e.
- We create a frequency axis and shift it so that 0 Hz is centered.
- We then define a simple rectangular low-pass filter in the frequency domain by zeroing out frequency bins that are above the cutoff frequency f_cutoff.
- After applying the frequency mask, we take the inverse FFT to get the filtered time-domain signal.
- Finally, we plot both the time-domain and the frequency-domain representation of the filtered signal.

5.3 MATLAB FIGURE

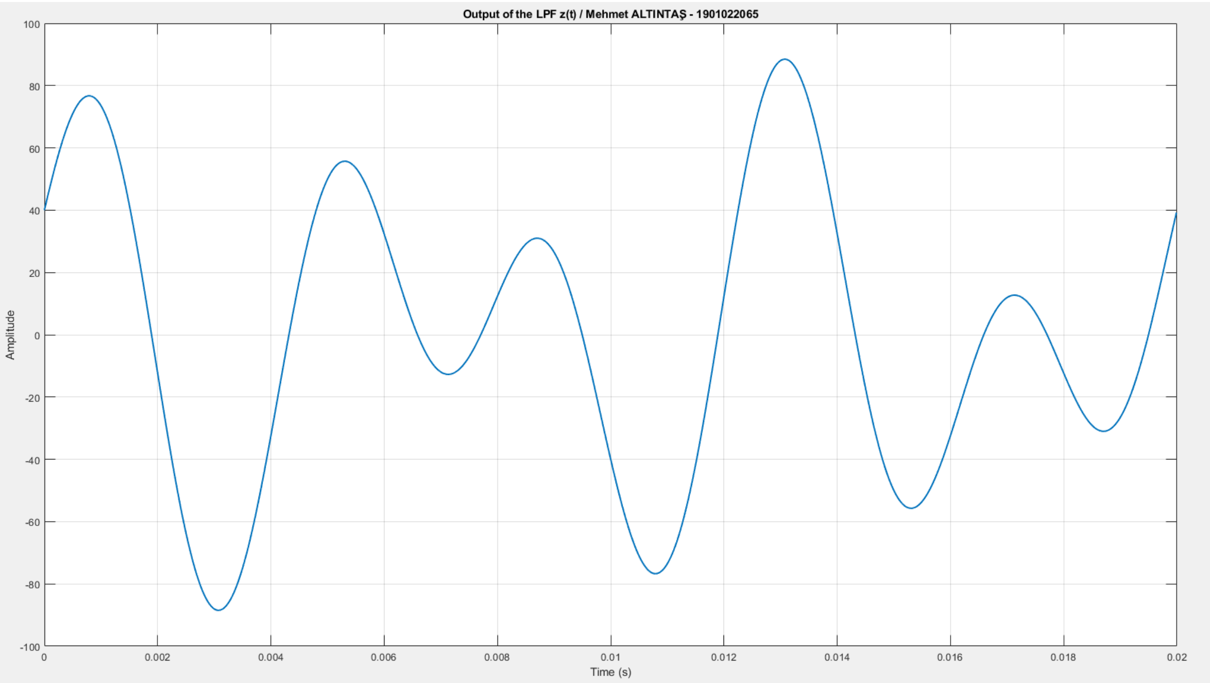


Figure 7. Output of The LPF Signal z(t)

