Surplus unit allocation

In this project, our goal is to identify the ideal surplus quantity to be added to the demand forecast of each product within the production plan, aiming to maximize profit margins. For each product, we are provided with several key pieces of data: estimated demand, the distribution reflecting demand variability, the margin per unit (calculated as net price minus cost), the cost of goods sold (COGS) representing production costs, capacity constraints (defined as the maximum percentage of demand that can be added as surplus), and a substitutability group number that indicates which products can be substituted for one another. The calculation of actual sales will proceed as follows:

$$targetDemand_p = demand_p * \widetilde{demandVar_p}$$

Product	Demand	Variance group	Margin	COGS	Capacity	Substitutability group
P1	100	2	165	40	-	1
P2	10	5	140	50	0	1
P3	20	2	130	20	<=15%	1
P4	50	3	170	40	<=15%	2
P5	100	3	160	45	-	2
P6	50	1	190	50	-	2
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Table 1: Product demand example data

Demand Var Group	distribution	c	d	loc	scale
0	burr12	2	25	-0	7
1	burr12	2	4	0	2
2	burr12	3	4	0	2
3	burr12	4	2	0	2
4	burr12	3	2	0	1
5	burr12	9	1	-1	2

Table 2: Distribution parameters for demand variance groups

The project must adhere to the following constraints to ensure optimal surplus allocation:

- The surplus quantity added to each product's demand must not surpass its designated capacity limit.
- The aggregate surplus quantity across all products should not exceed the total demand for all products, adjusted by a macro target percentage. This macro target percentage is an adjustable input parameter, ranging between 10% and 50%.
- For products classified within the same substitutability group, it's important to maintain adequate total surplus quantities. This approach aims to mitigate the risk of lost sales by leveraging the substitutability of products within these groups, ensuring that demand can be met even if specific products are over or undersupplied.

This problem can be formulated as a classic newsvendor problem with a few additional rules such as surplus capacity and some products can be substituted for one another. If you produce more than you need, you incur extra cost, but if you order less than you need, you lose profit.

Decision Variables:

y_i Regular production amount for product i.

$z_{i,j}$ Surplus production amount for product i for supplement product j if product i and j can be substituted, such that $j \in G(i)$ product j is in the same substitutability group with product i.

$s_{n,i}$ Shortage amount of quantity for product i in scenario n if demand of product i cannot be met by regular production of product i and surplus production from substitutions.

$k_{n,i}$ Excess amount of quantity for product i in scenario n if demand of product i can be met by regular production of product i and surplus production from substitutions.

Parameters:

$Demand_{n,i}$ Randomly generated demand for product i in scenario n using burr 12 distribution.

$Demand_{n,i} = Demand_i * Random\_Number(Burr12Dist_{c,d,loc,scale}^{n,i})$

Cap_i allowed capacity limit to be added to regular production amount for product i.

MTP is the macro target percentage parameter which constraints overall surplus production.

Objective:

Objective is to minimize the cost due to excess amount of production or shortage amount of production.

$ExcessAmount_{n,i} = \max(0, TotalProduction_{n,i} - Demand_{n,i})$

$ShortageAmount_{n,i} = \max(0, Demand_{n,i} - TotalProduction_{n,i})$

$$Cost = \sum_{n \in N} \sum_{i \in I} Margin_i * ShortageAmount_{n,i} + COGS\_i * ExcessAmount_{n,i}$$

$$Cost = \sum_{n \in N} \sum_{i \in I} Margin_i * s_{n,i} + COGS\_i * k_{n,i}$$

Constraints:

$$\sum_{j \in G(i)} z_{i,j} \leq Cap_i * y_i \qquad for\ every\ product\ i$$

Total surplus production amount from product i to product $j \in G(i)$ should be less than or equal to designated capacity limit.

$\sum_i \sum_{j \in G(i)} z_{i,j} \leq MTP * \sum_i y_i$

The total surplus production amount for all products to supply product j should be less than or equal to the macro target limit.

$s_{n,i} \geq Demand_{n,i} - y_i - \sum_{j \in G(i)} z_{i,j} \qquad for\ every\ product\ i\ and\ for\ every\ n$

$ShortageAmount_{n,i}\ should\ be\ greater\ than\ or\ equal\ to\ zero$

$k_{n,i} \geq y_i + \sum_{j \in G(i)} z_{i,j} - Demand_{n,i} \qquad for\ every\ product\ i\ and\ for\ every\ n$

$ExcessAmount_{n,i}\ should\ be\ greater\ than\ or\ equal\ to\ zero$