## Applied (Financial) Econometrics

## Lecture on Panel data

## Monique de Haan

(monique.dehaan@uva.nl)

Stock and Watson Chapter 10

## **OLS: The Least Squares Assumptions**

$$Y_i = \beta_0 + \beta_1 X_i + u_i$$

Assumption 1: conditional mean zero assumption:  $E[u_i|X_i] = 0$ 

Assumption 2:  $(X_i, Y_i)$  are i.i.d. draws from joint distribution

Assumption 3: Large outliers are unlikely

- Under these three assumption the OLS estimators are unbiased, consistent and normally distributed in large samples.
- Omitted variables that satisfy the following properties cause a violation of assumption 1
  - Omitted variable is a determinant of the outcome Y<sub>i</sub>
  - Omitted variable is correlated with regressor of interest X<sub>i</sub>
- In this lecture we discuss a method we can use in case of omitted variables

1

$$Y_i = \beta_0 + \beta_1 X 1_i + \beta_2 X 2_i + \beta_3 X 3_i + ... + \beta_k X k_i + u_i$$

- Even with multiple regression there is threat of omitted variables:
  - some factors are difficult to measure

problem of simple regression

- sometimes we are simply ignorant about relevant factors
- Multiple regression based on panel data may mitigate detrimental effect of omitted variables without actually observing them.

3

### Panel data

#### Cross-sectional data:

A sample of individuals observed in 1 time period

Panel data: same sample of individuals observed in multiple time periods

2010 (MINTON STATE AND STA

- Panel data consist of observations on n entities (cross-sectional units) and T time periods
- Particular observation denoted with two subscripts (i and t)

$$Y_{it} = \beta_0 + \beta_1 X_{it} + u_{it}$$

- Y<sub>it</sub> outcome variable for individual i in year t
- For balanced panel this results in nT observations

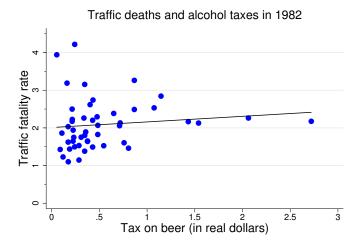
## Advantages of panel data

- More control over omitted variables.
- More observations.
- Many research questions typically involve a time component.

### The effect of alcohol taxes on traffic deaths

- About 40,000 traffic fatalities each year in the U.S.
- Approximately 25% of fatal crashes involve driver who drank alcohol.
- Government wants to reduce traffic fatalities.
- One potential policy: increase the tax on alcoholic beverages.
- We have data on traffic fatality rate and tax on beer for 48 U.S. states in 1982 and 1988.
- What is the effect of increasing the tax on beer on the traffic fatality rate?

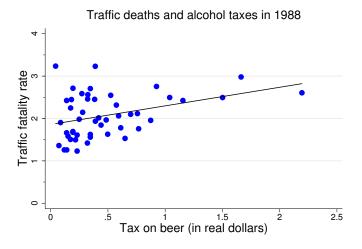
## Data from 1982



8

$$FatalityRate_{i,1982} = 2.01 + 0.15 BeerTax_{i,1982} (0.14) (0.18)$$

## Data from 1988



$$FatalityRate_{i,1988} = 1.86 + 0.44 BeerTax_{i,1988} \\ (0.11) (0.16)$$

## Panel data: before-after analysis

- Both regression using data from 1982 & 1988 likely suffer from omitted variable bias
- We can use data from 1982 and 1988 together as panel data
- Panel data with T = 2
- Observed are Y<sub>i1</sub>, Y<sub>i2</sub> and X<sub>i1</sub>, X<sub>i2</sub>
- Suppose model is

$$Y_{it} = \beta_0 + \beta_1 X_{it} + \beta_2 Z_i + u_{it}$$

and we assume  $E(u_{it}|X_{i1}, X_{i2}, Z_i) = 0$ 

- $Z_i$  are (unobserved) variables that vary between states but not over time
  - (such as local cultural attitude towards drinking and driving)
- Parameter of interest is β<sub>1</sub>

#### Panel data

#### browse state year fatalityrate beertax Data Editor (Browse) - [alcohol19821988.dta] File Edit View Data Tools 🌃 🖺 , 📑 🗎 🖨 🖺 🖺 💪 🤻 🗸 state[1] state vear fatalityrate beertax 1 1982 2.12836 1.539379 AL 2 AL 1988 2.49391 1.501444 3 ΑZ 1982 2.49914 .2147971 4 ΑZ 1988 2.70565 .346487 2.38405 5 AR 1982 .650358 AR 1988 2.54697 .5245429 CA 1982 1.86194 .1073986 1.90365 CA 1988 .0866218 9 CO 1982 2.17448 .2147971 10 CO 1988 1.5056 .1732435 CT 1982 1.64695 .2243437 11 12 CT 1988 1,49706 .2172185