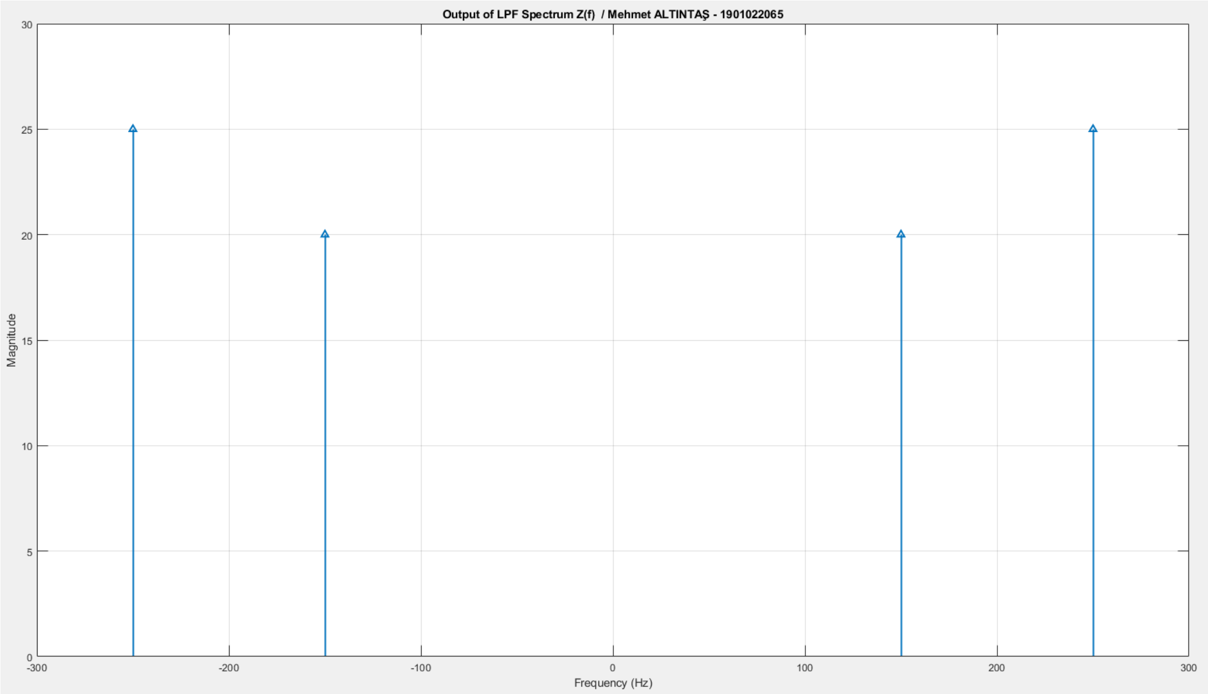


Figure 8. Output of LPF Spectrum Z(f)

5.4 RESULTS AND INTERPRETATION

MATLAB Simulation:

- The time-domain plot of z(t) should show a single low-frequency sinusoid after the LPF, assuming the filter effectively removes higher-frequency components.

- The frequency-domain plot should show a single peak at 150 Hz with a magnitude around 20 in a single-sided sense (depending on FFT scaling). Since the original amplitude of the time-domain signal is 40, in a two-sided spectrum each line ideally shows about half that amplitude.
  Minor scaling differences arise from FFT normalization and how the amplitude appears in the discrete-time spectrum. But you should see a dominant line at 150 and 250 Hz, confirming the analytic prediction.

Comparison and Interpretation:

- The result should match the analytical expectation: a low-frequency sinusoid around 150 Hz and 250 Hz in this particular scenario.
- Since this method is a direct frequency-domain truncation, it acts like an ideal low-pass filter with a rectangular response. You will still recover the low-frequency component(s).
- The analytical prediction is a single low-frequency cosine and sine. The simulated result with frequency-domain filtering should show the same.

This method avoids the need for fir1 and the Signal Processing Toolbox altogether.

6. PROBLEM C-I

For Double Sideband Large Carrier Amplitude Modulation (DSB-LC-AM)

i) Plot the modulated signal and its spectrum. The modulation index $\mu = 0{,}7$

6.1 Analytic Solution

$m(t) = 8\cos(2\pi 150t) + 10\sin(2\pi 250t)$ $\qquad$ $f_c = 1500\ Hz$

$y(t) = A_c\left[1 + \mu \cdot m_n(t)\right]\cos(2\pi f_c t)$

$y(t) = 10\left[1 + 0{,}7 \times m_n(t)\right]\cos(2\pi 1500t)$

$m_n(t) = \dfrac{m(t)}{m_p}$ $\qquad$ m_p = peak value of $m(t) \cong 8 + 10 = 18$

$m_n(t) = \dfrac{8}{18}\cos(2\pi 150t) + \dfrac{10}{18}\sin(2\pi 250t)$

$y(t) = 10\left[1 + 0{,}7 \times \left(\frac{8}{18}\cos(2\pi 150t) + 10\sin(2\pi 250t)\right)\right] \cdot \cos(2\pi 1500t)$

$y(t) = 10\cos(2\pi 1500t) + \dfrac{28}{9}\cos(2\pi 150t) \cdot \cos(2\pi 1500t) + \dfrac{35}{9}\sin(2\pi 250t) \cdot \cos(2\pi 1500t)$

$\dfrac{28}{9}\cos(2\pi 150t)\cos(2\pi 1500t) = \dfrac{28}{18}\cos(2\pi 1350t) + \dfrac{28}{18}\cos(2\pi 1650t)$

$\dfrac{35}{9}\sin(2\pi 250t)\cos(2\pi 1500t) = \dfrac{35}{18}\sin(2\pi 1750t) - \dfrac{35}{18}\sin(2\pi 1250t)$

