IE361 Case Study 2

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Introduction

In this case study, we are asked to maximize the expected daily profit of a production facility in a steady state. Two machines work independently and they produce a single item in this production facility. Furthermore, an item is being produced with an exponential rate of μ at each machine. In addition to this product information, we also know that each day customers' demand is distributed with rate λ, according to Poisson Process. In order not to reject and lost customers' demands, the facility has an allowable backorder level, B, for unsatisfied demands. Lastly, in this facility, a policy is applied for production/inventory control; when the inventory level reaches an upper limit, S, production stops until the inventory level diminishes to level S_1 again. A setup process is required at level S_1 in order to start production again. Another inventory level, S_2, which is less than S_1, also has to be determined since just a single machine works between this level and also above level S_1. On the other hand, if the inventory level drops to a level less than S_2, both machine work together to increase the inventory level to S_2. As I mentioned in the beginning, our aim is to maximize the expected daily profit of this facility in a steady state by considering the various cost and price parameters besides limitations on S and B parameters. While approaching this problem we modeled this problem as a birth and death process to get the asked performance measures and find the optimum parameters. In this report first, you will find our birth & death process model of the case and brief explanations about it. Secondly in the discussion; we first formalized the performance measures asked, and then in Part B of the discussion we computed these measures according to our optimum solution parameters. Lastly, in the conclusion part, we tried to summarize our findings and add our comments and opinions about the existing system.

Main Body:

Assumptions:

I. S_2 and S_1 values might be negative

II. Setup only occurs when the production stops at the level of S and decreased back to S_2

Birth & Death Model:

Sets:

i	Inventory level of state	{-B, 1-B, 2-B, ………………………., S}
e	State of production	{N, P}

Table-1 Index sets

N=production process stops

P=production process is on

Parameters:

- P: The unit selling price of the item
- K: Fixed production setup cost
- C_p: Fixed production setup cost
- H: Daily unit inventory holding cost for the item
- C_l: Unit lost sale cost
- C_B: daily unit backorder cost

Modelling as Birth and Death Process:

After briefly discussing the situation of the production facility, to obtain optimum values of B, S, s_1, s_2, we modeled the system as follows:

System State:

$N_{i,e}$= State of having i inventory while the production process is at state e (on or off).

State space:

S= {(-B, P), (1-B, P), (2-B, P) …., (S-1, P)} U {(s_1+1, N), (s_1+2, N), …., (S-1, N), (S, N)}

Birth and Death Rates at each state:

Birth $\longrightarrow$ Item gets done and put in inventory

Death $\longrightarrow$ Demand occurred and the item is either sent or backorder. In any case, the inventory level of the system decreases by one.

- If e=N since no production is in process, the birth rate is zero and only death occurs at rate λ per day.

- If e=P as long as the inventory level is between (–B, s_2) the birth rate is 2 μ since two Machine Works. After exceeding s_2 the birth rate is μ until the level of S.

Further information is given in the rate diagram below:

Rate Diagram:

