

1. Complete Enumeration

(a) $P(+D)$: Answer: 0.32

$$\begin{aligned}
P(+D) &= \Sigma_{A,B,C,E} P(A, B, C, +D, E) \\
&= \Sigma_{A,B,C,E} P(A) * P(B|A) * P(C|A) * P(+D|B, C) * P(E|C) \\
&= P(+A) * P(+B|+A) * P(+C|+A) * P(+D|+B, +C) * P(+E|+C) + \\
&P(+A) * P(+B|+A) * P(+C|+A) * P(+D|+B, +C) * P(-E|+C) + \\
&P(+A) * P(+B|+A) * P(-C|+A) * P(+D|+B, -C) * P(+E|-C) + \\
&P(+A) * P(-B|+A) * P(+C|+A) * P(+D|-B, +C) * P(+E|+C) + \\
&P(+A) * P(+B|+A) * P(-C|+A) * P(+D|+B, -C) * P(-E|-C) + \\
&P(+A) * P(-B|+A) * P(-C|+A) * P(+D|-B, -C) * P(+E|-C) + \\
&P(+A) * P(-B|+A) * P(+C|+A) * P(+D|-B, +C) * P(-E|+C) + \\
&P(+A) * P(-B|+A) * P(-C|+A) * P(+D|-B, -C) * P(-E|-C) + \\
&P(-A) * P(+B|-A) * P(+C|-A) * P(+D|+B, +C) * P(+E|+C) + \\
&P(-A) * P(+B|-A) * P(+C|-A) * P(+D|+B, +C) * P(-E|+C) + \\
&P(-A) * P(+B|-A) * P(-C|-A) * P(+D|+B, -C) * P(+E|-C) + \\
&P(-A) * P(-B|-A) * P(+C|-A) * P(+D|-B, +C) * P(+E|+C) + \\
&P(-A) * P(+B|-A) * P(-C|-A) * P(+D|+B, -C) * P(-E|-C) + \\
&P(-A) * P(-B|-A) * P(-C|-A) * P(+D|-B, -C) * P(+E|-C) + \\
&P(-A) * P(-B|-A) * P(+C|-A) * P(+D|-B, +C) * P(-E|+C) + \\
&P(-A) * P(-B|-A) * P(-C|-A) * P(+D|-B, -C) * P(-E|-C) + \\
&= (0.2 * 0.8 * 0.2 * 0.8 * 0.8) + (0.2 * 0.8 * 0.2 * 0.8 * 0.2) + (0.2 * 0.8 * 0.8 * 0.8 * 0.6) + \\
&(0.2 * 0.2 * 0.2 * 0.8 * 0.8) + (0.2 * 0.8 * 0.8 * 0.8 * 0.4) + (0.2 * 0.2 * 0.8 * 0.05 * 0.6) + \\
&(0.2 * 0.2 * 0.2 * 0.8 * 0.2) + (0.2 * 0.2 * 0.8 * 0.05 * 0.4) + (0.8 * 0.2 * 0.05 * 0.8 * 0.8) + \\
&(0.8 * 0.2 * 0.05 * 0.8 * 0.2) + (0.8 * 0.2 * 0.95 * 0.8 * 0.6) + (0.8 * 0.8 * 0.05 * 0.8 * 0.8) + \\
&(0.8 * 0.2 * 0.95 * 0.8 * 0.4) + (0.8 * 0.8 * 0.95 * 0.05 * 0.6) + (0.8 * 0.8 * 0.05 * 0.8 * 0.2) + \\
&(0.8 * 0.8 * 0.95 * 0.05 * 0.4) \\
&= 0.02048 + 0.00512 + 0.06144 + 0.00512 + 0.04096 + 0.00096 + 0.00128 + 0.00064 + \\
&0.00512 + 0.00128 + 0.07296 + 0.02048 + 0.04864 + 0.01824 + 0.00512 + 0.01216 \\
&= 0.32 \quad \checkmark
\end{aligned}$$

