Tuned Mass Damper Optimization

Single Degree of Freedom System with a Single Tuned Mass Damper

$m_1 := 1\ kg$ $k_1 := 1\ \frac{N}{m}$ $c_1 := 0.01\ N \cdot \frac{s}{m}$ $f_1 := 1\ N$

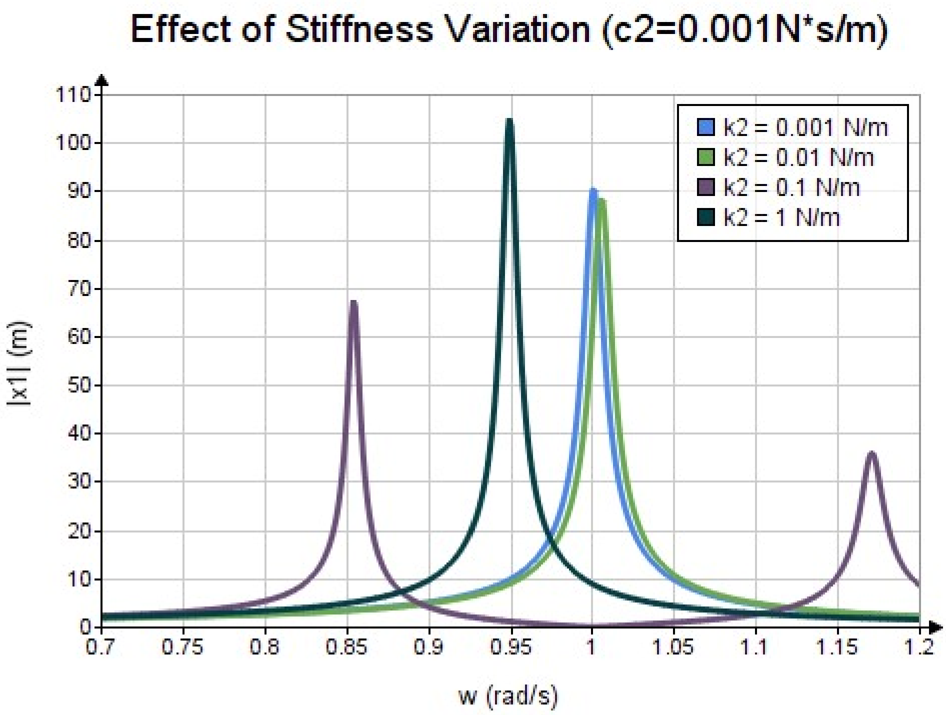
$m_2 := 0.1\ kg$

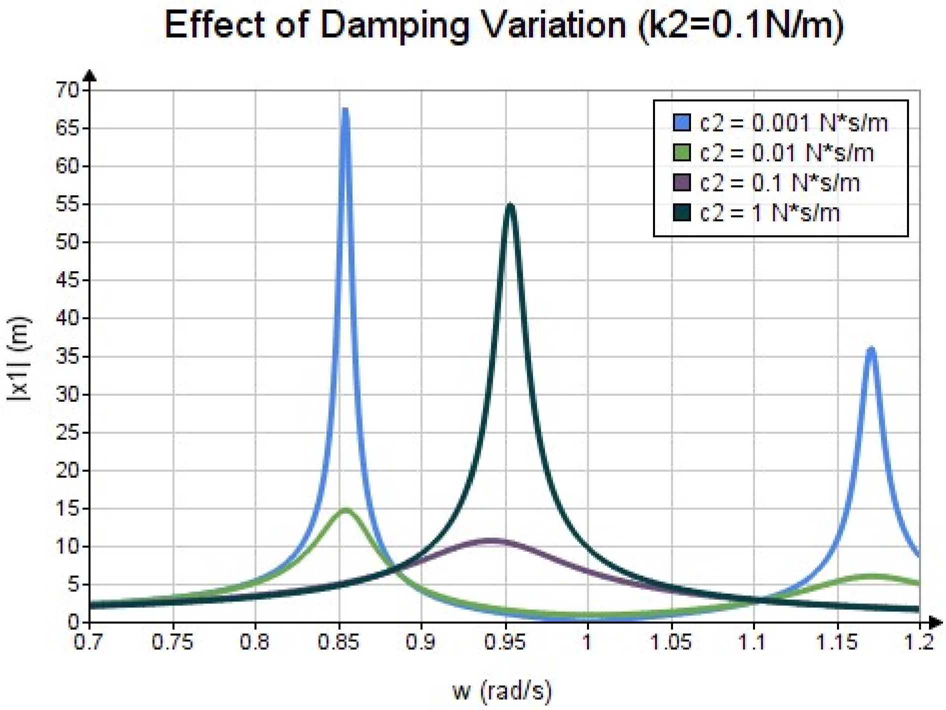
Parameters to be optimized: $k_2 := \begin{bmatrix} 0.001 & 0.01 & 0.1 & 1 \end{bmatrix}^{T} \frac{N}{m}$ $c_2 := \begin{bmatrix} 0.001 & 0.01 & 0.1 & 1 \end{bmatrix}^{T} N \cdot \frac{s}{m}$

