IE361 Case Study 1

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Introduction:

In this case, we are going to examine a repair shop that has a random number of arrivals and various limitations. To repair a car, all 3 steps of repairing must be done successfully. Two repairmen have single skills and additional multi-skilled temporary workers can be hired in need of help. The repair shop needs a temporary worker if the total man-hour requirement for a single skill exceeds 8 hours since each repairman works at most 8 hours in a day. Repairmen and temporary workers get paid differently. According to the repairman's abilities, repair operations could be successful and continue with the following repair operations or could be failed and send back to the customer with different probabilities. Lastly, while admitting the cars in the repair shop, the existing number of cars in the shop is considered. If there is more than 1 car in the repair shop, the arrivals are rejected.

Main Body:

Assumptions:

I. We assumed that each operation needs to different man-hour requirement for each different skill.

II. As stated in the question there is no limit on the temporary worker and skills of temporary workers in repair shops.

III. Workers can deal with the same operation simultaneously.

Mathematical Model:

Sets:

i	Arrival Cars	0,1,2
j	Operation	1,2,3,
k	Skill	1,2

Parameters:

- **w:** fixed cost of the repaired car
- **p:** revenue of successfully repaired car
- **a(j):** probability of sending to owner

- **r (j,k):** skills need of operation j for skill k
- **q(i):** the probability that cars arrive each morning

Modeling as a Markov Chain:

$\mathbf{X_t}$ = number of cars in each operation

State Space (S) = {(0,0,0), (1,0,0), (0,1,0), (0,0,1), (1,1,0), (1,0,1), (0,1,1), (2,1,0), (2,0,1), (0,2,1), (2,0,0), (0,2,0), (0,0,2)}

Transition Matrix (P)

P=

	(0,0,0)	(1,0,0)	(0,1,0)	(0,0,1)	(1,1,0)	(1,0,1)	(0,1,1)	(2,1,0)	(2,0,1)	(0,2,1)	(2,0,0)	(0,2,0)	(0,0,2)
(0,0,0)	q_0	q_1	0	0	0	0	0	0	0	0	q_2	0	0
(1,0,0)	a_1*q_0	a_1*q_1	$(1-a_1)*q_0$	0	$(1-a_1)*q_1$	0	0	$(1-a_1)*q_2$	0	0	a_1*q_2	0	0
(0,1,0)	a_2*q_0	a_2*q_1	0	$(1-a_2)*q_0$	0	$(1-a_2)*q_1$	0	0	$(1-a_2)*q_2$	0	a_2*q_2	0	0
(0,0,1)	q_0	q_1	0	0	0	0	0	0	0	0	q_2	0	0
(1,1,0)	$a_2*a_1*q_0$	$a_2*a_1*q_1$	$a_2*(1-a_1)*q_0$	$a_1*(1-a_2)*q_0$	$a_2*(1-a_1)*q_1$	$a_2*(1-a_1)*q_1$	$(1-a_1)*(1-a_2)$	$q_2*(1-a_1)*a_2$	$(1-a_2)*a_1*q_2$	0	$a_1*a_2*q_2$	0	0
(1,0,1)	a_1*q_0	a_1*q_1	$(1-a_1)*q_0$	0	$(1-a_1)*q_1$	0	0		0	0	a_1*q_2	0	0
(0,1,1)	a_2*q_0	a_2*q_1	0	$(1-a_2)*q_0$	0	$(1-a_2)*q_1$	0	0	$(1-a_2)*q_2$	0	a_2*q_2	0	0
(2,1,0)	$a_2*a_1^2*q_0$	$a_2*a_1^2*q_1$	$2*a_2*a_1*(1-a_1)*q_0$	$a_1^2*(1-a_2)*q_0$	$2*a_2*a_1*(1-a_1)*q_1$	$a_1^2*(1-a_2)*q_1$	$2*(1-a_1)*(1-a_2)*a_1$	$2*a_1*a_2*(1-a_1)*q_2$	$a_1^2*(1-a_2)*q_2$	$(1-a_2)*(1-a_1)^2$	$a_2*q_2*a_1^2$	$a_2*(1-a_1)^2$	0
(2,0,1)	$a_2^2*q_0$	$a_1^2*q_1$	$2*a_1*(1-a_2)*q_0$	0	$2*a_1*(1-a_1)*q_1$	0	0	$2*q_2*(1-a_1)*a_1$	0	0	$q_2*a_1^2$	$(1-a_1)^2$	0
(0,2,1)	$q_0*a_2^2$	$q_1*a_2^2$	0	$2*(1-a_2)*q_0*a_2$	0	$2*(1-a_2)*q_1*a_2$	0	0	$2*(1-a_2)*q_2*a_2$	0	$q_2*a_2^2$	0	$(1-a_2)^2$
(2,0,0)	$q_0*a_1^2$	$q_1*a_1^2$	$2*a_1*(1-a_1)*q_0$	0	$2*a_1*(1-a_1)*q_1$	0	0	$2*a_1*(1-a_1)*q_2$	0	0	$q_2*a_1^2$	$(1-a_1)^2$	0
(0,2,0)	$q_0*a_2^2$	$q_1*a_2^2$	0	$2*a_2*(1-a_2)*q_0$	0	$2*a_2*(1-a_2)*q_1$	0	0	$2*a_2*(1-a_2)*q_2$	0	$q_2*a_2^2$	0	$(1-a_2)^2$
(0,0,2)	q_0	q_1	0	0	0	0	0	0	0	0	q_2	0	0

