Undergraduate Research Project Report

Middle East Technical University

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1 Introduction

In this project we are interested in tuned mass damper method for reducing vibration amplitude of a single degree of freedom (SDOF) system in figure 1. We tried different number and configurations of tuned mass dampers (TMD) and optimized those configurations. The system parameters of the SDOF system are:

$$m_1 = 1\,kg$$
$$k_1 = 1\,N/m$$
$$c_1 = 0.01\,Ns/m$$
$$f_1 = 1\,N$$

Frequency response of the system without any TMDs can be seen at figure 2.

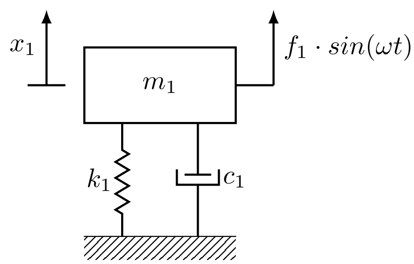

Figure 1: SDOF System

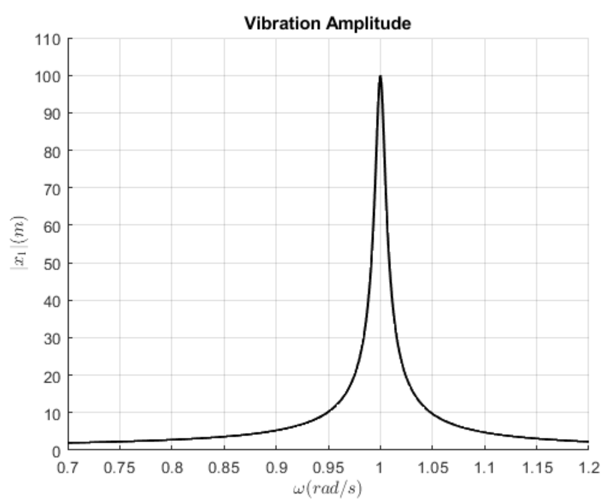

Figure 2: Frequency response of the SDOF system

2 Single Tuned Mass Damper

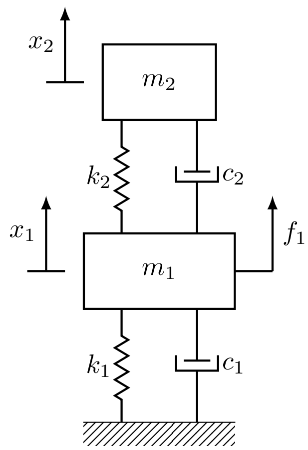

Figure 3: SDOF System with STMD

The first TMD we would like to analyse is a single tuned mass damper (STMD) attached to m_1. The only constraint that we have is $m_2 = 0.1\,kg$.

2.1 Analysis

Equation of motion (EOM) can be written as:

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \cdot \begin{Bmatrix} \ddot{x_1} \\ \ddot{x_2} \end{Bmatrix} + \begin{bmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix} \cdot \begin{Bmatrix} \dot{x_1} \\ \dot{x_2} \end{Bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \cdot \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} f_1 \\ 0 \end{Bmatrix}$$

In closed form:

$$[M] \cdot \{\ddot{X}\} + [C] \cdot \{\dot{X}\} + [K] \cdot \{X\} = \{F\}$$

Assuming $\{X\} = \{X^*\}e^{i\omega t}$ & $\{F\} = \{F\}sin(\omega t) = Im\{\{F\}e^{i\omega t}\}$, taking Fourier transform of the equation:

$$[-\omega^2[M] + i\omega[C] + [K])] \cdot \{X^*\} = \{F\}$$

$$\{X^*\} = [-\omega^2[M] + i\omega[C] + [K])]^{-1}\{F\}$$

By solving this equation for different frequencies, we can get the frequency response of the system for different TMD parameters. Effect of different stiffness and damping coefficients can be seen on figures 4 and 5.