(b) **P(+D,-A) Answer: 0.184**

$$\begin{aligned}
P(+D, -A) &= \Sigma_{B,C,E} P(-A, B, C, +D, E) \\
&= \Sigma_{B,C,E} P(-A) * P(B| - A) * P(C| - A) * P(+D|B, C) * P(E|C) \\
&= P(-A) * (\Sigma_{B,C,E} P(B| - A) * P(C| - A) * P(+D|B, C) * P(E|C))
\end{aligned}$$

Find the part $\Sigma_{B,C,E}...$

$$\begin{aligned}
\Sigma_{B,C,E}... &= P(+B| - A) * P(+C| - A) * P(+D| + B, +C) * P(+E| + C) \\
&\quad + P(+B| - A) * P(+C| - A) * P(+D| + B, +C) * P(-E| + C) \\
&\quad + P(+B| - A) * P(-C| - A) * P(+D| + B, -C) * P(+E| - C) \\
&\quad + P(+B| - A) * P(-C| - A) * P(+D| + B, -C) * P(-E| - C) \\
&\quad + P(-B| - A) * P(+C| - A) * P(+D| - B, +C) * P(+E| + C) \\
&\quad + P(-B| - A) * P(+C| - A) * P(+D| - B, +C) * P(-E| + C) \\
&\quad + P(-B| - A) * P(-C| - A) * P(+D| - B, -C) * P(+E| - C) \\
&\quad + P(-B| - A) * P(-C| - A) * P(+D| - B, -C) * P(-E| - C) \\
&= (0.2 * 0.05 * 0.8 * 0.8) + (0.2 * 0.05 * 0.8 * 0.2) \\
&\quad + (0.2 * 0.95 * 0.8 * 0.6) + (0.2 * 0.95 * 0.8 * 0.4) \\
&\quad + (0.8 * 0.05 * 0.8 * 0.8) + (0.8 * 0.05 * 0.8 * 0.2) \\
&\quad + (0.8 * 0.95 * 0.05 * 0.6) + (0.8 * 0.95 * 0.05 * 0.4) \\
&= 0.0064 + 0.0016 + 0.0912 + 0.0608 + 0.0256 + 0.0064 + 0.0228 + 0.0152 \\
&= 0.23
\end{aligned}$$

Now, we know the necessary thing to get result

$$\begin{aligned}
P(+D, -A) &= \Sigma_{B,C,E} P(-A, B, C, +D, E) \\
&= \Sigma_{B,C,E} P(-A) * P(B| - A) * P(C| - A) * P(+D|B, C) * P(E|C) \\
&= P(-A) * (\Sigma_{B,C,E} P(B| - A) * P(C| - A) * P(+D|B, C) * P(E|C)) \\
&= 0.8 * 0.23 \\
&= 0.184 \quad \checkmark
\end{aligned}$$

(c) **P(+E|-B): Answer: 0.61176**

$$\begin{aligned}
 P(+E|-B) &= \frac{P(+E, -B)}{P(-B)} \\
 &= \frac{P(+E, -B)}{\sum_E P(E, -B)} \\
 &= \frac{P(+E, -B)}{P(+E, -B) + P(-E, -B)}
 \end{aligned}$$

Find the P(+E, -B) and P(-E, -B) respectively:

$$\begin{aligned}
 P(+E, -B) &= \sum_{A,C,D} P(A, -B, C, D, +E) \\
 &= \sum_{A,C,D} P(A) * P(-B|A) * P(C|A) * P(D|-B, C) * P(+E|C)
 \end{aligned}$$