Discussion:

NOTE: After modeling the system of the repairman shop as a Markov-chain, we had to determine the steady-state probabilities to compute the performance measures asked. We calculated the steady-state probabilities as follows:

Let **PI** *be a 13x1 matrix consisting of steady-state probability values (π) of 13 states.*

$(P-I)^T*PI=PI$ → RANK=S-1=12 We need one more equation

$\sum_{k=0}^{13} PI(k)$ Normalization equation!!!

-By Solving this system of linear equations using MATLAB (for coding check appendix 2), we found the steady-state probabilities for each state defined earlier in terms of q_i's and a_j's. The solutions can be found in our MATLAB output, appendix 3.

A.1 Expected daily workload of the repairman with skill k=1,2

In order to determine the expected daily workload, we first defined a matrix that includes daily workloads regarding 13 different states for each skill, k This matrix DWL is derived from the parameter of $R_{j,k}$. Then, to satisfy the maximum 8 work hours for each worker limitation, we needed to find a minimum of 8 hours and the required daily workload. A minimum of 8 hours and a daily workload give us work daily workload of a repairman in the repair shop with skill k. In this way, we can multiply the steady-state probabilities of each state with the daily workload of the permanent repairman corresponding to this state. That multiplication gives us the expected daily workload of the repairman with skill k=1,2.

State\Skill	$K=1$	$K=2$
(0,0,0)	0	0
(1,0,0)	r_{11}	r_{21}
(0,1,0)	r_{12}	r_{22}
(0,0,1)	r_{13}	r_{23}
(1,1,0)	$r_{11}+r_{12}$	$r_{21}+r_{22}$
(1,0,1)	$r_{11}+r_{13}$	$r_{21}+r_{23}$
(0,1,1)	$r_{12}+r_{13}$	$r_{22}+r_{23}$
(2,1,0)	$2*r_{11}+r_{12}$	$2*r_{21}+r_{22}$
(2,0,1)	$2*r_{11}+r_{13}$	$2*r_{21}+r_{23}$
(0,2,1)	$2*r_{12}+r_{13}$	$2*r_{22}+r_{23}$
(2,0,0)	$2*r_{11}$	$2*r_{21}$
(0,2,0)	$2*r_{12}$	$2*r_{22}$
(0,0,2)	$2*r_{13}$	$2*r_{23}$

DWL= *13x2 matrix consist of daily workload of each skill in each steady state.*

PI= *13x1 matrix consisting of steady state probability values (π) of 13 states.*

ETDWL= *1x2 matrix of expected total daily workload for skill k.*

$$\mathbf{ETDWL= PI^T*(min(DWL,8))}$$

Or

$$\sum_{i=1}^{13} \pi i*(\min(DKL_i,8)) \text{ for } k=1,2$$

-We are taking the minimum of daily workload and 8 because if more than 8 hours any skill is require the repairman with that skill works for 8 hours and the rest is completed by the utility worker(s).

A.2 Expected daily workload of temporary workers

To get the expected daily workload of temporary workers, firstly we needed to find out the daily workload of temporary workers in each state. For this aim, we found the states in which the daily workload exceeds 8 hours. Then by using the "max" function on the daily workload matrix to find the exceed time precisely if it exists. After that, when we subtract 8 hours from the total daily workload of each day, we can get the daily workload of temporary workers if they were needed, if else that cell of the matrix becomes zero ((max(8,X)=8)-8). Therefore, we can get the expected daily workload of temporary workers as:

DWL= *13x2 matrix consists of the daily workload of each skill in each steady state*

ETDWLU= *expected total daily workload of temporary workers*

PI= *13x1 matrix consists of steady state probability values (π) of 13 states*

$$\mathbf{ETDWLU= PI^T*\sum_{k=1}^{2}(max(DWL,8)-8)}$$

Note that we summed up the DWL matrix for all k values because the temporary utility workers are multiskilled and their hour requirement can be calculated as one number.

