FE507 **Term Homework** **Fall 2021**

Data preparation: Using the time series in data file, calculate daily, weekly, and monthly rates of return ($r_t = \ln(P_t/P_{t-1})$ for all series.

1. Plot separate time graphs of *weekly* price indexes and returns for BIST100 and SP500 for the periods 1990 – 1999, 2000 – 2008, and 2009 – 2021. Explain what you "visually" discover in these graphs. Beware of currency differences.

2. Using *weekly* USD returns on SP500, BIST100, BISTALL, Gold and Bitcoin during 2015 – 2021,

    a. Calculate the sample mean, variance, standard deviation, maximum, minimum, the interquartile range, skewness, kurtosis, and first-order autocorrelation for each series,

    b. Calculate the covariance and correlation matrix for the five variables

    c. Calculate the geometric mean rate of return for each series

    d. Explain your findings.

3. Using *weekly* TL returns on BIST100 during 1990 – 2021,

    a. Calculate and plot time graphs of "moving" sample means and standard deviations:

$$m_s = \frac{1}{52}\sum_{t=s}^{s+51} r_t\,, \qquad \sigma_s = \sqrt{\frac{1}{52}\sum_{t=s}^{s+51}(r_t - m_s)^2}\,, \quad s = 1,2,\dots,T-51$$

    Explain what you discover.

    b. For the period 2001-2021, which empirical distributions do ***daily*** returns follow? Draw the histogram and perform tests of normality.

    c. Repeat Part (b) with ***monthly*** data. Do you find anything different?

4. Using *daily* returns for SP500 and BIST100 during 2018 – 2021, calculate sample autocorrelations in series of raw returns $\{r_t\}$ and also squared returns $\{r_t^2\}$ for lags up to 30 days. Plot the sample autocorrelation functions. Do you see any significant serial correlation in the series? Explain.

5. On December 17, 2021, BIST100 had a sharp drop of about -8.5%. During the time period in Part (b) of Question 3 above, what was the empirical probability $Prob(r_t \le -0.085)$?

Assuming daily returns are normally distributed with moments equalling the sample moments, what is the normal probability of such a tail event? How do you explain?

6. Background note: A market index is computed to represent the overall value performance of a market. A value-weighted stock market index like SP500 or BIST100 is basically calculated as

$$I_t = I_{t-1}\left(1 + \sum_{i=1}^{N} r_{it}\left(\frac{V_{it}^{(f)}}{V_t^{(f)}}\right)\right)$$

where $V_{it}^{(f)}$ is the market value of floating shares of stock i and $V_t^{(f)} = V_{1t}^{(f)} + \cdots + V_{Nt}^{(f)}$ is their total. (There are also adjustments for stock splits and dividends.) Since the float ratio is often less than one (average of about 30% in BIST and close to 90% in SP500), the total market value of a company's shares ("market cap" in the data file) will be greater than the traded value in float.

For SP500 and BIST100, analyze the time series behaviour of the indexes and that of the total market caps of their constituents. Do the indexes represent the value performance of their respective markets? In particular, does BIST100 (or, BISTALL) show what it is supposed to show? Elaborate.

7. During the period 2001 – 2021 and using annual data, calculate the equity risk premium for Borsa Istanbul (the difference between BIST100, or BISTALL, returns and bank deposit rates) for each year and plot its time series. What has been average risk premium for the period? Explain what you find.

8. Using your findings in Question 2, plot the five assets in a mean – standard deviation graph.

    a. Then, using data for the period 2016 – 2020, recalculate the sample means, variances and covariances

    b. Set up an equally-weighted portfolio of SP500, BIST100, Gold and Bitcoin (if you prefer, you may choose to construct a mean-variance optimal portfolio)

    c. Track the performance of your portfolio in 2021 (that is, out-of-sample testing) and report your findings.