

Making Classical (Threshold) Signatures Post-Quantum for Single Use on a Public Ledger

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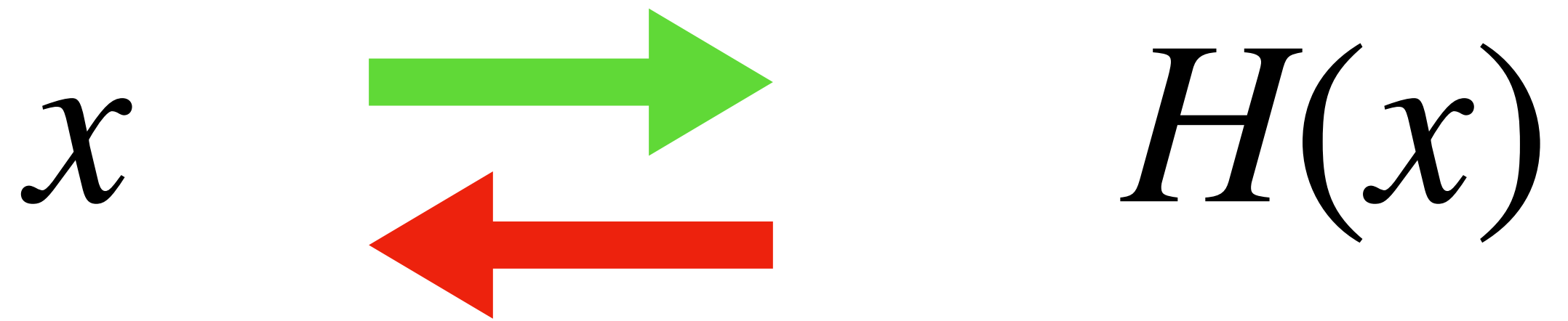
Background

Discrete Log Problem (DLP)

$$x \begin{array}{c} \xrightarrow{\text{green}} \\ \xleftarrow{\text{red}} \end{array} g^x \bmod p$$

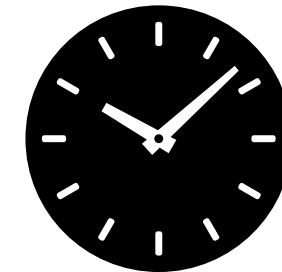
(p is a prime with some structure)

One-way Hash Functions




Commitment Schemes

$\text{com} \leftarrow \text{Commit}(x, \text{open})$



x, open



$\text{com} \stackrel{?}{=} \text{Commit}(x, \text{open})$

Making Classical (Threshold) Signatures Post-Quantum for Single Use on a Public Ledger

Making **Classical** (Threshold) **Signatures** Post-Quantum for Single Use on a Public Ledger

Digital Signatures

A digital signature scheme is a tuple of algorithms (KeyGen, Sign, Verify) with key space \mathcal{D} and message space \mathcal{M} such that:

$\text{KeyGen}(1^\lambda) \rightarrow (\text{sk}, \text{pk})$

$\text{Sign}(\text{sk}, \text{msg}) \rightarrow \sigma$

$\text{Verify}(\sigma, \text{pk}, \text{msg}) \rightarrow 0/1$

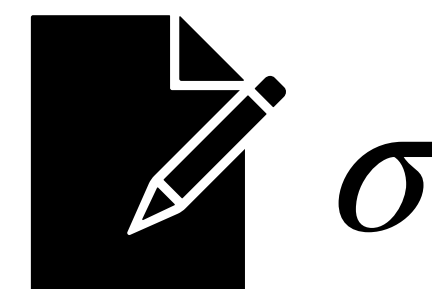
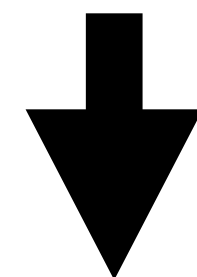
pk

$\text{Verify}(\sigma, \text{pk}, \text{msg})$

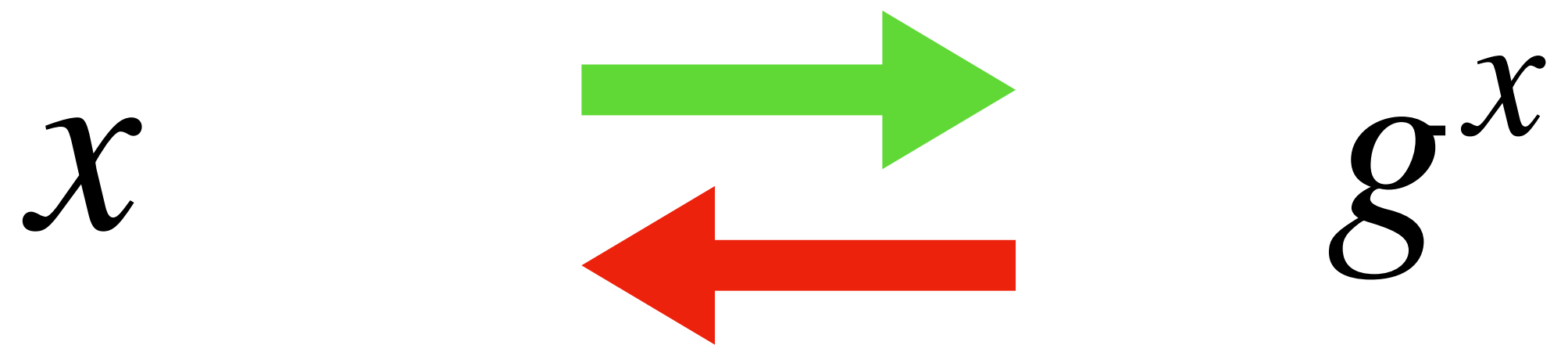
sk



$\text{Sign}(\text{sk}, \text{msg})$

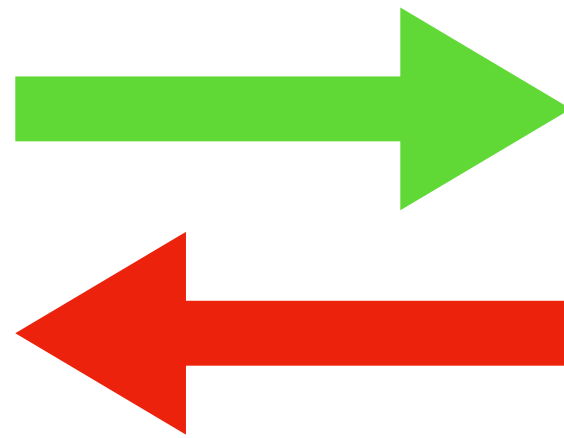


Discrete Log Problem (DLP)



KeyGen

x



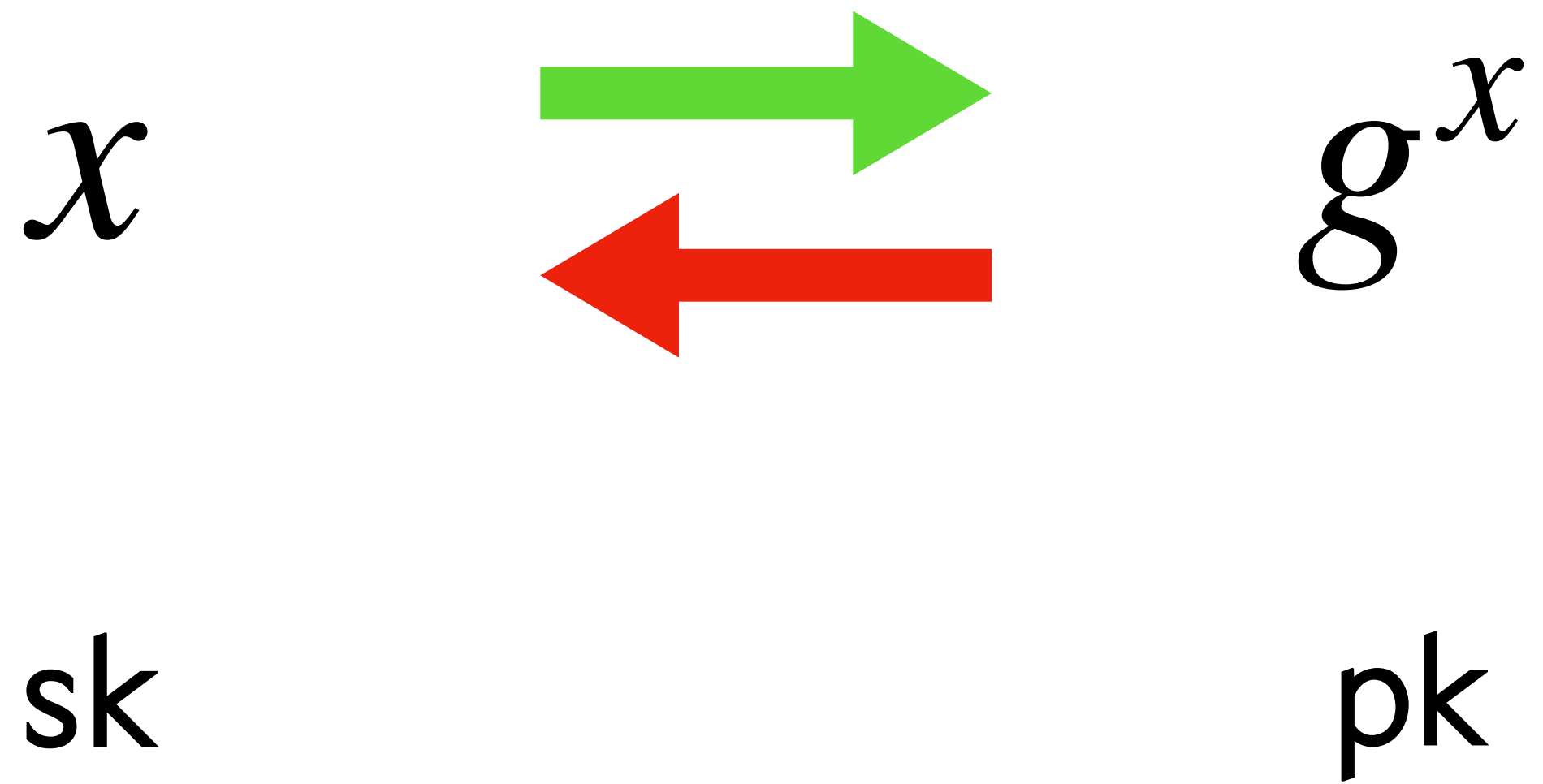
g^x

sk

pk

Making **Classical** (**Threshold**) **Signatures** Post-Quantum for Single Use on a Public Ledger

Threshold Keys



Threshold Keys

$$sk \quad x = x_1 + x_2 + x_3$$

$$sk_1 \quad x_1 \longrightarrow g^{x_1}$$

$$sk_2 \quad x_2 \longrightarrow g^{x_2}$$

$$sk_3 \quad x_3 \longrightarrow g^{x_3}$$

Threshold Keys

$$\text{sk} \quad x = x_1 + x_2 + x_3$$

$$\text{sk}_1 \quad x_1 \longrightarrow g^{x_1}$$

$$\text{sk}_2 \quad x_2 \longrightarrow g^{x_2}$$

$$\text{sk}_3 \quad x_3 \longrightarrow g^{x_3}$$

$$g^x$$

Threshold Keys

$$\text{sk} \quad x = x_1 + x_2 + x_3$$

$$\text{sk}_1 \quad x_1 \longrightarrow g^{x_1}$$

$$\text{sk}_2 \quad x_2 \longrightarrow g^{x_2}$$

$$\text{sk}_3 \quad x_3 \longrightarrow g^{x_3}$$

$$g^x$$

$$g^{x_1+x_2+x_3} = g^{x_1} \cdot g^{x_2} \cdot g^{x_3}$$

Threshold Keys

$$\text{sk} \quad x = x_1 + x_2 + x_3$$

$$\text{sk}_1 \quad x_1 \longrightarrow g^{x_1}$$

$$\text{sk}_2 \quad x_2 \longrightarrow g^{x_2}$$

$$\text{sk}_3 \quad x_3 \longrightarrow g^{x_3}$$

$$g^x$$

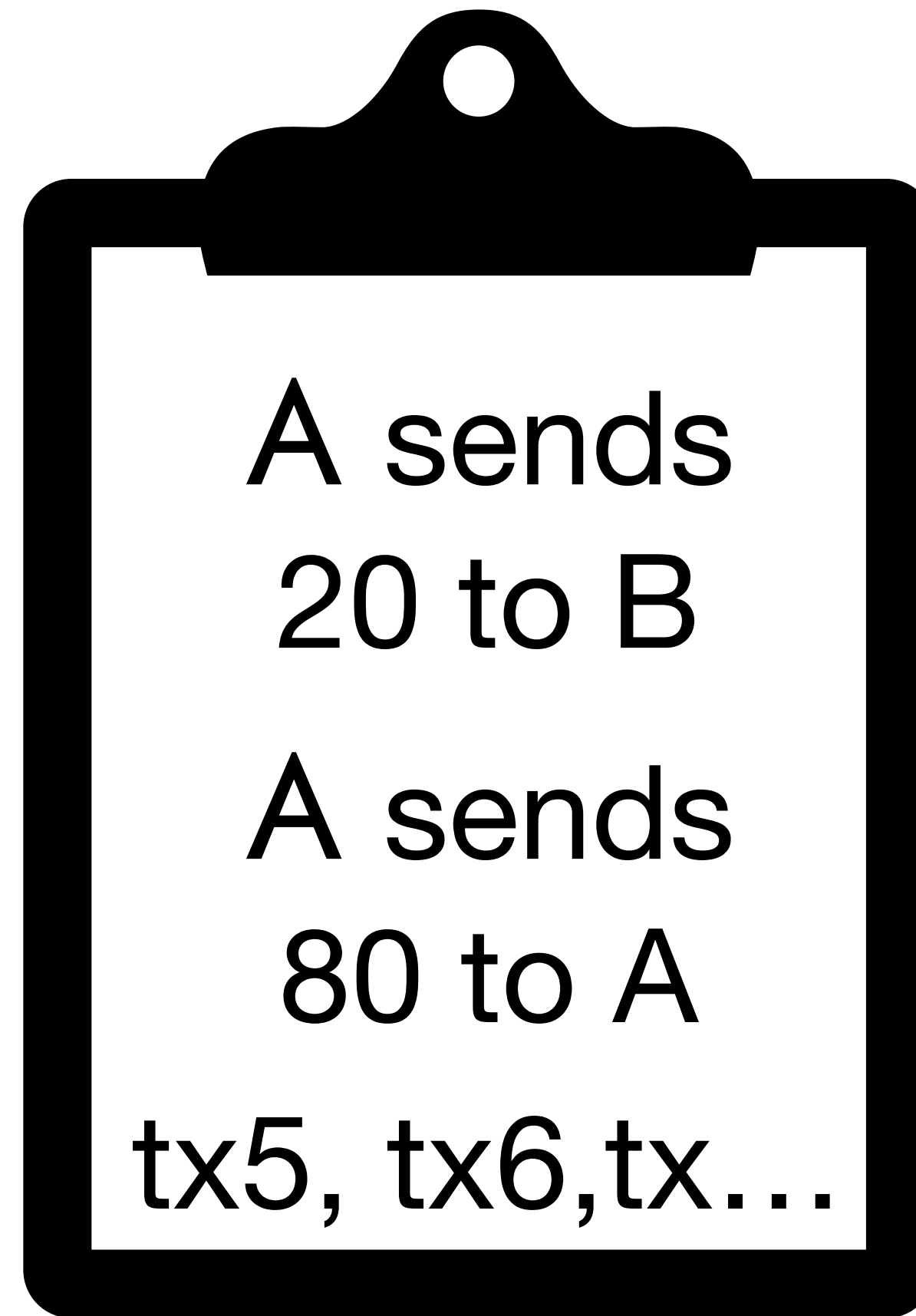
$$g^{x_1+x_2+x_3} = g^{x_1} \cdot g^{x_2} \cdot g^{x_3}$$

$$\text{pk}$$

Making **Classical** (**Threshold**) **Signatures** Post-Quantum for Single Use on a **Public Ledger**