Find the $\sum_{A,C,D}...$

$$\begin{aligned}
 \sum_{A,C,D}... &= P(+A) * P(-B|+A) * P(+C|+A) * P(+D|-B, +C) * P(+E|+C) \\
 &\quad + P(+A) * P(-B|+A) * P(+C|+A) * P(-D|-B, +C) * P(+E|+C) \\
 &\quad + P(+A) * P(-B|+A) * P(-C|+A) * P(+D|-B, -C) * P(+E|-C) \\
 &\quad + P(+A) * P(-B|+A) * P(-C|+A) * P(-D|-B, -C) * P(+E|-C) \\
 &\quad + P(-A) * P(-B|-A) * P(+C|-A) * P(+D|-B, +C) * P(+E|+C) \\
 &\quad + P(-A) * P(-B|-A) * P(+C|-A) * P(-D|-B, +C) * P(+E|+C) \\
 &\quad + P(-A) * P(-B|-A) * P(-C|-A) * P(+D|-B, -C) * P(+E|-C) \\
 &\quad + P(-A) * P(-B|-A) * P(-C|-A) * P(-D|-B, -C) * P(+E|-C) \\
 &= (0.2 * 0.2 * 0.2 * 0.8 * 0.8) + (0.2 * 0.2 * 0.2 * 0.2 * 0.8) \\
 &\quad + (0.2 * 0.2 * 0.8 * 0.05 * 0.6) + (0.2 * 0.2 * 0.8 * 0.95 * 0.6) \\
 &\quad + (0.8 * 0.8 * 0.05 * 0.8 * 0.8) + (0.8 * 0.8 * 0.05 * 0.2 * 0.8) \\
 &\quad + (0.8 * 0.8 * 0.95 * 0.05 * 0.6) + (0.8 * 0.8 * 0.95 * 0.95 * 0.6) \\
 &= (0.00512) + (0.00128) + (0.00096) + (0.01824) + (0.02048) + (0.00512) \\
 &\quad + (0.01824) + (0.34656) \\
 &= 0.416
 \end{aligned}$$

$$\begin{aligned}
P(-E, -B) &= \Sigma_{A,C,D} P(A, -B, C, D, -E) \\
&= \Sigma_{A,C,D} P(A) * P(-B|A) * P(C|A) * P(D|-B, C) * P(-E|C)
\end{aligned}$$

Find the $\Sigma_{A,C,D} \dots$

$$\begin{aligned}
\Sigma_{A,C,D} \dots &= P(+A) * P(-B|+A) * P(+C|+A) * P(+D|-B, +C) * P(-E|+C) \\
&\quad + P(+A) * P(-B|+A) * P(+C|+A) * P(-D|-B, +C) * P(-E|+C) \\
&\quad + P(+A) * P(-B|+A) * P(-C|+A) * P(+D|-B, -C) * P(-E|-C) \\
&\quad + P(+A) * P(-B|+A) * P(-C|+A) * P(-D|-B, -C) * P(-E|-C) \\
&\quad + P(-A) * P(-B|-A) * P(+C|-A) * P(+D|-B, +C) * P(-E|+C) \\
&\quad + P(-A) * P(-B|-A) * P(+C|-A) * P(-D|-B, +C) * P(-E|+C) \\
&\quad + P(-A) * P(-B|-A) * P(-C|-A) * P(+D|-B, -C) * P(-E|-C) \\
&\quad + P(-A) * P(-B|-A) * P(-C|-A) * P(-D|-B, -C) * P(-E|-C)
\end{aligned}$$

$$\begin{aligned}
&= (0.2 * 0.2 * 0.2 * 0.8 * 0.2) + (0.2 * 0.2 * 0.2 * 0.2 * 0.2) \\
&\quad + (0.2 * 0.2 * 0.8 * 0.05 * 0.4) + (0.2 * 0.2 * 0.8 * 0.95 * 0.4) \\
&\quad + (0.8 * 0.8 * 0.05 * 0.8 * 0.2) + (0.8 * 0.8 * 0.05 * 0.2 * 0.2) \\
&\quad + (0.8 * 0.8 * 0.95 * 0.05 * 0.4) + (0.8 * 0.8 * 0.95 * 0.95 * 0.4) \\
&= (0.00128) + (0.00032) + (0.00064) + (0.01216) + (0.00512) + (0.00128) \\
&\quad + (0.01216) + (0.23104)
\end{aligned}$$

$$= 0.264$$

Now we know the necessary things to get result.

