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Problem 1-A.

Signal & Systems for C.E. (BLG-354E)

Homework - 3

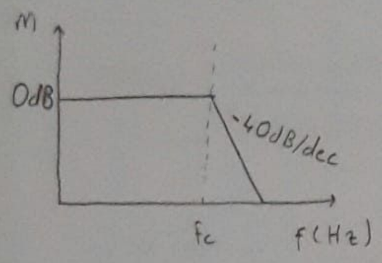

figure: 1 · LFP with 0 gain

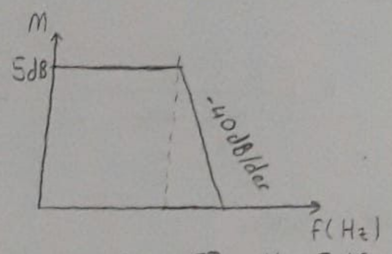

figure 2 · LFP with 5 dB gain

gain = K for 0dB

$20 \log K_0 = 0 \longrightarrow \boxed{K_0 = 1}$

gain = K for 5 dB

$20 \log K_5 = 5 \longrightarrow \boxed{K_5 = 1,778}$

There exists -40dB/dec at w_c (2 poles)

$$H_0(s) = \frac{K_0}{\left(1+\frac{s}{w_c}\right)^2} = \frac{K_0 \cdot w_c^2}{(s+w_c)^2} = \boxed{\frac{w_c^2}{(s+w_c)^2}} = H_0(s)$$

$$H_5(s) = \frac{K_5}{\left(1+\frac{s}{w_c}\right)^2} = \frac{K_5 \cdot w_c^2}{(s+w_c)^2} = \boxed{\frac{1,778 \cdot w_c^2}{(s+w_c)^2}} = H_5(s)$$

$$s = \frac{2}{T_s} \cdot \frac{z-1}{z+1} = \frac{2}{1/f_s} \cdot \frac{z-1}{z+1} = \boxed{2 \cdot f_s \cdot \frac{z-1}{z+1} = s}$$

using with $H_0(s)$ and $H_5(s)$ equalities we can get this equality for 0dB;

$$\frac{Y_0(z)}{X_0(z)} = \frac{w_c^2 \cdot (z+1)^2}{\left[(2f_s + w_c) \cdot z - (2f_s - w_c)\right]^2}$$

$f_s = 44100\text{Hz}$ sampling freq.
$f_c = 2000\text{ Hz}$ cut off freq.

$$X(z) \cdot \left[w_c^2 \cdot (z^2+2z+1)\right] = Y(z)\left[(2f_s+w_c)^2 \cdot z^2 - 2 \cdot (2f_s+w_c)(2f_s-w_c) \cdot z + (2f_s-w_c)^2\right]$$

$$y[n] = \left(\frac{w_c}{2f_s + w_c}\right)^2 \cdot x[n] + 2\left(\frac{w_c}{2f_s + w_c}\right)^2 \cdot x[n-1] + \left(\frac{w_c}{2f_s + w_c}\right)^2 \cdot x[n-2]$$

$$+ \left(\frac{2.(2f_s - w_c)}{2f_s + w_c}\right) \cdot y[n-1] - \frac{(2f_s - w_c)^2}{(2f_s + w_c)^2} \cdot y[n-2]$$

This solution was for 0dB

=> for 5dB, we can easily multiply, the terms with X, by K5 value. $\boxed{K_5 = 1.778}$

<u>Pseudo Code</u>

```
procedure LOW PASS FILTER
  Input fs
  Input fc
  Input gain
  c1 = (2π fc / (2fs + 2π fc))^2
  c2 = ((4fs - 4π.fc) / (2fs + 2π fc))
  c3 = ((2fs - 2π fc) / (2fs + 2π fc))^2
  k = 10^(gain/20)
  A = 0 , B = 0 , C = 0 , D = 0
  while input X
      y ← k c1 X + 2.k.c1 A + k.c1 B + c2.C - c3.D
      output Y
      B = A
      A = X
      D = C
      C = y
return y
```

You can find the code of other parts of the problem in the python file named hw3.py. The above pseudocode is implemented in hw3.py python file.

You can access the audio(wav) files created as output of this code from this link: https://drive.google.com/drive/folders/1_pHyhP4guFWKEa3k2B2jNW6t6SoNI-dg?usp=sharing