A.3 Expected number of cars in a day under each operation

In order to find the expected number of cars in a day under each operation, first we stored the number of existing cars in each operation for all states in a matrix. Since the PI matrix gives us the steady state probability, we multiplied these 3 matrices separately with the PI matrix, to get the expected number of cars in a day under each operation.

numberofcarso1= # of cars in operation 1 in each state	0	1	0	0	1	1	0	2	2	0	2	0	0
Numberofcarso2=# of cars in operation 2 in each state	0	0	1	0	1	0	1	1	0	2	0	2	0
Numberofcarso3=# of cars in operation 3 in each state	0	0	0	1	0	1	1	0	1	1	0	0	2

Σ

Expected # of cars in a day under operation 1 is expnumberofcarso1 = numberofcarso1*PI
Expected # of cars in a day under operation 2 is expnumberofcarso2 = numberofcarso2*PI
Expected # of cars in a day under operation 3 is expnumberofcarso3 =numberofcarso3*PI

The answers found by the terms of q_i's and a_i's by MATLAB. (Check appendix 2)

A.4 Expected Total Number of Cars in the Repair Shop in a Day

For calculating the expected number of cars in the repair shop in a day, it is necessary to find the total number of cars in the system for each state. After that calculation, just like the previous problem (A.3), we multiplied this total number of cars matrix with the steady state matrix, PI, in order to get the expected total number of cars in the repair shop in a day.

Numberofcarsinsystem(1x13) = Expected # of cars in system	0	1	1	1	2	2	2	3	3	3	2	2	2

Or

Numberofcarsinsystem(1x13) $=\sum_{j=1}^{3}$ expnumberofcarso(j)

Expected # of cars in the repair shop is **expnofc=Numberofcarsinsystem*PI**

A.5 Acceptance rate to repair shop

Since the words "acceptance" and "rate" are quite wide expressions, to give an output in this report we made two basic assumptions.

ASSUMTION 1: we assumed that the acceptance rate is the sum of states where the acceptance occurred per day.

ASSUMPTION 2: we assumed that the acceptance can be measured by just measuring the accepted arrivals (in this case the arrival of 2 cars is counted as one acceptance) or by measuring the accepted cars (in this case the arrival of 2 cars is counted as two acceptance).

CASE1: To find out the acceptance rate, we had to define a binary matrix showing in which state an acceptance occurred. The matrix is as follows:

NFS1(1x13) = # of cars accepted to the system in each state	0	1	0	0	1	1	0	1	1	0	1	0	0

AR1=NFS1*PI

CASE2: To find out the acceptance rate, we had to define a matrix showing in which state how many acceptances occurred. The matrix is as follows:

NFS2(1x13) = # of cars accepted to the system in each state	0	1	0	0	1	1	0	2	2	0	2	0	0

AR2=NFS2*PI

-For the Formulas of AR1 and AR2 check the appendix 3.

A.6 Expected Time Between Two Admissions

We followed a very simple approach to this question and assumed that if we think of expected time as the inverse of rate, we can find the expected time between two admissions by using the information we reached in A.5.

Expected Time Between Two Admissions=1/AR1 Days

-Note that we used AR1 instead of AR2 because of the condition explained in the question

A.7 Expected Daily Profit

We formularized the expected profit as.

Expected Profit=Expected. Revenue-Expected cost

Now let us show the components of expected revenue and expected cost:

Expected Revenue=Successful Departure rate*p

In order to calculate departure rate, we check each state's last operation and find the expected number of cars left successfully by following formula

If 0 car exists in the last operation — 0

If 1 car exists in the last operation — $(1-a_1)*1$

If 2 car exists in last operation — $((1-a_2)^2*2 + (1-a2)*a_1)$

for All states

store info in matrix D 1x13

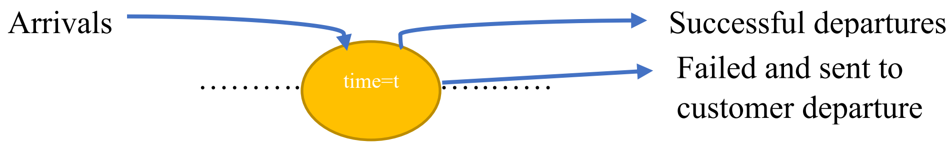
Expected successful departure rate=D*PI

Expected Revenue= D*PI *p

Expected Cost
regular workers=$\sum_{k=1}^{2} ETDWL(k) * c1$
utility workers= $ETDWLU * c2$
fixed cost for unrepaired=?