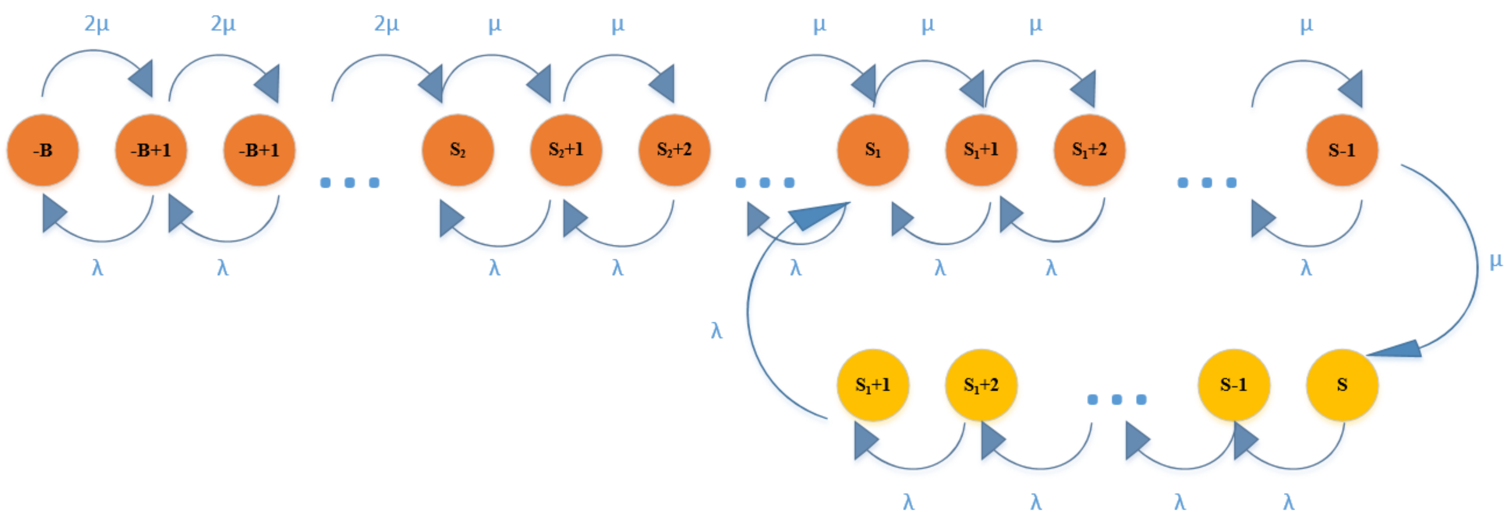

Figure 1 Rate Diagram

In the rate diagram, nodes represent states and arcs represent arrival and departures. We modeled the case as a birth and death process; therefore, rightward arrows show us the arrival rates and leftward arrows indicate departure rates. Because of our system state description, arrivals can be thought of as the entrance of products to the inventory as production occurs. Similarly, departures can be thought of as departures of products from inventory via the demand that occurred. Inventory levels can be observed in two different situations which are production (orange nodes) and no-production (yellow nodes). In addition, when the inventory level reaches the level of S, production will stop and there is no arrival between yellow nodes.

Discussion:

A)

A.1 Flow Balance Equations for Steady State Probabilities:

$2^* \mu ^*\pi_{-B, P} = \lambda ^*\pi_{1-B, P}$ *Special equation for first state, (-B, P).*

$(2^* \mu + \lambda) ^*\pi_{j, P} = 2^* \mu ^*\pi_{j-1, P} + \lambda ^*\pi_{j+1, P}$ *for* $\forall j=\{1-B, 2-B, \ldots\ldots\ldots, s_2-1\}$

$(\mu+\lambda) * \pi_{S2,P} = 2 * \mu * \pi_{S2-1,P} + \lambda * \pi_{S2+1,P}$ *Special equation for state (S_2, P).*

$(\mu+\lambda) * \pi_{j,P} = \mu * \pi_{j-1,P} + \lambda * \pi_{j+1,P}$ *for* $\forall$ *j={ S_2+1, S_2+2, ……….., S-2}-{ S_1}*

$(\mu+\lambda) * \pi_{S1,P} = \mu * \pi_{S1-1,P} + \lambda * (\pi_{S1+1,P} + \pi_{S1+1,N})$ *Special equation for state (S_1, P).*

$(\mu+\lambda) * \pi_{S-1,P} = \mu * \pi_{S-2,P}$ *Special equation for state (S-1, P).*

$\lambda * \pi_{S,N} = \mu * \pi_{S-1,P}$ *Special equation for state (S, N).*

$\lambda * \pi_{j,N} = \lambda * \pi_{j+1,N}$ *for* $\forall$ *j={ S_1+1, S_2+2, ……….., S-1}*

$\sum_j^S \pi(j)$ For all states in stat space S.

A.2 Performance Measures in Steady State as Functions of Steady State Probabilities:

After we set up our mathematical model, performance measures can be determined in terms of π.

A.2.a) Fraction of time that there is no production

$$\sum_{j=s1+1}^{S} (\pi_{j,N})$$ for ∀j= {s1+1, …. …., S}

Since π_j is equal to the fraction of time that system is in state j in a steady state; when we sum up π_j, we can find the desired fraction of time. In our model, between (s1+1, N) and (S, N) there is no production. Therefore, the sum of π_j between (s1+1, N) and (S, N) gives the fraction of time that there is no production.

A.2.b) Fraction of time that only one machine is working

$$\sum_{j=s2}^{S-1} (\pi_{j,P})$$ for ∀j= {s2, … … …, S-1}

Similarly, the sum of π_j between s2 and S-1 gives the fraction of the time that only one machine is working. In state S, production will stop, hence S state is not included in the summation above.

A.2.c) Average utilization of machines

$$\sum_{j=-B}^{s_2-1}(\pi_{j,P} * 1) + \sum_{k=s2}^{S-1}(\pi_{k,P}) * (.5)$$

When e=N no utilization

While formalizing the utilization of machines, we multiplied the steady state probability of each state by the ratio of working machines. (working machines/2 machines) Notice that in states with e=N, this ratio is 0 since no machine is working in those states. That is why they are not considered in our formulation as they do not contribute to the utilization of machines.