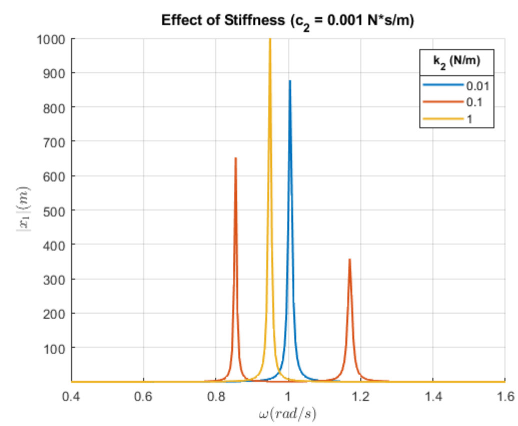

Figure 4: Effect of stiffness variation on STMD

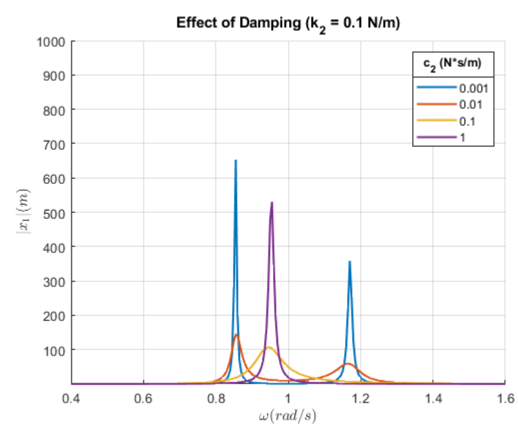

Figure 5: Effect of damping variation on STMD

2.2 Optimization

In this configuration of tuned mass damper, we have two parameters (k_2 and c_2) that need to be changed in order to optimize the response of the system. To find the optimized parameters, we need a quantitative measure of how well a system behaves. The first system measure that can be optimized is the area under the frequency response since area will be reduced as vibrations suppressed further. A gradient base method is used for finding the optimum point. Response of the optimized STMDs can be seen at figure 6.

The problem about this method is we can find different optimum values for different bounds of the integral. That prevents us to find the true optimum point. The other system measure that can be used to be optimized is the peak vibration amplitude of the system. To find the peak point, we can use a

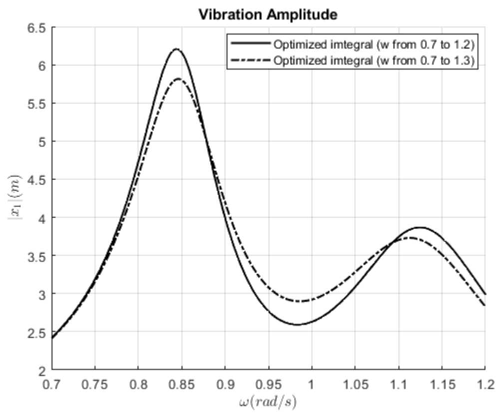

Figure 6: Frequency response of integral optimized STMDs

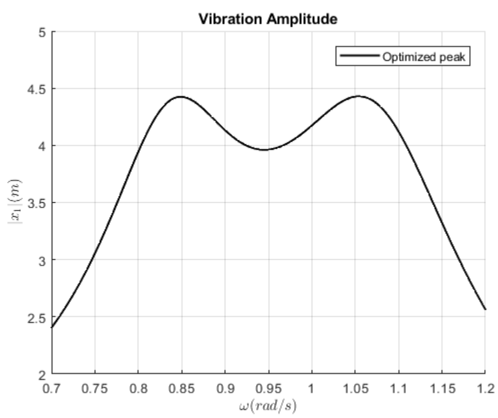

Figure 7: Frequency response of the peak optimized STMD

gradient based optimization technique. As initial guesses, resonance frequencies of undamped system (mode frequencies) can be used. Response of the optimized STMD can be seen at figure 7.

3 Double Tuned Mass Damper

In previous section, we investigated the simplest form of tuned mass damper, which is a mass connected with a spring and a damper. The optimized results were pretty promising. We can dig further into tuned mass dampers by using two masses instead of one. We have a similar constraint as before, which is $m_2 + m_3 \leq 0.1\,kg$. Two configurations of double tuned mass dampers (DTMD) are investigated following sections.