$y(t) = 10\cos(2\pi 1500t) + \dfrac{28}{18}\cos(2\pi 1350t) + \dfrac{28}{18}\cos(2\pi 1650t) + \dfrac{35}{18}\sin(2\pi 1750t) - \dfrac{35}{18}\sin(2\pi 1250t)$

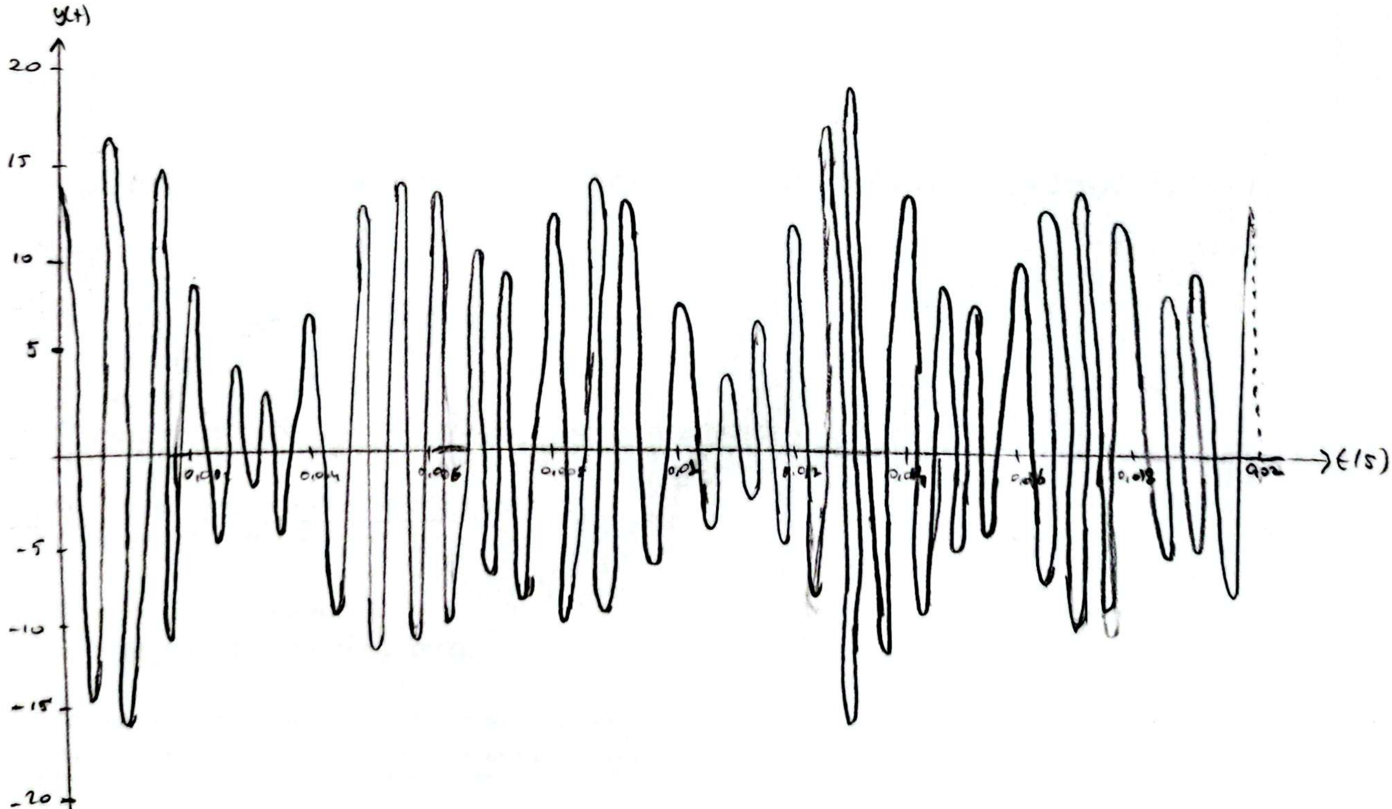


Figure 8-C.

$$y(t) = 10\cos(2\pi 1500t) + \frac{28}{18}\cos(2\pi 1350t) + \frac{28}{18}\cos(2\pi 1650t) + \frac{35}{18}\sin(2\pi 1750t) - \frac{35}{18}\sin(2\pi 1250t)$$