In order to determine the number of unrepaired and sent back to customers' cars, we made the following assumption:

In the long run, the system can be modeled considered as a system in which the net flow is 0.



fixed cost for unrepaired= (Arrival rate (AR2)- Expected successful departure rate) *w

-Note that we used AR2 instead of AR1 because this time the number of cars matters in the acceptance cases.

TP=TR-RWC-UWC-FC

B.1 Expected daily workload of the repairman with skill k=1,2

As distinct from part A.1, we can give 8 hours to some daily workload values directly, whose states that needed more than 8-man hours. We can get the daily workload of repairmen in the repair shop directly. Then when the daily workload of the repairman is multiplied with a steady-state matrix we get the results in below.

	Worker with Skill k=1	Worker with Skill k=2
Expected daily workload	5.8978 hours	6.2263 hours

Although the man-hour requirement for Skill 2 is less than Skill 1 in operations 2 and 3 (check table-3 in Appendix 1), the expected daily workload of Skill 1 is more than Skill 2. Regarding results, we can say that reason for this situation is about operation1 is more effective than operation2 and operation3 on the basis of contributing expected working hours. However, the effectiveness of operation 1 is expected because the expected number of cars in the first operation must be greater than 2 and 3 because we know that for passing to 2. and 3. operations a car should pass the 1. operation first and there is a failure probability for each operation. That is why the expected daily workload of skill1 is less than skill2.

B.2 Expected daily workload of temporary workers

After providing the numeric values of probabilities for arrivals and operation passing to the code we wrote in MATLAB we observed the following answer for the expected daily workload of temporary workers.

Expected daily workload for Temporary workers	2.4080 hours

This result shows that the temporary workers or a single worker must work expectedly 2.4080 hours per day in the repairman shop. This output was quite logical and consistent with the dynamics of the system considering that in 9 of 13 states those temporary workers are needed.

B.3 Expected number of cars in a day under each operation

As explained in part A.3, we calculated the expected number of cars in a day under each operation according to the given possibilities. The results can be found in the below table.

Expected # of cars in a day under operation 1 is expnumberofcarso1=0.745924
Expected # of cars in a day under operation 2 is expnumberofcarso2=0.522147
Expected # of cars in a day under operation 3 is expnumberofcarso3=0.417718

As seen in the table, the highest expected number of cars in a day belongs to operation 1 and the lowest one belongs to operation 3 (as it is guessed logically in part B.1). This is not an unexpected outcome, and it is consistent since follow-up operations are dependent on previous operation in other words if a repair is failed in operation 1, the car sends back to the customer and it would not have transferred to next operations. If we consider that repair starts with the first operation, each existing car(s) in the system requires this operation for once, a decrease of the expected numbers for the following operations is logical.

Σ

B.4 Expected total number of cars in the repair shop in a day

After the calculations are mentioned in part A.4, we found the expected number of cars in the repair shop.

Expected # of cars in the repair shop is expnofc=1.6858

It is also possible to forecast that the result will be between 0 and 3 since there can be a minimum of 0 and a maximum of 3 cars in the system. While analyzing this result, considering the number of car arrival probability and the failure of repair probability are important. Although there is a capacity limitation for a repair shop, arrival probability is important for fulfilling the capacity and serving maximum customers which increases our expected value. Furthermore, in the case of not fulfilling the capacity according to arrivals, failure probability is important to keep existing cars in the system.

B.5 Acceptance rate to repair shop

As we talked in part A.5 we analyzed this question under two cases and the results for each case are as follows:

CASE	Acceptance Rate
1	0.52215 per day
2	0.74592 per day

As it would be guessed, the rate of case 2 is larger than case 1 because of assuming the case of 2 car arrival as one. One other interesting think is if we had no capacity limitation in the repairman shop the expected acceptance per day would be as follows.

$$0*q_0+1*q_1+2*q_2=1$$

The acceptance rated found in this part are both smaller than 1 and it proves that the capacity limitation of the system squeezes the feasible region of acceptance and creates an unseen opportunity cost.

B.6 Expected Time Between Two Admissions

It is explained how the expected time between two admissions is calculated in part A.6, and the numeric solution is 1.9152 day.