3.1 Parallel Double Tuned Mass Damper

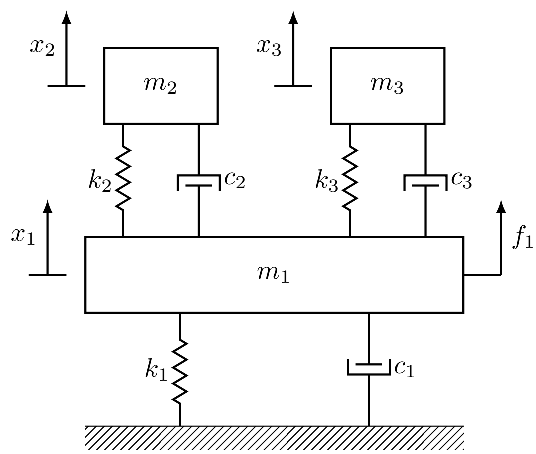

Figure 8: SDOF System with parallel DTMD

The first double tuned mass damper (DTMD) configuration is the parallel one. The parallel DTMD configuration can be seen at figure 8.

3.1.1 Analysis

EOM can be written as:

$$\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} \cdot \begin{Bmatrix} \ddot{x_1} \\ \ddot{x_2} \\ \ddot{x_3} \end{Bmatrix} + \begin{bmatrix} c_1+c_2+c_3 & -c_2 & -c_3 \\ -c_2 & c_2 & 0 \\ -c_3 & 0 & c_3 \end{bmatrix} \cdot \begin{Bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \end{Bmatrix} + \begin{bmatrix} k_1+k_2+k_3 & -k_2 & -k_3 \\ -k_2 & k_2 & 0 \\ -k_3 & 0 & k_3 \end{bmatrix} \cdot \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} f_1 \\ 0 \\ 0 \end{Bmatrix}$$

This equation has the same closed form as STMD. Therefore the equation can be solved using the same method as before.

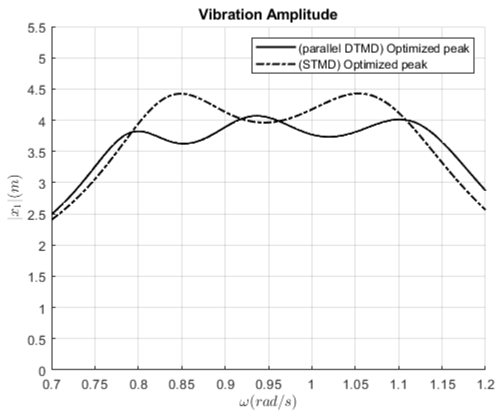

Figure 9: Frequency response of the peak optimized parallel DTMD

3.1.2 Optimization

There are 6 system parameters for DTMDs which are m_2, m_3, k_2, k_3, c_2 and c_3. However since the total mass added should be less than 0.1 kg, by equating m_3 to $0.1\,kg - m_2$, we can decrease the parameter number to 5.

Similar to STMD optimization, we can try to optimize peak value of the vibration amplitude. We can again use a gradient based optimization algorithm. Different from STMD, we can find multiple local optima using gradient based techniques, which are designed to find local extrema. Therefore finding the global optimum requires couple of different initial guesses using the gradient based approach. Therefore, an constrained genetic algorithm is used to find the region where the global optimum lies. Then, the result of the genetic algorithm can be used as an initial guess to the gradient based algorithm to refine the results.