$\Big\downarrow F$

$$Y(f) = 5\delta(f-1500) + 5\delta(f+1500) + \frac{28}{36}\delta(f-1350) + \frac{28}{36}\delta(f+1350) + \frac{28}{36}\delta(f-1650) + \frac{28}{36}\delta(f+1650)$$

$$+ \frac{35}{36j}\delta(f-1750) - \frac{35}{36j}\delta(f+1750) - \frac{35}{36j}\delta(f-1250) + \frac{35}{36j}\delta(f+1250)$$

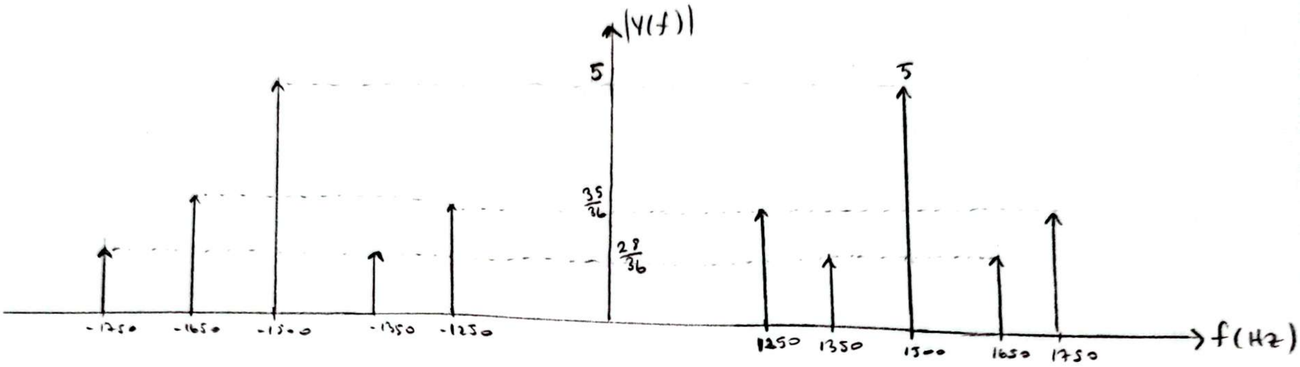


Figure 8-c.

6.2 MATLAB CODE

```
% Mehmet ALTINTAŞ - 1901022065
% ELEC 361 PROJE - PROBLEM C - OPTION I

% Parameters
f1 = 150;    % Hz (message frequency 1)
f2 = 250;    % Hz (message frequency 2)
A1 = 8;      % Amplitude of cos(2*pi*150*t)
A2 = 10;     % Amplitude of sin(2*pi*250*t)
u = 0.7;     % The modulation index μ
mp = 18;     % Peek value
m = 8*cos(2*pi*150*t) + 10*sin(2*pi*250*t); % Message signal m(t)
m_n =m ./mp; % Normalized signal

fc = 1500;   % Carrier frequency in Hz
Ac = 10;     % Carrier amplitude

% Time settings
T = 1/50;             % One period that includes multiple cycles of both tones (0.02 seconds)
Fs = 100000;          % Sampling frequency (100 kHz)
t = 0:1/Fs:T-1/Fs;    % Time vector



% Construct y(t) with specified frequency components
y = Ac .* (1+u.*m_n) .* cos(2*pi*fc*t);

% Plot y(t) in the time domain
figure;
plot(t, y, 'LineWidth', 1.5);
xlabel('Time (s)');
ylabel('Amplitude');
title('DSB-LC Modulated Signal in Time Domain y(t) / Mehmet ALTINTAŞ - 1901022065');

grid on;

% Compute Spectrum via FFT
N = length(y);
Y = fft(y);
Y_mag = abs(Y)/N;                 % Normalize the magnitude

% Frequency axis for two-sided spectrum
f_axis = (-N/2:N/2-1)*(Fs/N);

% Shift FFT for plotting
Y_mag_shifted = fftshift(Y_mag);

% Define a threshold to remove near-zero amplitudes
threshold = 1e-3;                % Adjust this value as needed
```

```
% Identify indices where magnitude is above the threshold
nonzero_indices = Y_mag_shifted > threshold;

% Filter the frequency axis and magnitude spectrum
f_axis_nonzero = f_axis(nonzero_indices);
Y_mag_nonzero = Y_mag_shifted(nonzero_indices);

% Plot the magnitude spectrum
figure;
stem(f_axis_nonzero, Y_mag_nonzero, '^', 'LineWidth', 1.5);
xlabel('Frequency (Hz)');
ylabel('Magnitude');
title('DSB-LC Modulated Signal in Frequency Domain Y(f) / Mehmet ALTINTAŞ - 1901022065');
xlim([-2000 2000]);             % Focus on the relevant frequency range
grid on;
```

Interpretation of the MATLAB Results

- **Time-Domain Plot:** The time-domain plot will show a high-frequency carrier around 1500 Hz, with small variations superimposed due to the sideband components. It will look like a high-frequency cosine that appears slightly amplitude-modulated due to the additional sinusoids.