B.7 Expected Daily Profit

After codding the logic, we mentioned in part A.7 to MATLAB, we are given the following result. is equal to.

Expected daily profit	Expected successful departure rate (daily)	Expected total revenue (daily)	Expected Total cost (daily)
$ 132.4191	0.3968	$317.4654	$185.0463

Those results show us that a car successfully leaves the system of repair shop one in 2.5 day expectedly in the long run. And one the repairman shop collects $800 for this departure car almost 70% of it is compensating the costs occurred in the system in the time period that this car is handled.

Conclusion

In this case study, we experienced modelling a stochastic system and deriving some performance measures by the example of a real socio-economic system. As we think of and practice the calculations of the measures asked in the report, we developed our skills of Markov chain applications and widen our perspectives to the probabilistic systems. To sum up, the measures found in part B give important details for the decision maker to consider while making decisions for the future of the repairman shop which contains randomness by its nature, and the logic used in modeling in part A can be used to simulate different situations of the repair shop's system.

Appendix

Appendix 1

Table 1. Man Hour Requirements for Each Operation (r_{jk})

Operation, j

Skill, k	1	2	3
1	3	4.5	5.9
2	4.4	3.8	5.3

Table 2. The failure of repair probability (a_j) after operation j=1,2,3

Operation, j	a_i
1	**.3**
2	**.2**
3	**.05**

Table 3. Probability (q_i) that i=0,1,2 cars arrive each morning

Number of Cars, i	q_i
0	.3
1	.4
2	.3

Appendix 2

MATLAB Codding

```
for DEVRIM=1:2
    if DEVRIM==1
syms q0 q1 q2 a1 a2 a3;
disp('PART A')
fprintf('\n\n')
    else
    q0=.3;q1=.4;q2=.3;
    a1=.3;a2=.2;a3=.05;
    disp('PART B')
fprintf('\n\n')
    end

P=sym('b%d',[13 13]);
for i=1:13
for j=1:13
P(i,j)=0;
end
end
clear j

P(1,1)=q0;
P(1,2)=q1;
P(1,11)=q2;

P(2,1)=a1*q0;
P(2,2)=a1*q1;
P(2,3)=(1-a1)*q0;
P(2,5)=(1-a1)*q1;
P(2,8)=(1-a1)*q2;
P(2,11)=a1*q2;

P(3,1)=a2*q0;
P(3,2)=a2*q1;
P(3,4)=(1-a2)*q0;
P(3,6)=(1-a2)*q1;
P(3,9)=(1-a2)*q2;
P(3,11)=a2*q2;
P(4,1)=q0;
P(4,2)=q1;
P(4,11)=q2;

P(5,1)=a1*a2*q0;
P(5,2)=a1*a2*q1;
P(5,3)=(1-a1)*a2*q0;
P(5,4)=a1*(1-a2)*q0;
P(5,5)=a2*(1-a1)*q1;
P(5,6)=a1*(1-a2)*q1;
P(5,7)=(1-a1)*(1-a2);
P(5,8)=a2*(1-a1)*q2;
P(5,9)=a1*(1-a2)*q2;
P(5,11)=a1*a2*q2;

P(6,1)=a1*q0;
P(6,2)=a1*q1;
P(6,3)=(1-a1)*q0;
P(6,5)=(1-a1)*q1;
```

```
P(6,8)=q2*(1-a1);
P(6,11)=q2*a1;

P(7,1)=a2*q0;
P(7,2)=a2*q1;
P(7,4)=(1-a2)*q0;
P(7,6)=(1-a2)*q1;
P(7,9)=q2*(1-a2);
P(7,11)=a2*q2;

P(8,1) = q0*a1*a1*a2;
P(8,2) = q1*a1*a1*a2;
P(8,3) = q0*a2*a1*(1-a1)*2;
P(8,4) = q0*a1*a1*(1-a2);
P(8,5) = q1*a1*(1-a1)*a2*2;
P(8,6) = q1*a1*a1*(1-a2);
P(8,7) = (1-a2)*(1-a1)*a1*2;
P(8,8) = q2*a1*(1-a1)*a2*2;
P(8,9) = q2*(1-a2)*a1*a1;
P(8,10) = (1-a1)*(1-a1)*(1-a2);
P(8,11) = q2*a2*a1*a1;
P(8,12) = (1-a1)*(1-a1)*a2;

P(9,1) = q0*a1*a1;
P(9,2) = q1*a1*a1;
P(9,3) = a1*(1-a1)*q0*2;
P(9,5) = q1*a1*(1-a1)*2;
P(9,8) = q2*(1-a1)*a1*2;
P(9,11) = q2*a1*a1;
P(9,12) = (1-a1)*(1-a1);

P(10,1) = a2*a2*q0;
P(10,2) = q1*a2*a2;
P(10,4) = a2*(1-a2)*q0*2;
P(10,6) = q1*(1-a2)*a2*2;
P(10,9) = q2*a2*(1-a2)*2;
P(10,11) = q2*a2*a2;
P(10,13) = (1-a2)*(1-a2);

P(11,1) = q0*a1*a1;
P(11,2) = a1*a1*q1;
P(11,3) = a1*(1-a1)*2*q0;
P(11,5) = a1*(1-a1)*q1*2;
P(11,8) = a1*(1-a1)*q2*2;
P(11,11) = a1*a1*q2;
P(11,12) = (1-a1)*(1-a1);