Public Ledger

A
100

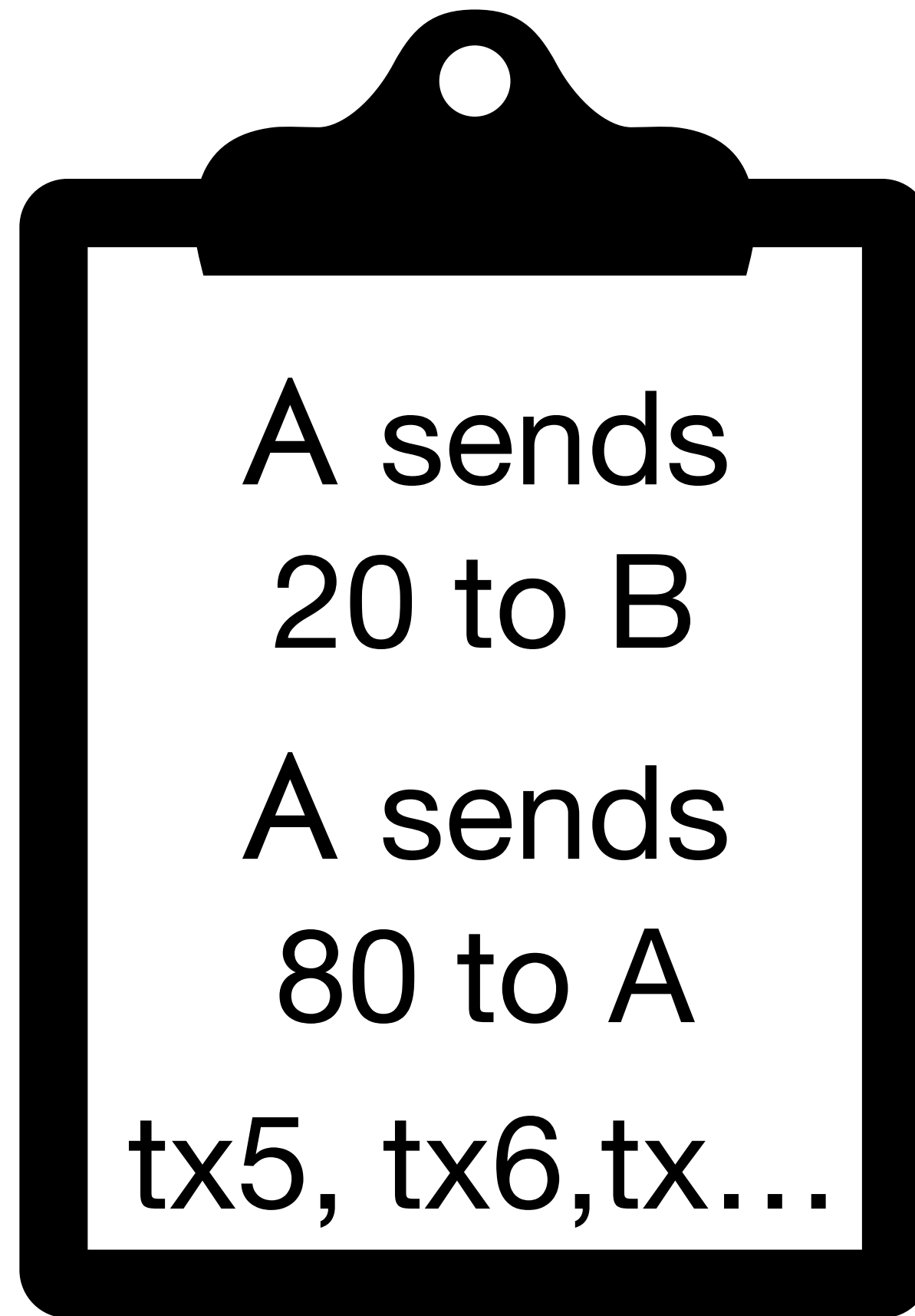


B

Public Ledger



A
100

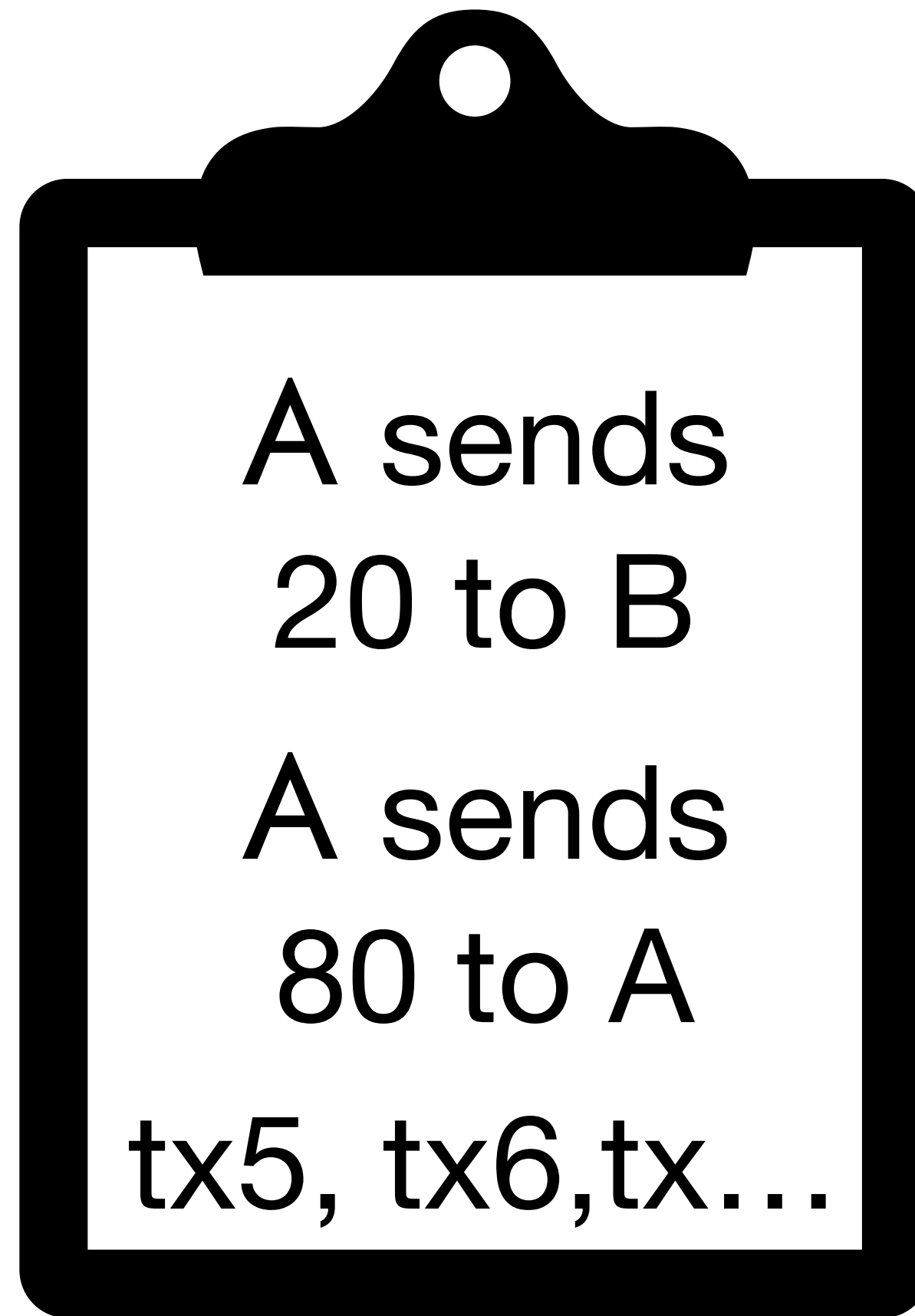


B

Public Ledger



A
100

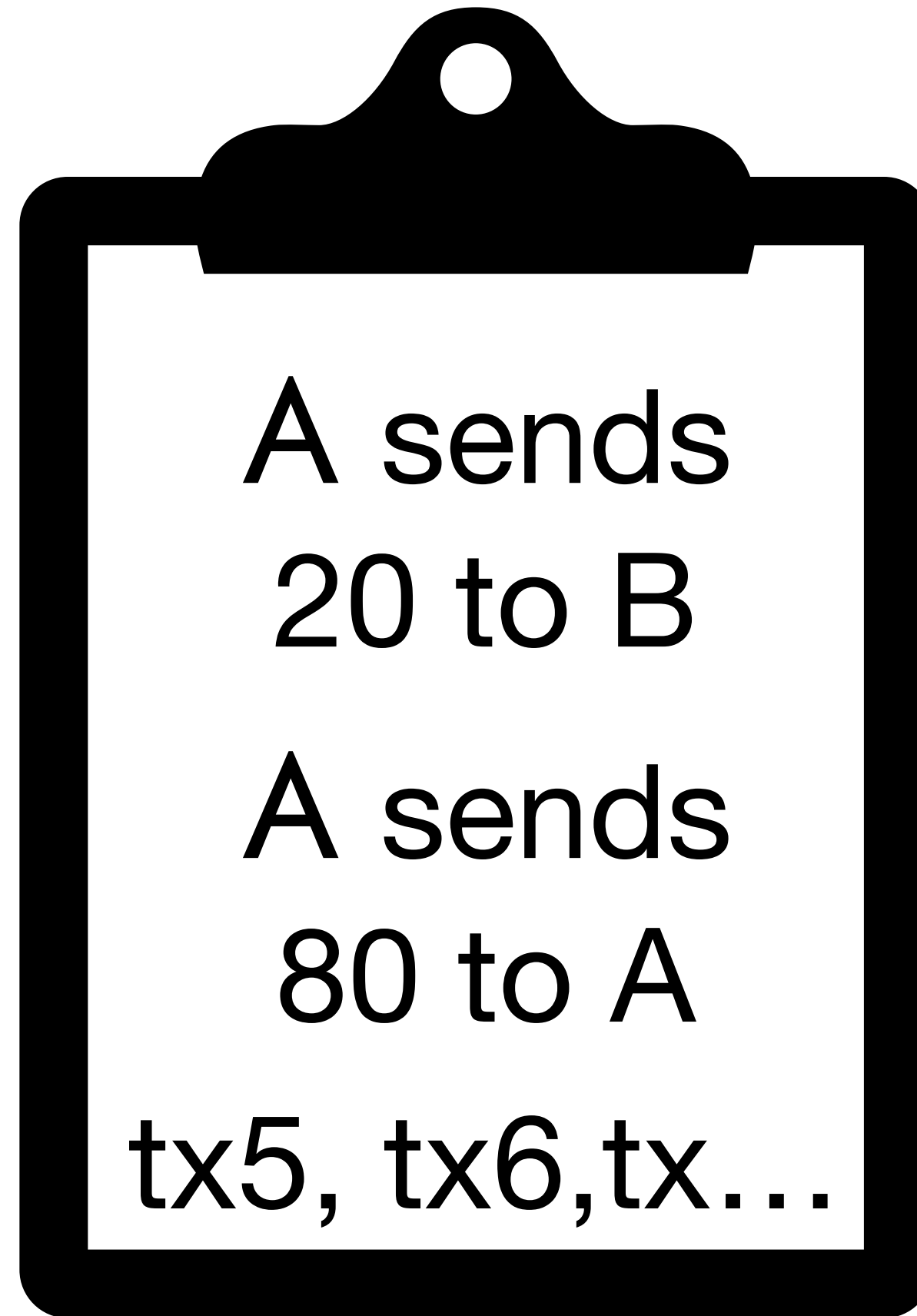


B

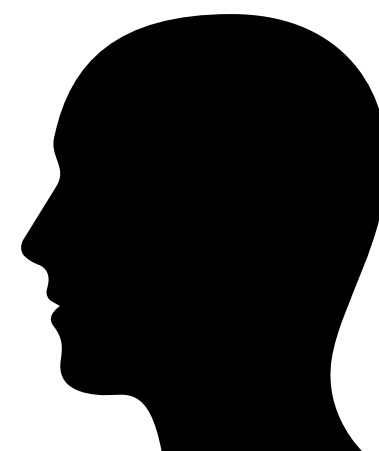
Public Ledger



A
100



B

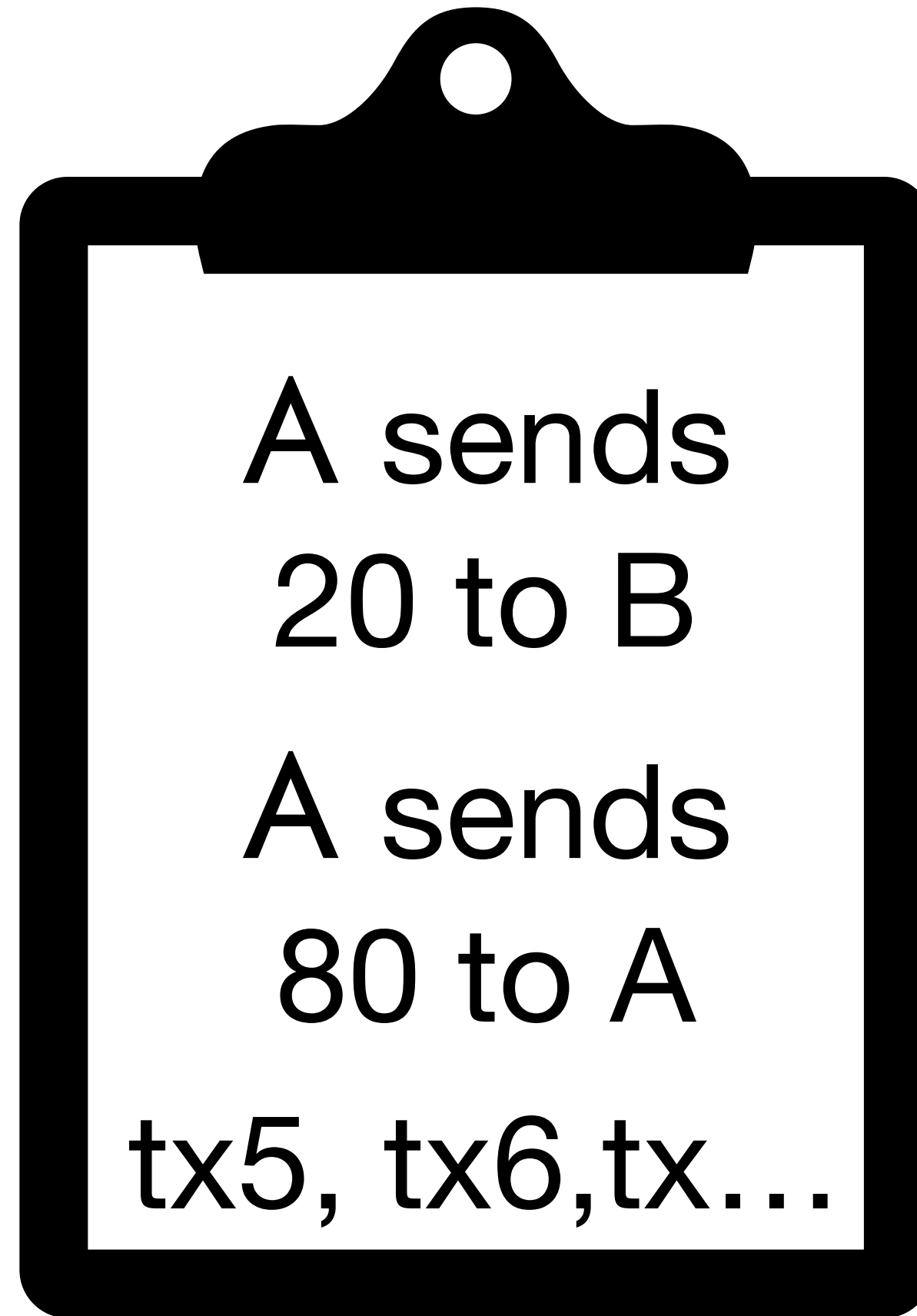


Public Ledger



A, sk_A , pk_A
100

$\sigma \leftarrow \text{Sign}(sk, tx)$



B, pk_B



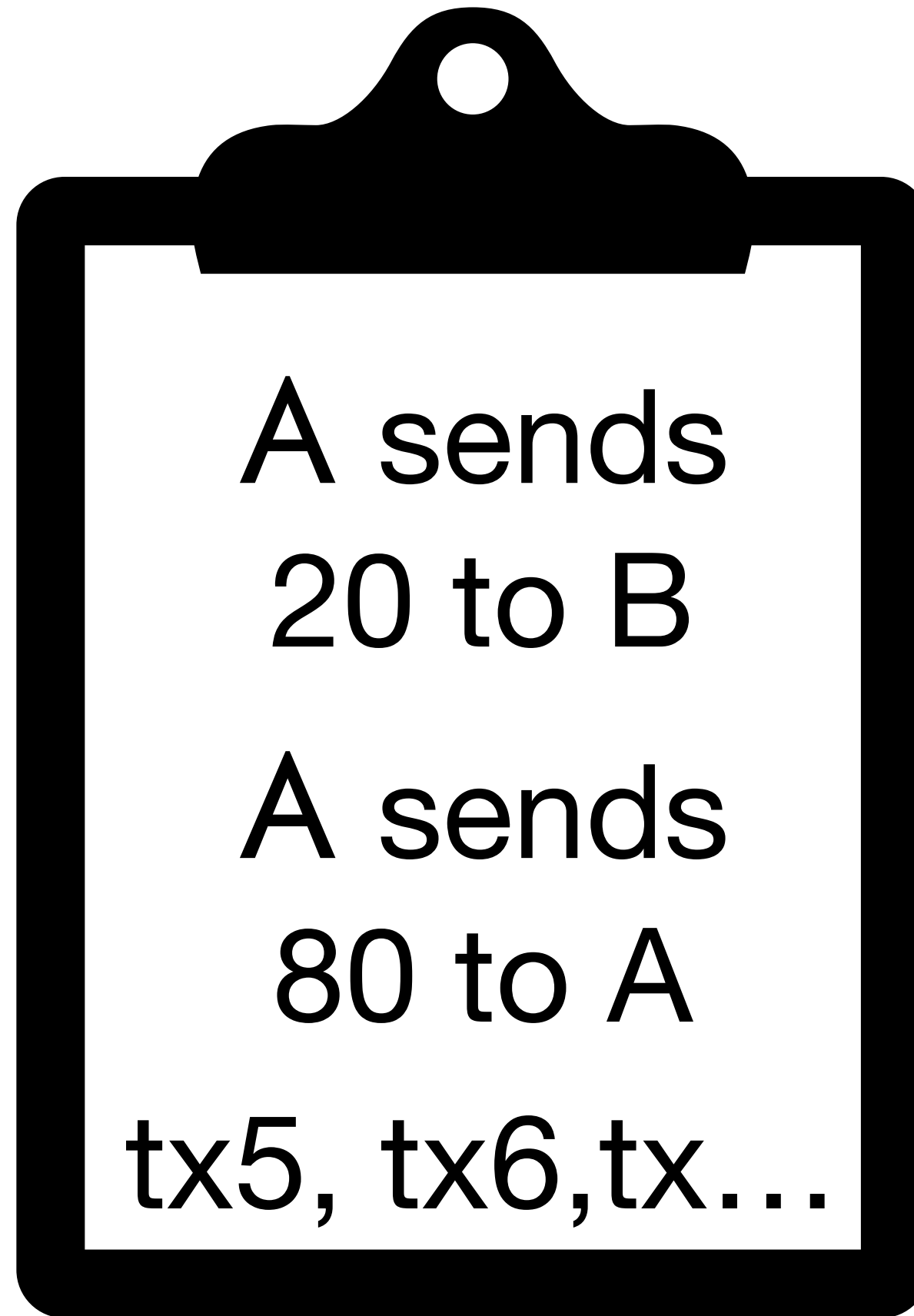
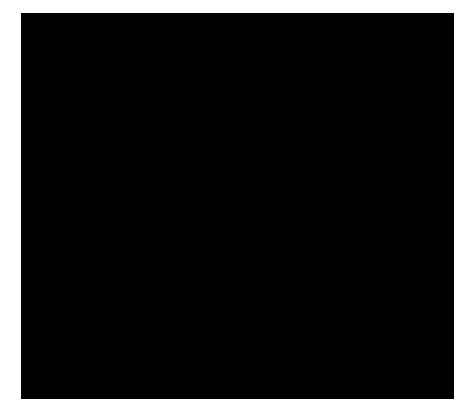
$\text{Verify}(\sigma, pk, tx)$