A.2.d) Expected inventory level

$$\sum_{j=0}^{S}(\pi_{j,P}) * j$$

for e=P, production is online

$$\sum_{j=s2+1}^{S-1}(\pi_{j,N}) * j$$

for e=N, production is offline

Each state's first index (i) is indicating the inventory level of that state so in order to find the expected daily level of inventory, the formulas above for both production and no-production states must be calculated.

A.2.e) Expected backorder level

$$\sum_{j=-B}^{1}(\pi_{j,P}) * (-j)$$

for e=P, Inventory is in negative levels

While the company is accepting backorder, it is sure that production is online (even with a rate of 2* μ), so we only considered the states with index e=P. In our model, back ordering is indicated by the negative i index of states. Therefore, we formulated the expected daily backorder level as given above.

A.2.f) Expected daily profit

Expected daily profit can be found with the following formula;

Expected Daily Profit = Expected Daily Revenue – Expected Total Daily Cost

Therefore, first, we should find expected daily revenue and expected daily cost. Elements of revenue and cost can be seen in the following table.

Revenue Elements		Cost Elements	
Symbol	**Meaning**	**Symbol**	**Meaning**
p	Unit selling price	h	Unit inventory holding cost
		k	Fixed production setup cost
		c_l	Unit lost sale
		c_b	Unit backorder cost
		c_p	Unit production cost

Table2 Revenue and Cost Parameters

NOTE THAT: h, c_b costs are daily.

There will be no departure in inventory only in –B state (check figure-1). Therefore, a sum of all states, except for –B state, gives the probability of states that departure of items is available. The demand rate is constant which is λ so the formula can be constructed as follows:

$$Expected\ Daily\ Revenue = \lambda * \left(1 - \pi_{-B,P}\right) * p$$

On the other hand, the expected daily cost includes different costs. These costs are formulated below separately.

$$Daily\ Fixed\ production\ setup\ cost = \pi_{s1+1,N} * \lambda * k$$

Daily fixed production setup cost occurs only when the transaction from state $N_{s1+1,N}$ to state $N_{s1,P}$ occurs. So we multiplied the steady state probability of $N_{s1+1,N}$, and demand rate λ to find # of daily expected setups. Then multiply it with the fixed setup cost parameter k.

$$Total\ Daily\ Variable\ Cost = \left(\sum_{j=-B}^{s2-1} \pi_{j,P} * 2\mu + \sum_{j=s2}^{S-1} \pi_{j,P} * \mu \right) * c_p$$

Total daily variable cost is applied in the states with e=P, production is online. Due to their production rate, we can classify these states under two. From –B to s_2-1 production rate is 2*μ and from s_2 to S-1 it is μ. Therefore; multiply the corresponding steady state probabilities with the corresponding production rate, we can find the total daily production rate. Then by multiplying this rate with the variable cost parameter c_p we can find the total daily variable cost.

$$Daily\ Holding\ Cost = \left(\sum_{j=1}^{S} \pi_{j,P} * j + \sum_{j=s1+1}^{S-1} \pi_{j,N} * j \right) * h$$

While computing daily holding cost we must multiply the expected inventory level (check Part-A.2.d) with its cost parameter h.

$$Daily\ Lost\ Demand\ Cost = \pi_{-B} * \lambda * c_l$$

The lost demand only occurs in the state of –B where the company stops getting back orders so if we multiply its steady state with the demand rate λ we will get the expected lost demand. In order to find its cost effective, we can multiply it with its cost parameter as above.

$$Daily\ Backorder\ Cost = \left(\sum_{j=-B}^{0} \pi_{j,P} * (j) \right) * c_b$$

While computing daily backorder cost we must multiply the expected backorder level (check Part-A.2.e) with its cost parameter c_b. Notice that the sign has changed to making a positive cost.

Expected Total Daily cost=

$$Daily\ Fixed\ production\ setup\ cost + Total\ Daily\ Variable\ Cost + Daily\ Holding\ Cost + Daily\ Lost\ Demand\ Cost + Daily\ Backorder\ Cost$$

B)

Parameter	Value	Parameter	Value
S_{max}	50	h	9
B_{max}	20	c_b	8
λ	16	c_l	20
μ	12	c_p	20
k	100	p	80

According to given parameters, optimal value of S, s_1, s_2 and B that maximize expected daily profit of the system in the steady state, are find using MATLAB. Until the values of S_{max} and B_{max}, expected daily profit calculated for every value of S and B by considering $-B<s_2<s_1<S$ rule. Maximum expected daily profit and value of S, s_1, s_2 and B for this profit are found.