$$X(\omega, k_2, c_2) := \overrightarrow{\left| \text{lsolve}\left(-\omega^2 \cdot \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} + \sqrt{-1} \cdot \omega \cdot \begin{bmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}, \begin{bmatrix} f_1 \\ 0 \end{bmatrix} \right) \right|}$$

$$x_1(\omega, k_2, c_2) := X(\omega, k_2, c_2)_0$$

$\omega_1 := 0.7\ \frac{rad}{s}$ $\omega_2 := 1.2\ \frac{rad}{s}$ $\omega_{inc} := 0.001\ \frac{rad}{s}$ $\omega := \omega_1, \omega_1 + \omega_{inc} .. \omega_2$





Optimizing the area under the curve

$\omega_1 := 0.7 \, \dfrac{rad}{s} \qquad \omega_2 := 1.2 \, \dfrac{rad}{s}$

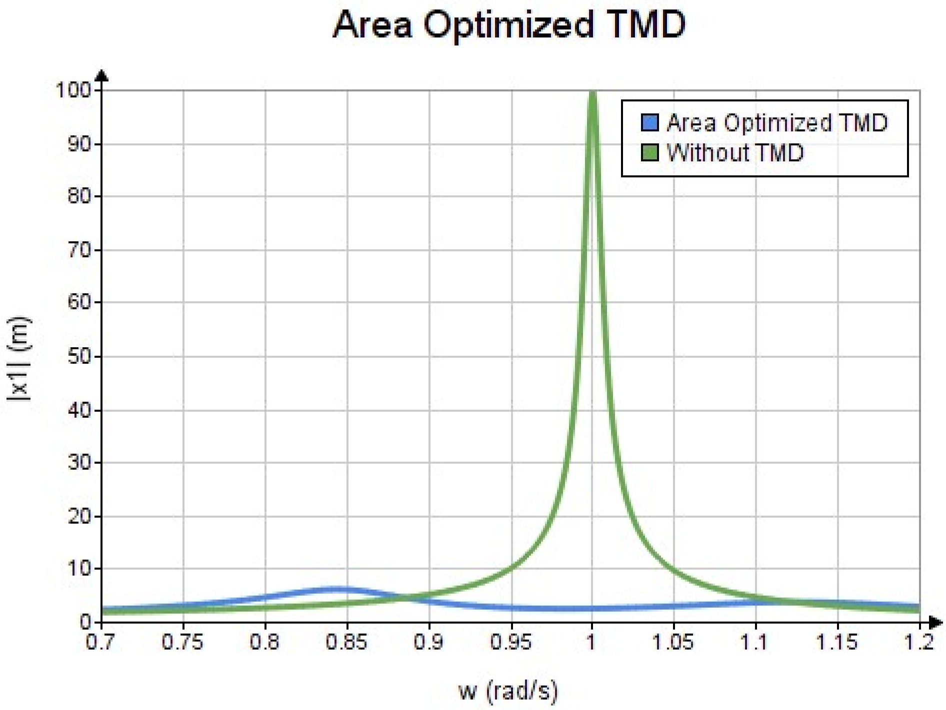
$$I\left(k_2, c_2\right) := \int_{\omega_1}^{\omega_2} X\left(\omega, k_2, c_2\right)_0 \, \mathrm{d}\omega$$

$$I\left(0.1 \, \frac{N}{m}, 0.001 \, N \cdot \frac{s}{m}\right) = 3.497 \, \frac{m}{s}$$