P(12,1) = a2*a2*q0;
P(12,2) = q1*a2*a2;
P(12,4) = a2*(1-a2)*q0*2;
P(12,6) = a2*(1-a2)*q1*2;
P(12,9) = q2*a2*(1-a2)*2;
P(12,11) = q2*a2*a2;
P(12,13) = (1-a2)*(1-a2);

P(13,1) = q0;
P(13,2) = q1;
P(13,11) = q2;
```

```
Coef=P.';
for i=1:13
  Coef(i,i)=Coef(i,i)-1;
end
 Coef(13,:)=1;
 Sol=zeros(13,1);
 Sol(13,1)=1;
PI=linsolve(Coef, Sol);
clear Sol
STATES=[0 0 0;1 0 0;0 1 0;0 0 1;1 1 0;1 0 1;0 1 1; 2 1 0; 2 0 1; 0 2 1; 2 0
0;0 2 0;0 0 2 ];

 for i=1:13
   fprintf('In the state when there is %d car in operation 1,%d car in
operation 2,%d car in operation 3\n',STATES(i,1),STATES(i,2),STATES(i,3))
     if DEVRIM==1
         disp(PI(i,1))
     fprintf('\n')
     else
      disp(double(PI(i,1)))
     fprintf('\n')
        end
 end
 %1
 disp('1')
 %Define r(j,k) man hour requirments for each operation
if DEVRIM==1
    R=sym('r',[2 3]);
else
R=[3 4.5 5.9; 4.4 3.8 5.3];
end
 %Define A matrix of DWL(s,k) daily workload of labour with skill k in any
 %day which state is s.
 DWL=sym('b%d',[13 2]);
 DWL2=sym('b%d',[13 2]);
 for s=1:13
     for k=1:2
         TP=0;
         for f=1:3
            TP= TP+R(k,f)*STATES(s,f);
         end
       if DEVRIM==2
      DWL(s,k)=min(8,TP);    %will be used if any data provided
      DWL2(s,k)=max(8,TP);      %will be used in part 2
       else
           DWL(s,k)=TP;
       end
     end
 end

 disp('Daily needed workload of labour with skill k in any day which state
is s is shown in matrix DWL')
 disp(DWL)
 if DEVRIM==1
 disp('Expected total daily workload of worker with skill k is
ETDWL(k)=Transpose(PI)*(min(DWL,8)) ')
 else
 disp('Expected total daily workload of worker with skill k is
ETDWL(k)=DWL*PI ')
 ETDWL=PI.'*DWL;
 disp(double(ETDWL))
```

```
 end
 fprintf('\n')
%  2
disp('2')
% If any value in DWL matrix esceeds 8 this excess time is assigned to
utility labour.
%  Also since utitity labours are unform and multiclear skilled no need to
think
%  operation based

%ETDWLU=sum((max(DWL,8)-8),2)*PI; will be used if any data provided
 if DEVRIM==1
 disp('Expected total daily workload of Utility workers is
ETDWLU=Transpose(PI)*sum((max(DWL,8)-8),2) ')
 else

   ETDWLU=PI.'*sum(((DWL2)-8),2);
   disp('Expected total daily workload of Utility workers is
ETDWLU=Transpose(PI)*sum((max(DWL,8)-8),2) ')
   disp(double(ETDWLU))
 end
% 3
disp('3')

numberofcarso1 = zeros(1,13);
numberofcarso2 = zeros(1,13);
numberofcarso3 = zeros(1,13);
     for i=1:13
         numberofcarso1(1,i) = STATES(i,1); %To find number of cars at a
day under 1st operation in each state.
         numberofcarso2(1,i) = STATES(i,2); %To find number of cars at a
day under 2nd operation in each state.
         numberofcarso3(1,i) = STATES(i,3); %To find number of cars at a
day under 3rd operation in each state.
     end
   expnumberofcarso1= numberofcarso1*PI;
   expnumberofcarso2= numberofcarso2*PI;
   expnumberofcarso3= numberofcarso3*PI;

   disp('# of cars at a day under 1st operation in each state is as
follows')
   disp(numberofcarso1)
     disp('# of cars at a day under 2nd operation in each state is as
follows')
   disp(numberofcarso2)
     disp('# of cars at a day under 3rd operation in each state is as
follows')
   disp(numberofcarso3)
 for k=1:3
     if DEVRIM==1
 fprintf('Expected # of cars in a day under operation %d is
expnumberofcarso1%d = numberofcarso%d*PI\n',k,k,k)
     else
         expnumberofcarso=zeros(1,3);