Public Ledger

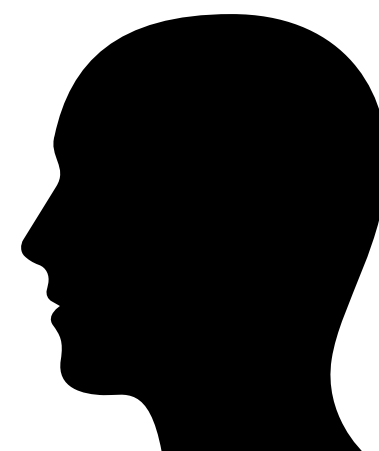


A, sk_A, pk_A
100

$\sigma \leftarrow \text{Sign}(sk, tx)$



B, pk_B



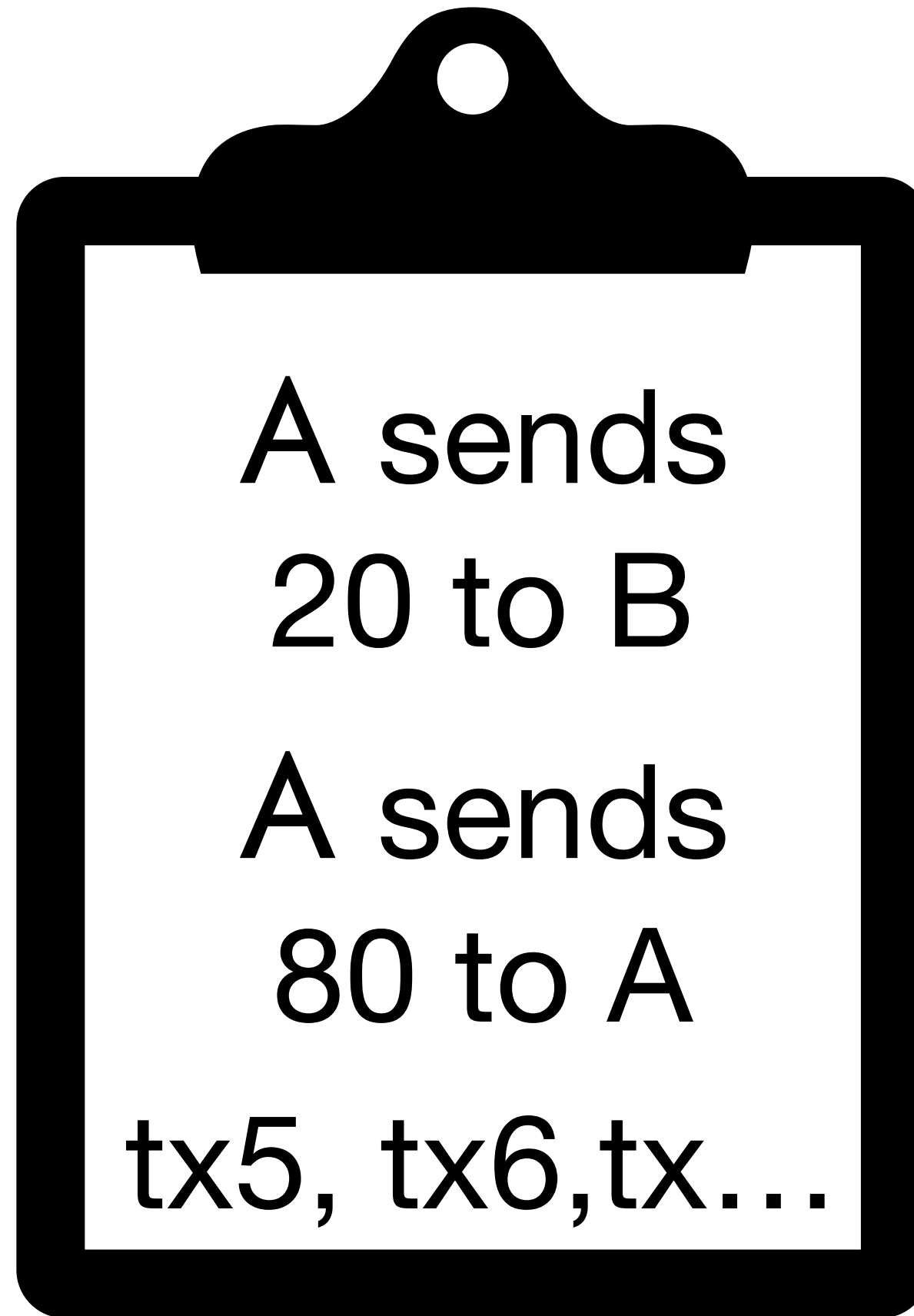
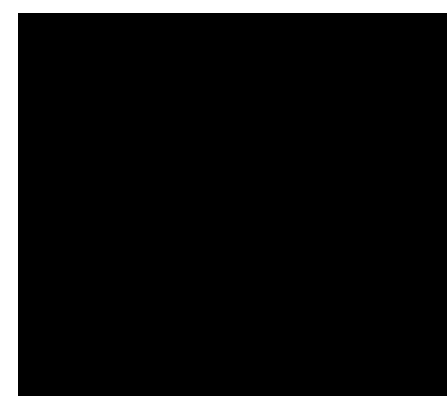
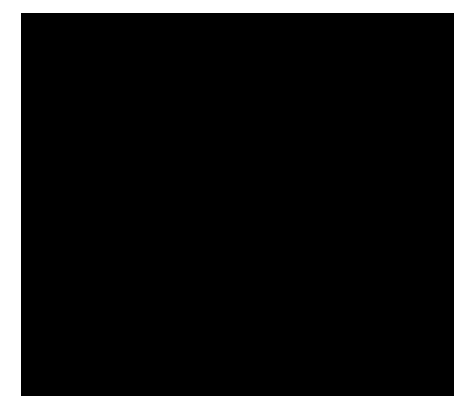
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Public Ledger



A, sk_A, pk_A
100

$\sigma \leftarrow \text{Sign}(sk, tx)$



B, pk_B



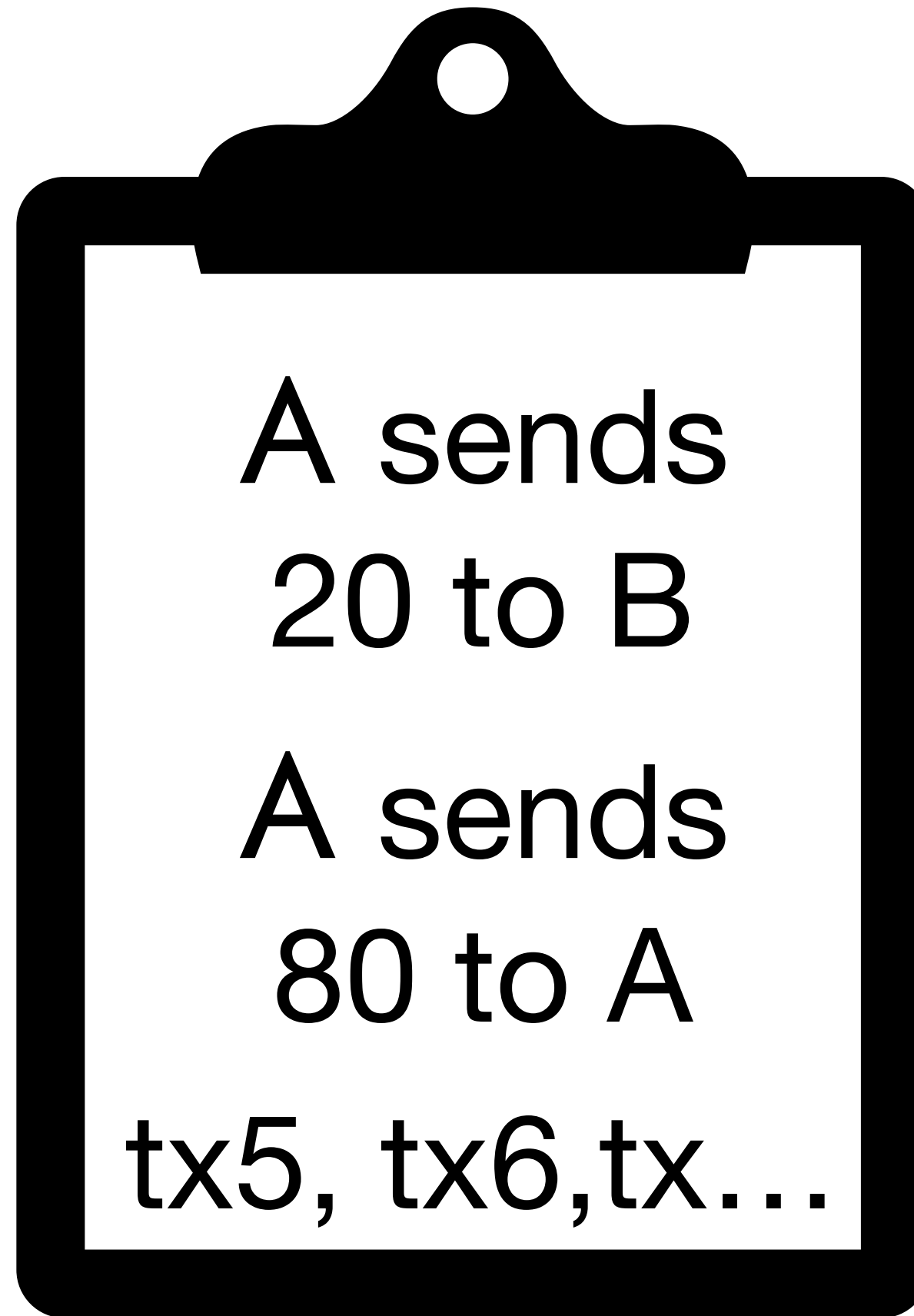
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Public Ledger

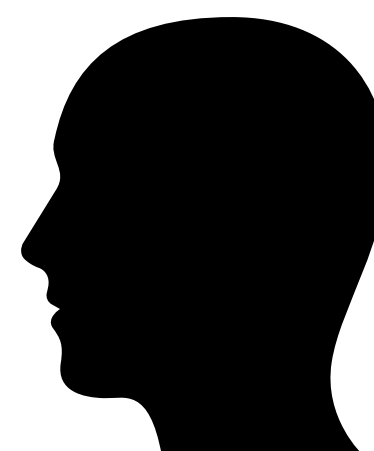
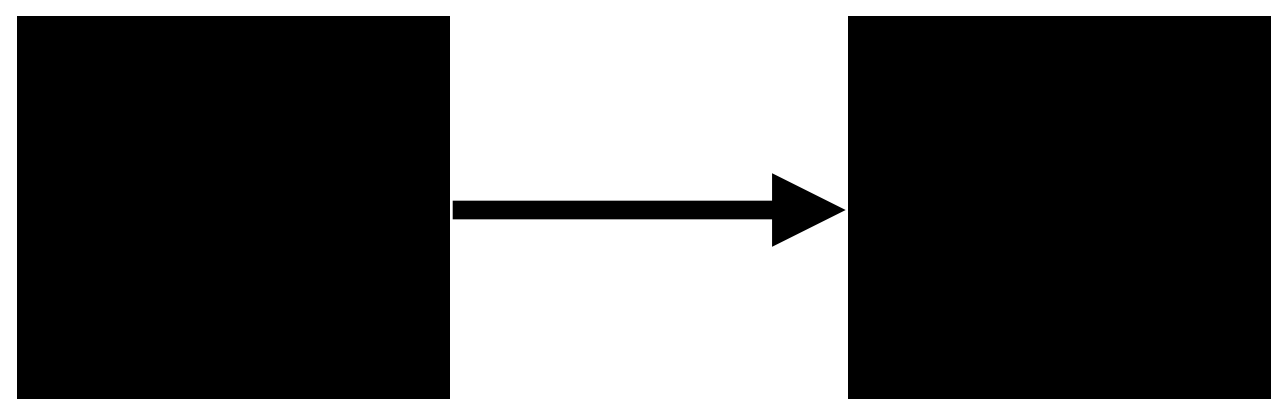