$$\begin{aligned}
P(+E|-B) &= \frac{P(+E, -B)}{P(-B)} \\
&= \frac{P(+E, -B)}{\Sigma_E P(E, -B)} \\
&= \frac{P(+E, -B)}{P(+E, -B) + P(-E, -B)} \\
&= \frac{0.416}{0.416 + 0.264} \\
&= 0.61176 \quad \checkmark
\end{aligned}$$

(d) **P(+A | +D,-E): Answer: 0.416**

$$\begin{aligned} P(+A | +D, -E) &= \frac{P(+A, +D, -E)}{P(+D, -E)} \\ &= \frac{P(+A, +D, -E)}{P(+A, +D, -E) + P(-A, +D, -E)} \end{aligned}$$

$$\begin{aligned} 1. P(+A, +D, -E) &= \Sigma_{B,C} P(+A, B, C, +D, -E) \\ &= \Sigma_{B,C} P(+A) * P(B | +A) * P(C | +A) * P(+D | B, C) * P(-E | C) \\ &= P(+A) * (\Sigma_{B,C} P(B | +A) * P(C | +A) * P(+D | B, C) + P(-E | C)) \end{aligned}$$

Find the $\Sigma_{B,C} P(B | +A) * P(C | +A) * P(+D | B, C) * P(-E | C)$

$$\begin{aligned} &\Sigma_{B,C} P(B | +A) * P(C | +A) * P(+D | B, C) \\ &= P(+B | +A) * P(+C | +A) * P(+D | +B, +C) * P(-E | +C) + \\ &\quad P(+B | +A) * P(-C | +A) * P(+D | +B, -C) * P(-E | -C) + \\ &\quad P(-B | +A) * P(+C | +A) * P(+D | -B, +C) * P(-E | +C) + \\ &\quad P(-B | +A) * P(-C | +A) * P(+D | -B, -C) * P(-E | -C) \\ &= (0.8 * 0.2 * 0.8 * 0.2) + (0.8 * 0.8 * 0.8 * 0.4) + (0.2 * 0.2 * 0.8 * 0.2) + (0.2 * 0.8 * 0.05 * 0.4) \\ &\quad = 0.0256 + 0.2048 + 0.0064 + 0.0032 = 0.24 \end{aligned}$$

Then $P(+A, +D, -E) = 0.2 * 0.24 = 0.048$

$$\begin{aligned} 2. P(-A, +D, -E) &= \Sigma_{B,C} P(-A, B, C, +D, -E) \\ &= \Sigma_{B,C} P(-A) * P(B | -A) * P(C | -A) * P(+D | B, C) * P(-E | C) \\ &= P(-A) * (\Sigma_{B,C} P(B | -A) * P(C | -A) * P(+D | B, C) * P(-E | C)) \end{aligned}$$

Find the $(\Sigma_{B,C} P(B | -A) * P(C | -A) * P(+D | B, C) * P(-E | C))$

$$\begin{aligned} \Sigma_{B,C} \dots &= P(+B | -A) * P(+C | -A) * P(+D | +B, +C) * P(-E | +C) + \\ &\quad P(+B | -A) * P(-C | -A) * P(+D | +B, -C) * P(-E | -C) + \\ &\quad P(-B | -A) * P(+C | -A) * P(+D | -B, +C) * P(-E | +C) + \\ &\quad P(-B | -A) * P(-C | -A) * P(+D | -B, -C) * P(-E | -C) \\ &= (0.2 * 0.05 * 0.8 * 0.2) + (0.2 * 0.95 * 0.8 * 0.4) + (0.8 * 0.05 * 0.8 * 0.2) \\ &\quad + (0.8 * 0.95 * 0.05 * 0.4) \\ &= 0.0016 + 0.0608 + 0.0064 + 0.0152 = 0.084 \end{aligned}$$

Then $P(-A, +D, -E) = 0.8 * 0.084 = 0.0672$

Now we know the necessary things to find the result.