#### Panel data: before

• Consider cross-sectional regression for first period (t = 1):

$$Y_{i1} = \beta_0 + \beta_1 X_{i1} + \beta_2 Z_i + u_{i1}$$
  $E[u_i | X_{i1}, Z_i] = 0$ 

- Z<sub>i</sub> observed: multiple regression of Y<sub>i1</sub> on constant, X<sub>i1</sub> and Z<sub>i</sub> leads to unbiased and consistent estimator of β<sub>1</sub>
- $Z_i$  not observed: regression of  $Y_{i1}$  on constant and  $X_{i1}$  only results in unbiased estimator of  $\beta_1$  when  $Cov(X_{i1}, Z_i) = 0$
- What can we do if we don't observe Z<sub>i</sub>?

### Panel data: after

• We also observe  $Y_{i2}$  and  $X_{i2}$ , hence model for second period is:

$$Y_{i2} = \beta_0 + \beta_1 X_{i2} + \beta_2 Z_i + u_{i2}$$

- Similar to argument before cross-sectional analysis for period 2 might fail
- Problem is again the unobserved heterogeneity embodied in Z<sub>i</sub>

## Before-after analysis (first differences)

We have

$$Y_{i1} = \beta_0 + \beta_1 X_{i1} + \beta_2 Z_i + u_{i1}$$

and

$$Y_{i2} = \beta_0 + \beta_1 X_{i2} + \beta_2 Z_i + u_{i2}$$

Subtracting period 1 from period 2 gives

$$Y_{i2} - Y_{i1} = (\beta_0 + \beta_1 X_{i2} + \beta_2 Z_i + u_{i2}) - (\beta_0 + \beta_1 X_{i1} + \beta_2 Z_i + u_{i1})$$

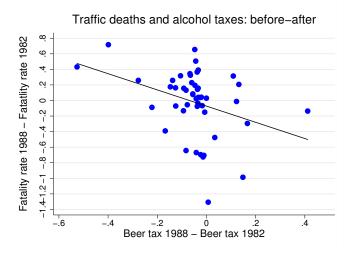
Applying OLS to:

$$Y_{i2} - Y_{i1} = \beta_1(X_{i2} - X_{i1}) + (u_{i2} - u_{i1})$$

will produce an unbiased and consistent estimator of  $\beta_1$ 

- Advantage of this regression is that we do not need data on Z
- By analyzing changes in dependent variable we automatically control for time-invariant unobserved factors

## Data from 1982 and 1988



$$\widehat{\textit{Fatality}}_{i,1988} - \widehat{\textit{Fatality}}_{i,1982} = \begin{array}{ccc} -0.07 & - & 1.04 & (\textit{BeerTax}_{i,1988} - \textit{BeerTax}_{i,1982}) \\ (0.06) & (0.42) \end{array}$$

## Panel data with more than 2 time periods



	state[1]	1		
	state	year	fatalityrate	beertax
1	AL	1982	2.12836	1.539379
2	AL	1983	2.34848	1.788991
3	AL	1984	2.33643	1.714286
4	AL	1985	2.19348	1.652542
5	AL	1986	2.66914	1.609907
6	AL	1987	2.71859	1.56
7	AL	1988	2.49391	1.501444
8	AZ	1982	2.49914	.2147971
9	AZ	1983	2.26738	.206422
10	AZ	1984	2.82878	.2967033
11	AZ	1985	2.80201	.3813559
12	AZ	1986	3.07106	.371517
13	AZ	1987	2.76728	.36
14	AZ	1988	2.70565	.346487

## Panel data with more than 2 time periods

Panel data with T ≥ 2

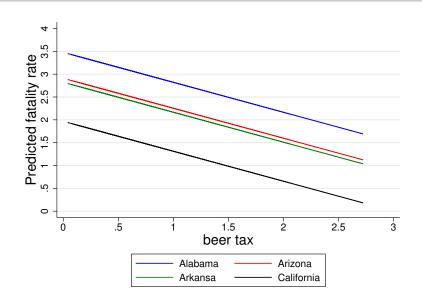
$$Y_{it} = \beta_0 + \beta_1 X_{it} + \beta_2 Z_i + u_{it}, \qquad i = 1, ..., n; \quad t = 1, ..., T$$