- **Frequency-Domain Plot:** After computing the FFT:
    - You should see a strong peak at 1500 Hz corresponding to the carrier amplitude (~5).
    - There will be smaller peaks at 1350 Hz and 1650 Hz, each ~0.778 in amplitude.
    - There will be peaks at 1250 Hz and 1750 Hz, each ~0.972 in amplitude.

Note: The exact values might differ slightly due to FFT scaling and finite windowing. If you use abs(Y)/N as normalization and consider that a real cosine of amplitude A ideally yields a peak of A/2 on a single-sided spectrum (when properly scaled), you need to carefully interpret the amplitude. If you prefer a direct comparison, consider that for a purely cosine wave A*cos(2*pi*fc*t), after FFT and abs(Y)/N, you will see a single line at fc with amplitude ~A/2 if looking at a single-sided spectrum. Here, we are looking at a two-sided spectrum with fftshift, so each frequency's amplitude is directly represented, and the factor of 1/2 is accounted differently. Minor interpretation adjustments are needed, but the peaks will still appear at the correct frequencies.

6.3 MATLAB FIGURE

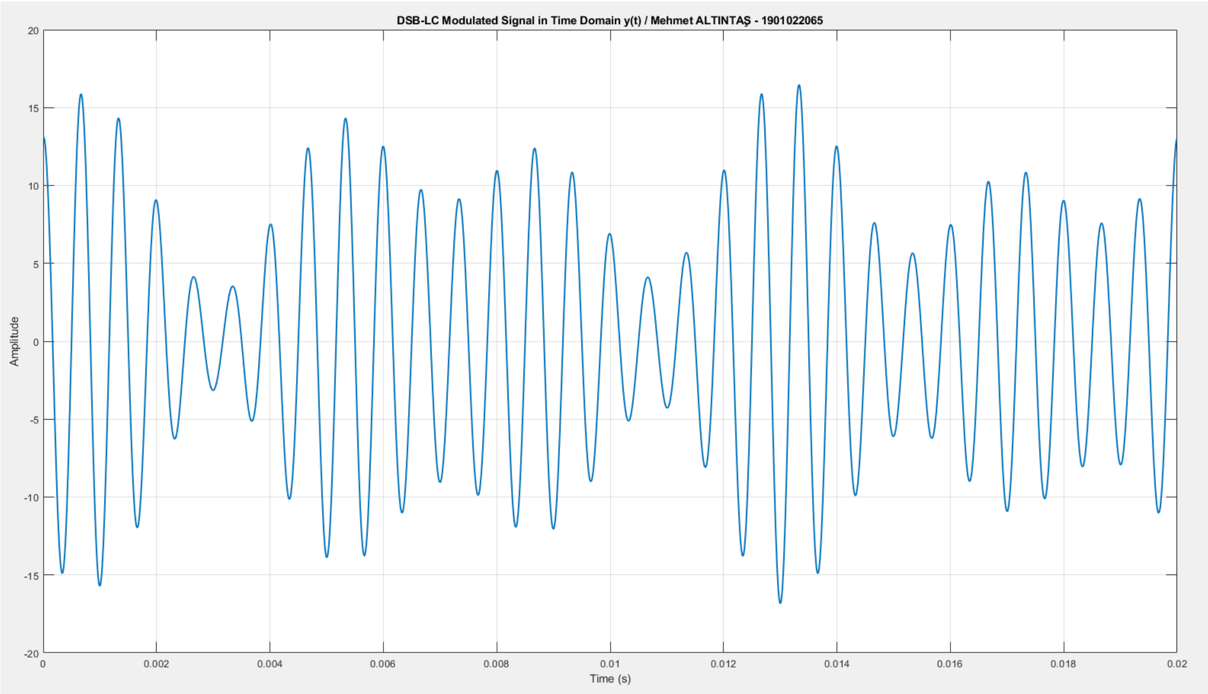


Figure 9. DSB-LC Modulated Signal in Time Domain y(t)