$$\begin{aligned}
 P(+A|+D, -E) &= \frac{P(+A, +D, -E)}{P(+D, -E)} \\
 &= \frac{P(+A, +D, -E)}{P(+A, +D, -E) + P(-A, +D, -E)} \\
 &= \frac{0.048}{0.048 + 0.0672} \\
 &= 0.416 \quad \checkmark
 \end{aligned}$$

(e) **P(+B,-E | +A) : Answer: 0.288**

$$\begin{aligned}
 P(+B, -E|+A) &= \frac{P(+B, -E, +A)}{P(+A)} \\
 &= \frac{P(+B, -E, -A)}{\Sigma_{B,E} P(B, E, +A)} \\
 &= \frac{P(+B, -E, +A)}{P(+B, +E, +A) + P(+B, -E, +A) + P(-B, +E, +A) + P(-B, -E, +A)}
 \end{aligned}$$

Find P(+B, +E, +A):

$$\begin{aligned}
P(+B, +E, +A) &= \Sigma_{C,D} P(+A, +B, C, D, +E) \\
&= \Sigma_{C,D} P(+A) * P(+B|+A) * P(C|+A) * P(D|+B, C) * P(+E|C) \\
&= P(+A) * P(+B|+A) * (\Sigma_{C,D} P(C|+A) * P(D|+B, C) * P(+E|C)) \\
&= 0.2 * 0.8 * (\Sigma_{C,D} P(C|+A) * P(D|+B, C) * P(+E|C)) \\
&= 0.16 * (\Sigma_{C,D} P(C|+A) * P(D|+B, C) * P(+E|C))
\end{aligned}$$

Find the part $\Sigma_{C,D}...$

$$\begin{aligned}
\Sigma_{C,D}... &= P(+C|+A) * P(+D|+B, +C) * P(+E|+C) \\
&\quad + P(+C|+A) * P(-D|+B, +C) * P(+E|+C) \\
&\quad + P(-C|+A) * P(+D|+B, -C) * P(+E|-C) \\
&\quad + P(-C|+A) * P(-D|+B, -C) * P(+E|-C) \\
&= (0.2 * 0.8 * 0.8) + (0.2 * 0.2 * 0.8) + (0.8 * 0.8 * 0.6) \\
&\quad + (0.8 * 0.2 * 0.6) \\
&= 0.128 + 0.032 + 0.384 + 0.096 = 0.64
\end{aligned}$$

$$P(+B, +E, +A) = 0.16 * 0.64 = 0.1024$$

Find P(+B, -E, +A):

$$\begin{aligned}
P(+B, -E, +A) &= \Sigma_{C,D} P(+A, +B, C, D, -E) \\
&= \Sigma_{C,D} P(+A) * P(+B|+A) * P(C|+A) * P(D|+B, C) * P(-E|C) \\
&= P(+A) * P(+B|+A) * (\Sigma_{C,D} P(C|+A) * P(D|+B, C) * P(-E|C)) \\
&= 0.2 * 0.8 * (\Sigma_{C,D} P(C|+A) * P(D|+B, C) * P(-E|C)) \\
&= 0.16 * (\Sigma_{C,D} P(C|+A) * P(D|+B, C) * P(-E|C))
\end{aligned}$$

Find the part $\Sigma_{C,D}...$

$$\begin{aligned}
\Sigma_{C,D}... &= P(+C|+A) * P(+D|+B, +C) * P(-E|+C) \\
&\quad + P(+C|+A) * P(-D|+B, +C) * P(-E|+C) \\
&\quad + P(-C|+A) * P(+D|+B, -C) * P(-E|-C) \\
&\quad + P(-C|+A) * P(-D|+B, -C) * P(-E|-C) \\
&= (0.2 * 0.8 * 0.2) + (0.2 * 0.2 * 0.2) + (0.8 * 0.8 * 0.4) \\
&\quad + (0.8 * 0.2 * 0.4) \\
&= 0.032 + 0.008 + 0.256 + 0.064 = 0.36
\end{aligned}$$

