Solution for Question 2 Mehmet Ozan Guven - 220201036

1. Variable Elimination

(a) **P(+D) : Answer: 0.32**

$$P(+D) = \Sigma_{A,B,C,E} P(A,B,C,+D,E)$$
$$= \Sigma_{A,B,C,E}\, P(A) * P(B|A) * P(C|A) * P(+D|B,C) * P(E|C)$$

Here is the factors: (ϕ : factor symbol that I have used)

$\phi_A, \phi_{B|A}, \phi_{C|A}, \phi_{+D|B,C}, \phi_{E|C}$

First Eliminate A:
Here is the factors list that include A:→ $\phi_A, \phi_{B|A}, \phi_{C|A}$

When we multiply these factors we will get:

A	B	C	$(\phi_A)(\phi_{B\|A})(\phi_{C\|A})$
+	+	+	0.2*0.8*0.2 = 0.032
+	+	-	0.2*0.8*0.8 = 0.128
+	-	+	0.2*0.2*0.2 = 0.008
+	-	-	0.2*0.2*0.8 = 0.032
-	+	+	0.8*0.2*0.05= 0.008
-	+	-	0.8*0.2*0.95= 0.152
-	-	+	0.8*0.8*0.05= 0.032
-	-	-	0.8*0.8*0.95= 0.608

Summing out A, then we have:

B	C	$\Sigma_A\ (\phi_A)(\phi_{B\|A})(\phi_{C\|A})$
+	+	0.032+0.008 = 0.04
+	-	0.128+0.152 = 0.28
-	+	0.008+0.032 = 0.04
-	-	0.608+0.032 = 0.64

Now our new factor is $\phi_{B,C}$

Here is the our factors now: $(\phi_{+D|B,C}), (\phi_{E|C}), (\phi_{B,C})$

Now Eliminate B:
Here is the factors list that include B:→ $(\phi_{+D|B,C}), (\phi_{B,C})$

When we multiply these factors we will get:

B	C	D	$(\phi_{+D\|B,C})(\phi_{B,C})$
+	+	+	0.8*0.04 = 0.032
+	-	+	0.8*0.28 = 0.224
-	+	+	0.8*0.04 = 0.032
-	-	+	0.05*0.64 = 0.032

Summing out B, then we have:

C	D	$\Sigma_B\ (\phi_{+D\|B,C})(\phi_{B,C})$
+	+	0.032+0.032 = 0.064
-	+	0.224+0.032 = 0.256

Now our new factor is $(\phi_{C,+D})$

Here is the our factors now: $(\phi_{E|C}), (\phi_{C,+D})$

Now Eliminate C:
Here is the factors list that include C:$\rightarrow (\phi_{E|C}), (\phi_{C,+D})$

When we multiply these factors we will get:

C	D	E	$(\phi_{E\|C})(\phi_{C,+D})$
+	+	+	0.8*0.064 = 0.0512
+	+	-	0.2*0.064 = 0.0128
-	+	+	0.6*0.256 = 0.1536
-	+	-	0.4*0.256 = 0.1024

Summing out C, then we have:

D	E	$\Sigma_C\ (\phi_{E\|C})(\phi_{C,+D})$
+	+	0.0512+0.1536 = 0.2048
+	-	0.0128+0.1024 = 0.1152

Now our new factor is $(\phi_{E,+D})$

Here is the our factors now: $(\phi_{E,+D})$

Now just summing out E:

D	$\Sigma_E\ (\phi_{E,+D})$
+	0.2048 + 0.1152 = 0.32

Finally we have found that our first equation equal to :

$$\begin{aligned} P(+D) &= \Sigma_{A,B,C,E} P(A, B, C, +D, E) \\ &= ... \\ &= \Sigma_E\ (\phi_{E,+D}) \\ &= 0.32 \end{aligned}$$