A, sk_A, pk_A
100

$\sigma \leftarrow \text{Sign}(sk, tx)$



B, pk_B



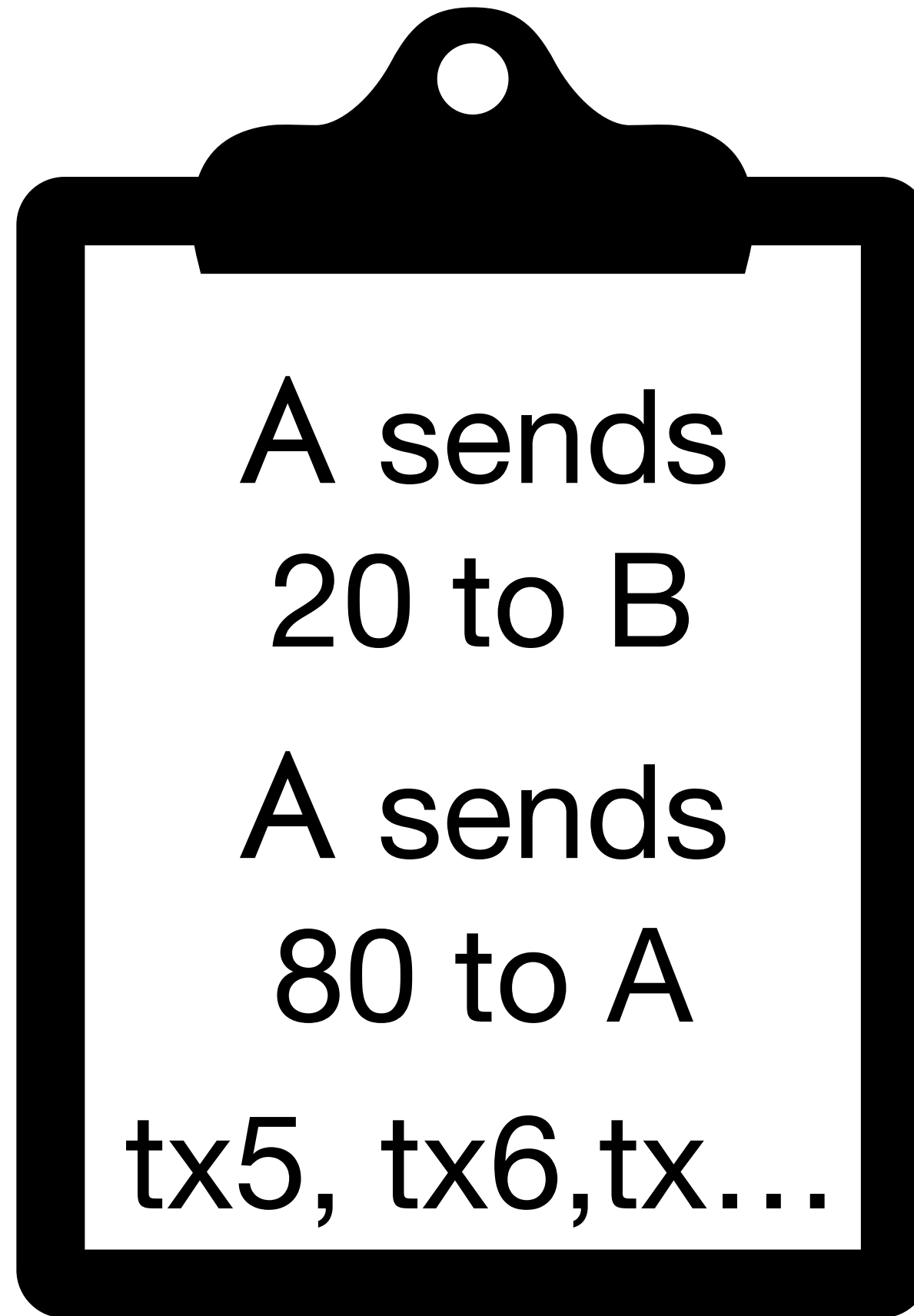
$\text{Verify}(\sigma, pk, tx)$

Public Ledger

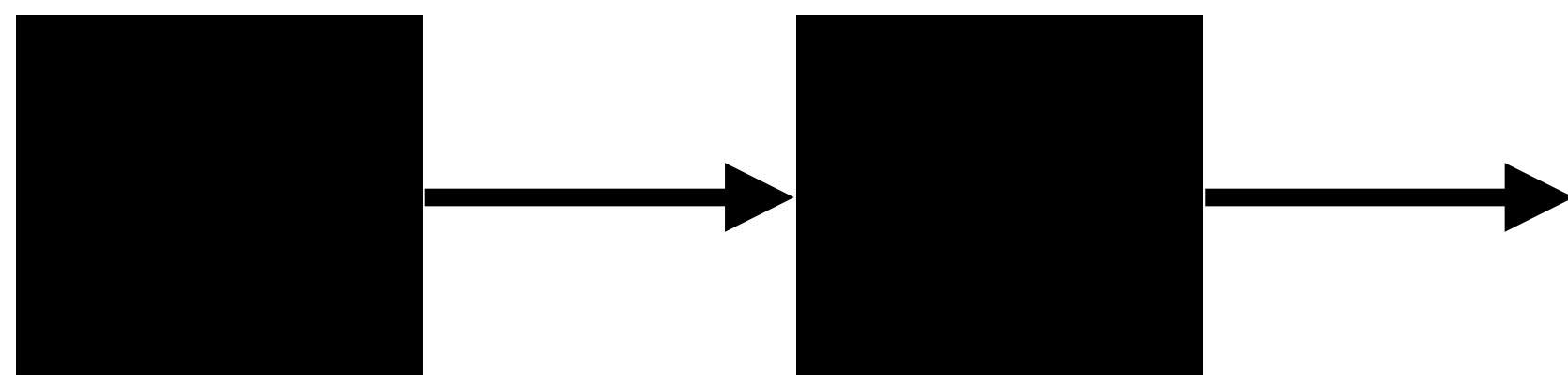


A, sk_A, pk_A
100

$\sigma \leftarrow \text{Sign}(sk, tx)$



B, pk_B



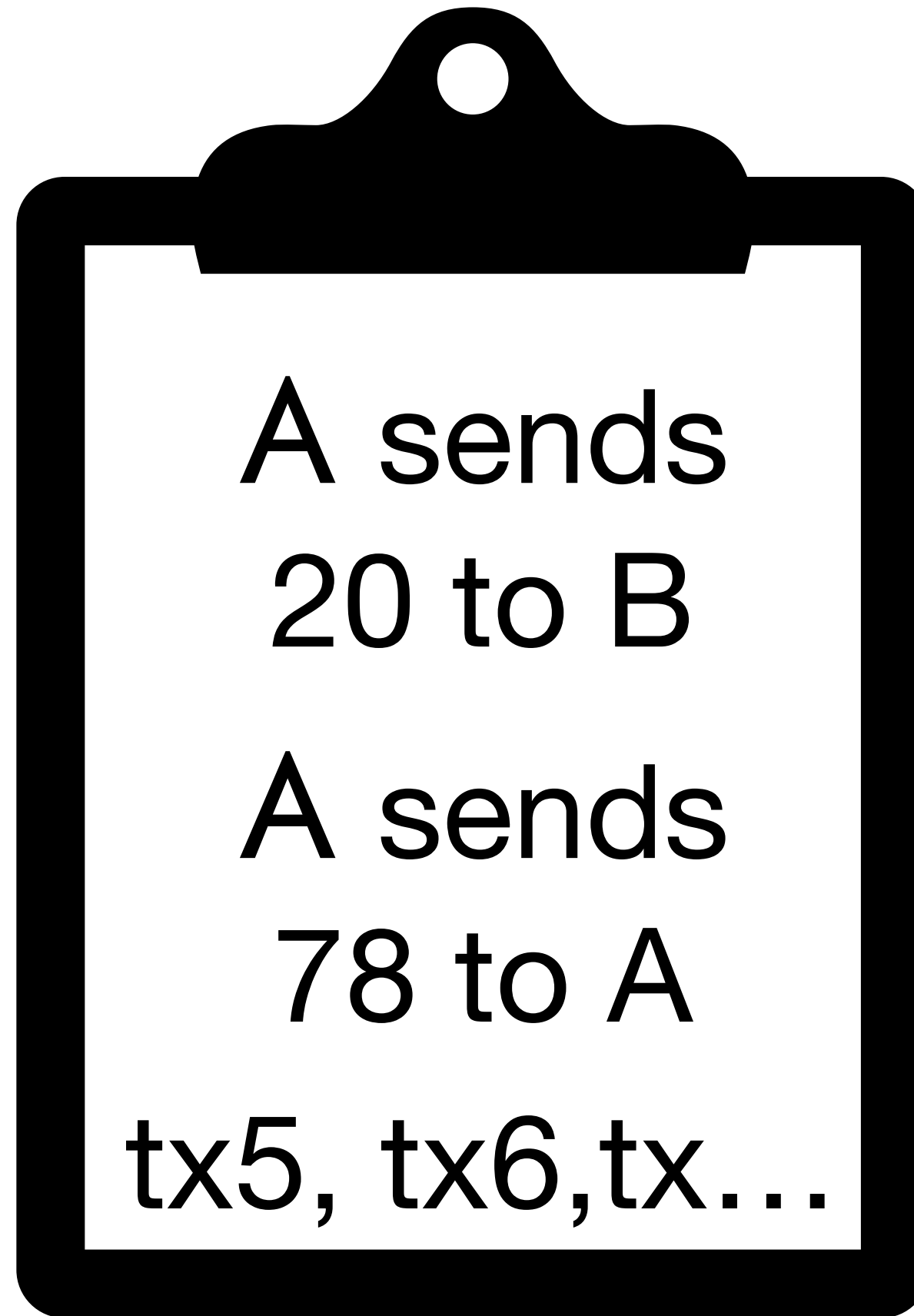
$\text{Verify}(\sigma, pk, tx)$

Public Ledger

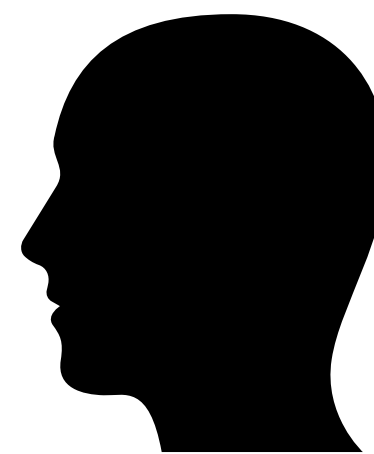
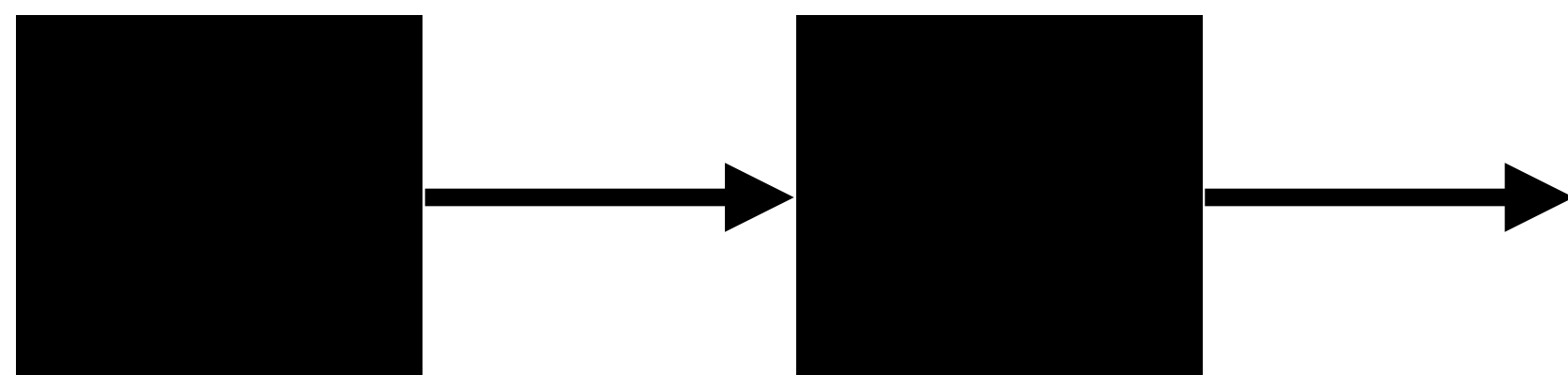


A, sk_A , pk_A
100\$

$\sigma \leftarrow \text{Sign}(sk, tx)$



B, pk_B

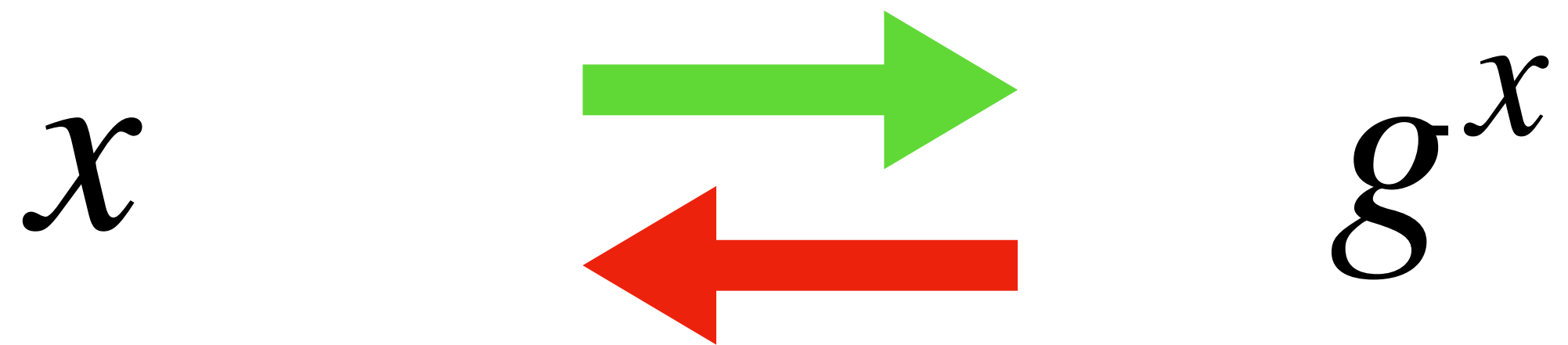


2\$

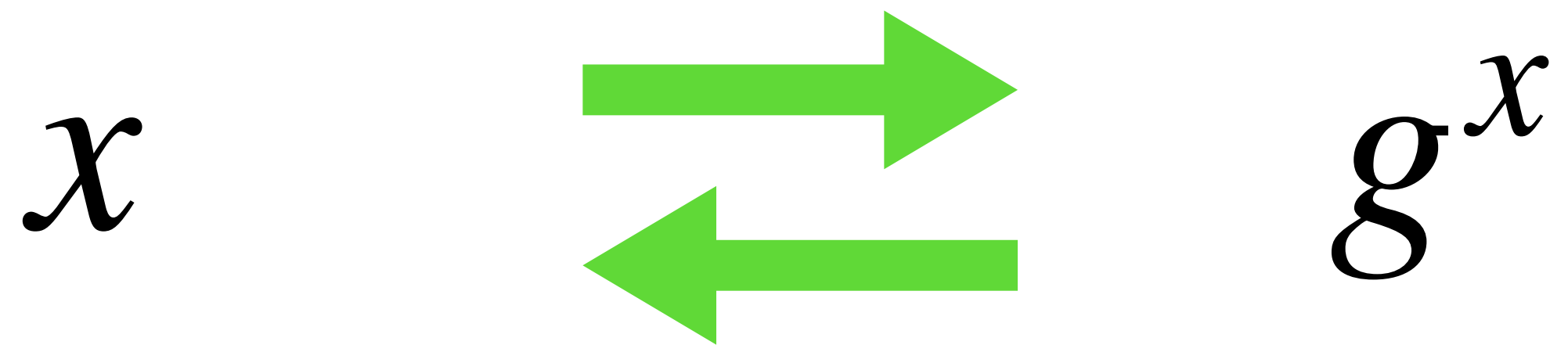
$\text{Verify}(\sigma, pk, tx)$

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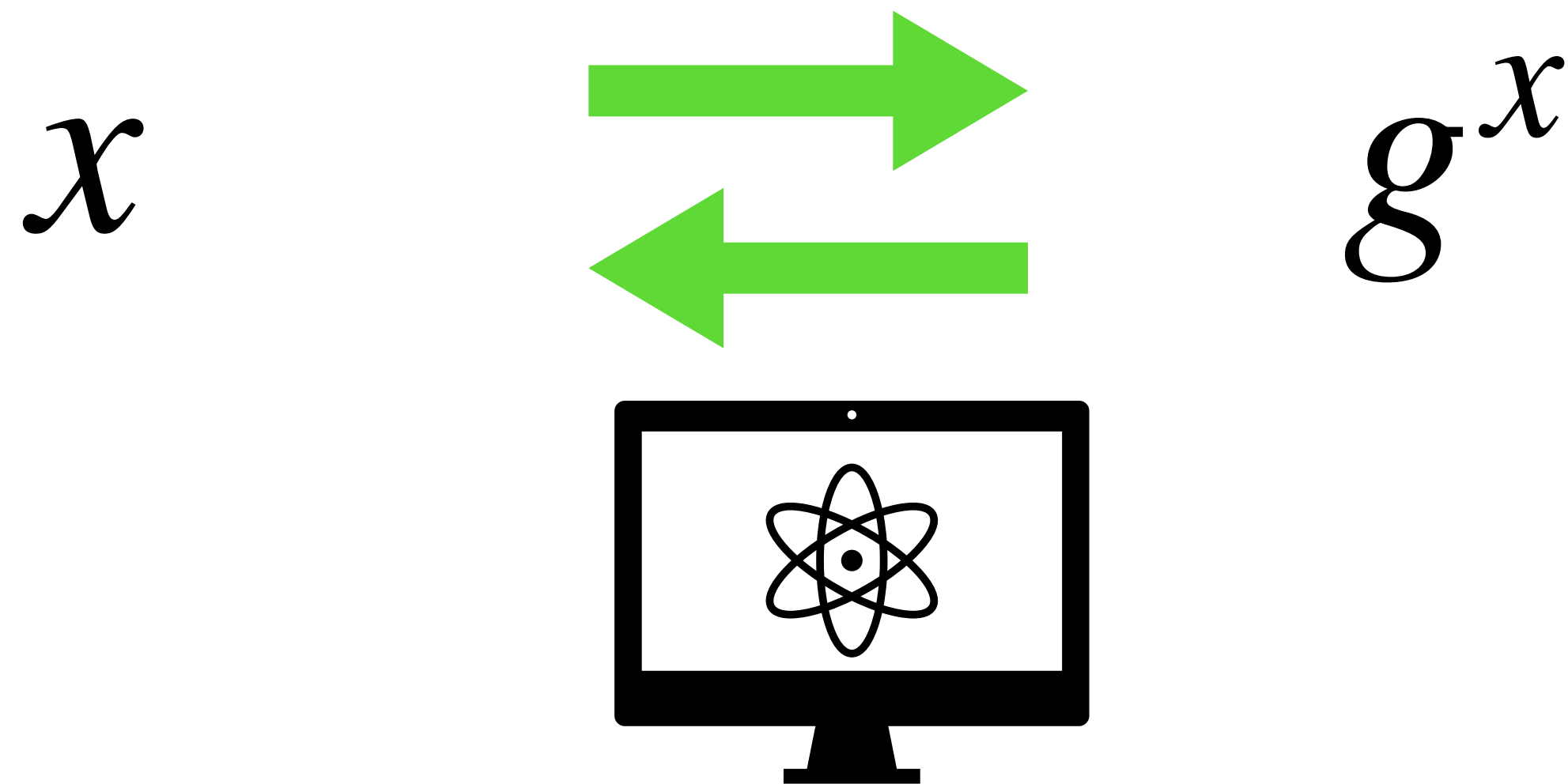
Discrete Log Problem (DLP)



Discrete Log Problem (DLP)



Discrete Log Problem (DLP)