NOTE THAT

Initial values of B, s_1, s_2, and S are selected as 0,1,2,3 respectively. When these values are chosen, we gave a chance to the company for no-backordering.

Optimal values of B, s_1, s_2, and S for the maximum profit can be seen in the following table.

Decision Var.	Optimal Value	Decision Var.	Optimal Value
S	13	s_1	0
B	20	s_2	-1

→ Expected Daily Profit = 935.41 $

Due to the lower level of backorder cost and the same level of all product prices, the company can back order more. Therefore, the B level is the same as B_{max}. Production with two machines has more advantages than production with one machine. Because of the higher value of fixed cost, when a company makes two machine setups, two machines should be running as much as possible. Therefore, the system tried to choose the least difference between s1 and s2. In addition, holding inventory is less costly than production. Hence, s1 should be far away from S.

In addition, performance measures in part A.2 were calculated with the optimal value of B, s_1, s_2, and S, using steady-state probabilities (see Appendix-A). They can be calculated as followings.

Fraction of time that there is no production

$$\sum_{j=s1+1}^{S}(\pi_{j,N}) = \sum_{j=1}^{13}(\pi_{j,N}) = \pi_{1,N} + \pi_{2,N} + \cdots + \pi_{13,N} = 0.01019 \qquad \text{for } \forall j = \{1,2, \ldots\ldots, 13\}$$

As it is seen above the fraction of time that the production process stops, is quite less compared to the fraction of time that production is online. The reason for this output is that the demand rate is greater than the production rate.

Fraction of time that only one machine is working

$$\sum_{j=s2}^{S-1}(\pi_{j,P}) = \sum_{j=-1}^{12}(\pi_{j,P}) = \pi_{-1,P} + \pi_{0,P} + \cdots + \pi_{12,P} = 0.6463900 \qquad \text{for } \forall j = \{-1,0, \ldots \ldots, 12\}$$

The high fraction of time that only one machine works, indicates that the inventory level of the facility is stable.

Average utilization of machines

$$\sum_{j=-B}^{s_2-1}(\pi_{j,P} * 1) + \sum_{k=s2}^{S-1}(\pi_{k,P}) * (.5) = \sum_{j=-20}^{-2}(\pi_{j,P} * 1) + \sum_{k=-1}^{12}(\pi_{k,P}) * (.5) = 0.6666150$$

Expected inventory level

$$\sum_{j=0}^{S}(\pi_{j,P}) * j + \sum_{j=s2+1}^{S-1}(\pi_{j,N}) * j = \sum_{j=0}^{13}(\pi_{j,P}) * j + \sum_{j=0}^{12}(\pi_{j,N}) * j = 1.20981 \text{ units}$$

Even though, the range for inventory level is simply [-20 13] we obtained a positive expected inventory level due to good managed production policy. (Increasing production rate after level s2)

Expected backorder level

$$\sum_{j=-B}^{1}(\pi_{j,P}) * (-j) = \sum_{j=-20}^{1}(\pi_{j,P}) * (-j) = \pi_{-20,P} * 20 + \pi_{-19,P} * 19 + \cdots + \pi_{1,P} * 1 = 1.54252$$

Due to the increased production rate starting from the level of s2 (-1), the facility obtained a good, expected backorder level considering it can descend to the level of -20.

NOTE: note that if the decision maker of the facility gets disappointed with one of the measures here, he/she can maximize his/her utilization by changing our mat lab code and combining it with the outputs of Sensitivity_Analyzer.

Also, we wanted to investigate the dual price of the backorder level which was found -20. We see that increasing S_{max} to 21 will contribute to the facility's profit by 0.0219 $.

Conclusion

In this report, we tried to analyze a production facility's performance and find optimal values for their decisions. For our side, it was joyful and full of learning activities. Even though the facility seems to be going well we also must state that the company's reliability may decrease due to the large back ordering range.

Appendix

Appendix 1

Pİ matrix

0.0001

0.0001

0.0002

0.0003

0.0004

0.0006

0.0009

0.0013

0.0020

0.0030

0.0045

0.0067

0.0101

0.0151

0.0226

0.0339

0.0509

0.0764

0.1145

0.1718

0.1288

0.0958

0.0711

0.0525

0.0386

0.0282

0.0204

0.0145

0.0101

0.0068

0.0043

0.0024

0.0010

0.0008

0.0008

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