$k_2 := 0.1 \, \dfrac{N}{m} \qquad c_2 := 0.01 \, N \cdot \dfrac{s}{m}$

$k_2 > 0 \, \dfrac{N}{m} \qquad c_2 > 0 \, \dfrac{N \cdot s}{m}$

$$\begin{bmatrix} k_{2imin} \\ c_{2imin} \end{bmatrix} := \mathbf{minimize}\left(I, k_2, c_2\right) = \begin{bmatrix} 0.091848 \, \dfrac{kg}{s^2} \\ 0.024254 \, \dfrac{kg}{s} \end{bmatrix}$$



Optimizing the peak point

$$\omega_{undamped\_peak}(k_2, c_2) := \sqrt{\text{eigenvals}\left(\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}^{-1} \cdot \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}\right)}$$

$$p(k_2, c_2) := \left\| \begin{array}{l} \omega \leftarrow \omega_{undamped\_peak}(k_2, c_2)_0 \\ x_{temp1}(\omega) \leftarrow x_1(\omega, k_2, c_2) \\ \omega_{peak1} \leftarrow \textbf{maximize}(x_{temp1}, \omega) \\ \omega \leftarrow \omega_{undamped\_peak}(k_2, c_2)_1 \\ x_{temp2}(\omega) \leftarrow x_1(\omega, k_2, c_2) \\ \omega_{peak2} \leftarrow \textbf{maximize}(x_{temp2}, \omega) \\ x_{peak} \leftarrow \max\left(x_{temp1}(\omega_{peak1}), x_{temp2}(\omega_{peak2})\right) \\ \text{return } x_{peak} \end{array} \right.$$

Guess Values

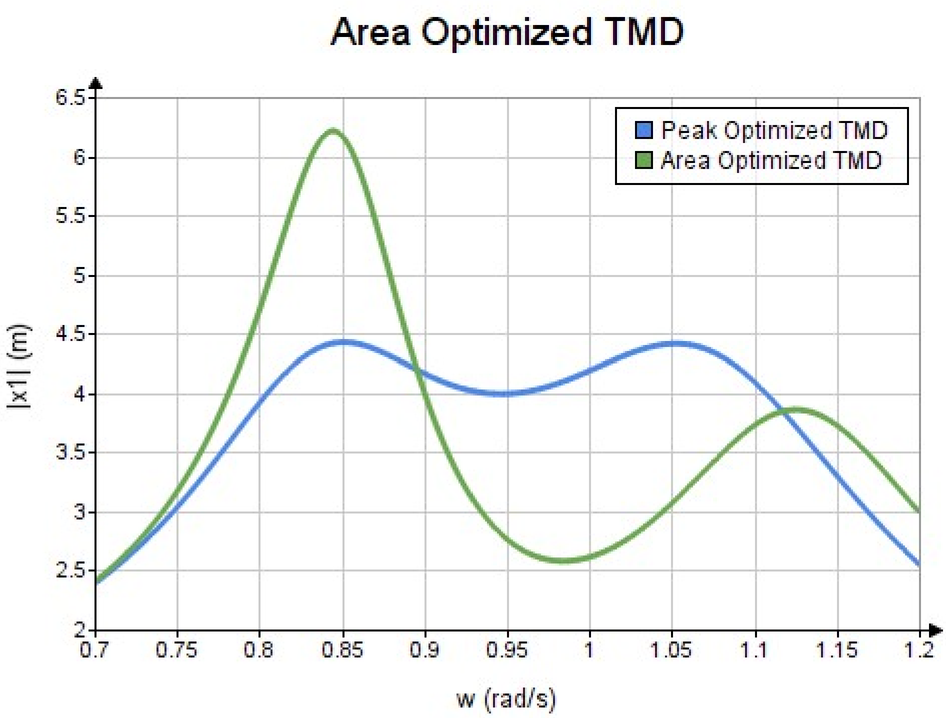
$$k_2 := 0.1 \ \frac{N}{m} \qquad c_2 := 0.01 \ N \cdot \frac{s}{m}$$