- Y<sub>it</sub> is dependent variable; X<sub>it</sub> is explanatory variable; Z<sub>i</sub> are state specific, time invariant variables
- Equation can be interpreted as model with n specific intercepts (one for each state)

$$Y_{it} = \beta_1 X_{it} + \alpha_i + u_{it},$$
 with  $\alpha_i = \beta_0 + \beta_2 Z_i$ 

- $\alpha_i$ , i = 1, ..., n are called entity fixed effects
- $\alpha_i$  models impact of omitted time-invariant variables on  $Y_{it}$

## State specific intercepts



Least squares with dummy variables

#### Having data on $Y_{it}$ and $X_{it}$ how to determine $\beta_1$ ?

- Population regression model:  $Y_{it} = \beta_1 X_{it} + \alpha_i + u_{it}$
- In order to estimate the model we have to quantify  $\alpha_i$
- Solution: create n dummy variables D1<sub>i</sub>,..., Dn<sub>i</sub>
  - with  $D1_i = 1$  if i = 1 and 0 otherwise,
  - with  $D2_i = 1$  if i = 2 and 0 otherwise,....
- Population regression model can be written as:

$$Y_{it} = \beta_1 X_{it} + \alpha_1 D1_i + \alpha_2 D2_i + \dots + \alpha_n Dn_i + u_{it}$$

Alternatively, population regression model can be written as:

$$Y_{it}=\beta_0+\beta_1X_{it}+\gamma_2D2_i+...+\gamma_nDn_i+u_{it}$$
 with  $\beta_0=\alpha_1$  and  $\gamma_i=\alpha_i-\beta_0$  for  $i>1$ 

- Interpretation of  $\beta_1$  identical for both representations
- Ordinary Least Squares (OLS): choose  $\hat{\beta}_0, \hat{\beta}_1, \hat{\gamma}_2..., \hat{\gamma}_n$  to minimize squared prediction mistakes (*SSR*):

$$\sum_{i=1}^{n} \sum_{t=1}^{T} \left( Y_{it} - \hat{\beta}_0 - \hat{\beta}_1 X_{it} - \hat{\gamma}_2 D 2_i - ... - \hat{\gamma}_n D n_i \right)^2$$

• *SSR* is function of  $\hat{\beta}_0, \hat{\beta}_1, \hat{\gamma}_2..., \hat{\gamma}_n$ 

Least squares with dummy variables

$$\sum_{i=1}^{n} \sum_{t=1}^{T} \left( Y_{it} - \hat{\beta}_0 - \hat{\beta}_1 X_{it} - \hat{\gamma}_2 D 2_i - \dots - \hat{\gamma}_n D n_i \right)^2$$

#### OLS procedure:

- Take partial derivatives of *SSR* w.r.t.  $\hat{\beta}_0, \hat{\beta}_1, \hat{\gamma}_2..., \hat{\gamma}_n$
- Equal partial derivatives to zero resulting in n + 1 equations with n + 1 unknown coefficients
- Solutions are the OLS estimators  $\hat{\beta}_0, \hat{\beta}_1, \hat{\gamma}_2..., \hat{\gamma}_n$

Least squares with dummy variables

- Analytical formulas require matrix algebra
- Algebraic properties OLS estimators (normal equations, linearity) same as for simple regression model
- Extension to multiple X's straightforward: n + k normal equations
- OLS procedure is also labeled Least Squares Dummy Variables (LSDV) method
- Dummy variable trap: Never include all n dummy variables and the constant term!

Within estimation

- Typically *n* is large in panel data applications
- With large n computer will face numerical problem when solving system of n + 1 equations
- OLS estimator can be calculated in two steps
- First step: demean Y<sub>it</sub> and X<sub>it</sub>
- Second step: use OLS on demeaned variables

#### Within estimation

We have

$$Y_{it} = \beta_1 X_{it} + \alpha_i + u_{it}$$

$$\bar{Y}_i = \beta_1 \bar{X}_i + \alpha_i + \bar{u}_i$$

- $\bar{Y}_i = \frac{1}{T} \sum_{t=1}^{T} Y_{it}$ , etc. is entity mean
- Subtracting both expressions leads to

$$Y_{it} - \bar{Y}_i = (\beta_1 X_{it} + \alpha_i + u_{it}) - (\beta_1 \bar{X}_i + \alpha_i + \bar{u}_i)$$

$$\tilde{Y}_{it} = \beta_1 \tilde{X}_{it} + \tilde{u}_{it}$$

- $\tilde{Y}_{it} = Y_{it} \bar{Y}_i$ , etc. is entity demeaned variable
- $\alpha_i$  has disappeared; OLS on demeaned variables involves solving one normal equation only!