As the result of the optimization we obtained a system with the resonance vibration amplitude of 4.0684 m. The vibration response of the optimized system can be seen at figure 9. The optimized DTMD parameters can be seen below:

$$
\begin{aligned}
m2 &= 0.0505\,kg \\
m3 &= 0.0495\,kg \\
k2 &= 0.0510\,N/m \\
k3 &= 0.0343\,N/m \\
c2 &= 0.0139\,N \cdot s/m \\
c3 &= 0.0093\,N \cdot s/m
\end{aligned}
$$

3.2 Series Double Tuned Mass Damper

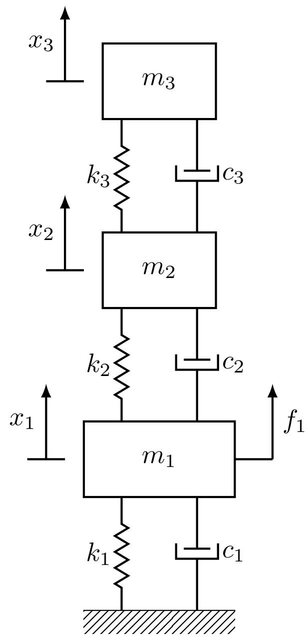

Figure 10: SDOF System with series DTMD

The other DTMD configuration we would like to investigate is the masses connected in series. The configuration can be seen at figure 10.

3.2.1 Analysis

EOM can be written as:

$$\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} \cdot \begin{Bmatrix} \ddot{x_1} \\ \ddot{x_2} \\ \ddot{x_3} \end{Bmatrix} + \begin{bmatrix} c_1 + c_2 & -c_2 & 0 \\ -c_2 & c_2 + c_3 & -c_3 \\ 0 & -c_3 & c_3 \end{bmatrix} \cdot \begin{Bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \end{Bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 & 0 \\ -k_2 & k_2 + k_3 & -k_3 \\ 0 & -k_3 & k_3 \end{bmatrix} \cdot \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} f_1 \\ 0 \\ 0 \end{Bmatrix}$$

This equation can be solved using the same method used in previous sections.

3.2.2 Optimization

Optimizing the series DTMD is mathematically same as the parallel one. Therefore we can use genetic algorithm to find the region of the global optimum and use a gradient based algorithm to find the optimum exactly.

The optimized TMD parameters and the frequency response can be seen below:

$$
\begin{aligned}
m2 &= 0.0030\,kg\\
m3 &= 0.0970\,kg\\
k2 &= 0.0981\,N/m\\
k3 &= 0.3164\,N/m\\
c2 &= 0.0282\,N\cdot s/m\\
c3 &= 0.4281\,N\cdot s/m
\end{aligned}
$$

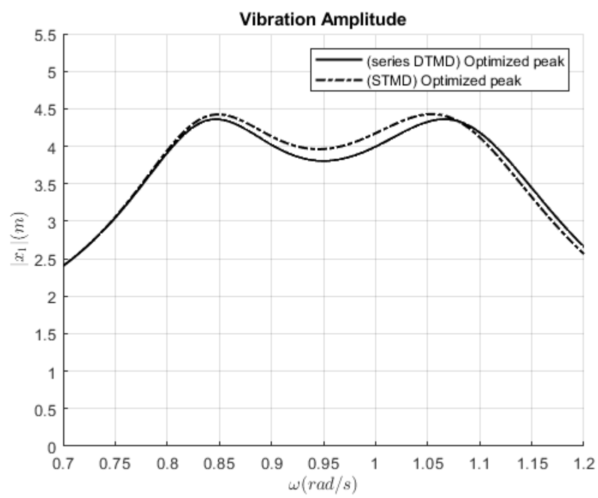

Figure 11: Frequency response of the peak optimized series DTMD

4 Non-Linear Single Tuned Mass Damper

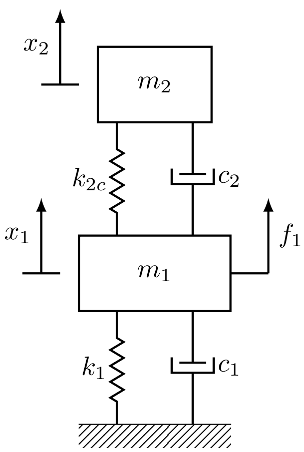

Figure 12: SDOF System with non-linear STMD

Up to that section, we assumed that all the springs and dampers as linear devices. However, in reality none of them is truly linear. In addition to that, introducing non-linearity in a TMD may increase its performance. Due to those reasons, it is important to analyse non-linear TMDs. In that example, non-linearity is introduced with changing the linear spring with the non-linear one. For that spring, force and displacement function is $F = x^3 \cdot k_{2c}$ instead of $F = x \cdot k_2$.