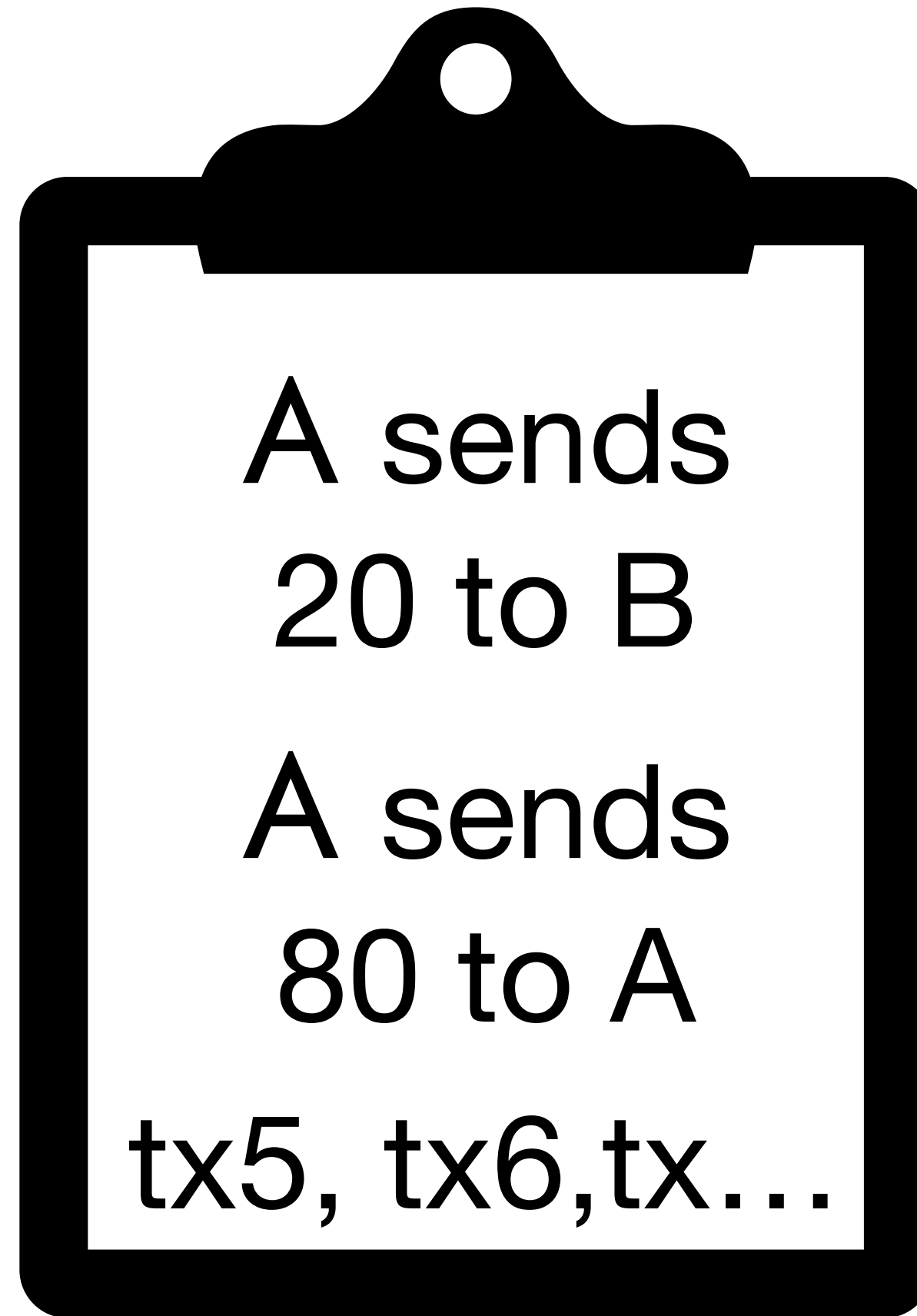
Quantum Computers

Public Ledger



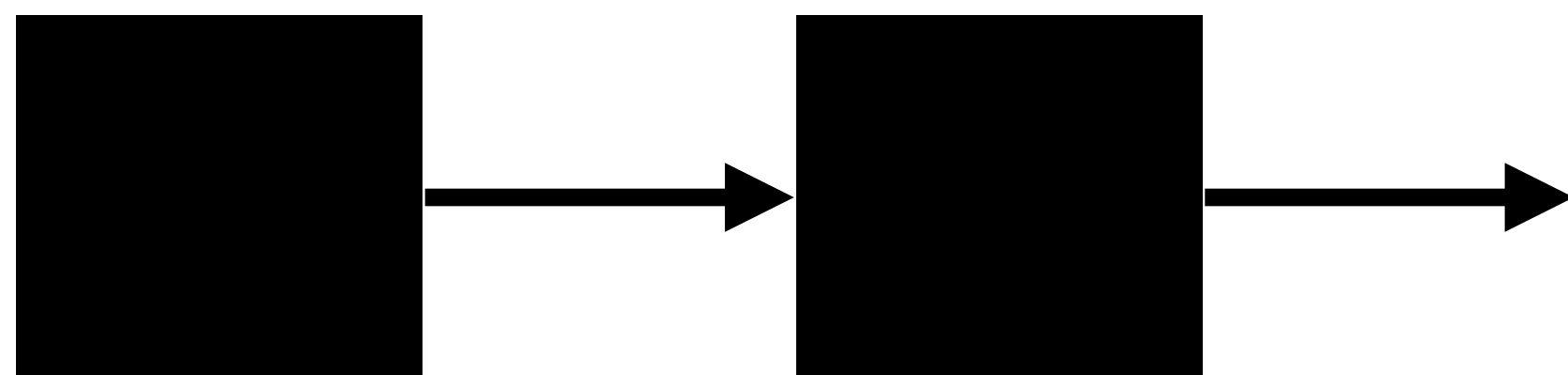
A, sk_A, pk_A
100

$\sigma \leftarrow \text{Sign}(sk, tx)$

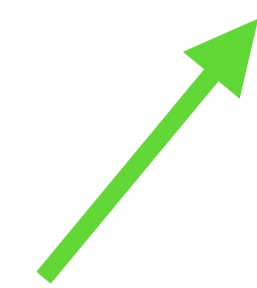


B, pk_B

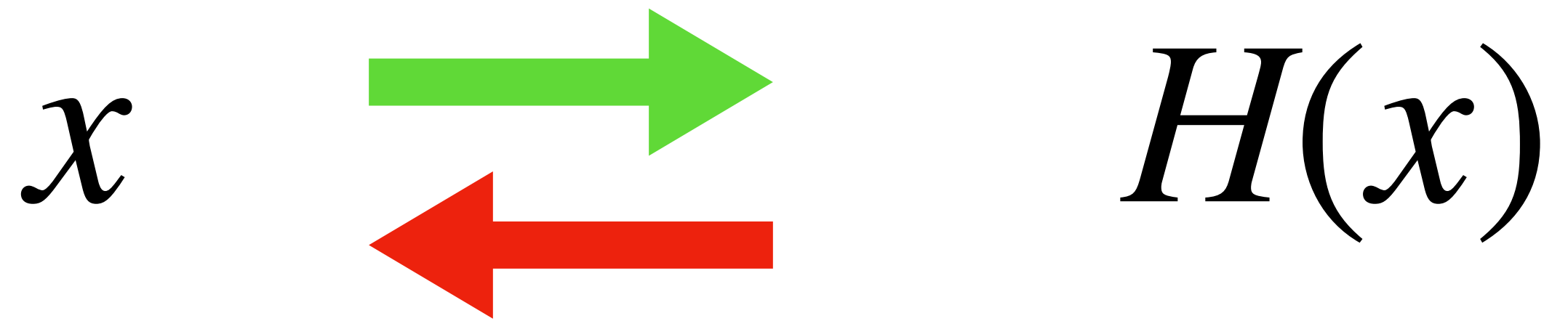
sk



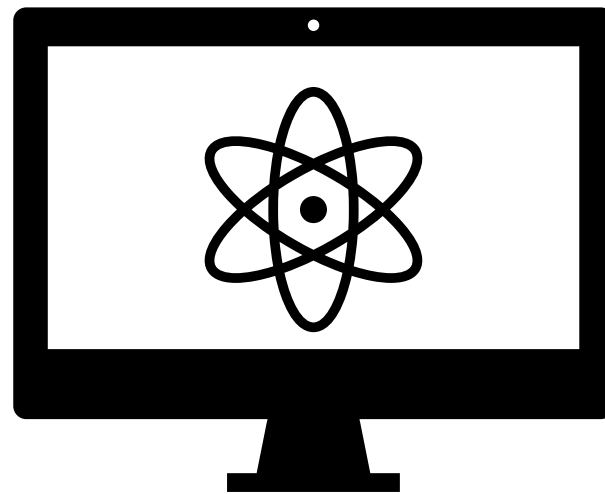
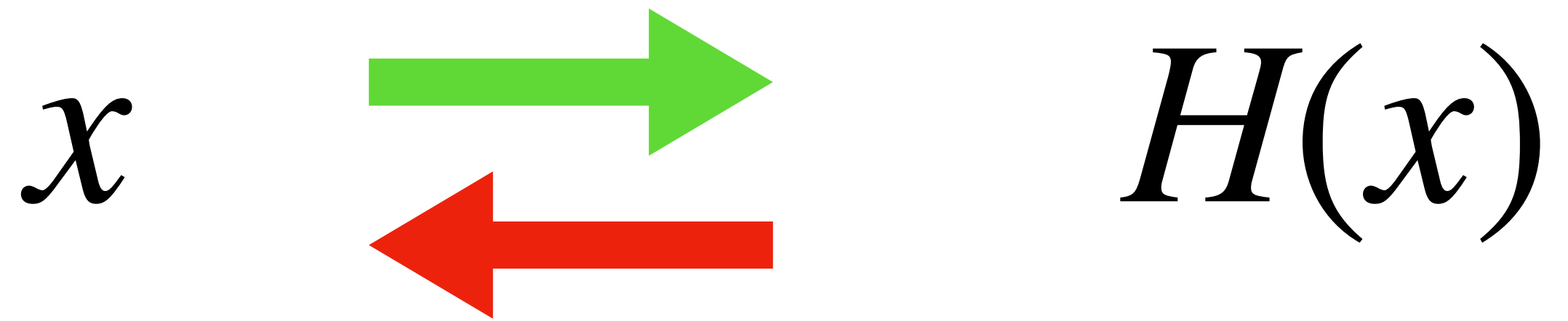
$\text{Verify}(\sigma, pk, tx)$



One-way Hash Functions



One-way Hash Functions

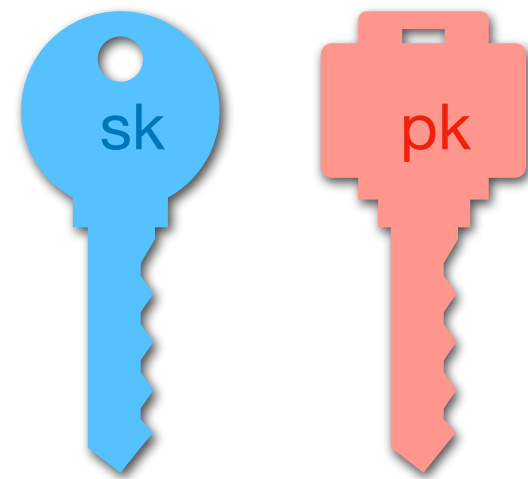


Quantum Computers

Making Classical (Threshold) Signatures Post-Quantum for Single Use on a Public Ledger

Motivation

Motivation



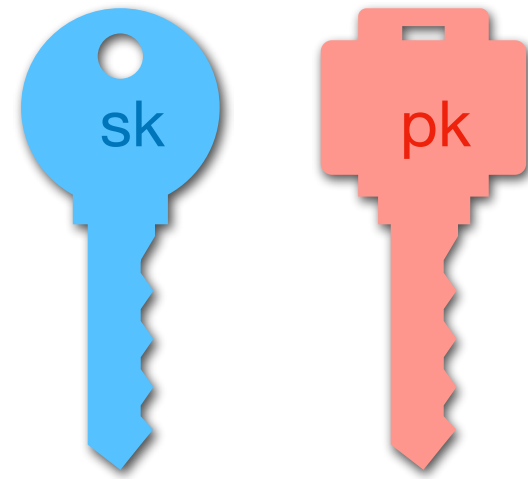
Public Key Cryptography

Motivation

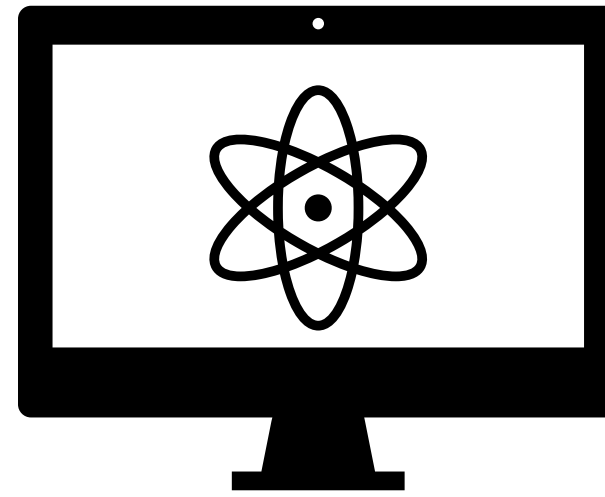


Public Key Cryptography

Motivation



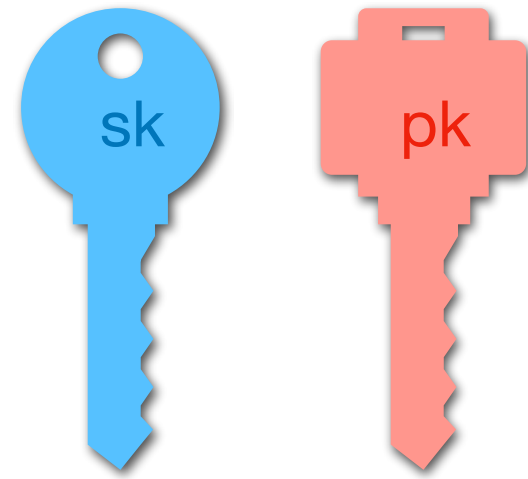
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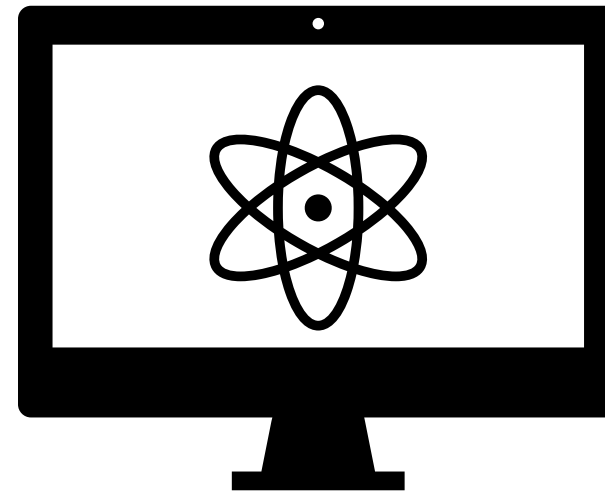
Public Key Cryptography

Quantum Computers

Motivation



+

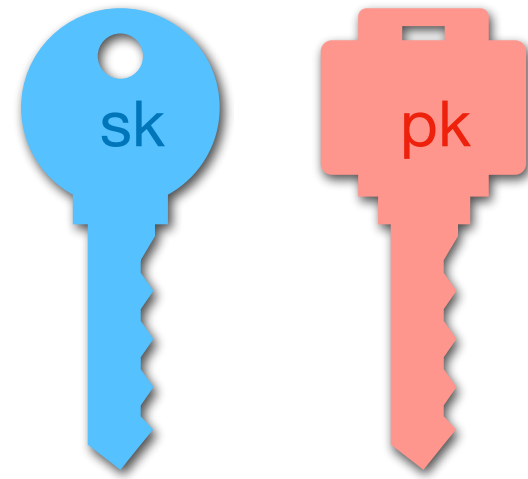


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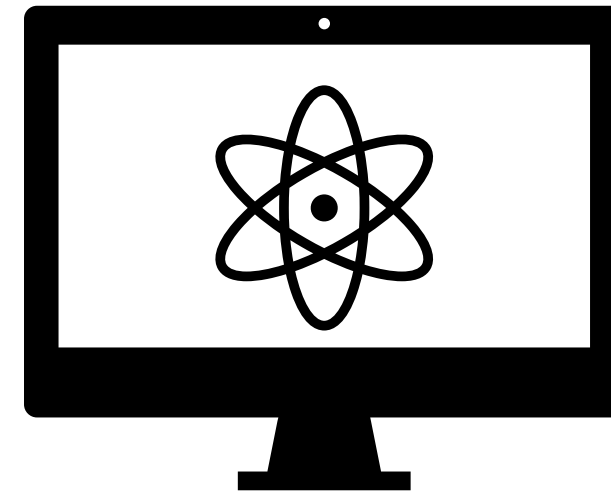
Public Key Cryptography

Quantum Computers

Motivation



+



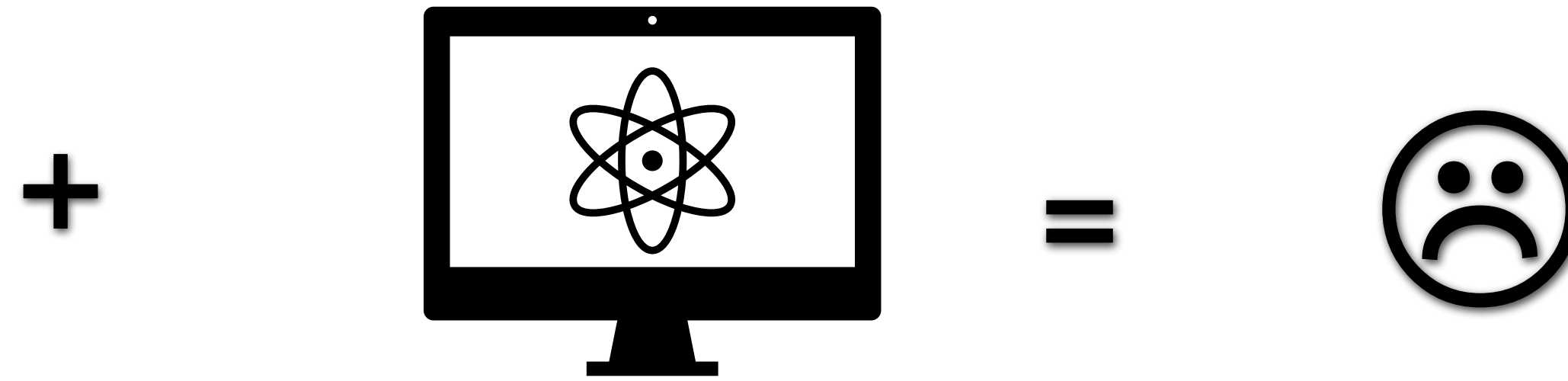
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Public Key Cryptography

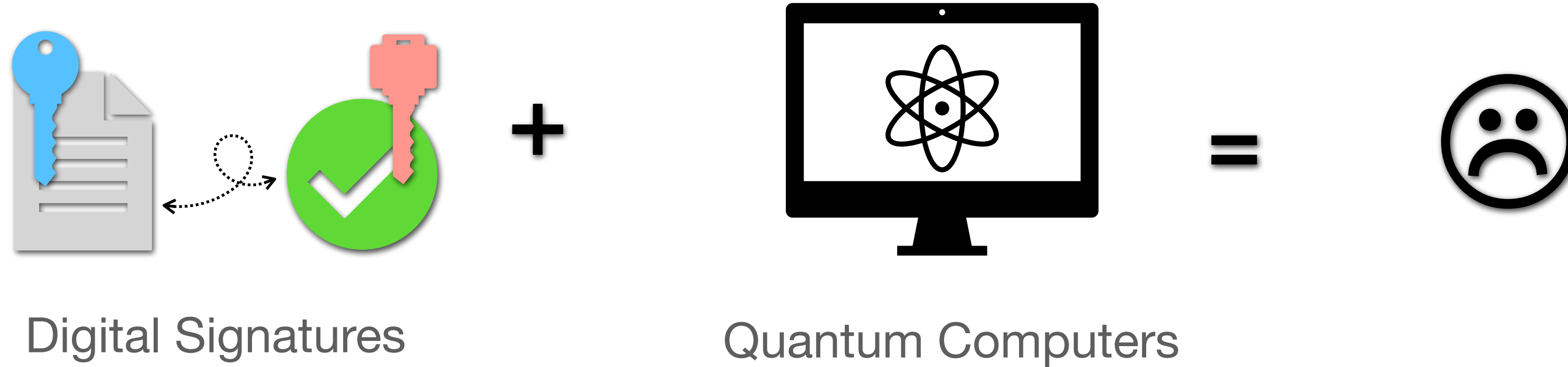
Quantum Computers

Motivation

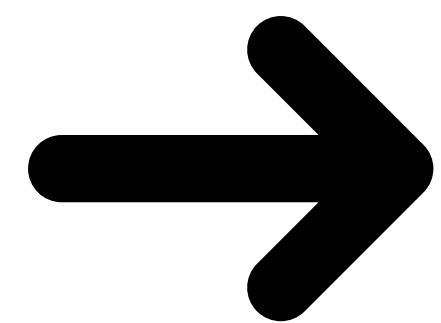
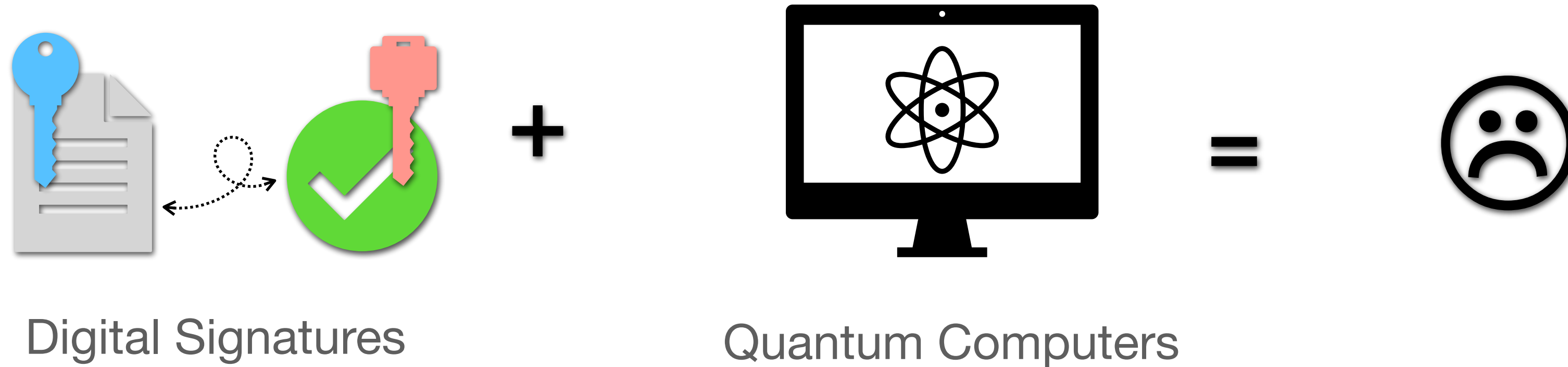


Quantum Computers

Motivation



Motivation



- Post-Quantum Signatures (NIST) : **secure**, **not yet widely deployed**
- Hiding the public key : **one time secure**, **ready to use with classical algorithms**

P2PKH

P2PKH hides the public key behind a **hash** → Generalised HashedPK transform

HashedPK-KeyGen(λ)	HashedPK-Sign(\tilde{sk} , msg)	HashedPK-Verify($\tilde{\sigma}$, $pk_{\mathcal{H}}$, msg)
1 : $(pk, sk) \leftarrow_{\$} \text{KeyGen}_{\Sigma}(\lambda)$	1 : $(sk, pk) \leftarrow \tilde{sk}$	1 : $(\sigma, pk) \leftarrow \tilde{\sigma}$
2 : $pk_{\mathcal{H}} \leftarrow H(pk)$	2 : $\sigma \leftarrow \text{Sign}_{\Sigma}(sk, msg)$	2 : if $pk_{\mathcal{H}} = H(pk)$:
3 : $\tilde{sk} \leftarrow (sk, pk)$	3 : $\tilde{\sigma} \leftarrow (\sigma, pk)$	return $\text{Verify}_{\Sigma}(\sigma, pk, msg)$
4 : return $(\tilde{sk}, pk_{\mathcal{H}})$	4 : return $\tilde{\sigma}$	3 : return 0

Hiding the public key

$\Sigma = (\text{KeyGen}_\Sigma, \text{Sign}_\Sigma, \text{Verify}_\Sigma)$

with additive public key
domain

+ hash function H

Generic Transform
→

Signature scheme
with hidden public
key

Hiding the public key

$\Sigma = (\text{KeyGen}_\Sigma, \text{Sign}_\Sigma, \text{Verify}_\Sigma)$

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Generic Transform
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Signature scheme
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HiddenPK-KeyGen(λ)