(b) **P(+D,-A) Answer: 0.184**

$$P(+D,-A) = \Sigma_{B,C,E}\ P(-A,B,C,+D,E)$$
$$= \Sigma_{B,C,E}\ P(-A) * P(B|-A) * P(C|-A) * P(+D|B,C) * P(E|C)$$
$$= P(-A) * (\Sigma_{B,C,E}\ P(B|-A) * P(C|-A) * P(+D|B,C) * P(E|C))$$

We will solve the part $\Sigma_{B,C,E}$... using variable elimination
Here is the factors: (ϕ : factor symbol that I have used)

$(\phi_{B|-A}), (\phi_{C|-A}), (\phi_{+D|B,C}), (\phi_{E|C})$

First Eliminate B:
Here is the factors list that include B:→ $(\phi_{+D|B,C}), (\phi_{B|-A})$

When we multiply these factors we will get:

B	C	$(\phi_{+D\|B,C})(\phi_{B\|-A})$
+	+	0.2*0.8=0.16
+	-	0.2*0.8=0.16
-	+	0.8*0.8=0.64
-	-	0.8*0.05=0.04

Summing out B, then we have:

C	$\Sigma_B(\phi_{+D\|B,C})(\phi_{B\|-A})$
+	0.16+0.64=0.8
-	0.16+0.04=0.2

Now our new factor is (ϕ_C)
Here is the our factors now: : $(\phi_{C|-A}), (\phi_{E|C}), (\phi_C)$

Now Eliminate C:
Here is the factors list that include C:→ $(\phi_{C|-A}), (\phi_{E|C}), (\phi_C)$

When we multiply these factors we will get:

C	E	$(\phi_{C\|-A})(\phi_{E\|C})(\phi_C)$
+	+	0.05*0.8*0.8=0.032
+	-	0.05*0.2*0.8=0.008
-	+	0.95*0.6*0.2=0.114
-	-	0.95*0.4*0.2=0.076

Summing out C, then we have:

E	$\Sigma_C(\phi_{C\|-A})(\phi_{E\|C})(\phi_C)$
+	0.032+0.114 = 0.146
-	0.008+0.076 = 0,084

Now our new factor is ϕ_E

Finally we have found that our equation equal to :

$$\begin{aligned} P(+D,-A) &= \Sigma_{B,C,E}\ P(-A,B,C,+D,E) \\ &= ... \\ &= P(-A) * (\Sigma_{B,C,E}\ P(B|-A) * P(C|-A) * P(+D|B,C) * P(E|C)) \\ &= P(-A) * \Sigma_E\ \phi_E \\ &= 0.8 * (0.146 + 0.084) \\ &= 0.184 \end{aligned}$$

(c) **P(+E | -B)**: **Answer: 0.61176**

$$\begin{aligned} P(+E|-B) &= \frac{P(+E,-B)}{P(-B)} \\ &= \frac{P(+E,-B)}{\Sigma_E\ P(E,-B)} \\ &= \frac{P(+E,-B)}{P(+E,-B) + P(-E,-B)} \end{aligned}$$

Find the part P(+E,-B) and P(-E,-B) respectively using variable elimination

1. $$\begin{aligned} P(+E,-B) &= \Sigma_{A,C,D}\ P(A,-B,C,D,+E) \\ &= \Sigma_{A,C,D}\ P(A) * P(-B|A) * P(C|A) * P(D|-B,C) * P(+E|C) \end{aligned}$$

Here is the factors: (ϕ : factor symbol that I have used)

$(\phi_A), (\phi_{-B|A}), (\phi_{C|A}), (\phi_{D|-B,C}), (\phi_{+E|C})$

First Eliminate A:
Here is the factors list that include A:→ $(\phi_A), (\phi_{-B|A}), (\phi_{C|A})$

When we multiply these factors we will get:

A	C	$(\phi_A)(\phi_{-B\|A})(\phi_{C\|A})$
+	+	0.2*0.2*0.2= 0.008
+	-	0.2*0.2*0.8= 0.032
-	+	0.8*0.8*0.05=0.032
-	-	0.8*0.8*0.95=0.608

Summing out A, then we have:

C	Σ_A $(\phi_A)(\phi_{-B\|A})(\phi_{C\|A})$
+	0.008+0.032=0.04
-	0.032+0.608=0.64