#### Within estimation

bys state: egen meanfatality=mean(fatality)

 ${\tt gen\ Dmeanfatality=fatality-meanfatality}$ 

bys state: egen meanbeertax=mean(beertax)

gen Dmeanbeertax=beertax-meanbeertax

Data Editor (Browse) - [alcohol.dta]

	state[1]							
	state	year	fatalityrate	meanfatality	Dmeanfatal~y	beertax	meanbeertax	Dmeanbeertax
1	AL	1982	2.12836	2.4126272	28426712	1.539379	1.6237927	0844132
2	AL	1983	2.34848	2.4126272	06414717	1.788991	1.6237927	.16519805
3	AL	1984	2.33643	2.4126272	07619708	1.714286	1.6237927	.09049293
4	AL	1985	2.19348	2.4126272	21914714	1.652542	1.6237927	.02874967
5	AL	1986	2.66914	2.4126272	.25651271	1.609907	1.6237927	01388565
6	AL	1987	2.71859	2.4126272	.30596287	1.56	1.6237927	06379274
7	AL	1988	2.49391	2.4126272	.08128292	1.501444	1.6237927	12234906
8	AZ	1982	2.49914	2.7059	20676	.2147971	.31104035	09624322
9	AZ	1983	2.26738	2.7059	43852002	.206422	.31104035	10461832
10	AZ	1984	2.82878	2.7059	.12287991	.2967033	.31104035	01433705
11	AZ	1985	2.80201	2.7059	.09611004	.3813559	.31104035	.07031559
12	AZ	1986	3.07106	2.7059	.36515992	.371517	.31104035	.06047668
13	AZ	1987	2.76728	2.7059	.06138008	.36	.31104035	.04895966
14	AZ	1988	2.70565	2.7059	00024993	.346487	.31104035	.03544666

#### Within estimation

- Entity demeaning is often called the Within transformation
- Within transformation is generalization of "before-after" analysis to more than  $\mathcal{T}=2$  periods
- Before-after:  $Y_{i2} Y_{i1} = \beta_1(X_{i2} X_{i1}) + (u_{i2} u_{i1})$
- Within:  $Y_{it} \bar{Y}_i = \beta_1 (X_{it} \bar{X}_i) + (u_{it} \bar{u}_i)$
- LSDV and Within estimators are identical:

$$FatalityRate_{it} = -0.66$$
  $BeerTax_{it} + State dummies$  (0.19)

$$(FatalityRate_{it} - \overline{FatalityRate}) = -0.66 (BeerTax_{it} - \overline{BeerTax})$$
 $(0.19)$ 

# Fixed effects regression model time fixed effects

- In addition to entity effects we can also include time effects in the model
- Time effects control for omitted variables that are common to all entities but vary over time
- Typical example of time effects: macroeconomic conditions or federal policy measures are common to all entities (e.g. states) but vary over time
- Panel data model with entity and time effects:

$$Y_{it} = \beta_1 X_{it} + \alpha_i + \lambda_t + u_{it}$$

#### time fixed effects

- OLS estimation straightforward extension of LSDV/Within estimators of model with only entity fixed effects
- LSDV: create T dummy variables B1<sub>t</sub>....BT<sub>t</sub>

$$Y_{it} = \beta_0 + \beta_1 X_{it} + \gamma_2 D 2_i + \dots + \gamma_n D n_i$$
$$+ \delta_2 B 2_t + \delta_3 B 3_t + \dots + \delta_T B T_t + u_{it}$$

- Within estimation: Deviating Y<sub>it</sub> and X<sub>it</sub> from their entity and time-period means
- The effect of the tax on beer on the traffic fatality rate:

$$FatalityRate_{it} = -0.64$$
  $BeerTax_{it} + State dummies + Time dummies$  (0.20)

# Fixed effects regression model statistical properties OLS

$$Y_{it} = \beta_1 X_{it} + \alpha_i + \lambda_t + u_{it}$$

statistical assumptions are:

ASS #1:  $E(u_{it}|X_{i1},...,X_{iT},\alpha_i,\lambda_t) = 0$ 

ASS #2:  $(X_{i1},...,X_{iT},Y_{i1},...,Y_{iT})$  are i.i.d. over the cross-section