1 : $(\text{sk}, \text{pk}) \leftarrow \$ \text{KeyGen}_\Sigma(\lambda)$

2 : $\rho \leftarrow \$ \mathcal{D}$

3 : $\tilde{\text{sk}} \leftarrow (\text{sk}, \rho)$

4 : $\tilde{\text{pk}} \leftarrow (H(\rho), \text{pk} + \rho)$

5 : **return** $(\tilde{\text{sk}}, \tilde{\text{pk}})$

Hiding the public key

$\Sigma = (\text{KeyGen}_\Sigma, \text{Sign}_\Sigma, \text{Verify}_\Sigma)$
with additive public key
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Generic Transform
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HiddenPK-KeyGen(λ)

```
1 : (sk, pk)  $\leftarrow$   $\text{KeyGen}_\Sigma(\lambda)$ 
2 :  $\rho \leftarrow \mathcal{D}$ 
3 :  $\tilde{\text{sk}} \leftarrow (\text{sk}, \rho)$ 
4 :  $\tilde{\text{pk}} \leftarrow (H(\rho), \text{pk} + \rho)$ 
5 : return ( $\tilde{\text{sk}}, \tilde{\text{pk}}$ )
```


Hiding the public key

$\Sigma = (\text{KeyGen}_\Sigma, \text{Sign}_\Sigma, \text{Verify}_\Sigma)$

with additive public key
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Generic Transform
→

Signature scheme
with hidden public
key

HiddenPK-KeyGen(λ)	HiddenPK-Sign($\tilde{\text{sk}}, \text{msg}$)
1 : $(\text{sk}, \text{pk}) \leftarrow_{\$} \text{KeyGen}_\Sigma(\lambda)$	1 : $(\text{sk}, \rho) \leftarrow \tilde{\text{sk}}$
2 : $\rho \leftarrow_{\$} \mathcal{D}$	2 : $\sigma \leftarrow \text{Sign}_\Sigma(\text{sk}, \text{msg})$
3 : $\tilde{\text{sk}} \leftarrow (\text{sk}, \rho)$	3 : $\tilde{\sigma} \leftarrow (\sigma, \rho)$
4 : $\tilde{\text{pk}} \leftarrow (H(\rho), \text{pk} + \rho)$	4 : return $\tilde{\sigma}$
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Hiding the public key

$\Sigma = (\text{KeyGen}_\Sigma, \text{Sign}_\Sigma, \text{Verify}_\Sigma)$
 with additive public key
 domain
 + hash function H

Generic Transform
 \longrightarrow

Signature scheme
 with hidden public
 key

HiddenPK-KeyGen(λ)	HiddenPK-Sign($\tilde{\text{sk}}, \text{msg}$)	HiddenPK-Verify($\tilde{\sigma}, \tilde{\text{pk}}, \text{msg}$)
1 : $(\text{sk}, \text{pk}) \leftarrow_{\$} \text{KeyGen}_\Sigma(\lambda)$	1 : $(\text{sk}, \rho) \leftarrow \tilde{\text{sk}}$	1 : $(\sigma, \rho) \leftarrow \tilde{\sigma}$
2 : $\rho \leftarrow_{\$} \mathcal{D}$	2 : $\sigma \leftarrow \text{Sign}_\Sigma(\text{sk}, \text{msg})$	2 : $(\tilde{\text{pk}}_1, \tilde{\text{pk}}_2) \leftarrow \tilde{\text{pk}}$
3 : $\tilde{\text{sk}} \leftarrow (\text{sk}, \rho)$	3 : $\tilde{\sigma} \leftarrow (\sigma, \rho)$	3 : if $H(\rho) = \tilde{\text{pk}}_1$
4 : $\tilde{\text{pk}} \leftarrow (H(\rho), \text{pk} + \rho)$	4 : return $\tilde{\sigma}$	4 : $\text{pk} = \tilde{\text{pk}}_2 - \rho$
5 : return $(\tilde{\text{sk}}, \tilde{\text{pk}})$		5 : return $\text{Verify}_\Sigma(\sigma, \text{pk}, \text{msg})$
		6 : return 0

Hiding the public key

$\Sigma = (\text{KeyGen}_\Sigma, \text{Sign}_\Sigma, \text{Verify}_\Sigma)$

with additive public key
domain

+ hash function H

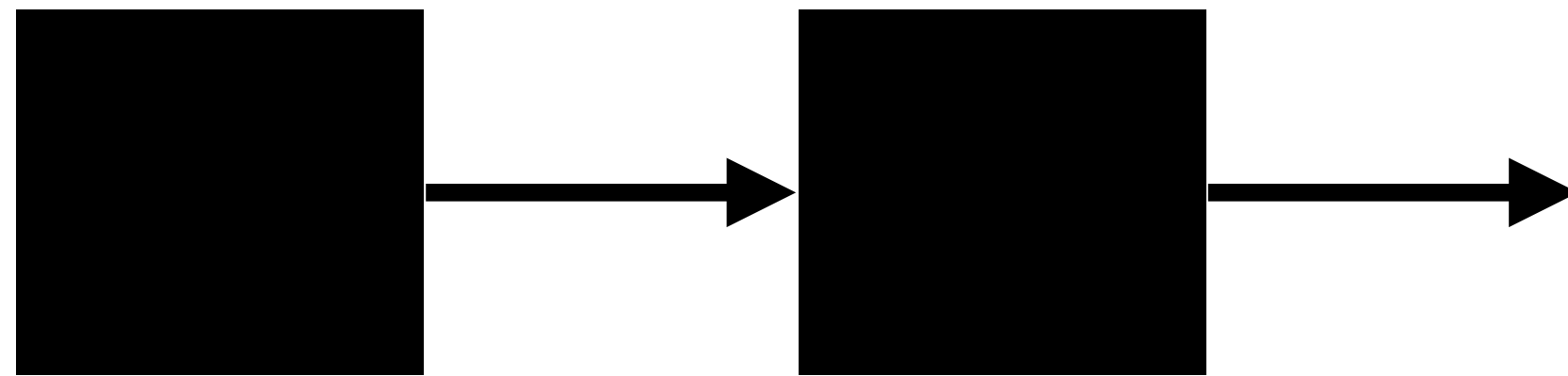
Generic Transform
→

Signature scheme
with hidden public
key

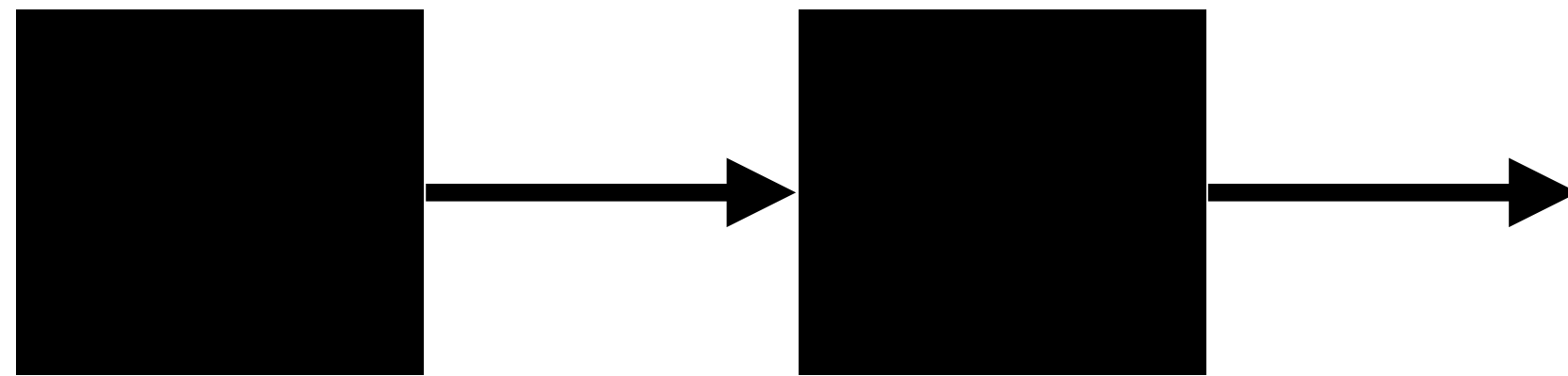
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1 : $(\text{sk}, \text{pk}) \leftarrow_{\$} \text{KeyGen}_\Sigma(\lambda)$	1 : $(\text{sk}, \rho) \leftarrow \tilde{\text{sk}}$	1 : $(\sigma, \rho) \leftarrow \tilde{\sigma}$
2 : $\rho \leftarrow_{\$} \mathcal{D}$	2 : $\sigma \leftarrow \text{Sign}_\Sigma(\text{sk}, \text{msg})$	2 : $(\tilde{\text{pk}}_1, \tilde{\text{pk}}_2) \leftarrow \tilde{\text{pk}}$
3 : $\tilde{\text{sk}} \leftarrow (\text{sk}, \rho)$	3 : $\tilde{\sigma} \leftarrow (\sigma, \rho)$	3 : if $H(\rho) = \tilde{\text{pk}}_1$
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5 : return $(\tilde{\text{sk}}, \tilde{\text{pk}})$		5 : return $\text{Verify}_\Sigma(\sigma, \text{pk}, \text{msg})$
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Problem Front Running