Constraints

$$k_2 > 0 \ \frac{N}{m} \qquad c_2 > 0 \ \frac{N \cdot s}{m}$$

Solver

$$\begin{bmatrix} k_{2pmin} \\ c_{2pmin} \end{bmatrix} := \textbf{minimize}\left(p, k_2, c_2\right) = \begin{bmatrix} 0.082239 \ \frac{kg}{s^2} \\ 0.034885 \ \frac{kg}{s} \end{bmatrix}$$



Single Degree of Freedom System with Double Tuned Mass Damper

$m_1 := 1\ kg$ $\qquad k_1 := 1\ \frac{N}{m}$ $\qquad c_1 := 0.01\ N \cdot \frac{s}{m}$ $\qquad f_1 := 1\ N$

$$X(\omega, m_2, k_2, k_3, c_2, c_3) := \overrightarrow{\left| \text{lsolve}\left(-\omega^2 \cdot \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & 0.1\ kg - m_2 \end{bmatrix} + \sqrt{-1} \cdot \omega \cdot \begin{bmatrix} c_1+c_2+c_3 & -c_2 & -c_3 \\ -c_2 & c_2 & 0 \\ -c_3 & 0 & c_3 \end{bmatrix} + \begin{bmatrix} k_1+k_2+k_3 & -k_2 & -k_3 \\ -k_2 & k_2 & 0 \\ -k_3 & 0 & k_3 \end{bmatrix}, \begin{bmatrix} f_1 \\ 0 \\ 0 \end{bmatrix} \right) \right|}$$

$$x_1(\omega, k_2, c_2) := X(\omega, k_2, c_2)_0$$

$$\omega_{undamped\_peak}(m_2, k_2, k_3, c_2, c_3) := \sqrt{\text{eigenvals}\left(\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & 0.1\ kg - m_2 \end{bmatrix}^{-1} \cdot \begin{bmatrix} k_1+k_2+k_3 & -k_2 & -k_3 \\ -k_2 & k_2 & 0 \\ -k_3 & 0 & k_3 \end{bmatrix} \right)}$$

$$p(m_2, k_2, k_3, c_2, c_3) := \left\| \begin{array}{l} \omega \leftarrow \omega_{undamped\_peak}(m_2, k_2, k_3, c_2, c_3)_0 \\ x_{temp1}(\omega) \leftarrow x_1(\omega, m_2, k_2, k_3, c_2, c_3) \\ \omega_{peak1} \leftarrow \textbf{maximize}(x_{temp1}, \omega) \\ \omega \leftarrow \omega_{undamped\_peak}(m_2, k_2, k_3, c_2, c_3)_1 \\ x_{temp2}(\omega) \leftarrow x_1(\omega, m_2, k_2, k_3, c_2, c_3) \\ \omega_{peak2} \leftarrow \textbf{maximize}(x_{temp2}, \omega) \\ \omega \leftarrow \omega_{undamped\_peak}(m_2, k_2, k_3, c_2, c_3)_2 \\ x_{temp3}(\omega) \leftarrow x_1(\omega, m_2, k_2, k_3, c_2, c_3) \\ \omega_{peak3} \leftarrow \textbf{maximize}(x_{temp3}, \omega) \\ x_{peak} \leftarrow \max(x_{temp1}(\omega_{peak1}), x_{temp2}(\omega_{peak2}), x_{temp3}(\omega_{peak3})) \\ \text{return } x_{peak} \end{array} \right.$$

Guess Values

$m_2 := 0.05\ kg$ $\qquad k_2 := 0.1\ \frac{N}{m}$ $\qquad k_3 := 0.1\ \frac{N}{m}$ $\qquad c_2 := 0.001\ N \cdot \frac{s}{m}$ $\qquad c_3 := 0.001\ N \cdot \frac{s}{m}$

Constraints

$m_2 > 0\ kg$ $\qquad k_2 > 0\ \frac{N}{m}$ $\qquad k_3 > 0\ \frac{N}{m}$ $\qquad c_2 > 0\ \frac{N \cdot s}{m}$ $\qquad c_3 > 0\ \frac{N \cdot s}{m}$

Solver

$\textbf{minimize}(p, m_2, k_2, k_3, c_2, c_3) = ?$