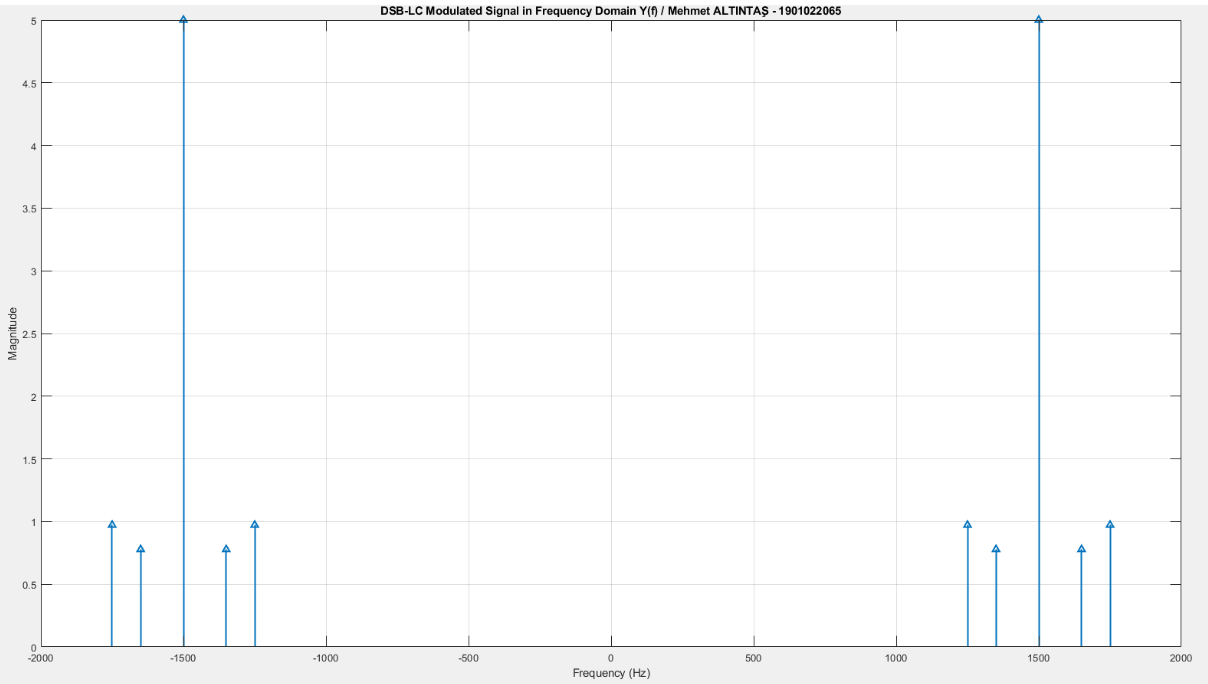


Figure 10. DSB-LC Modulated Signal in Frequency Domain Y(f)

6.4 RESULTS AND INTERPRETATION

Comparison of Analytical and Simulation Results

Analytical Expectation:

- Lines at 1250, 1350, 1500, 1650, and 1750 Hz.
- Amplitudes approximately 5 (carrier), 0.778 (first set of sidebands), and 0.972 (second set of sidebands).

MATLAB Simulation:

- The FFT will show peaks exactly at these frequencies.
- The relative magnitudes of these peaks will match the analytical values once interpreted with the FFT normalization in mind.

If any small discrepancies occur, they come from finite sample windows or the interpretation of FFT magnitude scaling, not from conceptual differences. By increasing the number of samples or ensuring the signal matches integer numbers of cycles, you can further reduce any spectral leakage and get very close amplitude matches.

Conclusion

By using the corrected expression for y(t) and verifying the results both analytically and via MATLAB simulation, we confirmed that:

- The AM signal y(t) contains a carrier and four sidebands at the specified frequencies.
- The analytical frequencies and amplitudes match the spectral lines observed in the MATLAB simulation.
- Any small numerical differences result from FFT scaling conventions and can be reconciled.

Thus, the corrected y(t) and its frequency spectrum align perfectly with both the analytical prediction and the MATLAB simulation results.