Front Running

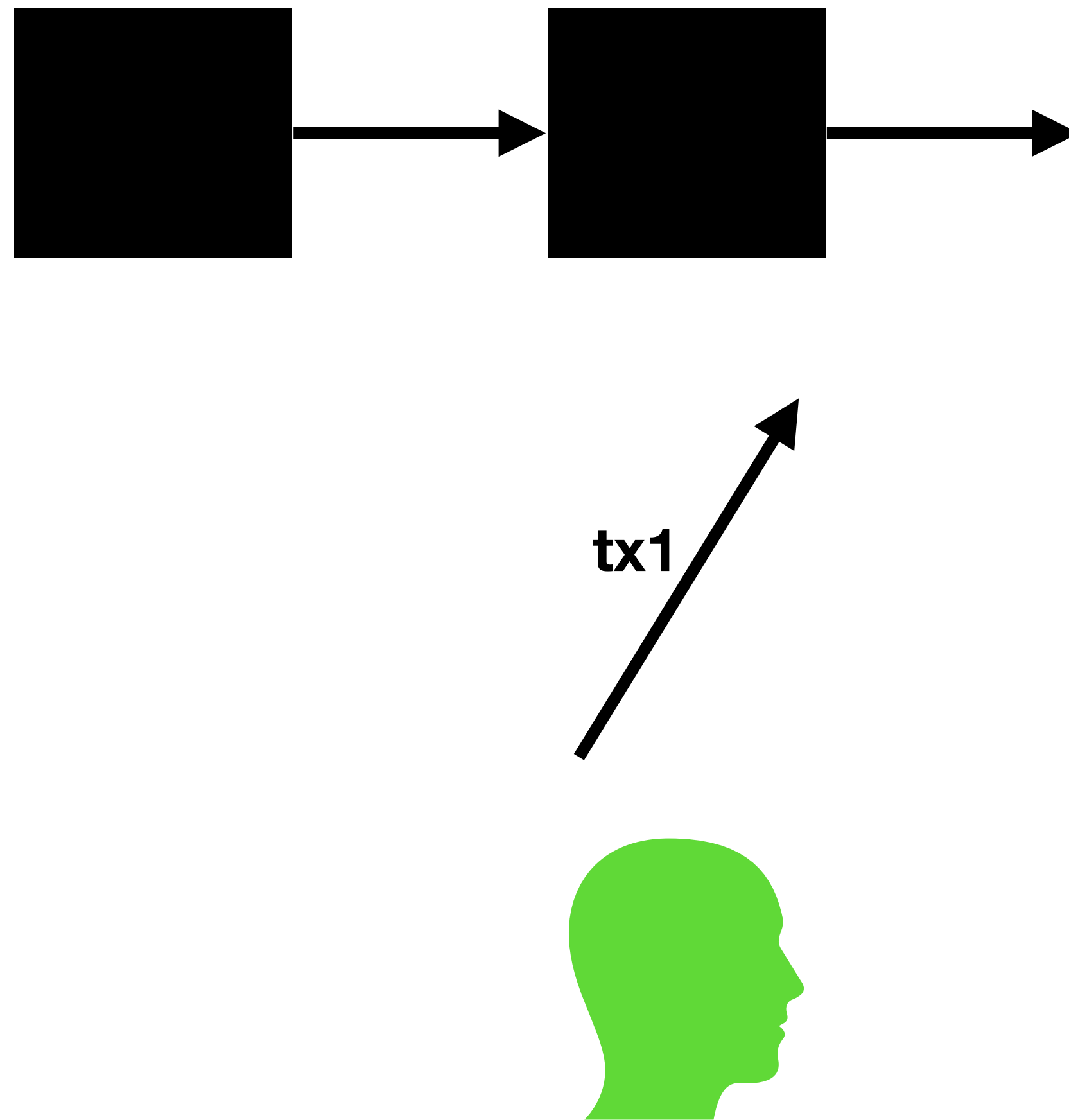


Front Running



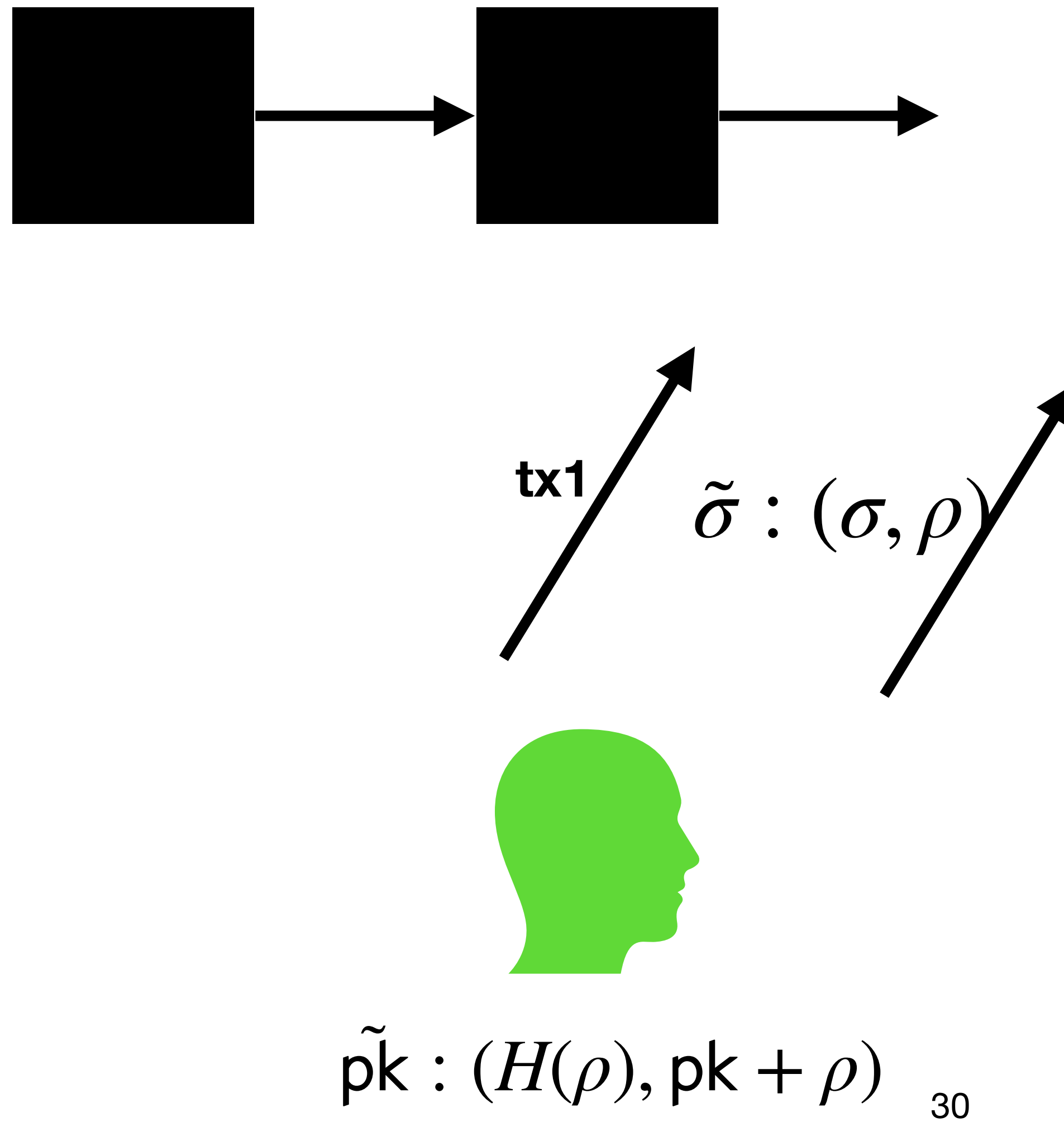
$$\tilde{p}k : (H(\rho), pk + \rho)$$

Front Running

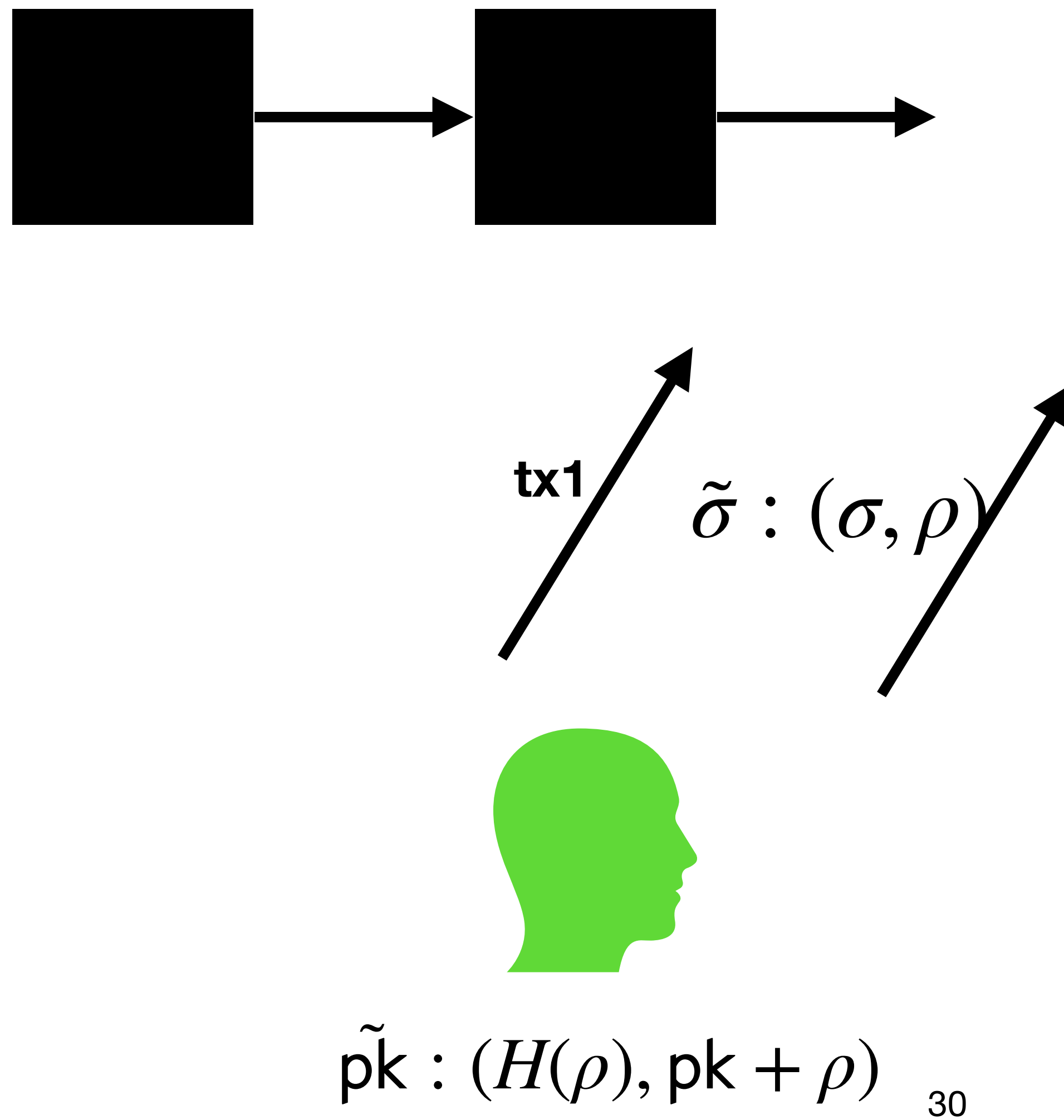


$$\tilde{pk} : (H(\rho), pk + \rho)$$

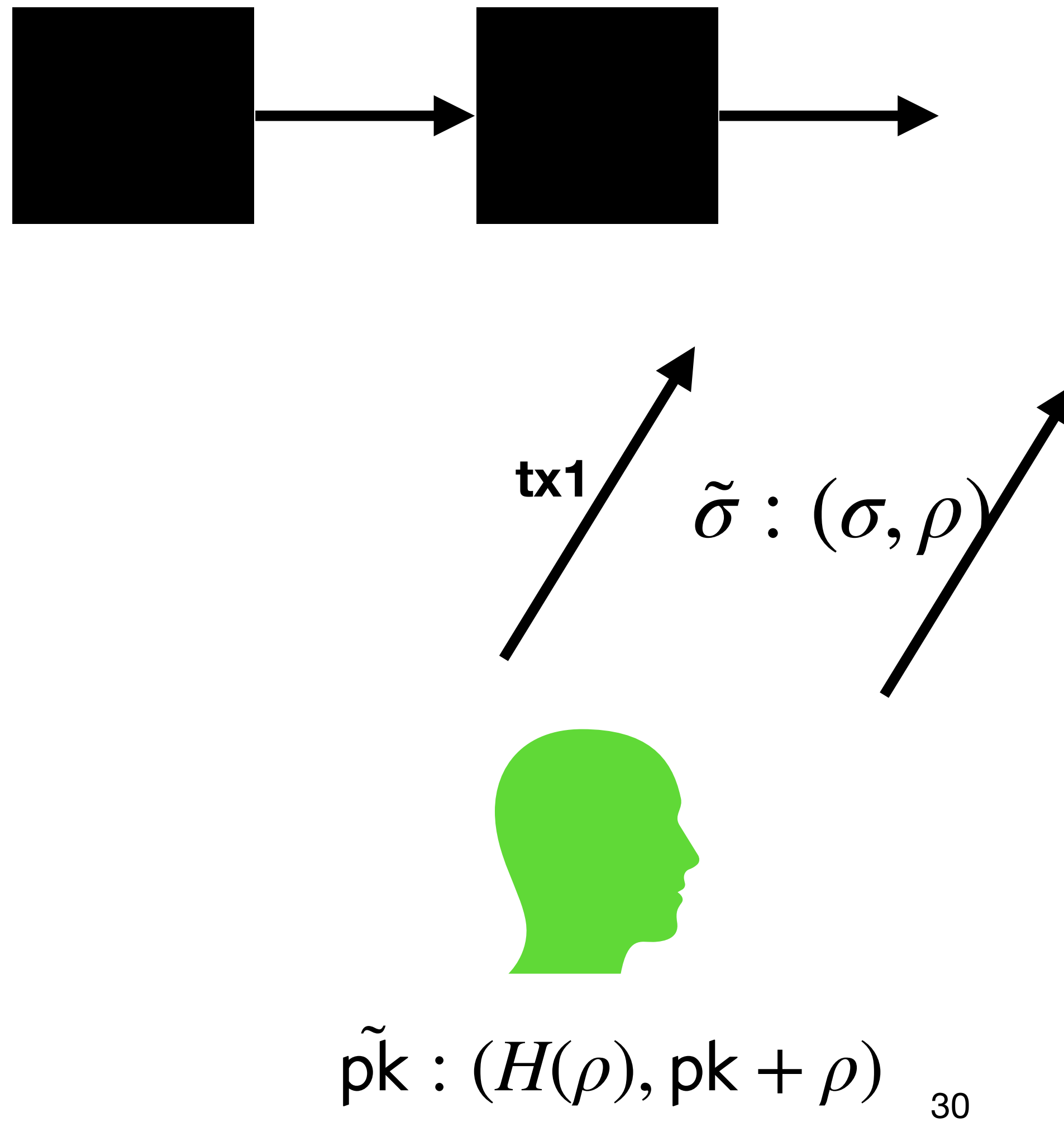
Front Running



Front Running



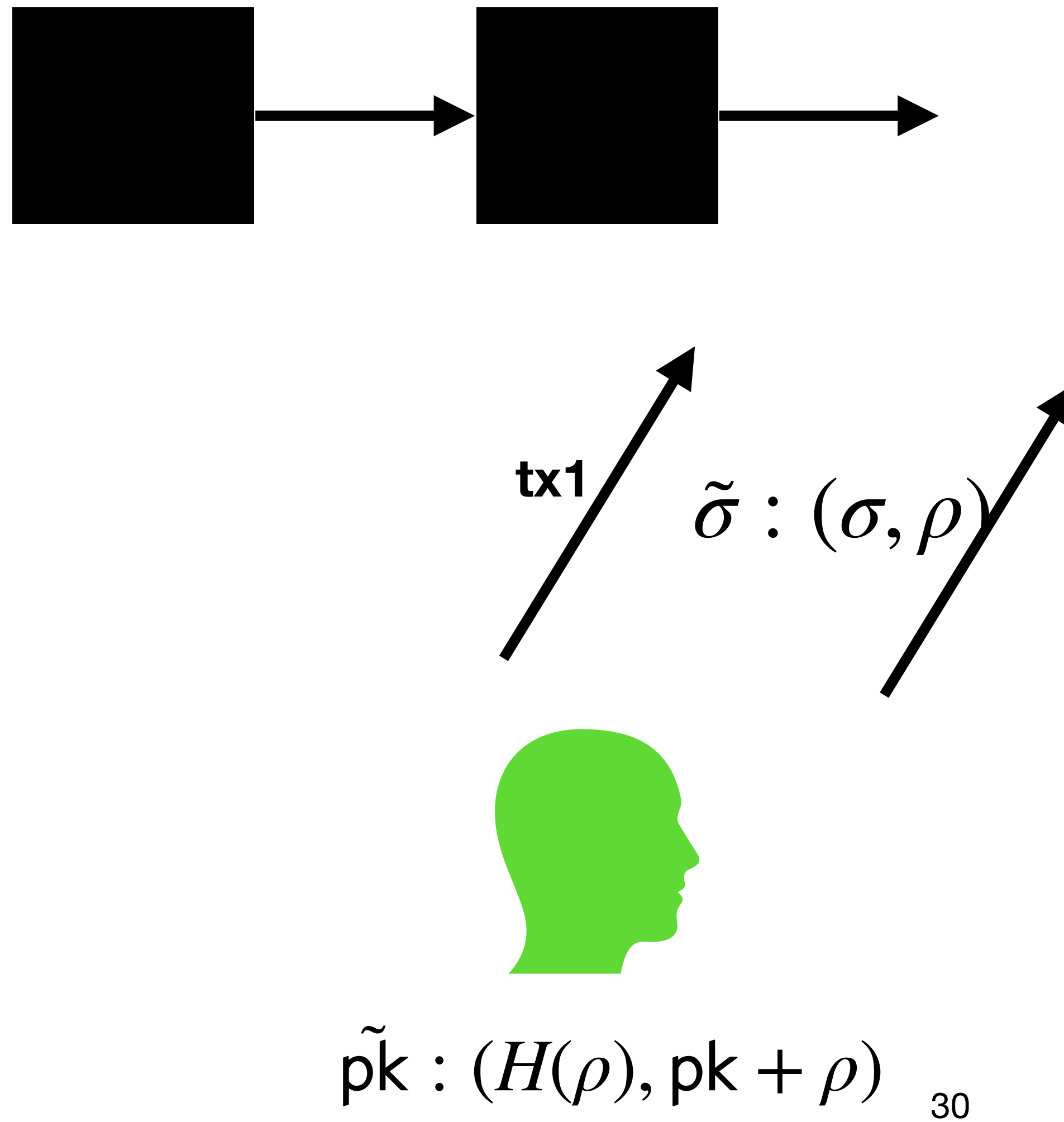
Front Running



$\tilde{\sigma} : (\sigma, \rho)$



Front Running

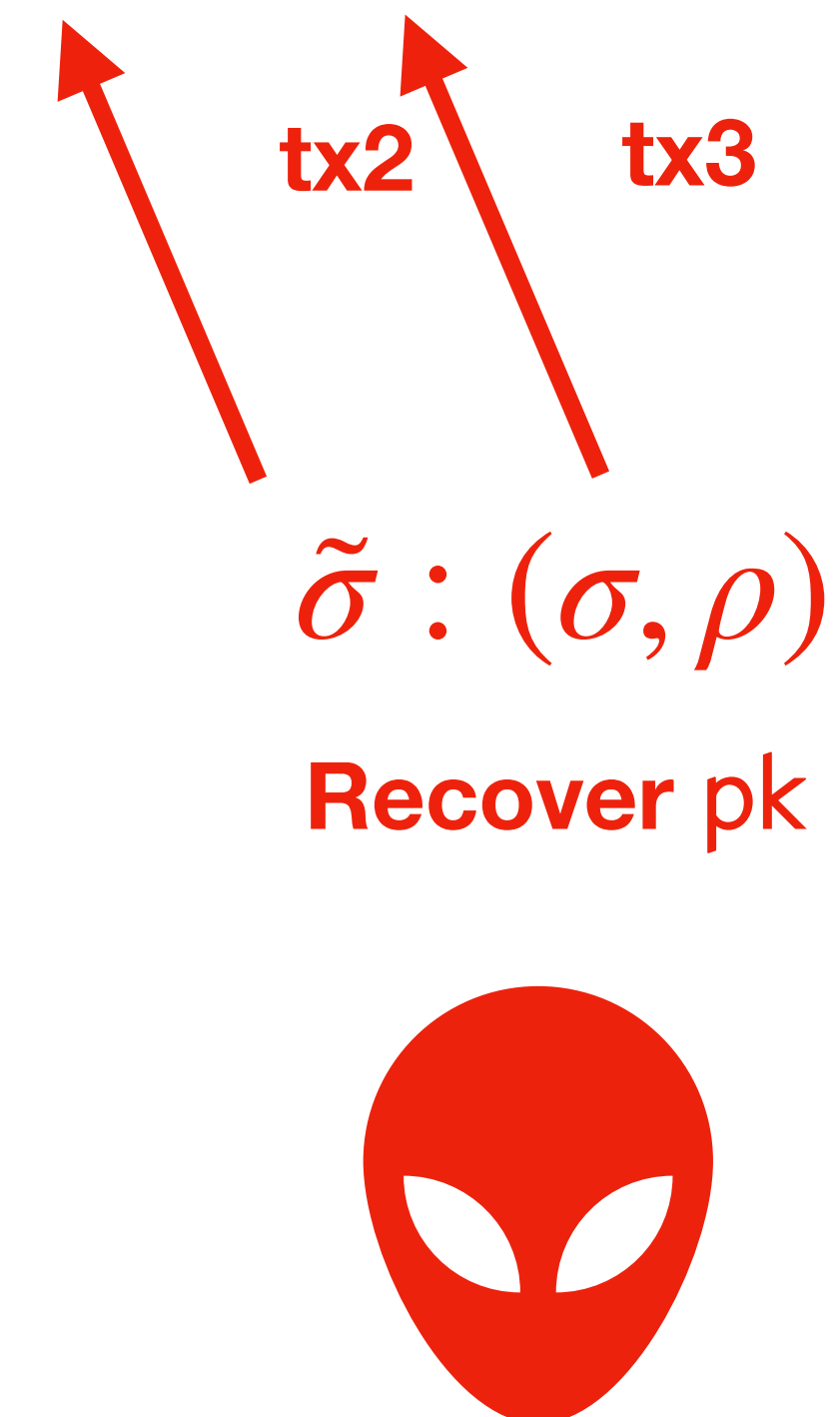
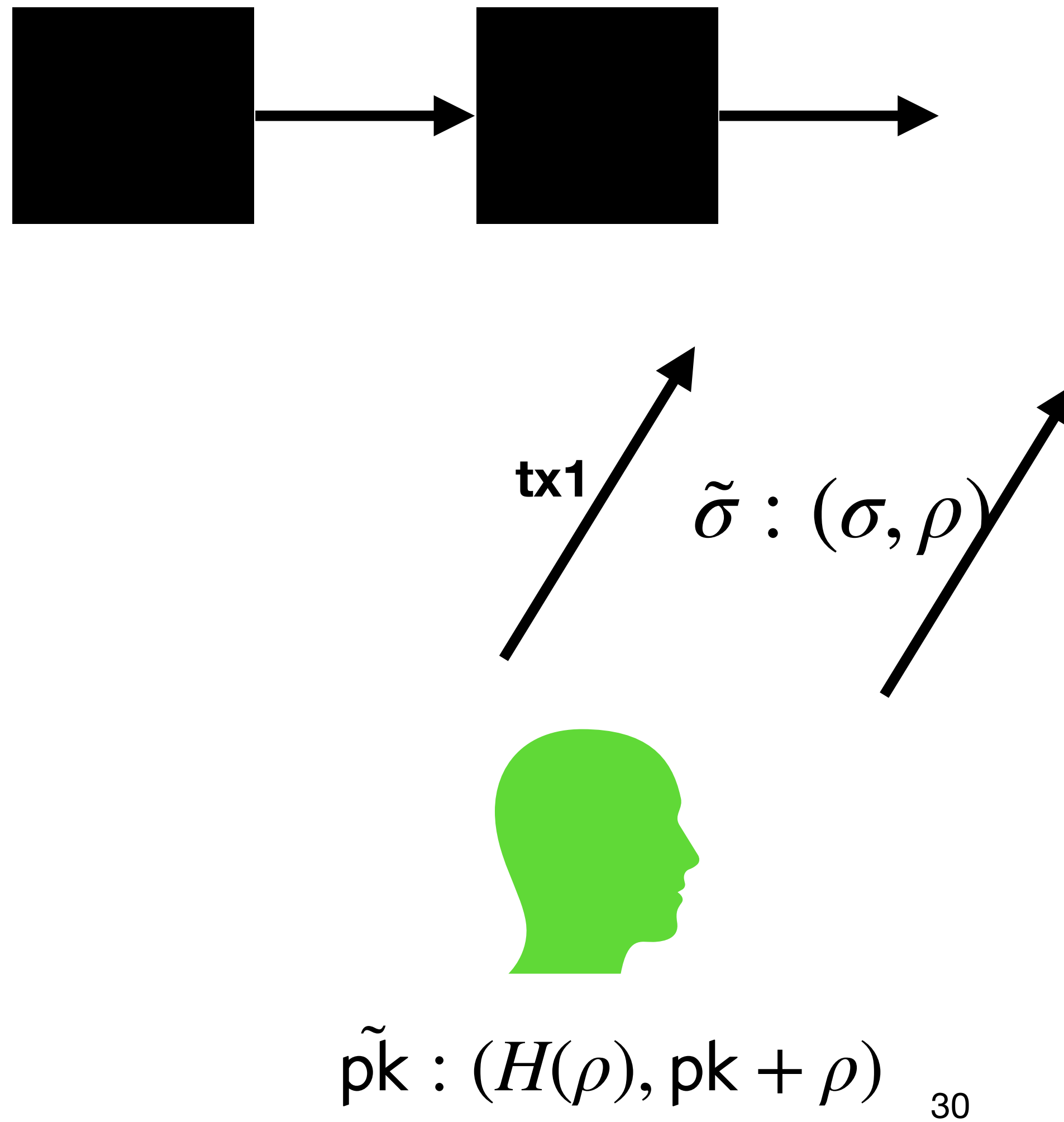