         expnumberofcarso=[expnumberofcarso1  expnumberofcarso2
expnumberofcarso3 ];
  fprintf('Expected # of cars in a day under operation %d is
expnumberofcarso%d=%f\n',k,k,expnumberofcarso(k))
     end
```

```
    end

%  4
 disp('4')

 numberofcarsinsystem = zeros(1,13); %To find number of cars in each
states.
 for i=1:13
     for j=1:3
         numberofcarsinsystem(1,i) = STATES(i,j) +
numberofcarsinsystem(1,i);
     end
 end
 expnofc=numberofcarsinsystem*PI; %Expected number of cars in a repair shop
in a day

 disp('# of cars in system in state s is given in numberofcarsinsystem(s)')
 disp('Expected # of cars in system is given in below')
 disp('expnofc=numberofcarsinsystem*PI')
 disp(expnofc);

% 5
disp('5')
% ASSUMPTION: We assumed that if two cars arrives at the same time they
% counted s two acceptance
disp(' ASSUMPTION: We assumed that if two cars arrives at the same time
they counted s two acceptance')
disp('ASSUMTION FOR CASE 1: we assumed that the acceptance rate is the sum
of states that the acceptance occured per day and  We assumed that if two
cars arrives at the same time they counted as one acceptance ')
%In order to determine the acceptance rate we need to check the # of cars
%in first operation

% NFS(s)=number of cars in state s, under operation 1
NFS=STATES(:,1);
Ad=((NFS==0)-1)*(-1);
disp('# of cars in first state is given in NFS(s) matrix')
disp(NFS)
disp('Daily acceptance rate for Case 1 can be found as Transpose(NFS)*PI');
AR1=(Ad.')*PI;
disp(AR1)
disp('ASSUMTION FOR CASE 2:we assumed that the acceptance rate is the sum
of states that the acceptance occured per day and  We assumed that if two
cars arrives at the same time they counted s two acceptance , per day ')
ARM=[STATES(:,1) (STATES(:,2)+STATES(:,3))];
PIT=sym('b%d',[1 13]);
AR2=(NFS.')*PI;
disp('Daily acceptance rate for Case 2 can be found as
AR2=Transpose(NFS)*PI');
disp(AR2)




%6
disp('6')
%In order to determine the expected time between two admission we need to
%find the admission rate.
disp('In order to determine the expected time between two admission we need
to find the admission rate.')
NFS=STATES(:,1);
```

```
disp('Admission is occured if the element of Ad is 1, not occured is it is
0')
disp('Ad')
disp(Ad)
disp('Daily acceptance rate had been found as Transpose(Ad)*PI');
ARR=(Ad.')*PI;
Ans=1/ARR;
disp('expected time between two admission');
disp(Ans)

% 7
disp('7')
% Expected profit=revenue-cost
% cost factors: regular workers, utility workers, fixed cost for unrepaired
% cars.
if DEVRIM==1
syms c1 c2 p w ETDWL ETDWLU;
UWC=ETDWLU*c2;
else
   c1=10;
   c2=12;
   p=800;
   w=100;
end
RWC=sum(ETDWL)*c1;
UWC=ETDWLU*c2;
TD=0;% total departures
for i=1:13
    if STATES(i,3)==0
        TD=TD+0;
    elseif STATES(i,3)==1
        TD=TD+(1-a3)*PI(i);
    else
       TD=TD+(1-a3)*a3*2*PI(i)+((1-a3)^2)*2*PI(i); %2*(1-a3) because 2 car
left successfully
    end
end
TR=p*TD; %total revenue
% Total cars left because they didn't fixed is found by the difference
% between total arrival rate and total succesful departure
TCL=AR2-TD; %total car left
fc=TCL*w;
TP=TR-fc-RWC-UWC;
disp(' Expected profit=revenue-cost')
disp('Cost factors: regular workers, utility workers, fixed cost for
unrepaired cars')
disp('TP=TR-fc-RWC-UWC')
disp('total profit is equal to:')
if DEVRIM==1
disp(TP)
else
    disp(double(TP))
end

fprintf('\n\n')
% clear all
end
```