Now our new factor is (ϕ_C)

Here is the our factors now: $(\phi_{D|-B,C}), (\phi_{+E|C}), (\phi_C)$

Now Eliminate C:
Here is the factors list that include C:$\rightarrow (\phi_{D|-B,C}), (\phi_{+E|C}), (\phi_C)$

When we multiply these factors we will get:

C	D	$(\phi_{D\|-B,C})(\phi_{+E\|C})(\phi_C)$
+	+	0.8 *0.8*0.04=0.0256
+	-	0.2*0.8*0.04=0.0064
-	+	0.05*0.6*0.64=0.0192
-	-	0.95*0.6*0.64=0.3648

Summing out C, then we have:

D	Σ_C $(\phi_{D\|-B,C})(\phi_{+E\|C})(\phi_C)$
+	0.0256+0.0192=0.0448
-	0.0064+0.3648=0.3712

Now our new factor is (ϕ_D)

Here is the our factors now: (ϕ_D)

Finally we have found that our equation equal to :

1. $P(+E, -B) = \Sigma_{A,C,D}\ P(A, -B, C, D, +E)$
$= \Sigma_{A,C,D}\ P(A) * P(-B|A) * P(C|A) * P(D| - B, C) * P(+E|C)$
$= \Sigma_D\ \phi_D$
$= 0.0448 + 0.3712 = 0.416$

2. $P(-E, -B) = \Sigma_{A,C,D}\ P(A, -B, C, D, -E)$
$= \Sigma_{A,C,D}\ P(A) * P(-B|A) * P(C|A) * P(D| - B, C) * P(-E|C)$

Find the $\Sigma_{A,C,D}$... using variable elimination
Here is the factors: (ϕ : factor symbol that I have used)

$(\phi_A), (\phi_{-B|A}), (\phi_{C|A}), (\phi_{D|-B,C}), (\phi_{-E|C})$

First Eliminate A:
Here is the factors list that include A:→ $(\phi_A), (\phi_{-B|A}), (\phi_{C|A})$

When we multiply these factors we will get:

A	C	$(\phi_A)(\phi_{-B\|A})(\phi_{C\|A})$
+	+	0.2*0.2*0.2= 0.008
+	-	0.2*0.2*0.8= 0.032
-	+	0.8*0.8*0.05=0.032
-	-	0.8*0.8*0.95=0.608

Summing out A, then we have:

C	$\Sigma_A(\phi_A)(\phi_{-B\|A})(\phi_{C\|A})$
+	0.008+0.032=0.04
-	0.032+0.608=0.64

Now our new factor is (ϕ_C)

Here is the our factors now: $(\phi_{D|-B,C}), (\phi_{-E|C}), (\phi_C)$

Now Eliminate C:
Here is the factors list that include C:→ $(\phi_{D|-B,C}), (\phi_{-E|C}), (\phi_C)$

When we multiply these factors we will get:

C	D	$(\phi_{D\|-B,C})(\phi_{-E\|C})(\phi_C)$
+	+	0.8*0.2*0.04=0.0064
+	-	0.2*0.2*0.04=0.0016
-	+	0.05*0.4*0.64=0.0128
-	-	0.95*0.4*0.64=0.2432

Summing out C, then we have:

D	$\Sigma_C\ (\phi_{D\|-B,C})(\phi_{-E\|C})(\phi_C)$
+	0.0064+0.0128=0.0192
-	0.0016+0.2432=0.2448

Now our new factor is (ϕ_D)

Here is the our factors now: (ϕ_D)

Finally we have found that our equation equal to :

$$\begin{aligned}
2.\ P(-E,-B) &= \Sigma_{A,C,D}\ P(A,-B,C,D,-E) \\
&= \Sigma_{A,C,D}\ P(A) * P(-B|A) * P(C|A) * P(D|-B,C) * P(-E|C) \\
&= \Sigma_{D}\ \phi_D \\
&= 0.0192 + 0.2448 = 0.264
\end{aligned}$$