4.1 Analysis

EOM can be written as:

$$m_1\ddot{x_1} + (c_1 + c_2)\dot{x_1} - c_2\dot{x_2} + k_1 x_1 + k_{2c}(x_1 - x_2)^3 = f_1$$
$$m_2\ddot{x_2} - c_2\dot{x_1} + c_2\dot{x_2} - k_{2c}(x_1 - x_2)^3 = 0$$

We can write the state-space representation as follows:

$$\frac{d}{dt}\begin{Bmatrix} x_1 \\ x_2 \\ \dot{x_1} \\ \dot{x_2} \end{Bmatrix} = \begin{Bmatrix} \dot{x_1} \\ \dot{x_2} \\ \ddot{x_1} \\ \ddot{x_2} \end{Bmatrix} = \begin{Bmatrix} \dot{x_1} \\ \dot{x_2} \\ \frac{-(c_1+c_2)\dot{x_1}+c_2\dot{x_2}-k_1x_1-k_{2c}(x_1-x_2)^3+f_1 sin(\omega t)}{m_1} \\ \frac{c_2\dot{x_1}-c_2\dot{x_2}+k_{2c}(x_1-x_2)^3}{m_2} \end{Bmatrix}$$

For a given k_{2c}, c_2 and ω t is possible to solve that equation numerically using Runge Kutta Fehlberg method. Time domain solutions of the nonlinear system for the parameters $k_{2c} = 0.1N/m^3$, $c_2 = 0.05N \cdot s/m$ and $\omega = 0.8rad/s$ and the optimum linear STMD can be seen at figure 13. In that particular example, the response amplitude of the non-linear TMD is less than the response amplitude of the linear TMD.

As seen on the figure 13, both linear and non-linear systems are reaching the

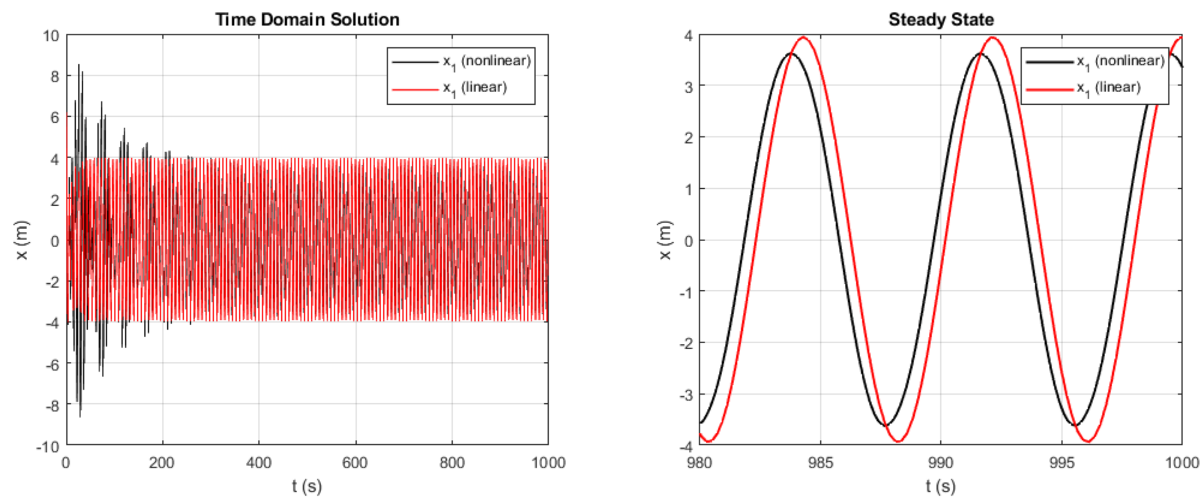

Figure 13: Nonlinar STMD with optimum linear STMD

steady state after some time. We can find the frequency response of the non-linear system by finding the steady state amplitudes for a range of ω values. The frequency response of the system can be seen at figure 14.