ASS #3: large outliers are unlikely

ASS #4: no perfect multicollinearity

ASS #5:  $cov(u_{it}, u_{is}|X_{i1}, ..., X_{iT}, \alpha_i, \lambda_t) = 0$  for  $t \neq s$ 

# Fixed effects regression model statistical properties OLS

#### ASS #1 to ASS #4 imply that:

- OLS estimator  $\hat{\beta}_1$  is *unbiased* and *consistent* estimator of  $\beta_1$
- OLS estimators approximately have a normal distribution

#### remarks:

- ASS #1 is most important
- extension to multiple X's straightforward

$$Y_{it} = \beta_1 X \mathbf{1}_{it} + \beta_2 X \mathbf{2}_{it} + \dots + \beta_k X k_{it} + \alpha_i + \lambda_t + u_{it}$$

 additional assumption ASS #5 implies that error terms are uncorrelated over time (no autocorrelation)

#### Clustered standard errors

- Violation of assumption #5: error terms are correlated over time:  $(Cov(u_{it}, u_{is}) \neq 0)$
- u<sub>it</sub> contains time-varying factors that affect the traffic fatality rate (but that are uncorrelated with the beer tax)
- These omitted factors might for a given entity be correlated over time
- Examples: downturn in local economy, road improvement project
- Not correcting for autocorrelation leads to standard errors which are often too low

#### Clustered standard errors

- Solution: compute HAC-standard errors (clustered se's)
  - robust to arbitrary correlation within clusters (entities)
  - robust to heteroskedasticity
  - assume no correlation across entities
- Clustered standard errors valid whether or not there is heteroskedasticity and/or autocorrelation
- Use of clustered standard errors problematic when number of entities is below 50 (or 42)
- In stata: command, cluster(entity)

## The effect of a tax on beer on traffic fatalities

Dependent variable: traffic fatality rate (number of deaths per 10 000)									
Beer tax	0.36*** (0.06)	-0.66*** (0.19)	-0.64*** (0.20)	-0.59*** (0.18)	-0.59* (0.33)				
State fixed effects	-	yes	yes	yes	yes				
Time fixed effects	-	-	yes	yes	yes				
Additional control variables	-	-	-	yes	yes				
Clustered standard errors	-	-	-	-	yes				
N	336	336	336	336	336				

Note: \* significant at 10% level, \*\* significant at 5% level, \*\*\* significant at 1% level. Control variables: Unemployment rate, per capita income, minimum legal drinking age.

# Panel data: an example

returns to schooling

$$Y_{it} = \beta_1 X_{it} + \alpha_i + U_{it}$$

- Y<sub>it</sub> is logarithm of individual earnings; X<sub>it</sub> is years of completed education
- $\alpha_i$  unobserved ability
- Likely to be cross-sectional correlation between  $X_{it}$  and  $\alpha_i$ , hence standard cross-sectional analysis with OLS fails
- However, in this case panel data does not solve the problem because X<sub>it</sub> typically lacks time series variation (X<sub>it</sub> = X<sub>i</sub>)
- We have to resort to cross-sectional methods (instrumental variables) to identify returns to schooling

• Is there an effect of cigarette taxes on smoking behavior?

$$Y_{it} = \beta_1 X_{it} + \alpha_i + U_{it}$$

- Y<sub>it</sub> number of packages per capita in state i in year t, X<sub>it</sub> is real tax on cigarettes in state i in year t
- $\alpha_i$  is a state specific effect which includes state characteristics which are constant over time
- Data for 48 U.S. states in 2 time periods: 1985 and 1995

Lpackpc = log number of packages per capita in state i in year t
 rtax = real avr cigarette specific tax during fiscal year in state i
 Lperinc = log per capita real income

. regress lpackpc rtax lperinc, robust

Linear regression

Number of obs = 96 F(2, 93) = 26.84 Prob > F = 0.0000 R-squared = 0.3137 Root MSE = .20401

lpackpc	Coef.	Robust Std. Err.	t	P> t	[95% Conf.	Interval]
rtax	0156393	.0025676	-6.09	0.000	020738	0105405
lperinc	0139094	.1509266	-0.09	0.927	3136198	.2858009
_cons	5.206615	.3593867	14.49	0.000	4.492944	5.920285

```
diff_lpackpc=lpackpc1995 - lpackpc1985diff_rtax=rtax1995 - rtaxt1985diff_lperinc=lperinc1995 - lperinc1985
```