Now we know the P(+E,-B) and P(-E, -B) using variable elimination.
Then, we can get the answer:

$$\begin{aligned}
P(+E|-B) &= \frac{P(+E,-B)}{P(-B)} \\
&= \frac{P(+E,-B)}{\Sigma_E\ P(E,-B)} \\
&= \frac{P(+E,-B)}{P(+E,-B) + P(-E,-B)} \\
&= \frac{0.416}{0.416 + 0.26} \\
&= 0.6117
\end{aligned}$$

(d) **P(+A | +D,-E)**: **Answer: 0.416**

$$\begin{aligned}
P(+A|+D,-E) &= \frac{P(+A,+D,-E)}{P(+D,-E)} \\
&= \frac{P(+A,+D,-E)}{P(+A,+D,-E) + P(-A,+D,-E)}
\end{aligned}$$

$$\begin{aligned}
1.\ P(+A,+D,-E) &= \Sigma_{B,C}\ P(+A,B,C,+D,-E) \\
&= \Sigma_{B,C}\ P(+A) * P(B|+A) * P(C|+A) * P(+D|B,C) * P(-E|C) \\
&= P(+A) * (\Sigma_{B,C}\ P(B|+A) * P(C|+A) * P(+D|B,C) + P(-E|C))
\end{aligned}$$

We will solve the part $\Sigma_{B,C}$... using variable elimination
Here is the factors: (ϕ : factor symbol that I have used)

$(\phi_{B|+A}), (\phi_{C|+A}), (\phi_{+D|B,C}), (\phi_{-E|C})$

First Eliminate B:
Here is the factors list that include B:→ $(\phi_{B|+A}), (\phi_{+D|B,C})$

When we multiply these factors we will get:

A	B	C	D	$(\phi_{B\|+A})(\phi_{+D\|B,C})$
+	+	+	+	0.8*0.8 = 0.64
+	+	-	+	0.8*0.8 = 0.64
+	-	+	+	0.2*0.8 = 0.16
+	-	-	+	0.2*0.05 = 0.01

Summing out B, then we have

C	$\Sigma_B\ (\phi_{B\|+A})(\phi_{+D\|B,C})$
+	0.64+0.16 = 0.8
-	0.64+0.01 = 0.65

Now our new factor is (ϕ_C)

Here is the our factors now: $(\phi_{C|+A}), (\phi_{-E|C}), (\phi_C)$

Now Eliminate C:
Here is the factors list that include C:→ $(\phi_{C|+A}), (\phi_{-E|C}), (\phi_C)$

When we multiply these factors we will get:

A	C	E	$(\phi_{C\|+A})(\phi_{-E\|C})(\phi_C)$
+	+	-	0.2*0.8*0.2 = 0.032
+	-	-	0.8*0.65*0.4 = 0.208

Summing out C, then we have:

A	E	$\Sigma_C\ (\phi_{C\|+A})(\phi_{-E\|C})(\phi_C)$
+	-	0.032+0.208=0.24

Finally we have found that our equation equal to:

$$
\begin{aligned}
P(+A,+D,-E) &= \Sigma_{B,C}\, P(+A,B,C,+D,-E) \\
&= ... \\
&= P(+A) * (\Sigma_{B,C}\, P(B|+A) * P(C|+A) * P(+D|B,C) + P(-E|C)) \\
&= P(+A) * \Sigma_C\ (\phi_{C|+A})(\phi_{-E|C})(\phi_C) \\
&= 0.2 * 0.24 \\
&= 0.048
\end{aligned}
$$

2. $P(-A,+D,-E) = \Sigma_{B,C}\, P(-A,B,C,+D,-E)$
$= \Sigma_{B,C}\, P(-A) * P(B|-A) * P(C|-A) * P(+D|B,C) * P(-E|C)$
$= P(-A) * (\Sigma_{B,C}\, P(B|-A) * P(C|-A) * P(+D|B,C) * P(-E|C))$