In previous sections, we tried to optimize the peak value of the frequency response of the systems. Unfortunately, for the nonlinear system, we can't find the exact maximum since the frequency response function is unknown. However, what we can do is to take the maximum calculated value as the maximum in order to understand the behavior of the system for different TMD parameters. Finding the peak values for a grid of different TMD parameters (k_{2c} and c_2) we can create a plot. It can be seen at figure 15.

The system shows a chaotic behavior especially at the top left part of the plot. At other areas it has more predictable behaviour. However, it is important to point out that this plot has some significant drawbacks due to the assumptions made at each step while creating it. This should be considered while interpreting the results.

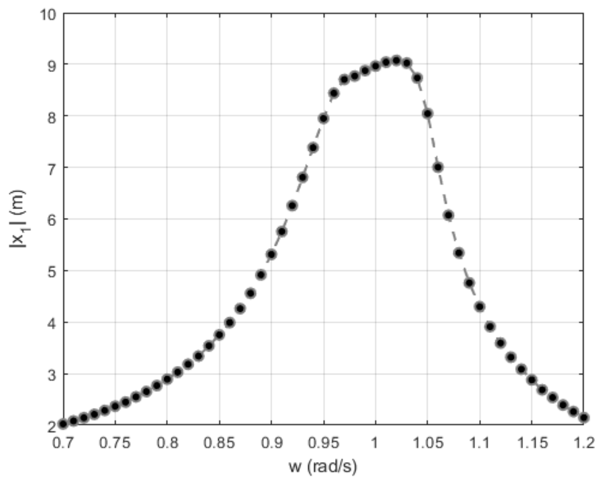

Figure 14: Frequency response of the nonlinear system

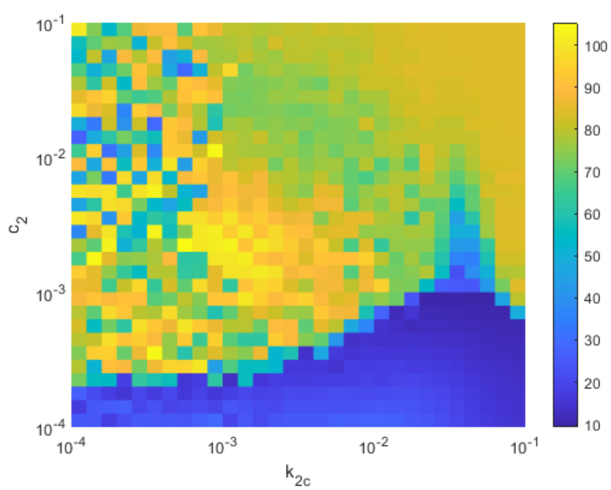

Figure 15: Peak values of the frequency response of the nonlinear TMD

5 Conclusion

Tuned mass dampers are used when the original system parameters cannot be optimized for reducing the mechanical vibrations. Even if the parameters is optimized, the desired vibration behaviour might not be achieved. It is a simple and relatively cheap solution. Due to those reasons, TMD's are used at a wide range of applications from skyscrapers to high performance airplanes.

In that project, the effects of different types of tuned mass dampers are investigated. Even the simplest form of tuned mass damper can reduce the vibration amplitude by more than 95 %. It can be seen from the results that double TMD's are more successful at reducing the vibration than the single TMD's. In addition to linear TMD's a nonlinear TMD is also analysed using stepped sine analysis technique.

Vast majority of the calculations in that project are made using MATLAB R2020b, Optimization Toolbox and Global Optimization Toolbox. In addition to MATLAB, Mathcad Prime 5.0.0.0 is also used for checking some of the calculations.

All of the files created for this project (including MATLAB, Mathcad and LaTeX files for creating that document) can be seen at this GitHub page.