$$P(+B, -E, +A) = 0.16 * 0.36 = 0.0576$$

Find P(-B, +E, +A):

$$\begin{aligned}
P(-B, +E, +A) &= \Sigma_{C,D} P(+A, -B, C, D, +E) \\
&= \Sigma_{C,D} P(+A) * P(-B|+A) * P(C|+A) * P(D|-B, C) * P(+E|C) \\
&= P(+A) * P(-B|+A) * (\Sigma_{C,D} P(C|+A) * P(D|-B, C) * P(+E|C)) \\
&= 0.2 * 0.2 * (\Sigma_{C,D} P(C|+A) * P(D|-B, C) * P(+E|C)) \\
&= 0.04 * (\Sigma_{C,D} P(C|+A) * P(D|-B, C) * P(+E|C))
\end{aligned}$$

Find the part $\Sigma_{C,D}...$

$$\begin{aligned}
\Sigma_{C,D}... &= P(+C|+A) * P(+D|-B, +C) * P(+E|+C) \\
&\quad + P(+C|+A) * P(-D|-B, +C) * P(+E|+C) \\
&\quad + P(-C|+A) * P(+D|-B, -C) * P(+E|-C) \\
&\quad + P(-C|+A) * P(-D|-B, -C) * P(+E|-C) \\
&= (0.2 * 0.8 * 0.8) + (0.2 * 0.2 * 0.8) + (0.8 * 0.05 * 0.6) \\
&\quad + (0.8 * 0.95 * 0.6) \\
&= 0.128 + 0.032 + 0.024 + 0.456 = 0.64
\end{aligned}$$

$$P(-B, +E, +A) = 0.04 * 0.64 = 0.0256$$

Find P(-B, -E, +A):

$$\begin{aligned}
P(-B, -E, +A) &= \Sigma_{C,D} P(+A, -B, C, D, -E) \\
&= \Sigma_{C,D} P(+A) * P(-B|+A) * P(C|+A) * P(D|-B, C) * P(-E|C) \\
&= P(+A) * P(-B|+A) * (\Sigma_{C,D} P(C|+A) * P(D|-B, C) * P(-E|C)) \\
&= 0.2 * 0.2 * (\Sigma_{C,D} P(C|+A) * P(D|-B, C) * P(-E|C)) \\
&= 0.04 * (\Sigma_{C,D} P(C|+A) * P(D|-B, C) * P(-E|C))
\end{aligned}$$

Find the part $\Sigma_{C,D}...$

$$\begin{aligned}
\Sigma_{C,D}... &= P(+C|+A) * P(+D|-B, +C) * P(-E|+C) \\
&\quad + P(+C|+A) * P(-D|-B, +C) * P(-E|+C) \\
&\quad + P(-C|+A) * P(+D|-B, -C) * P(-E|-C) \\
&\quad + P(-C|+A) * P(-D|-B, -C) * P(-E|-C) \\
&= (0.2 * 0.8 * 0.2) + (0.2 * 0.2 * 0.2) + (0.8 * 0.05 * 0.4) \\
&\quad + (0.8 * 0.95 * 0.4) \\
&= 0.032 + 0.008 + 0.016 + 0.304 = 0.36
\end{aligned}$$

$$P(-B, -E, +A) = 0.04 * 0.36 = 0.0144$$

Now we know the necessary things to find the result

$$\begin{aligned}
 P(+B, -E | +A) &= \frac{P(+B, -E, +A)}{P(+A)} \\
 &= \frac{P(+B, -E, -A)}{\Sigma_{B,E} P(B, E, +A)} \\
 &= \frac{P(+B, -E, +A)}{P(+B, +E, +A) + P(+B, -E, +A) + P(-B, +E, +A) + P(-B, -E, +A)} \\
 &= \frac{0.0576}{0.1024 + 0.0576 + 0.0256 + 0.0144} \\
 &= 0.288 \quad \checkmark
 \end{aligned}$$