We will solve the part $\Sigma_{B,C}$... using variable elimination
Here is the initial factors: (ϕ : factor symbol that I have used)

$(\phi_{B|-A}), (\phi_{C|-A}), (\phi_{+D|B,C}), (\phi_{-E|C})$

First Eliminate B:
Here is the factors list that include B:→ $(\phi_{B|-A}), (\phi_{+D|B,C})$

When we multiply these factors we will get:

A	B	C	D	$(\phi_{B\|-A})(\phi_{+D\|B,C})$
-	+	+	+	0.2*0.8 = 0.16
-	+	-	+	0.2*0.8 = 0.16
-	-	+	+	0.8*0.8 = 0.64
-	-	-	+	0.8*0.05 = 0.04

Summing out B, then we have:

C	$\Sigma_B(\phi_{B\|-A})(\phi_{+D\|B,C})$
+	0.16+0.64 = 0.8
-	0.16+0.04 = 0.2

Now our new factor is (ϕ_C)
Here is the our factors now: $(\phi_{C|-A}), (\phi_{-E|C}), (\phi_C)$

Now Eliminate C:
Here is the factors list that include B:→ $(\phi_{C|-A}), (\phi_{-E|C}), (\phi_C)$

When we multiply these factors we will get:

A	C	E	$(\phi_{C\|-A})(\phi_{-E\|C})(\phi_C)$
-	+	-	0.05*0.2*0.8 = 0.008
-	-	-	0.95*0.4*0.2 = 0.076

Summing out C, then we have:

A	E	$\Sigma_C\, (\phi_{C\|+A})(\phi_{-E\|C})(\phi_C)$
-	-	0.008 + 0.076 = 0.084

Finally we have found that our equation equal to:

$$\begin{aligned} P(-A,+D,-E) &= \Sigma_{B,C}\, P(-A,B,C,+D,-E) \\ &= ... \\ &= P(-A) * (\Sigma_{B,C}\ P(B|-A) * P(C|-A) * P(+D|B,C) * P(-E|C)) \\ &= P(-A) * \Sigma_{C}\ (\phi_{C|+A})(\phi_{-E|C})(\phi_{C}) \\ &= 0.0672 \end{aligned}$$

Now we can find the result

$$\begin{aligned} P(+A|+D,-E) &= \frac{P(+A,+D,-E)}{P(+D,-E)} \\ &= \frac{P(+A,+D,-E)}{P(+A,+D,-E)+P(-A,+D,-E)} \\ &= \frac{0.048}{0.0672+0.048} \\ &= 0.416 \end{aligned}$$

(e) **P(+B,-E|+A)**: **Answer: 0.288**

$$P(+B,-E|+A) = \frac{P(+B,-E,+A)}{P(+A)}$$

Now find the P(+B, -E, +A) and P(+A) using variable elimination

1. $P(+B,-E,+A) = P(+A) * P(+B|+A) * (\Sigma_{C,D}\ P(C|+A) * P(D|+B,C) * P(-E|C))$

Find the part $\Sigma_{C,D}$... using variable elimination
Here is the factors: (ϕ : factor symbol that I have used)

$(\phi_{C|+A}), (\phi_{D|+B,C}), (\phi_{-E|C})$

Firstly Eliminate C:
Here is the factors list that include C:→ $(\phi_{C|+A}), (\phi_{D|+B,C}), (\phi_{-E|C})$

When we multiply these factors we will get:

A	B	C	D	E	$(\phi_{C\|+A})(\phi_{D\|+B,C})(\phi_{-E\|C})$
+	+	+	+	-	0.2*0.8*0.2 = 0.032
+	+	+	-	-	0.2*0.2*0.2 = 0.008
+	+	-	+	-	0.8*0.8*0.4 = 0.256
+	+	-	-	-	0.8*0.2*0.4 = 0.064

Summing out C, then we have:

D	$(\Sigma_C\ (\phi_{C\|+A})(\phi_{D\|+B,C})(\phi_{-E\|C})$
+	0.032+0.256 = 0.288
-	0.008+0.064 = 0.072

Now our new factor is (ϕ_D)