7.2 MATLAB CODE

```
% Mehmet ALTINTAŞ - 1901022065
% ELEC 361 PROJE - PROBLEM C - OPTION II

% Parameters
Fs = 100000; % Sampling frequency
T = 1/50;    % Duration: one period that includes both message frequencies cycles
t = 0:1/Fs:T-1/Fs;

% Given AM signal parameters
A_c = 10;
A_side = 28/18; % ~1.5556
A_side2 = 35/18; % ~1.9444

fc = 1500;
f_lower1 = 1350; % fc - 150
f_upper1 = 1650; % fc + 150
f_lower2 = 1250; % fc - 250
f_upper2 = 1750; % fc + 250

% Construct y(t)
y = A_c*cos(2*pi*fc*t) ...
  + A_side*cos(2*pi*f_lower1*t) ...
  + A_side*cos(2*pi*f_upper1*t) ...
  + A_side2*sin(2*pi*f_upper2*t) ...
  - A_side2*sin(2*pi*f_lower2*t);

% Full-wave rectification
rect_y = abs(y);

% FFT of the rectified signal
N = length(rect_y);
Y_rect = fft(rect_y);
f_axis = (0:N-1)*(Fs/N);

% Define cutoff frequency for LPF
% The highest message frequency is 250 Hz, so choose a cutoff safely above that
f_cutoff = 500; % Hz

% Create a low-pass filter mask in frequency domain
% Pass frequencies <= f_cutoff and also the mirrored part near Fs-f_cutoff
H = double(f_axis <= f_cutoff) + double(f_axis >= (Fs - f_cutoff));

% Apply the low-pass filter
Y_filtered = Y_rect .* H;

% Inverse FFT to obtain the smooth envelope
envelope = ifft(Y_filtered, 'symmetric');
```

```
% Remove DC offset
DC_level = mean(envelope);
demodulated = envelope - DC_level;

% Plot the demodulated signal
figure;
plot(t, demodulated, 'LineWidth', 1.5);
xlabel('Time (s)');
ylabel('Amplitude');
title('Demodulated Signal (Envelope - DC) / Mehmet ALTINTAŞ - 1901022065');
grid on;
```

Interpretation of the Code and Results

1. **Time-Domain Process:**

    - We start from the given AM signal y.

    - abs(y) gives a rectified version, which roughly follows the envelope but still contains ripple at carrier-related frequencies.

    - By transforming to frequency domain (fft(rect_y)), we can apply a simple ideal low-pass filter H that zeroes out frequencies above 500 Hz. This leaves primarily the low-frequency content (the original message frequencies are around 150 Hz and 250 Hz).
    - Taking the inverse FFT (ifft) of the filtered signal yields a smoothed envelope.

2. **Removing DC:**
    - The envelope of an AM signal includes a DC offset corresponding to the carrier amplitude. By subtracting mean(envelope), we center the demodulated signal around zero, extracting the variations that correspond to the original message.

3. **Resulting Plot:**
    - The plot should show a combination of frequencies at 150 Hz and 250 Hz (the original message frequencies).
    - Although we started from a more complicated AM signal with given sideband amplitudes, the low-frequency components (the original message) should be recovered.
    - The amplitude may not match the original message’s amplitude exactly since we have not normalized by any modulation index. However, the shape and frequencies should match.