$\tilde{\sigma} : (\sigma, \rho)$

Recover pk

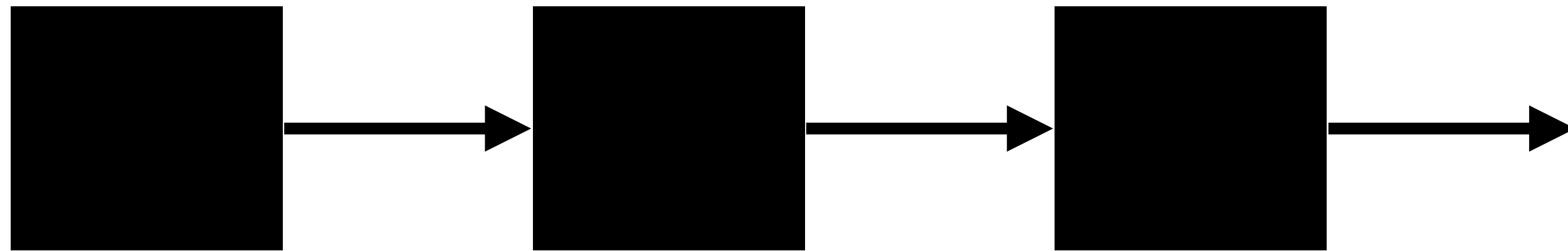


Front Running



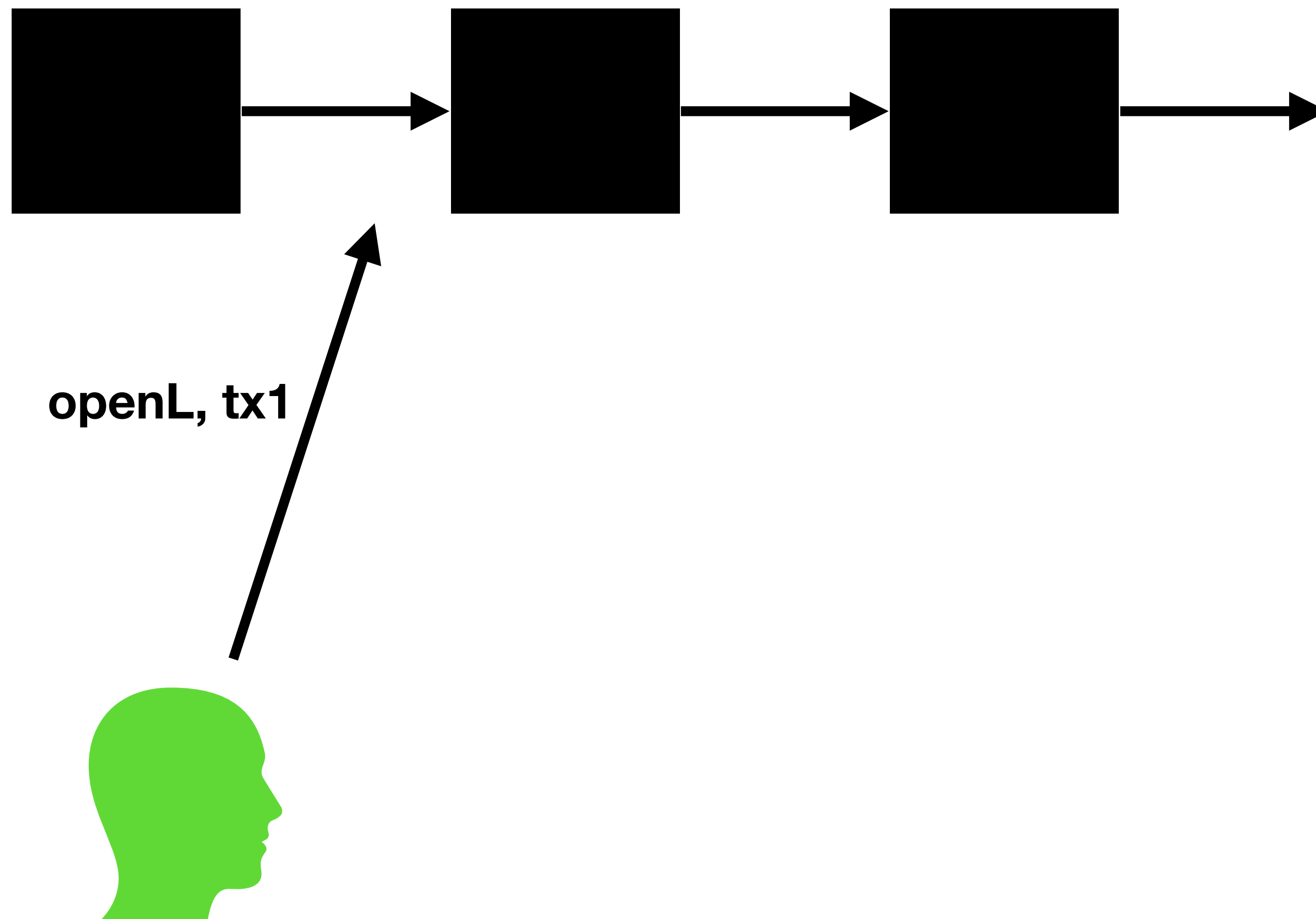
One-Time Signature

with Delayed Ledger

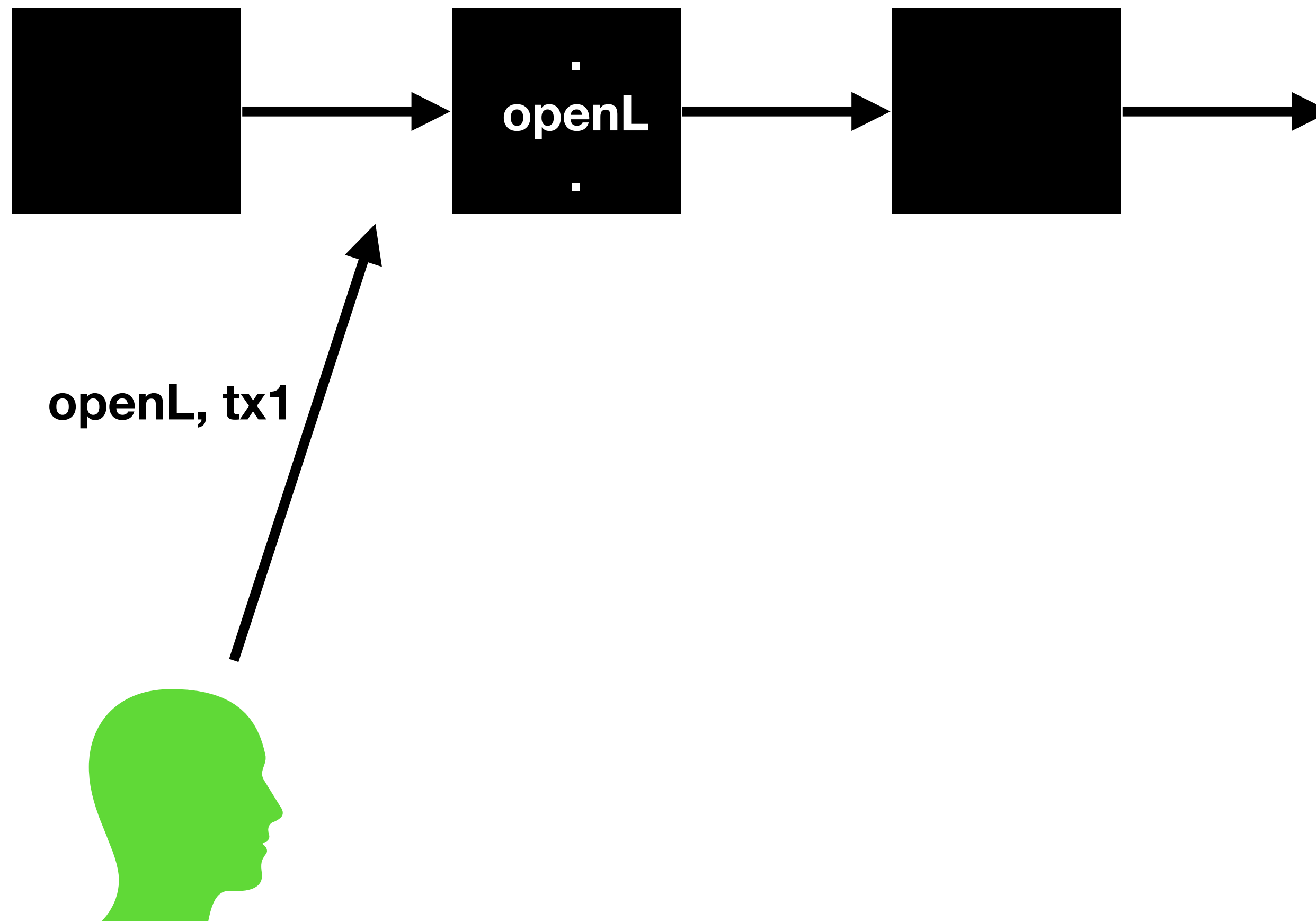


One-Time Signature

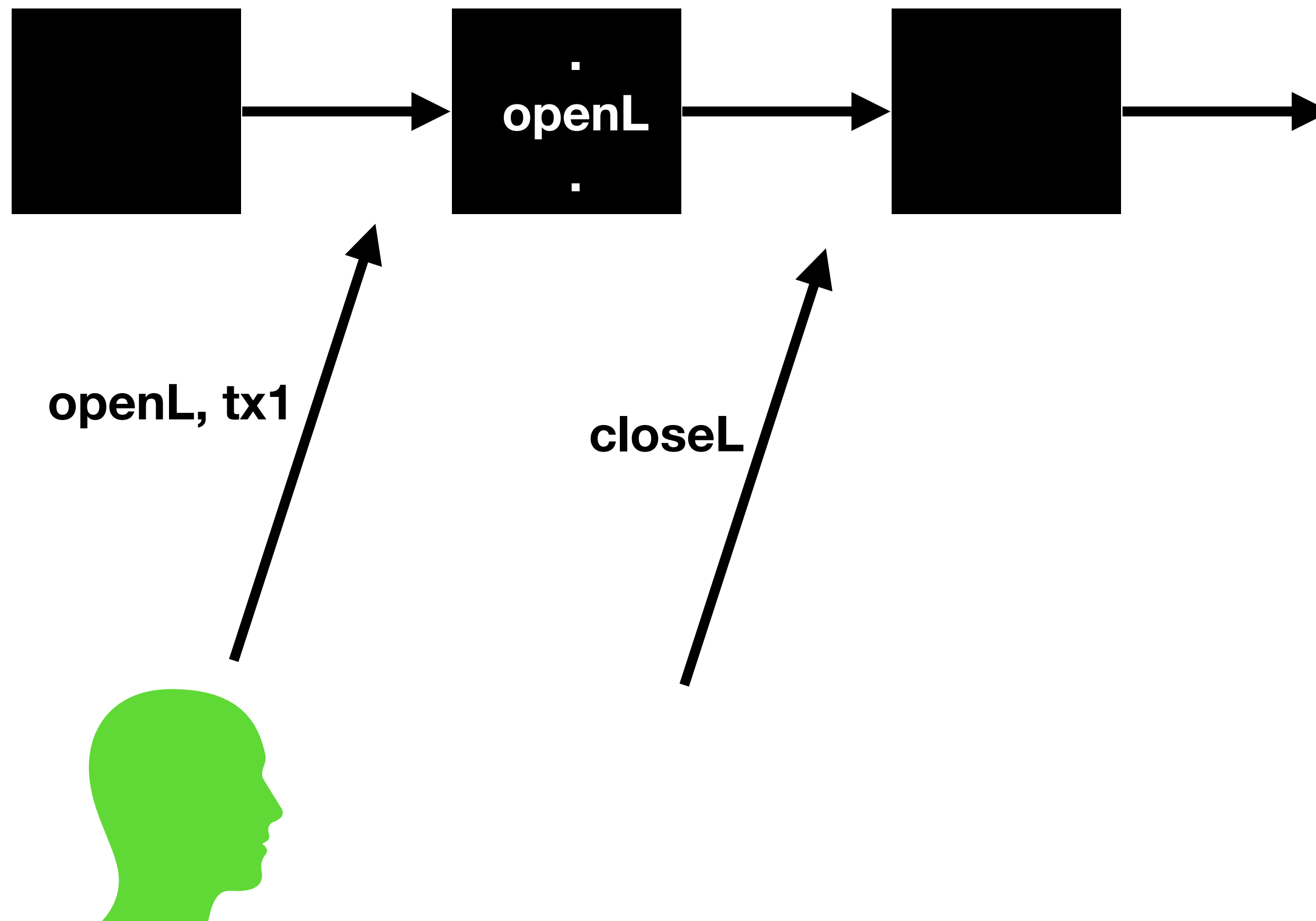
with Delayed Ledger



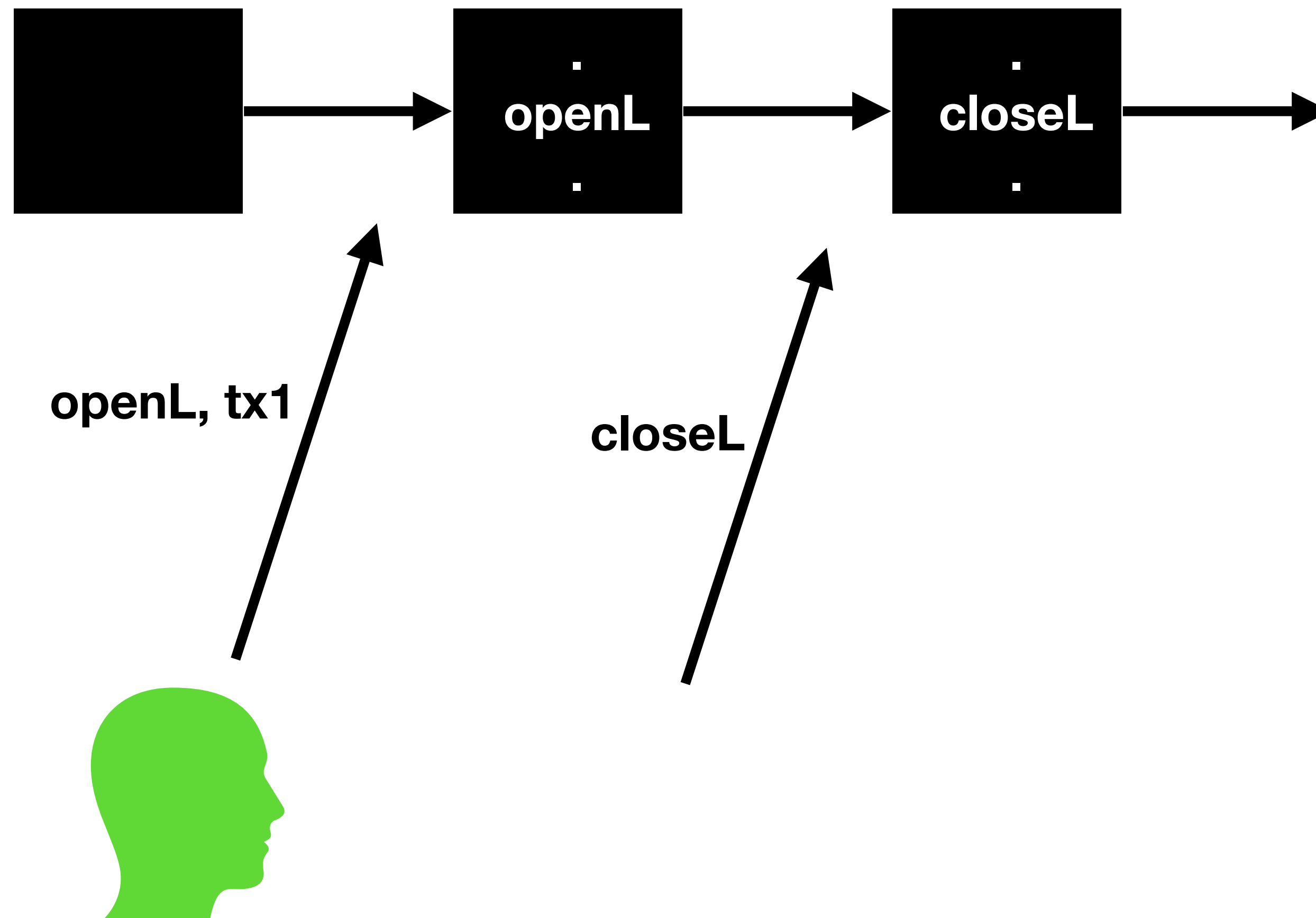
One-Time Signature with Delayed Ledger



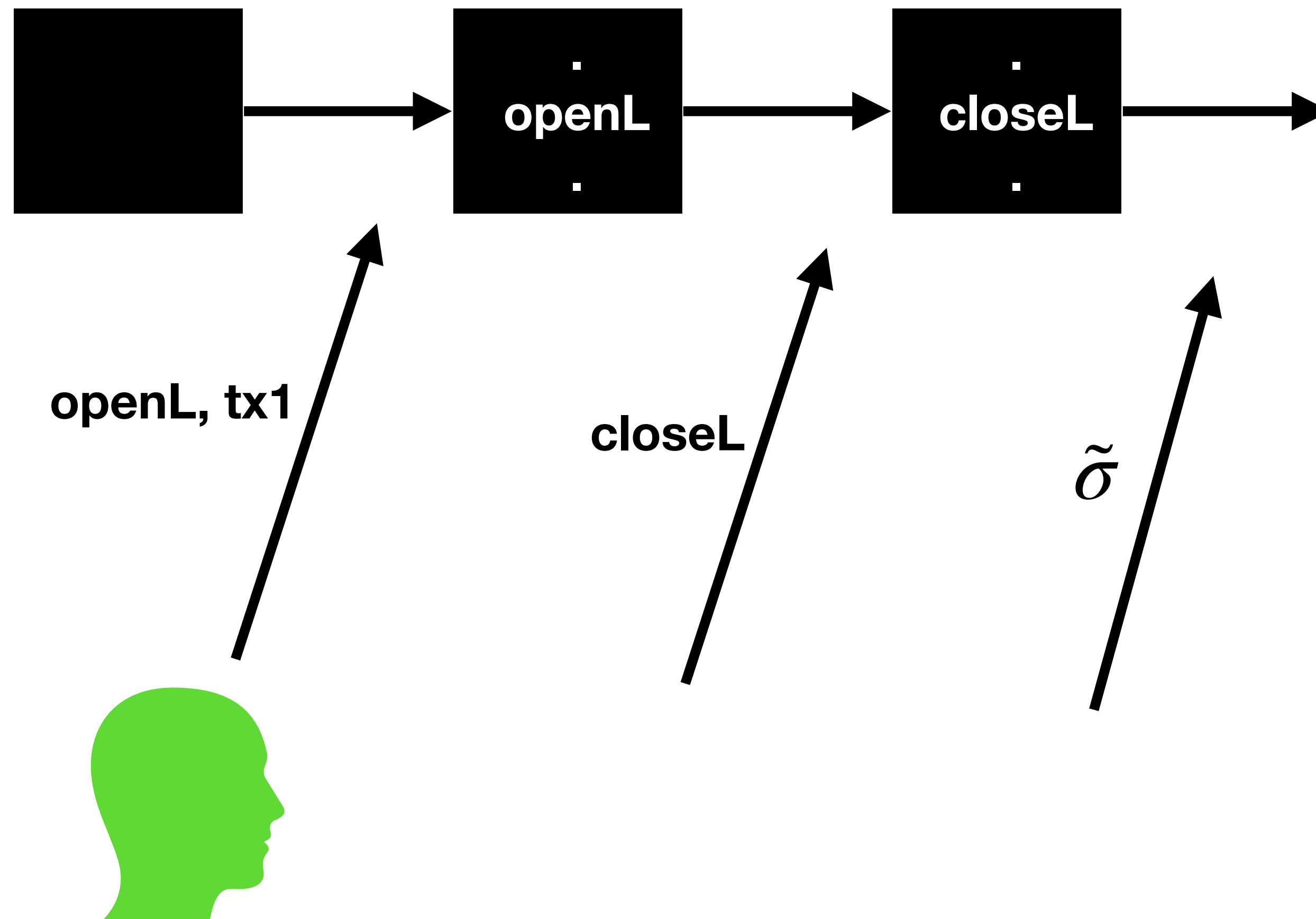
One-Time Signature with Delayed Ledger



One-Time Signature with Delayed Ledger



One-Time Signature with Delayed Ledger



Conclusion & Takeaway

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- Formal study of hiding a public key in a public ledger setting.
- HiddenPK is easier to extend to threshold setting compared to P2PKH.
- Practical instantiations require improvement in DDoS protection & miner motivation.

Thanks!