

Given:

- Replacement Algorithm: Intel 486 pseudo-LRU (Least Recently Used)
- Cache Organization: 128 sets
- Each set has: 4 lines (L0, L1, L2, L3)
- Replacement Metadata: Each set is associated with 3 bits (B0, B1, B2)
- Replacement Strategy:
 - Cache first distinguishes between pairs: (L0, L1) vs (L2, L3)
 - Then within the selected pair, identifies the least recently used block to evict

Solution:

a/ In what way are bits B0, B1, B2 used

In the pseudo-LRU replacement algorithm, these three bits (B0, B1, B2) form a binary tree structure to determine which line should be replaced:

- **B0 bit:** Makes the selection between lines L0 and L1 versus lines L2 and L3
 - If $B0 = 0$, it means lines L0 and L1 were more recently used (and L2, L3 were less recently used)
 - If $B0 = 1$, it means lines L2 and L3 were more recently used (and L0, L1 were less recently used)
- **B1 bit:** Makes the selection between lines L0 and L1
 - If $B1 = 0$, it means line L0 was more recently used than L1
 - If $B1 = 1$, it means line L1 was more recently used than L0
- **B2 bit:** Makes the selection between lines L2 and L3
 - If $B2 = 0$, it means line L2 was more recently used than L3
 - If $B2 = 1$, it means line L3 was more recently used than L2

The replacement algorithm works as follows:

1. First, the B0 bit is checked
2. If $B0 = 0$, the B2 bit is used to choose between L2 and L3 (if $B2 = 0$, replace L3; if $B2 = 1$, replace L2)

- 3.** If $B0 = 1$, the $B1$ bit is used to choose between $L0$ and $L1$ (if $B1 = 0$, replace $L1$; if $B1 = 1$, replace $L0$)

When a line is accessed, the bits are updated as follows:

- When $L0$ is accessed: $B0 = 0, B1 = 0$
- When $L1$ is accessed: $B0 = 0, B1 = 1$
- When $L2$ is accessed: $B0 = 1, B2 = 0$
- When $L3$ is accessed: $B0 = 1, B2 = 1$

b/ How many bits need each set to implement full LRU algorithm

A full LRU implementation for a 4-way set requires tracking the exact order of usage for all 4 lines. The number of possible permutations of 4 lines is:

- $4! = 24$ unique usage orders
- To encode 24 states, you need: $\log_2(24) \approx 4.58 \Rightarrow$ at least 5 bits

Answer: 5 bits are required per set to implement a true/full LRU algorithm for 4 lines.