Here is the our factors now: (ϕ_D)

Finally we have found that our equation equal to :

1. $P(+B,-E,+A) = P(+A) * P(+B|+A) * (\Sigma_{C,D}\ P(C|+A) * P(D|+B,C) * P(-E|C))$
$= 0.2 * 0.8 * \Sigma_D\ \phi_D$
$= 0.0576$

2. $P(+A) = P(+A) * (\Sigma_{B,C,D,E}\ P(B|+A) * P(C|+A) * P(D|B,C) *$
$P(E|C))$

Find the part $\Sigma_{B,C,D,E}$... using variable elimination

Here is the factors: (ϕ : factor symbol that I have used)

$(\phi_{B|+A}), (\phi_{C|+A}), (\phi_{D|B,C}), (\phi_{E|C})$

Firstly Eliminate B:
Here is the factors list that include B:→ $(\phi_{B|+A}), (\phi_{D|B,C})$

When we multiply these factors we will get:

A	B	C	D	$(\phi_{B\|+A}), (\phi_{D\|B,C})$
+	+	+	+	0.8*0.8 = 0.64
+	+	+	-	0.8*0.2 = 0.16
+	+	-	+	0.8*0.8 = 0.64
+	+	-	-	0.8*0.2 = 0.16
+	-	+	+	0.2*0.8 = 0.16
+	-	-	+	0.2*0.05= 0.01
+	-	+	-	0.2*0.2 = 0.04
+	-	-	-	0.2*0.95= 0.19

Summing out B, then we have:

C	D	$\Sigma_B\ (\phi_{B\|+A}), (\phi_{D\|B,C})$
+	+	0.64+0.16 = 0.8
+	-	0.16+0.04 = 0.2
-	+	0.64+0.01 = 0.65
-	-	0.16+0.19 = 0.35

Now our new factor is $(\phi_{C,D})$

Here is the our factors now: $(\phi_{C|+A}), (\phi_{E|C}), (\phi_{C,D})$

Now Eliminate C:
Here is the factors list that include C:$\rightarrow (\phi_{C|+A}), (\phi_{E|C}), (\phi_{C,D})$

When we multiply these factors we will get:

A	C	D	E	$(\phi_{C\|+A})(\phi_{E\|C})(\phi_{C,D})$
+	+	+	+	0.2*0.8*0.8 = 0.128
+	+	+	-	0.2*0.8*0.2 = 0.032
+	+	-	+	0.2*0.2*0.8 = 0.032
+	+	-	-	0.2*0.2*0.2 = 0.008
+	-	+	+	0.8*0.65*0.6= 0.312
+	-	-	+	0.8*0.35*0.6= 0.168
+	-	+	-	0.8*0.65*0.4 = 0.208
+	-	-	-	0.8*0.35*0.4 = 0.112

Summing out C, then we have:

D	E	$\Sigma_C\ (\phi_{C\|+A})(\phi_{E\|C})(\phi_{C,D})$
+	+	0.128+0.312 = 0.44
+	-	0.032+0.208 = 0.24
-	+	0.032+0.168 = 0.2
-	-	0.008+0.112 = 0.12

Now our new factor is $(\phi_{D,E})$

Here is the our factors now: $(\phi_{D,E})$

Summing out D, then we have:

E	$\Sigma_D(\phi_{D,E})$
+	0.44+0.2 = 0.64
-	0.24+0.12 = 0.36

Now our new factor is (ϕ_E)

Here is the our factors now: (ϕ_E)

Finally we have found that our equation equal to :

2. $P(+A) = P(+A) * (\Sigma_{B,C,D,E}\ P(B|+A) * P(C|+A) * P(D|B,C) * P(E|C))$
$= P(+A) * \Sigma_E\ \phi_E$
$= 0.2 * 1$
$= 0.2$

Finally we know the all the things to get answer:

$$P(+B, -E|+A) = \frac{P(+B, -E, +A)}{P(+A)}$$

$$= \frac{0.0576}{0.2}$$

$$= 0.288$$