Appendix 3

MATLAB Output

PART A

-Part A is 28 pages so in order to save paper and the planet we only provided output of part B. run the code in appendix 2 for symbolic solutions of Part A.

PART B

In the state when there is 0 car in operation 1,0 car in operation 2,0 car in operation 3

0.1023

In the state when there is 1 car in operation 1,0 car in operation 2,0 car in operation 3

0.1364

In the state when there is 0 car in operation 1,1 car in operation 2,0 car in operation 3

0.0686

In the state when there is 0 car in operation 1,0 car in operation 2,1 car in operation 3

0.0529

In the state when there is 1 car in operation 1,1 car in operation 2,0 car in operation 3

0.0914

In the state when there is 1 car in operation 1,0 car in operation 2,1 car in operation 3

0.0705

In the state when there is 0 car in operation 1,1 car in operation 2,1 car in operation 3

0.0742

In the state when there is 2 car in operation 1,1 car in operation 2,0 car in operation 3

0.0686

In the state when there is 2 car in operation 1,0 car in operation 2,1 car in operation 3

0.0529

In the state when there is 0 car in operation 1,2 car in operation 2,1 car in operation 3

0.0269

In the state when there is 2 car in operation 1,0 car in operation 2,0 car in operation 3

0.1023

In the state when there is 0 car in operation 1,2 car in operation 2,0 car in operation 3

0.0828

In the state when there is 0 car in operation 1,0 car in operation 2,2 car in operation 3

0.0702

1

Daily needed workload of labour with skill k in any day which state is s is shown in matrix DWL

[0, 0]

[3, 22/5]

[9/2, 19/5]

[59/10, 53/10]

[15/2,   8]
[8,   8]
[8,   8]
[8,   8]
[8,   8]
[8,   8]
[6,   8]
[8, 38/5]
[8,   8]

Expected total daily workload of worker with skill k is ETDWL(k)=DWL*PI

5.8978   6.2263

2

Expected total daily workload of Utility workers is

ETDWLU=Transpose(PI)*sum((max(DWL,8)-8),2)

2.4080

3

# of cars at a day under 1st operation in each state is as follows

0	1	0	0	1	1	0	2	2	0	2	0	0

# of cars at a day under 2nd operation in each state is as follows

0	0	1	0	1	0	1	1	0	2	0	2	0

# of cars at a day under 3rd operation in each state is as follows

0	0	0	1	0	1	1	0	1	1	0	0	2

Expected # of cars in a day under operation 1 is expnumberofcarso1=0.745924

Expected # of cars in a day under operation 2 is expnumberofcarso2=0.522147

Expected # of cars in a day under operation 3 is expnumberofcarso3=0.417718

4

# of cars in system in state s is given in numberofcarsinsystem(s)

Expected # of cars in system is given in below

expnofc=numberofcarsinsystem*PI

23391000/13875403

5

ASSUMPTION: We assumed that if two cars arrives at the same time they counted s two acceptance

ASSUMTION FOR CASE 1: we assumed that the acceptance rate is the sum of states that the acceptance occured per day and We assumed that if two cars arrives at the same time they counted as one acceptance

# of cars in first state is given in NFS(s) matrix

0
1
0
0
1
1
0
2
2
0
2
0
0

Daily acceptance rate for Case 1 can be found as Transpose(NFS)*PI

7245000/13875403

ASSUMTION FOR CASE 2:we assumed that the acceptance rate is the sum of states that the acceptance occured per day and We assumed that if two cars arrives at the same time they counted s two acceptance , per day

Daily acceptance rate for Case 2 can be found as AR2=Transpose(NFS)*PI

10350000/13875403

6

In order to determine the expected time between two admission we need to find the admission rate.

Admission is occured if the element of Ad is 1, not occured is it is 0

Ad

0
1
0
0
1
1
0
1
1
0
1
0
0

Daily acceptance rate had been found as Transpose(Ad)*PI

expected time between two admission

13875403/7245000

7

Expected profit=revenue-cost

Cost factors: regular workers, utility workers, fixed cost for unrepaired cars

TP=TR-fc-RWC-UWC

total profit is equal to:

132.4191