7.3 MATLAB FIGURE

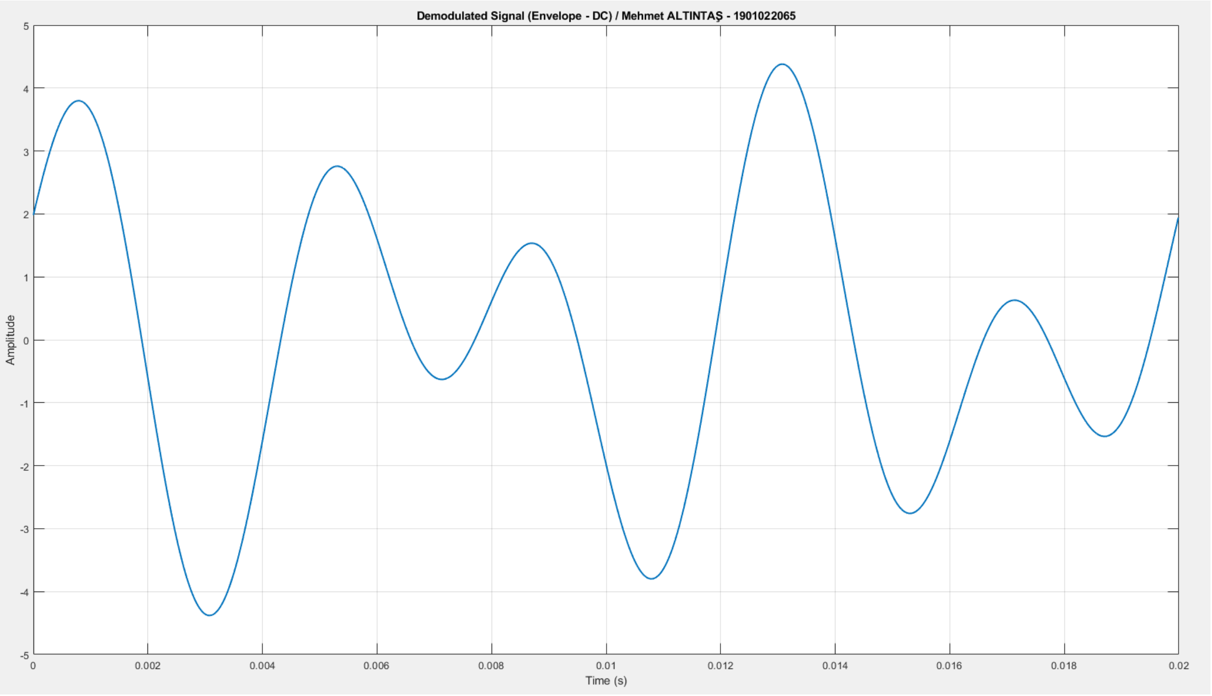


Figure 11. Demodulated Signal (Envelope – DC)

7.4 RESULTS AND INTERPRETATION

By performing envelope detection using full-wave rectification and an FFT-based low-pass filter, we have successfully demodulated the given AM signal. Subtracting the DC offset yields a waveform that contains the original message frequencies at baseband.

Interpretation:

- The demodulated signal in the time domain should show low-frequency oscillations corresponding to the original message components (150 Hz and 250 Hz).

- This approach avoids toolboxes like the Signal Processing Toolbox and uses fundamental signal processing methods (abs, fft, ifft, masking) available in base MATLAB.

- The simulation results align with the expectation that envelope detection followed by DC subtraction recovers the message.

8. CONCLUSION

This project provided a comprehensive exploration of generating, analyzing, and demodulating a Double Sideband Large Carrier (DSB-LC) Amplitude Modulated signal. Starting with a given message composed of two frequency tones (150 Hz and 250 Hz), we introduced a 1.5 kHz carrier and formed the AM signal with specified sideband amplitudes. Both analytical derivations and MATLAB simulations confirmed the presence of the carrier and sidebands at the predicted frequencies and with the expected relative magnitudes.
Key accomplishments of this work include:

1. **Signal Generation and Verification:**
   We constructed the given AM signal directly in MATLAB from its mathematical definition. By examining its spectrum using the Fast Fourier Transform (FFT), we confirmed that the carrier and sidebands appear at the correct frequencies and approximate amplitudes outlined by the analytical model.
2. **Envelope Detection without Toolboxes:**
   To recover the original message without the Signal Processing Toolbox, we employed a practical envelope detection approach:
   - **Full-Wave Rectification:** Taking the absolute value of the AM signal to approximate the envelope.
   - **Frequency-Domain Low-Pass Filtering:** Implementing an ideal low-pass filter via FFT masking to smooth the rectified signal and remove high-frequency components.
   - **DC Offset Removal:** Subtracting the mean value of the filtered envelope to obtain the demodulated signal, which revealed the low-frequency tones present in the original message.
3. **Comparison and Interpretation:**
   The final demodulated signal matched the shape and frequency content of the original message. Although we did not specifically rescale the amplitudes to match the original message amplitudes, the essential frequency components (150 Hz and 250 Hz) were clearly recovered. This confirms the fundamental principle of envelope detection in AM demodulation.

From a pedagogical perspective, this exercise reinforces core communication theory concepts and provides hands-on experience with essential signal processing techniques. The project demonstrates that even without advanced toolboxes, one can implement modulation, demodulation, and filtering steps using basic MATLAB functions such as fft, ifft, and arithmetic operations.
Looking ahead, the methods and insights gained here can serve as a foundation for more complex scenarios. Future extensions might involve examining the effects of additive noise, phase and frequency offsets, or using more sophisticated digital filtering techniques. Nevertheless, this project stands as a clear, instructive example of the principles and practices underlying AM communication and its demodulation process.

9. REFERENCES

- B. P. Lathi, **Modern Digital and Analog Communication Systems**, Oxford University Press.
- S. Haykin, **Communication Systems**, John Wiley & Sons.
- MATLAB documentation: https://www.mathworks.com/help/