. regress diff\_lpackpc diff\_rtax diff\_lperinc, noconstant robust

```
Linear regression Number of obs = 48 F(2, 46) = 149.39 Prob > F = 0.0000 R-squared = 0.8636 Root MSE = 10699
```

diff_lpackpc	Coef.	Robust Std. Err.	t	P> t	[95% Conf.	Interval]
<pre>diff_rtax diff_lperinc</pre>	0169369	.0028016	-6.05	0.000	0225762	0112975
	-1.011625	.1343813	-7.53	0.000	-1.282121	74113

Least squares with dummy variables (no intercept)

- . qui tab state, gen(STATE)
- . regress lpackpc rtax lperinc STATE\*, noconstant robust

Root MSE = **.07565** 

		Robust				
lpackpc	Coef.	Std. Err.	t	P> t	[95% Conf.	Interval]
rtax	0169369	.0028016	-6.05	0.000	0225762	0112975
lperinc	-1.011625	.1343813	-7.53	0.000	-1.282121	74113
STATE1	7.66369	.2826764	27.11	0.000	7.094692	8.232688
STATE2	7.83445	.2677568	29.26	0.000	7.295483	8.373416
STATE3	7.678434	.2894531	26.53	0.000	7.095795	8.261073
			:			
			:			
STATE42	7.045478	.2827088	24.92	0.000	6.476415	7.614541
STATE43	7.816717	.3381646	23.12	0.000	7.136027	8.497408
STATE44	7.992471	.2945996	27.13	0.000	7.399473	8.585469
STATE45	7.84436	.2894223	27.10	0.000	7.261783	8.426937
STATE46	7.926662	.2882333	27.50	0.000	7.346478	8.506845
STATE47	7.644742	.2774148	27.56	0.000	7.086335	8.20315
STATE48	7.825945	.3116665	25.11	0.000	7.198592	8.453297

#### Least squares with dummy variables (with intercept)

. regress lpackpc rtax lperinc STATE\*, robust note: STATE43 omitted because of collinearity

Linear regression

Number of obs = 96 F(49, 46) = 391.27 Prob > F = 0.0000 R-squared = 0.9533 Root MSE = .07565

lpackpc	Coef.	Robust Std. Err.	t	P> t	[95% Conf.	Interval]
rtax	0169369	.0028016	-6.05	0.000	0225762	0112975
lperinc	-1.011625	.1343813	-7.53	0.000	-1.282121	74113
STATE1	1530274	.0834036	-1.83	0.073	3209102	.0148554
STATE2	.0177322	.1031251	0.17	0.864	189848	.2253123
STATE3	1382833	.0970212	-1.43	0.161	3335768	.0570103
STATE42 STATE43	7712392 0	.0961022 (omitted)	-8.03	0.000	964683	5777955
STATE44	.1757536	.0815627	2.15	0.036	.0115763	.3399308
STATE45	.0276426	.0995756	0.28	0.783	1727927	.228078
STATE46	.1099443	.0929477	1.18	0.243	0771497	.2970382
STATE47	171975	.1059073	-1.62	0.111	3851552	.0412053
STATE48	.009227	.0696938	0.13	0.895	1310594	.1495134
_cons	7.816717	.3381646	23.12	0.000	7.136027	8.497408

96

## Panel data: Cigarette taxes and smoking

#### Within estimation

```
Group variable: state
                                                Number of groups =
                                                                            48
R-sa:
                                                Obs per group:
     within = 0.8636
                                                             min =
     between = 0.0896
                                                                           2.0
                                                              avg =
     overall = 0.2354
                                                              max =
                                                F(2,47)
                                                                        149.42
corr(u_i, Xb) = -0.5687
                                                Prob > F
                                                                        0.0000
```

(Std. Err. adjusted for 48 clusters in state)

Number of obs =

Coef.	Std. Err.	t	P> t	[95% Conf.	Interval]
0169369	.0028013	-6.05	0.000	0225724	0113013
-1.011625	.1343659	-7.53	0.000	-1.281935	7413162
7.856715	. 295989	26.54	0.000	7.261262	8.452168
	0169369 -1.011625	Coef. Std. Err. 0169369 .0028013 -1.011625 .1343659	Coef. Std. Err. t 0169369 .0028013 -6.05 -1.011625 .1343659 -7.53	Coef. Std. Err. t P> t  0169369 .0028013 -6.05 0.000 -1.011625 .1343659 -7.53 0.000	0169369 .0028013 -6.05 0.0000225724 -1.011625 .1343659 -7.53 0.000 -1.281935